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A Contact Characteristic of Roller Bearing with Palm Oil-based Grease Lubrication

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ABSTRACT

Grease lubricants are widely used in rolling contact applications to reduce friction between two rolling surfaces. Improper lubrication may cause high contact stress and deformation to the bearings and lead to machine failure The purpose of this study is to investigate the coefficient of friction produced by newly developed palm oil-based grease and to investigate the contact characteristics in lubricated roller bearings. In this work, the coefficient of friction of new greases was evaluated experimentally and the values were compared with the conventional mineral oil-based grease to investigate the friction performance. The friction test was performed using a four-ball tester. The finite element model was developed based on the roller bearing geometry and the simulation was carried out the evaluate the contact characteristic. The experimental result shows that the palm oil grease formulation A had the least coefficient of friction, followed by palm oil grease formulation B, mineral grease and food grade grease. This indicates that palm oil-based grease has the potential to be applied in rolling contact applications due to low friction characteristics. Finite element analysis shows that the maximum von Mises stress and total deformation for frictional contact are higher than the frictionless contact. For the frictional contact analysis with various lubricant COF, similar values were obtained with von Mises stress at 400.69 MPa and 3.4033×10-4 mm deformation. The finding shows that the small difference in grease COF did not affect the rolling contact. The finding also shows that the newly developed biodegradable grease has a similar performance in terms of rolling contact friction and contact characteristic in a condition that the bearing is operating in normal condition.

1. Introduction

Lubrication is widely applied in sliding and rolling contact applications to minimize friction and prevent excessive stress and deformation. Different types of lubricants such as oil and grease are designed for specific operating conditions. Grease lubricant is normally used due to its unique characteristics such as maintaining a stable viscosity over a wide range of temperatures, providing

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good film strength which is able to support loads and provides a barrier against moisture and contaminant. This type of lubricant is widely used in rolling contact applications, such as in roller bearing. Due to the growing concern over environmental sensitivity, biodegradable greases are being considered to be used as lubricants for industrial and transportation applications. The ability of biodegradable grease to be decomposed into harmless products provide a renewable source of environmentally friendly lubricants. To solve this issue, research and development efforts have been carried out to determine the appropriate type of lubrication for the bearings.

Researches have been done to investigate the performance of vegetable grease include the viscosity and friction of newly developed vegetable oil-based lubricant [1-18]. A comprehensive review of palm oil and the challenges of the use of vegetable oil as lubricant base-oil has been presented by Dandan *et al.,* [1]. Sutaria *et al.,* [2] carried out ASTM performance tests for different blending ratios of oil samples using the four-ball tribotester and found out that as the blending ratio of oil-based with based oil increases, the viscosity of oil decreases, which results in decreasing coefficient of friction and increasing wear scar diameter. Pillay and Sidik [3] investigated the tribological properties of biodegradable nano-lubricant with various additives and concluded that the performance of vegetable lubricants can be improved with hybrid additives. Syahrullail *et al.,* [4] performed a tribological analysis of modified RBD palm kernel containing anti-oxidant additive and compared the effect of load on the tribological performance of refined bleached, deodorized RBD palm olein and paraffinic mineral oil. The authors concluded that RBD palm olein had better friction reduction and wear resistance than paraffinic mineral oil [5]. A similar study was performed by Chiong Ing *et al.,* [6] and concluded that RBD palm olein had a lower coefficient of friction but had had larger wear scar compared to paraffinic mineral oil at various normal loads. Laurentis *et al.,* [7] discovered that an increase in temperature leads to a decrease in the friction coefficient of grease as a result of the base oil viscosity of the grease drives its frictional behavior under high-temperature conditions. Asadauskas *et al.,* [8] compared three types of nanoparticles used in grease synthesis and found that copper nanoparticles have higher friction reduction compared to cobalt and iron nanoparticles. Further work by Fatima *et al.,* [9] found that copper nanoparticle is a good additive that gives an impact on the friction and wear reduction of the oil but for the oil that has higher polarity such as synthetic ester it is not suitable because it only worsened the contact between the metal surfaces. Rajubhai *et al.,* [10] investigated the use of copper nanoparticles as an additive in pongamia base oil and observed a reduction of coefficient of fraction and wear scar diameter. They also found that the increase in the concentration of copper nanoparticles may reduce friction and wear.

The coefficient of friction is very important as it may affecting lubrication regime and surface wear as exhibited in the well-known Stribeck curve. The Stribeck curve illustrated in a review by Halme and Anderson [11] explained that high friction may induce high vibration and surface roughening. Numbers of finite element analyses have been conducted to investigate the contact behavior in rolling contact, however, the studies are limited to the dry frictionless contact model. The finite element analysis for the elastic-plastic rolling contact is generally simplified to the contact between the sphere and flat rigid surface [19-23]. The contact stress and deformation in dry contact were presented, however, the simulation with lubricated roller contact is rarely reported. Toumi *et al.,* [24] developed a 3D finite element model of the frictional rolling in wheel-rail contact and found that the normal contact solution in elasticity from different FE methods is in good agreement with Hertzian theory for the maximum contact pressure and the contact shape. The effect of rolling friction on the surface crack [25] and rolling contact fatigue and thermal cracks damage [26] of the railway track model also has been studied. Kudra and Awrejcewicz [27] developed the approximate models of coupled friction and rolling resistance in the case of elliptic contact and concluded that generally rotational motion of the deformation zone can also have some influence on the contact stress distribution. The simulation on the grease mixing also has been investigated to evaluate the suitable stirring method to produce the homogeneous mixture of high viscous fluid by considering the viscous force [28].

In this work, the palm oil-based grease has been produced and the friction is measured and compared with available commercial grease. The friction coefficient value of the different types of grease is applied in the finite element simulation. The finite element model is developed based on available roller bearing geometry and the contact characteristics included contact stress produced and deformation is investigated.

2. Methodology

2.1 Preparation of Grease

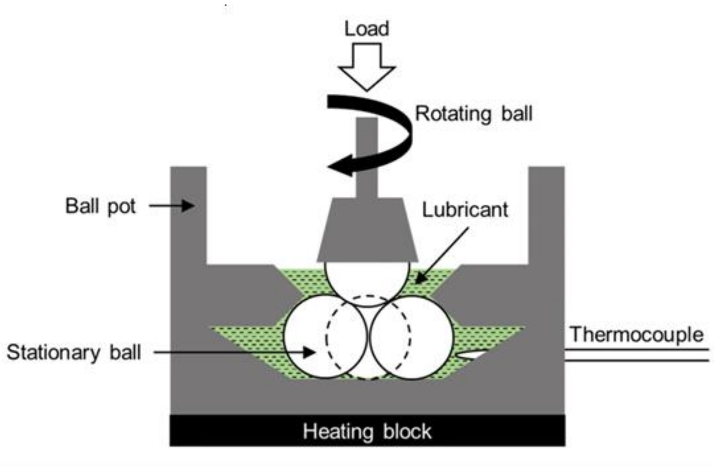
In this study, the grease samples were prepared by mixing palm oil as a base oil with lithium stearate as thickener and copper nanoparticles as additives. Each material was carefully weighed separately based on the compositional formulation given in Table 1. The contents were mixed in a beaker, then heated to a temperature of 90 - 120 °C while being stirred using a magnetic stirrer for about 4 hours. After that, the heating plate was turned off, but the stirring was continued until the contents were cooled to room temperature. The grease was covered with a plastic wrap and left overnight to check for its stability. The steps were repeated for a different composition formulation. Based on the table, the palm oil A formulation has a higher percentage of palm base oil compared to formulation B, while the palm oil formulation B has a higher percentage of thickener and additive.

Table 1
Composition formulations of grease samples

Material	Composition (%)	
	A	B
Cooking oil	93.5	87.6
Lithium stearate	5	11
Copper nanoparticles	1.5	11.4

2.2 Lubricant Friction Test

The friction coefficient of the greases was measured using a four-ball tester. For comparison, two types of conventional grease with mineral oil-based were tested, which are Biogrease Marlin-9 Food Grade Grease and SKF LGGB 2/0.4 Biodegradable Mineral Grease. Figure 1 shows a (a) four-ball test set up in the lab and (b) a schematic diagram of the four-ball tester [29]. The test was performed according to the standard testing procedure ASTM D2266 with the test parameters shown in Table 2.



(a) (b)

Fig. 1. (a) Four-ball test in the lab and (b) Schematic diagram of the four-ball tester [29]

Table 2
Test parameters according to ASTM D2266

Parameter	Condition
Load applied	40 kg (392 N)
Duration	60 min
Temperature	75±2 °C
Speed	1200±60 rpm

Four standard steel balls were used in this experiment, with three balls held firmly in a pot containing the grease and one ball on the top pressed against them, as shown in Figure 1 (b). The steel ball was cleaned using acetone and wiped dry to make sure the steel balls were free of contaminants. The average friction coefficient for each test was calculated according to IP-239, which is expressed as follows [11]

$$\mu = \frac{T\sqrt{6}}{3Wr} \tag{1}$$

where μ is the coefficient of friction, T is the friction torque in kg mm, W is the applied load in kg and r is the distance from the center of the contact surface on the lower balls to the axis of rotation, which was determined to be 3.67 mm. Hence, the equation to calculate the friction coefficient can be simplified as follows

$$\mu = 0.22248\,\frac{T}{W} \tag{2}$$

2.3 Contact Analysis Using Finite Element Analysis

The finite element analysis (FEA) was carried out in two-dimensional (2D) using ANSYS Static Structural. The flowchart of the FEA is summarized in Figure 2.

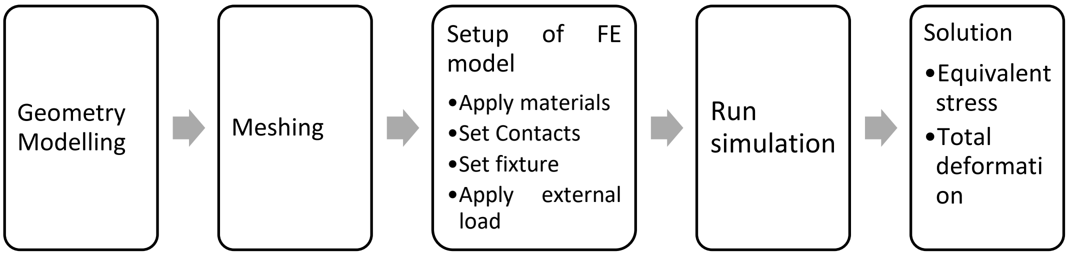


Fig. 2. Flowchart of finite element analysis of roller bearing in ANSYS

The 2D model of the cylindrical roller bearing (NU205E.TVP2.C3) was simplified as shown in Figure 3. In this study, both inner ring and roller were considered to be in contact as per plane stress theory criteria to improve the accuracy of the results [30]. The material properties of the roller bearing, which is low-alloy, thorough hardened chromium steel (AISI 5210) was inserted in the ANSYS engineering data and its material properties were updated. Initially, the contact between the edges at the inner race and the roller was set at frictionless. The numerical contact behavior of frictionless contact was compared with the calculated stress using the Hertzian contact theory. The boundary conditions of the bearing were specified as shown in Figure 3, with a normal load of 100 N on the roller acting downwards. The displacements of both inner ring and roller were fixed in the x-direction and only allowed in the y-direction.

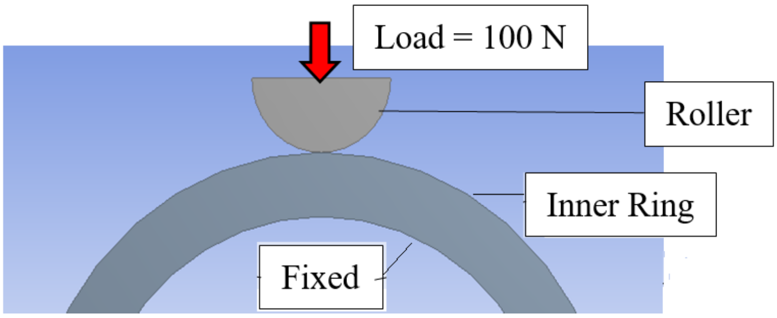
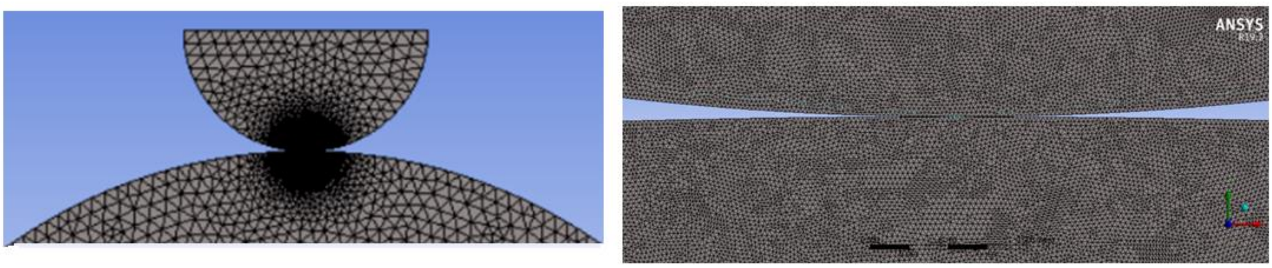


Fig. 3. 2D model of the partial roller bearing and its boundary condition for simulation

Figure 4(a) shows the meshing of the finite element model, with an element size of 0.4 mm using the triangles method. Finer meshing size was set at the contact area which is at the inner race and roller surface with 0.0055 mm element size. The generated mesh was consisting 132788 elements as shown in Figure 4(b). The simulation was then repeated by changing the contact from frictionless to frictional contact with the coefficient of frictions (COF) value of four grease samples. The equivalent stress and total displacement of the bearing model were evaluated.



(a) (b)

Fig. 4. (a) Meshing of the 2D roller bearing model and (b) detailed mesh at the contact area between roller (top) and inner ring (bottom)

2.4 Hertzian Contact Theory

Hertzian contact stress refers to the localized stresses which develop as two curved surfaces come in contact and deform slightly under imposed load. The contact stress and deformation are functions of the normal contact force, the radii of curvature of both bodies, Young's modulus and the Poisson's ratio of the two bodies. In this work, the frictionless contact simulation is validated with Hertzian contact theory. The Hertzian equation for line contact was used for the case of roller bearing. Based on the 3D model, the line contact between the contacting components is formed as shown in Figure 5.

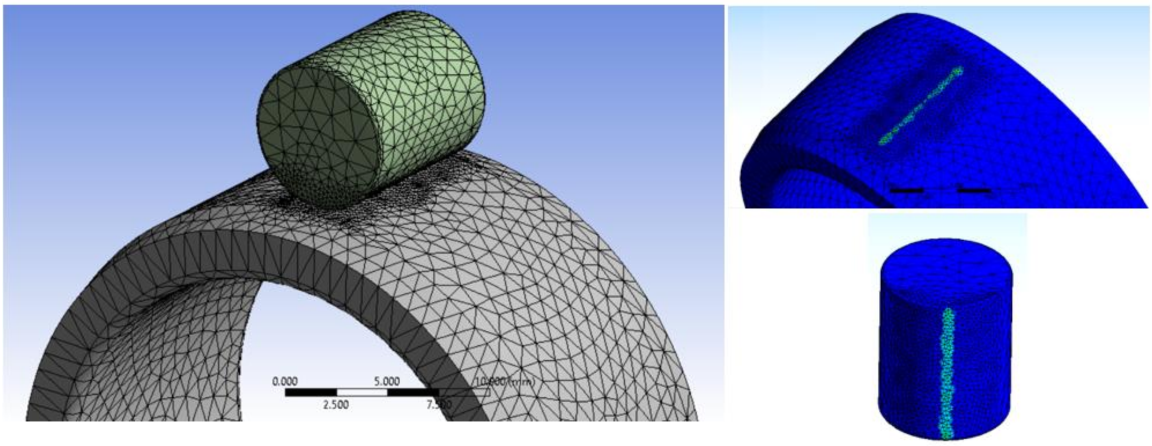


Fig. 5. The 3D model for Hertzian line contact

Based on the theory, the effective radius of curvature, R is given by [31-33]

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2} \tag{3}$$

where R_1: radius of the roller, R_2: radius of the inner race.

The effective Young's modulus, E^* on the Hertzian contact is determined by

$$\frac{1}{E^*}=\left(\frac{1-v_1^{\ 2}}{E_1}+\frac{1-v_2^{\ 2}}{E_2}\right) \tag{4}$$

where E_1, E_2: elastic moduli associated to the roller and the inner race respectively. v_1, v_2: Poisson's ratios associated to the roller and the inner race respectively.

The contact radius between the two cylindrical surfaces under load is given by

$$a=\left(\frac{4PR}{\pi E^*}\right)^{\frac{1}{2}} \tag{5}$$

where P: load per unit length (N/m).

The maximum Hertzian contact stress, P_o is given by

$$P_o=\frac{2P}{\pi a}=\left(\frac{PE^*}{\pi R}\right)^{\frac{1}{2}} \tag{6}$$

and the interference/ deformation, δ is given by

$$\delta = \frac{P}{\pi E^*}\left[\ln\left(\frac{L^2 \pi E^*}{2RP}+1\right)\right] \quad (7)$$

3. Results and Discussion

3.1 Lubricant Friction Coefficient

Figure 6 shows the variation of COF for different types of greases. The initial COF values are almost similar for all greases. Both palm oil greases and mineral grease shows almost similar characteristic for the first 1600 seconds with an average value of 0.1. After that, the COF value of both palm oil-based greases is reduced consistently to an average value of 0.5 until the end of the experiment. For mineral grease, the COF started to fluctuate after 1600 seconds while the COF value for the food grade grease was peak up after 400 seconds and fluctuates until the end of the experiment with an average COF of 0.16. The average COF calculated based on the measured data in 3600 seconds is shown in Table 3. The coefficient of friction for the food grade grease was determined to be the highest at 0.096311, followed by mineral grease at 0.063654, palm oil grease formulation B at 0.057769 and finally palm oil grease formulation A at 0.046842. The finding shows that the newly developed palm oil-based grease has a good performance and has the potential to be applied in rolling contact applications due to low friction characteristics.

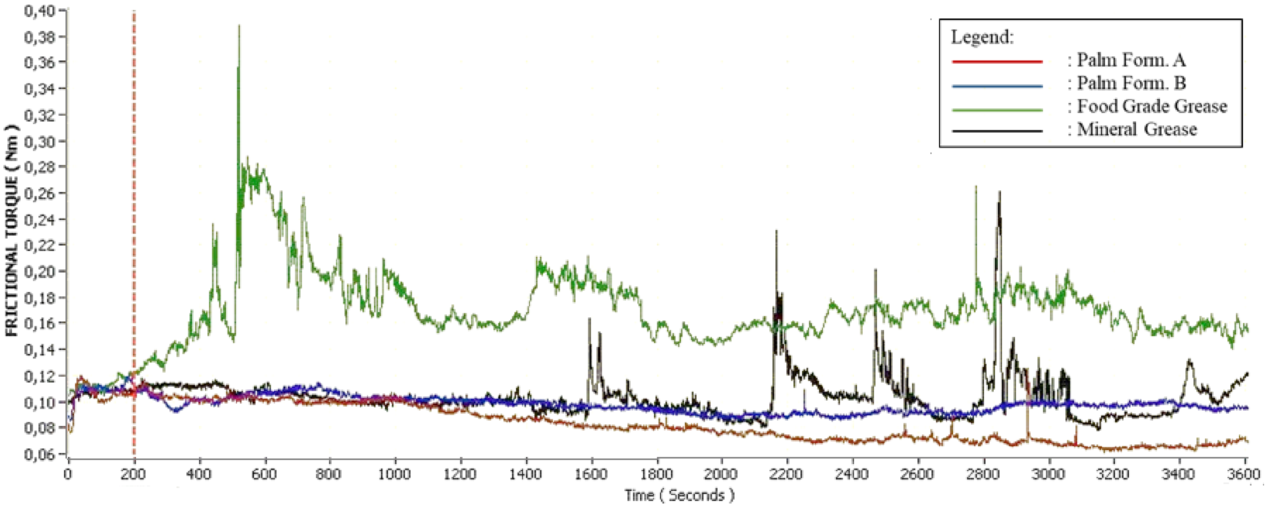


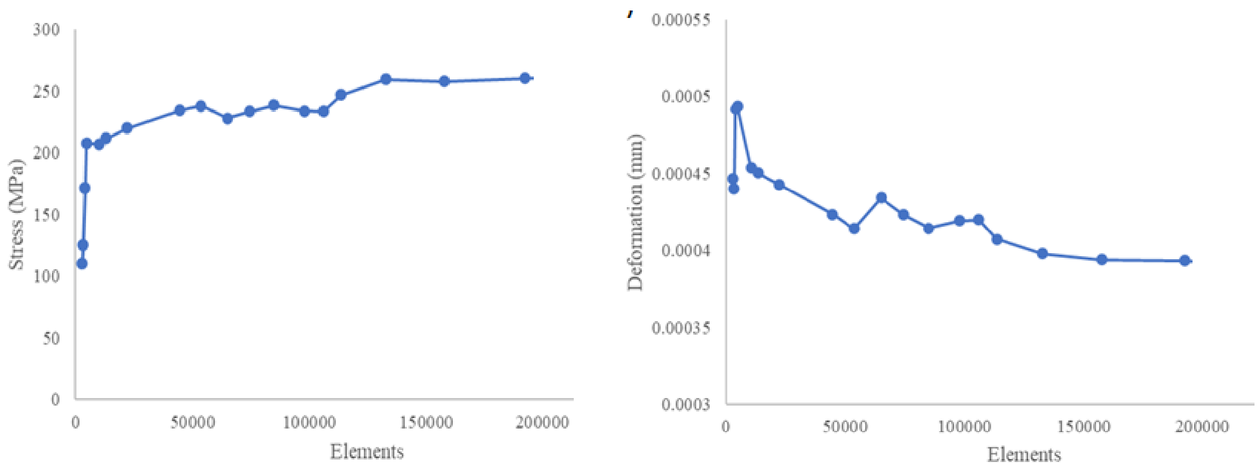
Fig. 6. Variation of coefficient of friction for different types of greases

Table 3
The average coefficient of friction of four different grease

Grease	Coefficient of Friction
Palm oil formulation A	0.046842
Palm oil formulation B	0.057769
Food grade	0.096311
Mineral	0.063654

3.2 Contact Analysis

The result of the meshing convergence test is shown in Figure 7 for both (a) stress and (b) deformation analysis. The mesh numbers ranging from 2900 to 243374 elements were evaluated. It is observed that the simulation results started to be consistent after 130000 meshing elements, therefore the mesh value with 132788 elements was selected for the simulation.



(a) (b)

Fig. 7. Variation of (a) equivalent stress and (b) total deformation against mesh number

The von Mises stress contours together with the resulting maximum stress P_{max} are shown in Figure 8. It could be seen that the maximum stress occurred at the center of the contact between the inner ring and the roller, which was at 259.35 MPa. Compared to the calculated maximum stress value based on the Hertz Contact Theory, which was 367.1 MPa, the percentage difference between calculated and FE methods was 29.35%. When frictional contact was introduced into the FE model, the von Mises stress increase to 400. 69 MPa. The von Mises stress for four greases with different COF showed identical values of 400.69 MPa (Table 4). This finding is due to the differences in COF values of these four types of grease are very small.

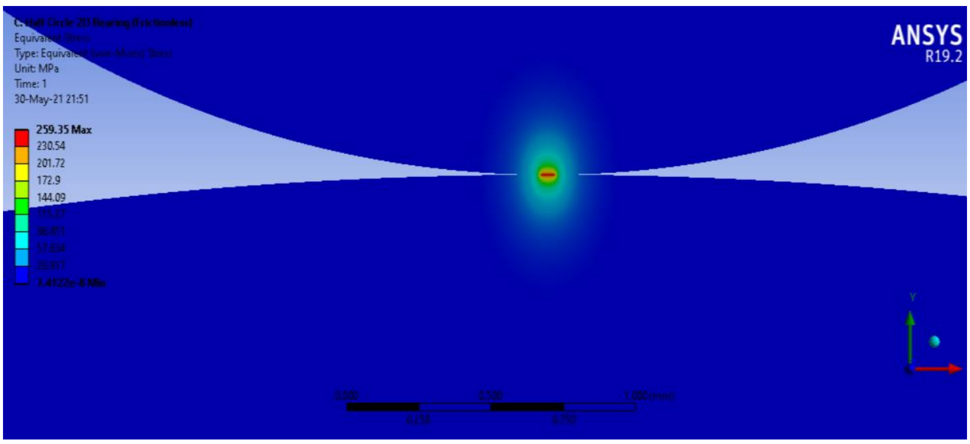


Fig. 8. Von Mises stress of the roller (top) and inner ring (bottom) in 2D bearing model for frictionless contact

Table 4
Von Mises stress of the roller bearing model for different COF

Grease	Coefficient of Friction, μ	Equivalent Stress, P_o (MPa)
Frictionless	0.00	259.35
Palm oil grease formulation A	0.046842	400.69
Palm oil grease formulation B	0.057769	400.69
Food grade grease	0.096311	400.69
Mineral grease	0.063654	400.69

The total roller bearing surface deformation at the contact area was evaluated and the value is shown in Table 5. It could be seen that the maximum displacement recorded for the contact between inner race and roller for frictionless contact was 3.9788×10^{-4} mm. The calculated total deformation/displacement value based on the Hertzian contact theory was obtained 3.9806×10^{-4} mm, therefore the percentage difference between calculated and FE methods was only 0.045%. With frictional contact, the total displacement for all greases showed identical values of 3.4033×10^{-4} mm, due to the small differences between COF between these four types of grease. The finding shows that the small difference in grease COF did not affect the rolling contact. In the research work by Mai *et al.,* and Caprioli [25,26], the initial crack has been seeded in the finite element model of railway track to see the effect of rolling friction on the surface damage. In contrast, the roller bearing model in this study is between the smooth surfaces of roller bearing geometry, representing the contact in real conditions. The finding shows that the newly developed palm oil-based grease has a similar performance in terms of rolling contact friction and contact stress with a condition that the bearing is operating in normal condition.

Table 5
Total deformation of the roller bearing surface for different COF

Grease	Coefficient of Friction, μ	Total Deformation, δ (mm)
Frictionless	0.00	3.9788×10^{-4}
Palm oil grease formulation A	0.046842	3.4033×10^{-4}
Palm oil grease formulation B	0.057769	3.4033×10^{-4}
Food grade grease	0.096311	3.4033×10^{-4}
Mineral grease	0.063654	3.4033×10^{-4}

4. Conclusion

In this project, the coefficients of friction of new formulated palm oil-based grease and mineral oil-based greases were determined experimentally and their contact characteristic was evaluated by finite element analysis. From the finding, the following conclusion can be made

i. The palm oil-based grease formulation A had the least coefficient of friction, followed by palm oil grease formulation B, mineral grease and food grade grease. This indicates that the palm oil-based grease has the potential to be applied in rolling contact applications due to low friction characteristics.
ii. The contact analysis shows that the maximum von Mises stress and total deformation for frictional contact are higher than the frictionless contact. With various lubricant COF, similar values were obtained with Von Mises stress at 400.69 MPa and 3.4033×10-4 mm deformation. The finding shows that the small difference in grease COF did not affect the rolling contact.

The finding also shows that the newly developed biodegradable grease has a similar performance with conventional mineral oil-based grease in terms of rolling contact friction and contact characteristic in a condition that the bearing is operating in normal condition.

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