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Numerical Investigation of Heat Transfer Inside Participating Media: Built Thermal Environment Application

Raoudha Chaabane[1,*], Nor Azwadi Che Sidik[2]

[1] Laboratory of Thermal and Energetic Systems Studies (LESTE) at the National School of Engineering of Monastir,University of Monastir, Tunisia
[2] Malaysia-Japan International Institute of Technology (MJIIT), Universiti Teknologi Malaysia, Jalan Sultan Yahya Petra,54100 Kuala Lumpur, Malaysia

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ABSTRACT

This paper is devoted to studying the effect of volumetric radiation on coupled convection and radiation heat transfer in a participating two-dimensional enclosure with emitting absorbing and scattering medium which is a crucial criterion in design of built thermal environment. Radiative information is gotten by resolving RTE (the radiative transfer equation) by means of CVFEM (the Control Volume Finite Element Method). Temperature, density and velocity fields are calculated using the mesoscopic two double population lattice Boltzmann approach. Results are validation of results with existing works in literature and the proposed approach was found to be numerically stable, efficient and accurate.

1. Introduction

The transient coupled convective and radiative heat transfer was the object of many numerical engineering studies in the area of solar radiation, built thermal environment, building comfort, fire protection, industrial furnaces, fibrous and foam insulations, fiber processing of optical textile, thermal insulation, combustors, high temperature-heat exchangers, cylindrical metal-hydrogen reactor, glass manufacturing, heat pipes, jet engines, gas turbine combustors, combustion chambers design, fibrous insulation and energy conservation [1-9]. In the present paper, we highlight numerical study of combined convection in the presence of radiation in a rectangular gray, absorbing, emitting and isotropically scattering enclosure. The vertical lids are perfectly conducting and the remaining two side walls are maintained at hot and cold temperature respectively. The aim of this study is to increase better understanding related to heat transfer and fluid flow characteristics in a such combined process. The used numerical search based on two approaches (LBM-VBFEM); the Lattice Boltzmann [10-13] and Control Volume Finite Element Methods [14-24] via an in house made Fortran 90 code. LBM is applied to solve flow and thermal model equations, however CVFEM is incorporated to handle the radiative information.

* *Corresponding author.*
E-mail address: raoudhach@gmail.com

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In one hand, LBM has been progressively used in a diversity of fluid mechanics engineering applications. Obtained results in fluid and heat transfer problems has been inspiring. Owing to computational simplicity, efficiency, ability, parallel architecture efficient implementation and boundary condition's implementation simplicity. besides, LBM is the best alternative to CFD traditional conventional computational fluid dynamics solvers which essentially solve transport equations of fluid flow, mass and heat transfer by directly discretizing them. In the other hand, recently, CVFEM demonstrated to be successful, promising and accurate in the solution of radiative transfer equation (RTE) in multidimensional participating configurations [14-24].

2. Formulation

Figure 1 and 2 represents the considered physical system which is composed of a rectangular enclosure limited by two vertical insulating lids. The lower horizontal isothermal wall is subjected to hot temperature Th and the upper wall is maintained at cold temperature Tc. The fluid in the enclosure is emitting, absorbing and isotopically scattering medium. Except density which vary in the Boussinesq sense, thermo-physical properties are supposed to be constant. The boundaries of the cavity are considered grey and diffuse. Governing equation are considered to be two-dimensional, incompressible and laminar. Energy and Navier-Stokes equations are solved using hybrid approach namely the Bhatangar–Gross–Krook (BGK) approximation of the lattice Boltzmann [10-13] and CVFEM formulation [14-24].

In space and time, the Boltzmann equation is given by

$$f_k(\vec{r}+\overrightarrow{c_k}\Delta t, t+\Delta t) = f_k(\vec{r},t) - \frac{\Delta t}{\tau_v}[f_k(\vec{r},t) - {f_k}^{eq}(\vec{r},t)] + \Delta tF, k = 0,..,8 \tag{1}$$

where

$$\tau_v = \frac{1}{2} + \frac{3\nu}{c^2\Delta t} \tag{2}$$

$$f_k^{eq} = w_k\rho(1 + \frac{c_k u}{c_s^2} + \frac{(c_k u)^2}{2c_s^4} - \frac{uu}{2c_s^2}) \tag{3}$$

f_k are the PDF (Particle Distribution Function) defined for discrete particle velocity vectors. ${f_k}^{eq}$ is the LEDF (Local Equilibrium Distribution Function) that has an appropriately prescribed functional dependence on the local hydrodynamic properties. F represents the external force and τ_v is the relaxation time.

Viscosity ν must be selected to ensure that Mach number Ma satisfy the limit of incompressible flow [10-13]. The macroscopic density ρ and the velocity $\vec{u}$are written as

$$\rho(\vec{r},t) = \sum_k f_k(\vec{r},t) \tag{4}$$

$$u(\vec{r},t) = \sum_k c_k f_k(\vec{r},t)/\rho(\vec{r},t) \tag{5}$$

No slip boundary condition is executed along all the side walls for resolving hydrodynamic fields. Bounce-back boundary conditions were applied on all solid boundaries of the density distribution functions.

Considering volumetric radiation, the governing LB equation for the thermal field is written as [13-14, 18-20, 23-24]

$$g_k(x + \Delta x, y + \Delta y, t + \Delta t) = (1 - (\frac{\Delta t}{\tau_T}))g_k(x, y, t) + (\frac{\Delta t}{\tau_T})g_k^{eq}(x, y, t) + S \quad (6)$$

$$g_k^{eq} = w_k T(1 + \frac{c_k u}{c_s^2}) \quad (7)$$

g_k is the (PDF) Particle Distribution Function that denote internal energy evolution, g_k^{eq} is the (EDF) Equilibrium Distribution Function for thermal energy, τ_T is the relaxation time, S is the term source designed for simulate the radiative heat flux $\overrightarrow{q_R}$.the temperature is assumed as:

$$T(\vec{r}, t) = \sum_k g_k\,(\vec{r}, t) \quad (8)$$

We consider two vertical side walls maintained at constant temperatures and two adiabatic horizontal side walls. So, to determine these isothermal temperatures, the normal equilibrium condition was used for the adiabatic boundaries, the temperatures to be specified on the north boundary, for example, are expressed as:

$$g_k(x, n, t) = g_k(x, n - 1, t), k = 0,1..,8 \quad (9)$$

where n-1 is the lattice in the domain neighbor to the boundary lattice n.

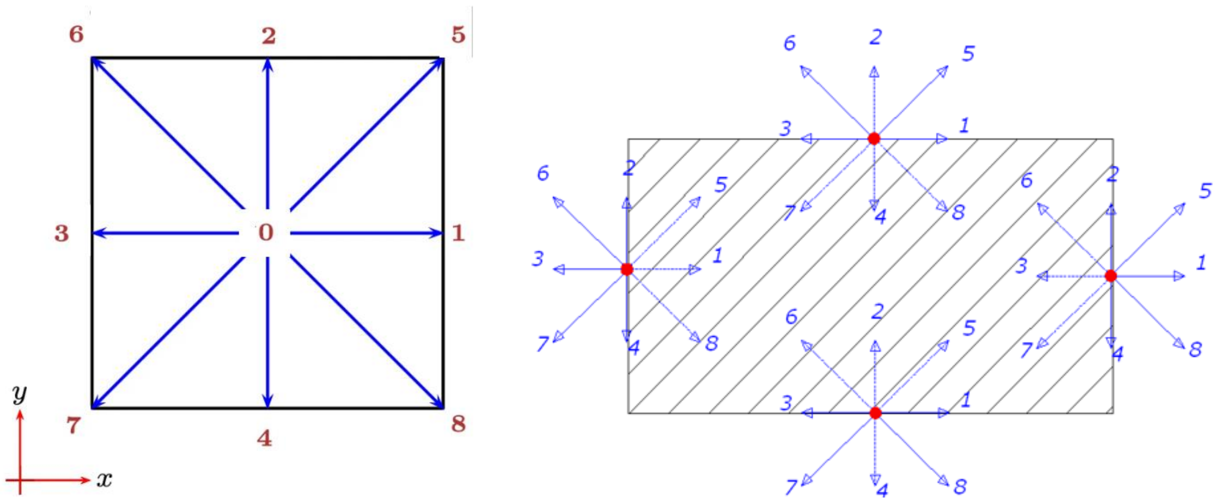


Fig.1. D2Q9 model.

The radiative heat flux$\vec{q}_R$ and $\vec{\nabla}.\vec{q}_R$ the divergence of radiative heat flux are:

$$\vec{q}_R = \int_{4\pi} I\vec{\Omega}\, d\Omega \quad (10)$$

$$\vec{\nabla}.\vec{q}_R = k_a(4\pi I_b - G) \quad (11)$$

Radiative intensity I is the solution of RTE (Radiative Transfer Equations). $I_b = \sigma T^4/\pi$ is the blackbody intensity, $k_a$is the absorption coefficient and $G$is the incident radiation.

For an absorbing, emitting and scattering grey medium, The RTE can be inscribed via the gradient of the intensity in the direction$\vec{\Omega}$ as

$$\vec{\nabla}.(I(s,\vec{\Omega}).\vec{\Omega}) = -(k_a + k_d)I(s,\vec{\Omega}) + k_a I_b(s) + \frac{k_d}{4\pi}\int_{\Omega'=4\pi} I(s,\overrightarrow{\Omega'})\, \Phi(\overrightarrow{\Omega'} \to \vec{\Omega})d\Omega' \tag{12}$$

Radiative intensity $I(s,\vec{\Omega})$ depend on position s and the direction $\vec{\Omega}$, k_d is the scattering coefficient and$\Phi(\overrightarrow{\Omega'} - \vec{\Omega})$ is the scattering phase function.

Wall bounding the physical domain is assumed grey and emits and reflects diffusely and radiative boundary condition for Eq. (12) is expressed as

$$I_w(\vec{\Omega}) = \frac{\varepsilon_w \sigma T_w^4}{\pi} + \frac{1-\varepsilon_w}{\pi}\int_{\overrightarrow{\Omega'}.\vec{n}_w<0} I_w(\overrightarrow{\Omega'})\left|\overrightarrow{\Omega'}.\vec{n}_w\right|d\Omega' \quad \text{if} \quad \overrightarrow{\Omega'}.\vec{n}_w > 0 \tag{13}$$

$\varepsilon_w$represents the wall emissivity and $\vec{n}_w$ is the unit normal vector on the wall

The CVFEM is used to discretize the RTE. In this numerical method, angular and spatial domains are divided into a finite number of control solid angles and control volumes, respectively. The direction of propagation$\vec{\Omega}$is defined by (θ,φ) where$\theta$and$\varphi$are, respectively, the polar and azimuthal angles.

As shown in Figure 2, total solid angle is subdivided into$N_\theta \times N_\varphi$control solid angles. These$N_\varphi N_\theta$control solid angles are non-overlapping, and their sum is4π. The control solid angle $\Delta\Omega^{mn}$is given by (Figure 2)

$$\Delta\Omega^{mn} = \int_{\Delta\theta}\int_{\Delta\varphi} \sin\theta \, d\theta d\varphi \tag{14}$$

The spatial domain is subdivided into three-node triangular elements.

As shown in Figure 2(b), a control volume$\Delta V_{ij}$is created around each node$N$by enjoining the centroids$G_l$of the elements to midpoints $M_l$and $M_{l+1}$of the corresponding sides. Each element has two faces, $M_l G_l$ and$G_l M_{l+1}$; bounding the sub-control volume around$N$; and each control volume is constructed by adding all sub-volumes $N M_l G_l M_{l+1} N$, Figure 2(c).

The obtained mesh is composed of$N_x N_y$control volumesΔV_{ij}. The$N_x$and$N_y$represent numbers of nodes in x and y direction, respectively. After integrating the RTE over both control volume and control solid, the final algebraic equation of the RTE is given by the following expression [18]

$$\beta_{ij}^{mn} = \gamma_{1ij}^{mn} I_{ij-1}^{mn} + \gamma_{2ij}^{mn} I_{i+1j}^{mn} + \gamma_{3ij}^{mn} I_{i+1j+1}^{mn} + \sum_{(m',n')=(1,1)}^{(N_\theta,N_\varphi)} \alpha_{ij}^{mnm'n'} I_{ij}^{m'n'} + \gamma_{4ij}^{mn} I_{ij+1}^{mn} + \gamma_{5ij}^{mn} I_{i-1j}^{mn} + \gamma_{6ij}^{mn} I_{i-1j-1}^{mn} \tag{15}$$

At that time, the algebraic Eq. (15) is written in an adequate matrix form [16,18] and the resulting equation set is resolved using the CCGS (conditioned conjugate gradient squared method).

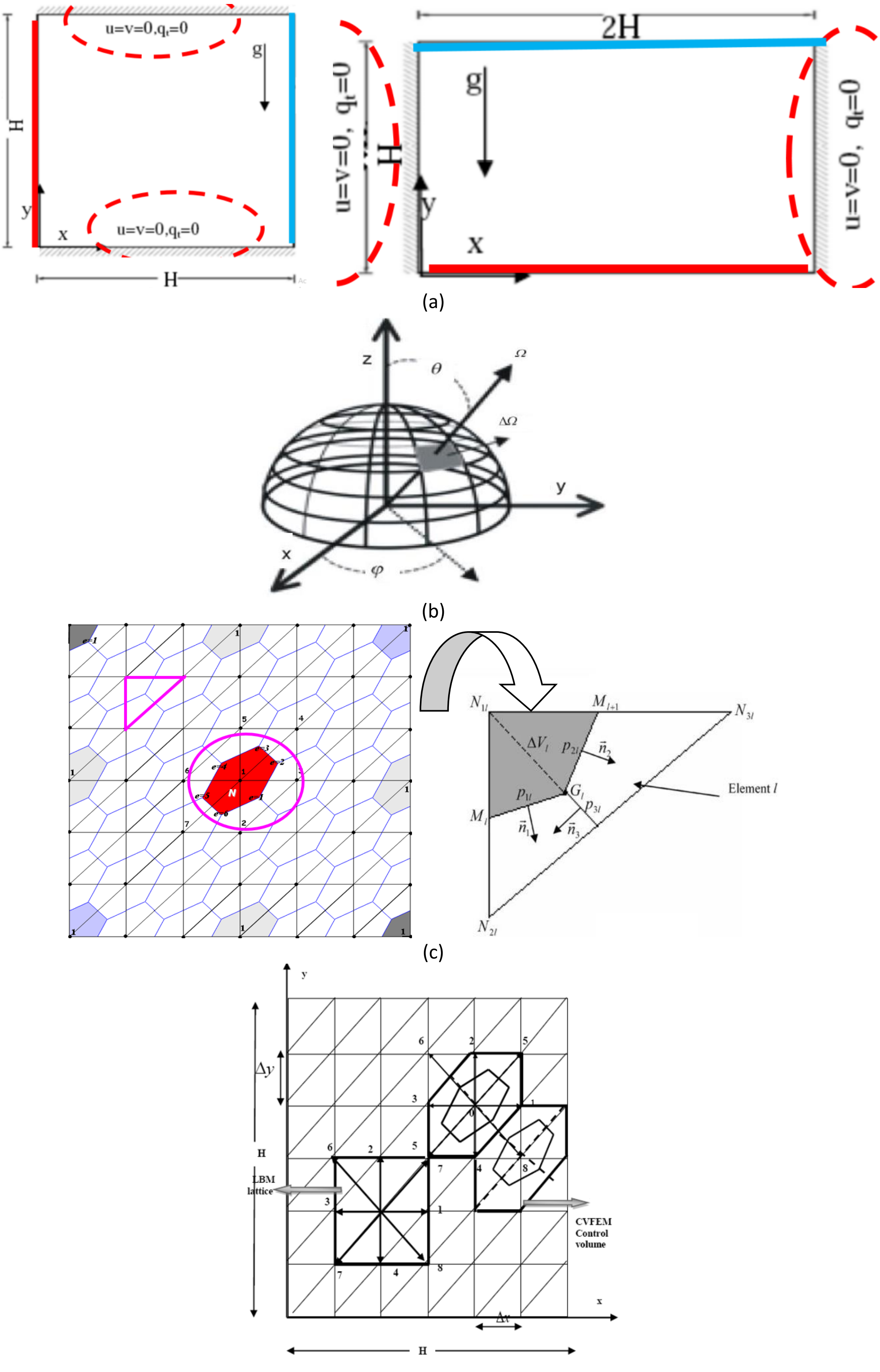


Fig. 2. (a) Geometries of the cavities, (b) Angular and spatial discretization, (c) Control volume and sub-volume and (d) coupled lattices

3. Validation and Results

For validation, isotherms profiles were studied at first in a differentially heated cavity without volumetric radiation by comparing literature results [13].

The first two-dimensional Rayleigh Benard convection simulation consider the case of $Pr = 0.71$with a characteristic velocity$V^2 = \beta_T g_y \Delta TH = 0.085$. For validation the steady state solution of vertical velocity component profile against y at Ra =510[4] is highlighted in Figure 3. A very good concordance is depicted with literature [13].

We treat also the effect of volumetric radiation on free convection. Figure 4 shows that when volumetric radiation is present, the temperature distributions are no longer anti-symmetric as in the pure natural convection case, because of the nonlinearity of the energy equation.

Figure 5 highlight the effect of volumetric radiation on free convection. We notice that the isotherms present a significant change and become almost parallel to the hot and cold side walls. Besides, dense isotherms occur in a region close to the cold side, demonstrating a high temperature gradient there

The transient Benard convection with volumetric radiation spanning an aspect ratio (Ar=2) is discussed. The problem at issue involves the effect of the (convection radiation parameter) N on velocity profiles. Figure 6 show the evolution of the streamlines that present a significant change as N increase.

Horizontal side wall creates a resistance to the convection flow because boundary layers are formed near the horizontal side walls, horizontal side wall creates a resistance to the convection flow.

We notice also that, the viscous forces increase, resulting in a higher critical Rayleigh number for the onset of convection due to the no-slip condition near the side wall causes a hindrance to the flow.

Volumetric radiation consequences in heat transfer from the horizontal sidewalls into the fluid domain. This reduces the temperature gradient inside the enclosure and drives the Rayleigh-Benard convection. Consequently, the onset of convection is deferred attributable to volumetric radiation.

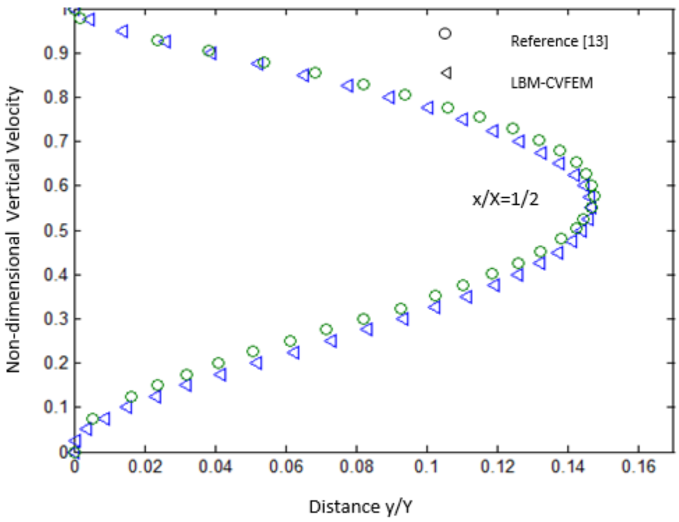


Fig. 3. Steady state (SS) y- velocity profile against y-direction at Ra =5.10^5 for Pr=0.71

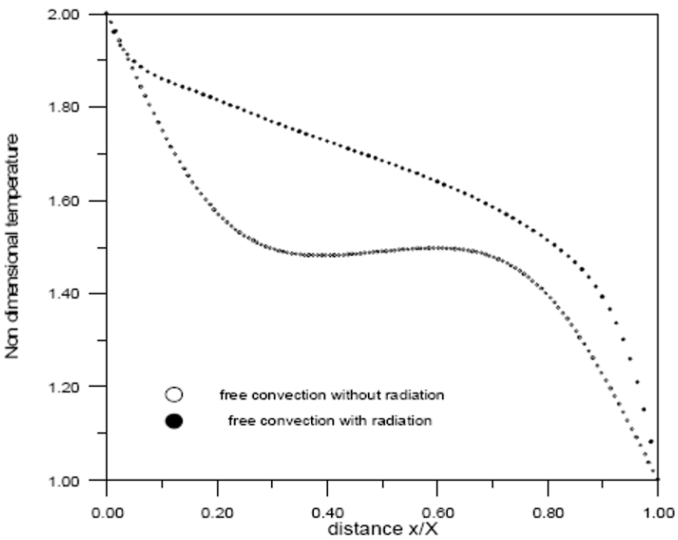
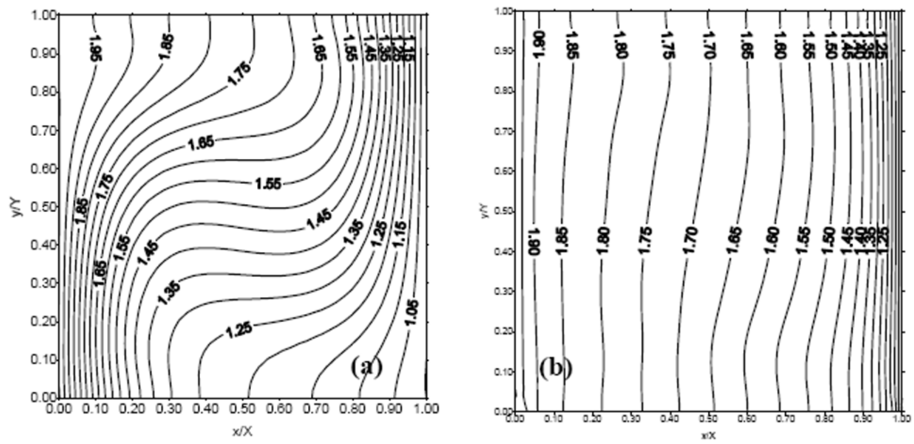


Fig. 4. The dimensionless steady state temperature (y/Y=0.5) for $N = 0.0 and N = 0.1, \varepsilon = 1, \beta = 1, \omega = 0 and Ra = 10^4$



N=0 N=0.1

Fig. 5. The dimensionless steady state isotherms$N = 0.0 and N = 0.1, \varepsilon = 1,$ $\beta = 1, \omega = 0 and Ra = 10^4$

Figure 7 show the evolution of the isotherms which change significantly N increase and become almost parallel to the cold and hot boundaries. Besides, we notice that dense isotherms take place in a region near the horizontal cold lid, showing a high temperature gradient there. Consequently, as N increases, volumetric radiation effect increases and the flow is therefore stabilized by the presence of the volumetric radiative source.

As shown in Figure 6 and 7, as the convection radiation parameter N increases, the effect of radiation increases and the flow is therefore stabilized by the presence of the radiative source.

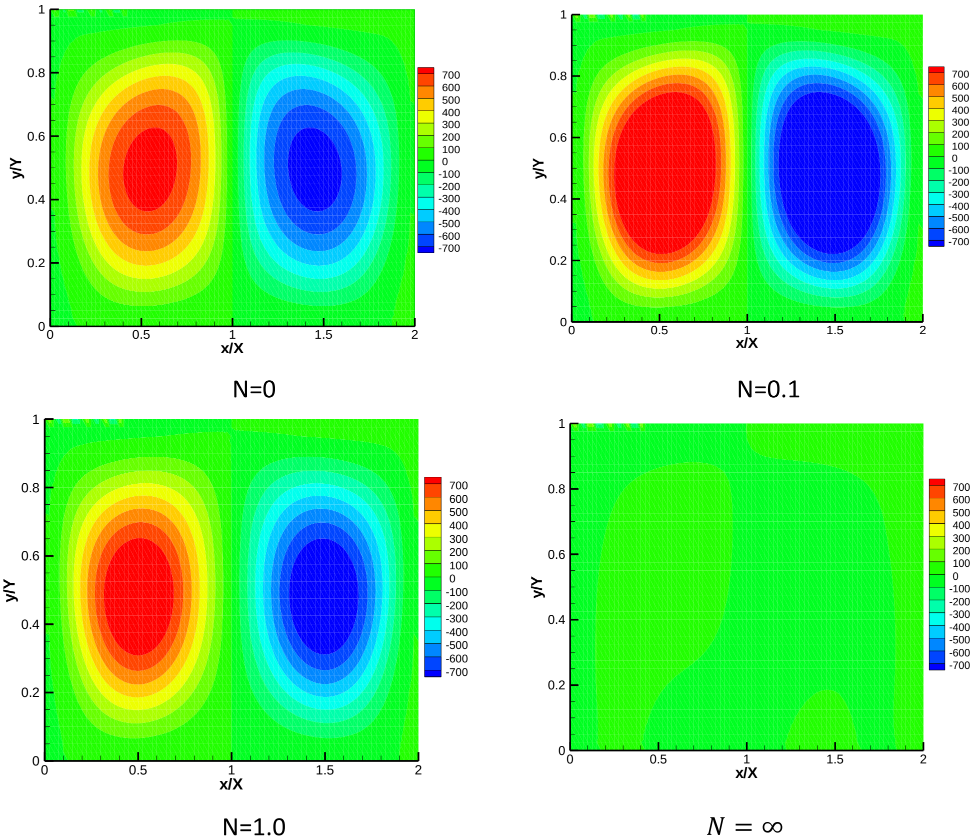


Fig. 6. Dimensionless steady state (SS) streamlines$Ra = 510^4$, $\omega = 0$and $Pr = 0.71$

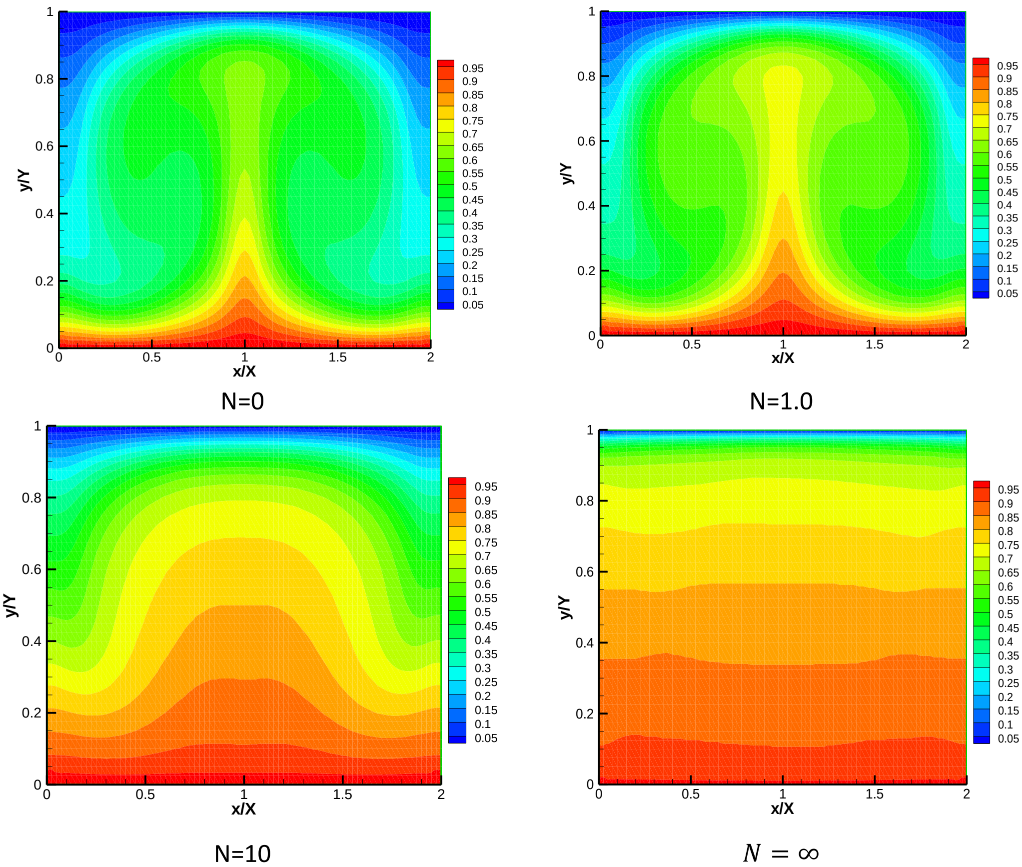


Fig. 7. Dimensionless steady state (SS) isotherms$Ra = 510^4$, $\omega = 0$and $Pr = 0.71$

4. Conclusions

In this work, a novel hybrid algorithm for combined transient convection and radiation systems has been explored with an in house made Fortran 90 code. The convection coupled to the volumetric radiation in a participating enclosure with absorbing, emitting and scattering medium is considered. In the absence of volumetric radiation, the results for free convection were compared with literature. Good agreements are highlighted. It has been shown that the new hybrid algorithm duplicates all the acknowledged features of the coupled convection-volumetric radiation phenomena in a participating media which make of the present numerical tool to predict heat transfer when dealing with built thermal environment and thermal comfort.

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