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# Influence Of Thermal Radiation on Unsteady Mixed Convection Flow of Hybrid Nanoliquid Past an Elongated Sheet

A. Mahdy[1], Taghreed H. Al-Arabi[2], Ahmed M. Rashad[3], Wafaa Saad[3]

[1] Mathematics Department, Faculty of Science, South Valley University, Qena, Egypt
[2] Department of Mathematics, Faculty of Science, Taibah University, Al-Madinah Al-Munawwarah, Saudi Arabia
[3] Mathematics Department, Faculty of Science, Aswan University, Aswan, Egypt

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**ABSTRACT**

Actually, the article spotlights an aspect of thermal radiative on free-forced convective of a hybrid nanomaterial flow over a vertical elongated plate. Viscous dissipation with convective condition is exhibited. The primary PDEs governing the case paradigm is converted into a non-dimensional system due to feasible transformations. The acquired mathematical differential equations is solved using a the very vigorous computer algebra software MATLAB code. Graphs were presented to analyze the influence of multiple physical impacts of involving factors on the flow fluctuations of both hybrid nanoliquid velocity and temperature. Through these factors, both of Nusselt number and drag factor are manifested and argued amply. Comparison with earlier published data for steady and unsteady states flow is provided and it noticed to be in completely agreement. The outcomes point out that Nusselt number is an ascending function of unsteadiness and mixed convection factors.

## 1. Introduction

Through the present era, nanoliquids have gained prime and worthy attention by engineers and researchers due to its capability to support the thermal conductivity of the traditional liquids and thus elevate the rate heat transmission congruous to those conventional liquids. Regarding to these features of nanoliquids, it was lucid that nanofluids provided in miscellaneous scopes and implementations like as in the industrialization and designing of petroleum engineering, electronic devices, material composition, pharmaceutical manufacture and biomechanical field, etc. Choi [1] is pathfinder in scrutinizing the main endeavor to utilize nanofluids to amend the thermal characteristic of traditional fluids. Next this active attempt, which opened unprecedented horizons for researchers through this area a number of investigations have been achieved to scrutinize the nanofluids synthesis and miscellaneous empirical samples were submitted by a number of authors to inspect the influence of thermophysical characteristics on several contributions of nanomaterials flow viz the Twari-Das model [2] and Buongiornio paradigm [3]. Mankinde and Aziz [4] investigated the nanofluid

*Corresponding author.
*E-mail address: tagr61@yahoo.com*

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flow with convective boundary condition through a moving plate. However, theoreticians focused on the comparable arena [5-11].

The main defiance for engineers and scientists of immediate epoch is to get the novel tools in nanotechnology with low cost; long-lasting and great performance. Thus, to produce cost-effective and sustainable, an innovative type of hybrid nanoliquid are utilized. The primary fluid conducting capacity can considerably enhance by combination of indicator whole of metal nanotubes/nanoparticles into metal/oxide nanoparticles which actually are dispersed in the primary liquid. This kind of hybrid nanoliquids attracts its engineering implementations through the area of thermal administration in cooling of generators, biomedical, vehicles, and atomic frame cooling, thermal capacity, sunlight founded heating, transformer cooling, coolant in machining, warming and cooling in structures, and so on. The above aforesaid industrial application motivates various innovators and investigators to do on these particular attributes of hybrid nanoliquids. Jana *et al.,* [12] performed an empirical and explored that thermal conductivity of various nanolquids were proportionally more than notional computations and it's noted that steadiness is perfectly depend on the characteristics of dispersed nanoparticles. Suresh *et al.,* [13] structured hybrid nanofluid and elucidated that thermal and mechanical attribute of the pure liquid enhance extremely. Momin [14] performed an empirical study on hybrid nanoliquid flow through an oblique cylinder. Asghar and Ying [15] The examined the 3D hybrid nanoliquid flow with rotating stretching/shrinking sheet under the impact of magnetic field and Joule heating due to Tiwari-Das model. Recently, various investigators numerically examined the hybrid nanofluids flow and its heat transfer properties several physical cases [16-20].

In branches science and several industrial procedures such as ocean circulation, electronic equipment and thermal distribution in buildings, ventilation and air conditioning systems in atmospheric. Aforementioned industrial significance encouraged the study the notion of mixed convection flow in different geometries. The first study was experimental work on the impact of various factors on the rate of heat transfer from a circular cylinder by Hatton *et al.,* [21] after that, Sparrow and Lee [22] compared the analytical results that he obtained with the experimental results [21] and found that the effect of the mixed load is greater on the back side of the cylinder. Recently, this concept has been found in several contributions [23-31].

The importance of the influence of thermal radiation on the thermal and mass transfer of moving fluids lies in many industrial and engineering fields, including but not limited to metal waste, oil tanks, catalytic reactors [32,33]. Therefore, many researchers have investigated many works related to thermal radiation with different geometric shapes, to name a few, Akaje and Olajuwon [34], Hayat *et al.,* [35], Rusdi *et al.,* [36], Khan *et al.,* [37] and Patil and Goudar [38]. The effect of heat radiation and distribution with mixed load on mass transfer processes and heat in a moving fluid medium becomes worthy of attention because of its contributions to various sources of energy conversion in many engineering and industrial processes such as satellites, nuclear power plants, spacecraft, solar fans, etc. Therefore, many researchers were interested in their study Patil and Kulkarni [39], Khan *et al.,* [40,42] and Khan *et al.,* [41].

The current study has many important industrial applications in the manufacturing of polymer sheets, coating of wires, fibre sheets, optical fibres and many more. Anyway, the importunity of our argumentation is to deduce examination of unsteady mixed convection flow of viscous dissipative hybrid nanoliquid past a vertical radiative stretchable sheet with convective condition theoretically. The novelties in our contribution, analysis of unsteady mixed convective flow over a stretching sheet, influence of radiation and Biot number impact, impact of Cu-$Al_2O_3$ hybrid nanofluid on the flow characteristics. The arithmetical pattern is calculated for our contribution and resolved numerically after utilizing the proper transformations using the function bvp4c from MATLAB. Due to there isn't almost empirical data, the selection of the values of the factors were transcribed by the values selected by earlier authors. The effectiveness of abundant pertinent factors on the momentum and thermal fields due to drag friction and heat transmission are examined with the assistance of graphs elucidation.

## 2. Equations Formulation

Present the laminar, time dependent, 2D free-forced convection-radiation interaction boundary layer movement through a stretched vertical sheet in an incompressible hybrid nanofluid with viscous dissipation and Newtonian heating aspects, portrayed in Figure 1. The base liquid is chosen to be $H_2O$ loaded with nanoparticles of Copper Cu and Alumina $Al_2O_3$. The pure liquid and in thermal equilibrium solid volume nanoparticles were considered to be, besides non slip exists among them. The thermo-physical attributes of the nanomaterial were shown in Table 1 as Oztop and Abu-Nada [10]. Additionally, let us assume that before initial time, i.e., when $t < 0$ the hybrid nanofluid and heat flows represent non-time dependent case. At $t = 0$, the time-dependent state of flow and heat transmit begin, the surface being elongated due to the speed $\bar{U}_{\mathrm{w}}(x, t)$ through the horizontal axis $x$. The sheet temperature $T_{\mathrm{f}}(x, t)$ was postulated to obey a linear mathematical formula with variable $x$ and an inverse square formula with it reduce in time. Moreover, the origin of stationary Cartesian coordinate was situated at the front edge of the elongated sheet with the plus $x$-axis expanding through the plate in the upwards trend, whereas the $y$-axis is selected to be perpendicular to the elongated plate and is plus in the trend from the sheet to the fluid. Both cooling ($T_{\mathrm{f}} < T_\infty$) and warming ($T_{\mathrm{f}} > T_\infty$) of the elongated plate were recognized, that referring to opposing and assisting flows, respectively. Regarding to the case of assisting flow, the stretching generated movement and the thermal buoyant flow support each other, and the reverse occurs for the opposing case flow. Boussinesq approximations and boundary layer are postulated to be adequate. The primary time-dependent mass conservation, momentum and thermal energy formulas for hybrid nanoliquids may be expressed in the formulas [1,2, 8,23].

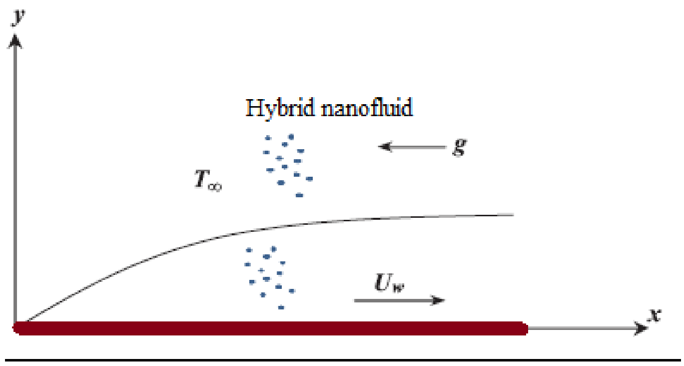


**Fig. 1.** Flow configuration model

$$\frac{\partial \bar{U}}{\partial x} + \frac{\partial \bar{V}}{\partial y} = 0 \tag{1}$$

$$\rho_{\mathrm{hnf}}\left(\frac{\partial \bar{U}}{\partial t} + \bar{U}\frac{\partial \bar{U}}{\partial x} + \bar{V}\frac{\partial \bar{U}}{\partial y}\right) = \mu_{\mathrm{hnf}}\frac{\partial^2 \bar{U}}{\partial y^2} + g(\rho\beta)_{\mathrm{hnf}}(T - T_\infty) \tag{2}$$

$$(\rho c_p)_{\mathrm{hnf}}\left(\frac{\partial T}{\partial t} + \bar{U}\frac{\partial T}{\partial x} + \bar{V}\frac{\partial T}{\partial y}\right) = k_{\mathrm{hnf}}\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + \mu_{\mathrm{hnf}}\left(\frac{\partial \bar{U}}{\partial y}\right)^2 \tag{3}$$

Associated with the following appropriate boundary conditions

$$\bar{U} = \bar{U}_{\mathrm{w}}, \quad \bar{V} = 0, \quad -k_{\mathrm{hnf}}\frac{\partial T}{\partial y} = h_{\mathrm{f}}(T_{\mathrm{f}} - T) \qquad \text{as} \quad y = 0$$
$$\bar{U} \to 0, \quad T \to T_{\infty}, \qquad \text{as} \quad y \to \infty \tag{4}$$

wherein, $\bar{U}$ and $\bar{V}$ stand for the hybrid nanoliquid speed components through the $x$ and $y$ axes, respectively, $\rho_{\mathrm{hnf}}$ means the active density of the hybrid nanoliquid, $\mu_{\mathrm{hnf}}$ indicates the effective dynamic viscosity of the hybrid nanoliquid, $T$ gives the temperature of the hybrid nanoliquid, $\beta_{\mathrm{hnf}}$ denotes the thermal expansion of the hybrid nanomaterial, $g$ points out the acceleration due to gravity.

Note the term of radiation in energy equation is obtained utilizing Rosseland approximation $q_r = -\frac{4\sigma}{3k^\star}\frac{\partial T^4}{\partial y}$ here $\sigma$ gives Stefan-Boltzmann fixed value and $k^\star$ means absorption factor. Extension $T^4$ suing Taylor formula about $T_\infty$ with ignoring terms of higher order, one gets $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.

Now, for hybrid nanofluids, let us present the following expression for $\rho_{\mathrm{hnf}}, \mu_{\mathrm{hnf}}, (\rho c_p)_{\mathrm{hnf}}$ and $(\rho\beta)_{\mathrm{hnf}}$ of the hybrid nanofluid

$$\rho_{\mathrm{hnf}} = \varphi_1\rho_1 + \varphi_2\rho_2 + (1-\varphi)\rho_{\mathrm{f}} \tag{5}$$

$$\mu_{\mathrm{hnf}} = \frac{\mu_{\mathrm{f}}}{\left(1-(\varphi_1+\varphi_2)\right)^{2.5}} \tag{6}$$

$$(\rho c_p)_{\mathrm{hnf}} = \varphi_1(\rho c_p)_1 + \varphi_2(\rho c_p)_2 + (1-\varphi)(\rho c_p)_{\mathrm{f}} \tag{7}$$

$$(\rho\beta)_{\mathrm{hnf}} = \varphi_1(\rho\beta)_1 + \varphi_2(\rho\beta)_2 + (1-\varphi)(\rho\beta)_{\mathrm{f}} \tag{8}$$

$$\frac{k_{\mathrm{hnf}}}{k_{\mathrm{f}}} = \left(\frac{(\varphi_1 k_1 + \varphi_2 k_2)}{\varphi} + 2k_{\mathrm{f}} + 2(\varphi_1 k_1 + \varphi_2 k_2) - 2\varphi k_{\mathrm{f}}\right)$$
$$\times\left(\frac{(\varphi_1 k_1+\varphi_2 k_2)}{\varphi} + 2k_{\mathrm{f}} - (\varphi_1 k_1 + \varphi_2 k_2) + \varphi k_{\mathrm{f}}\right)^{-1} \tag{9}$$

wherein the symbol $\varphi$ denotes the overall volume concentration of two miscellaneous sorts of nanoparticles suspended in hybrid nanoliquid ($\varphi_1 = \varphi_{\mathrm{Al_2O_3}}, \varphi_2 = \varphi_{\mathrm{Cu}}$) and is evaluated as; $\varphi = \varphi_{\mathrm{Al_2O_3}} + \varphi_{\mathrm{Cu}}$

The attributes of the hybrid nanomaterial were given in the relations above are estimated from water and solid volume nanoparticles attributes at mean bulk temperature. One postulate that both of t $\bar{U}_{\mathrm{w}}(x,t)$ and $T_{\mathrm{f}}(x,t)$ were presented as $\bar{U}_{\mathrm{w}}(x,t) = \frac{ax}{1-\tilde{\gamma}t}$, $\quad T_{\mathrm{f}}(x,t) = T_\infty + \frac{bx}{(1-\tilde{\gamma}t)^2}$ wherein $a$ and $\tilde{\gamma}$ indicate fixed values (where $a > 0$ and $\tilde{\gamma} \geqslant 0$, with $\tilde{\gamma}t < 1$), and both associated with dimension s $^{-1}$, whereas $b$ stands for a fixed value and of dimension Km $^{-1}$, wherein $b > 0$ and $b < 0$ indicating to the movements of assisting and opposing, respectively, and absence of $b$ related to forced convective limit (ignoring of buoyancy force).

**Table 1**
Thermo-physical attributes of $H_2O$, Cu and $Al_2O_3$

| Property | $H_2O$ | Cu | $Al_2O_3$ |
|---|---|---|---|
| $\boldsymbol{\rho}$ (kg m $^{-3}$) | 997.1 | 8933 | 3970 |
| $\boldsymbol{c_p}$ (J kg $^{-1}$ K $^{-1}$) | 4179 | 385 | 765 |
| $\boldsymbol{k}$ (W m $^{-1}$ K $^{-1}$) | 0.613 | 401 | 40 |
| $\boldsymbol{\beta \times 10^5}$ (K $^{-1}$) | 21 | 1.67 | 0.85 |

At this stage, one inserts the next non-dimensional functions $F$, $\theta$ and $\eta$ like Andersson *et al.,* [43], Mahdy [23,44].

$$\eta = \left(\frac{a}{\nu_{\mathrm{f}}(1-\tilde{\gamma}t)}\right)^{1/2} y, \qquad \psi = \left(\frac{a\nu_{\mathrm{f}}}{1-\tilde{\gamma}t}\right)^{1/2} xF(\eta), \qquad \theta(\eta) = \frac{T-T_\infty}{T_{\mathrm{f}}-T_\infty} \tag{10}$$

$\psi$ stands for stream function, in terms of previous mathematical formulas the velocity components may be given by, $\bar{U} = \frac{ax}{1-\tilde{\gamma}t}F'(\eta), \qquad \bar{V} = -\left(\frac{a\nu_{\mathrm{f}}}{1-\tilde{\gamma}t}\right)^{1/2} F(\eta)$

The mutated momentum and heat mathematical formulas associated with the boundary conditions expressed by Eq. (2)-(4) may be stated as

$$\frac{\mu_{\mathrm{hnf}}}{\mu_f}F''' + \frac{\rho_{\mathrm{hnf}}}{\rho_f}\left(FF'' - F'^2 - A\left(F' + \frac{1}{2}\eta F''\right)\right) + \frac{(\rho\beta)_{\mathrm{hnf}}}{(\rho\beta)_f}\gamma\theta = 0 \tag{11}$$

$$\left(\frac{k_{\mathrm{hnf}}}{k_{\mathrm{f}}}\right)(1+Rd)\theta'' + \mathrm{Pr}\left(\frac{(\rho c_p)_{\mathrm{hnf}}}{(\rho c_p)_{\mathrm{f}}}\right)\left(F\theta' - F'\theta - A\left(2\theta + \frac{1}{2}\eta\theta'\right)\right) + \frac{\mu_{\mathrm{hnf}}}{\mu_{\mathrm{f}}}EcF''^2 = 0 \tag{12}$$

Subjected to the hybrid nanofluid mixed convection flow boundary conditions

$$F(0) = 0, \quad F'(0) = 1, \quad \frac{k_{\mathrm{hnf}}}{\partial k_{\mathrm{f}}}\theta' = Bi(\theta - 1), \qquad F'(\infty) \to 0, \quad \theta(\infty) \to 0 \tag{13}$$

where, primes refer to differentiation regard to similarity variable $\eta$ and $A = \tilde{\gamma}a^{-1}$ gives a factor that indicates the unsteadiness, $Rd = \frac{16\sigma T_\infty^3}{3k^* k_{\mathrm{f}}}$ indicates radiation factor, $Bi = \frac{h_{\mathrm{f}}}{k_{\mathrm{f}}}\sqrt{\frac{\nu_{\mathrm{f}}(1-\tilde{\gamma}t)}{a}}$ points out Biot number, $Ec = \frac{\bar{U}_{\mathrm{w}}^2}{(c_p)_{\mathrm{f}}(T_{\mathrm{f}}-T_\infty)}$ represents Eckert number and $\mathrm{Pr} = \nu_{\mathrm{f}}/\alpha_{\mathrm{f}}$ stands for the Prandtl number. Addationally, $\gamma$ means the buoyancy or free-forced convection factor known as $\gamma = Gr_x Re_x^{-2}$, where $Gr_x$ and $Re_x$ represent local Grashof and Reynolds numbers, respectively, and are expressed as $Gr_x = \frac{g\beta_{\mathrm{f}}(T_{\mathrm{f}}-T_\infty)x^3}{\nu_{\mathrm{f}}^2}, \qquad Re_x = \frac{x\bar{U}_{\mathrm{w}}}{\nu_{\mathrm{f}}}$. It is clear that $\gamma$ is a non-dimensional fixed value with $\gamma > 0$ and $\gamma < 0$ referring to assisting and opposing movements, respectively, whereas $\gamma = 0$ gives the case of forced convective movement.

To reveal the heat transmission characteristics of the model of current hybrid nanofluid movement, the quantities of physical important were pointed as the local skin friction factor $C_f$ and the local heat transmission factors that formulated as local Nusselt number $Nu_x$ and those were expressed as Eq. (14).

$$C_f = \frac{2\tau_w}{\rho_{\mathrm{f}}\bar{U}_{\mathrm{w}}^2}, \qquad Nu_x = \frac{xq_w}{k_{\mathrm{f}}(T_{\mathrm{f}}-T_\infty)} \tag{14}$$

Here the drag friction $\tau_w$ as well as the heat transmission from the elongated surface $q_w$ were evaluated by Eq. (15)

$$\tau_w = \mu_{\mathrm{hnf}} \frac{\partial \bar{U}}{\partial y}\bigg|_{y=0}, \qquad q_w = -k_{\mathrm{hnf}} \frac{\partial T}{\partial y}\bigg|_{y=0} \tag{15}$$

Invoking the non-dimensional transformations (10), one gets Eq. (16).

$$C_f Re_x^{1/2} = 2\left(\frac{\mu_{\mathrm{hnf}}}{\mu_{\mathrm{f}}}\right) F''(0), \qquad Nu_x Re_x^{-1/2} = -\left(\frac{k_{\mathrm{hnf}}}{k_{\mathrm{f}}}\right) \theta'(0) \tag{16}$$

## 3. Results and Discussion

The gained system of finally established non-linear flow of ODE (11) and (12) with convenient boundary conditions expressed in Eq. (13) exhibit a bit of mathematical complication in finding the exact analytical solution. Therefore, these stated mathematical expressions were simulated numerically, by means of the function bvp4c strategy from MATLAB software for representative values of the non-dimensional factors. The imposed bvp4c code was promoted employing an approach of finite-difference that implement the 3-stage Lobatto IIIa formula. This represents a combination technique of 4 $^{\text{th}}$-order precision. In the imposed approach, the ODE (10-11) were first altered to a system of 1 $^{\text{st}}$-order by inserting novel set of transformations i.e., initial value problem (IVP). The mesh selected and error governing were related to the residual of the continuous solution. Inclusive numerical computations were estimated for some values of involved factors describing the flow characteristics for obtaining an evident physical view of the motion and heat transmission case and the outcomes were graphically portrayed. The characteristic of the hybrid nanoliquid speed and heat besides the rate of heat transmission and drag friction at boundaries were plotted. Table 2 illustrates a comparison between the outcomes that we gained and the previous computations given by Ishak *et al.,* [45] for pure fluid when $Rd = Ec = 0$. We verified that the results are very close, which indicates that the accuracy of the used approach is excellent. The plotted hybrid nanofluid temperature and velocity variations through Figure 2 until Figure 5 verify that the imposed boundary conditions were met, that advocacy the computation outcomes.

Figure 2 exhibits the impact of time-dependent factor $A$ on the hybrid nanoliquid speed and heat fluctuations respectively considering the cases of $Bi = 0.5$ and constant wall temperature $\beta \rightarrow \infty$. It's notice that raising in time-dependent factor $A$ yields decrease boundary and thermal layer thickness, that can be regarded to the truth that when the unsteadiness factor $A$ boosts, the speed of the stretched surface diminishes as a result in the transmission of little amount of heat and mass from the plate to the hybrid nanoliquid in the boundary layer region. In addition, with greater values of Biot number $Bi$, hybrid nanofluid velocity distribution boosts. Figures 3 and 4 disclose the impact of free-forced convection factor $\gamma$ and Eckert number $Ec$ on the hybrid nanoliquid speed and temperature variations respectively. Note, higher values of $\gamma$ refers to more vigorous natural (free) convection, i.e., higher values of Grashof number. A vigorous considerable strengthen in velocity profiles is obtained within the boundary layer transverse to the sheet plate with rising $\gamma$. It is evident that an increment in the value of both parameters yield strengthen the thickness of momentum boundary layer. A larger buoyancy force is generated by the greater amount of $\gamma$, which award the highest transmission energy and such energy produce confrontation through the motion, Figure 3. Mixed convection parameter leads to reduce the thermal boundary layer thickness as depicted in Figure 3. Unlike the mixed convection factor, Figure 4 illustrates that hybrid nanoliquid temperature enhances with higher values of Eckert number $Ec$. An increment of $Ec$ leads to stock the heat energy in the hybrid nanomaterial due to the forces of the friction which strengthen the hybrid nanofluid temperature profile.

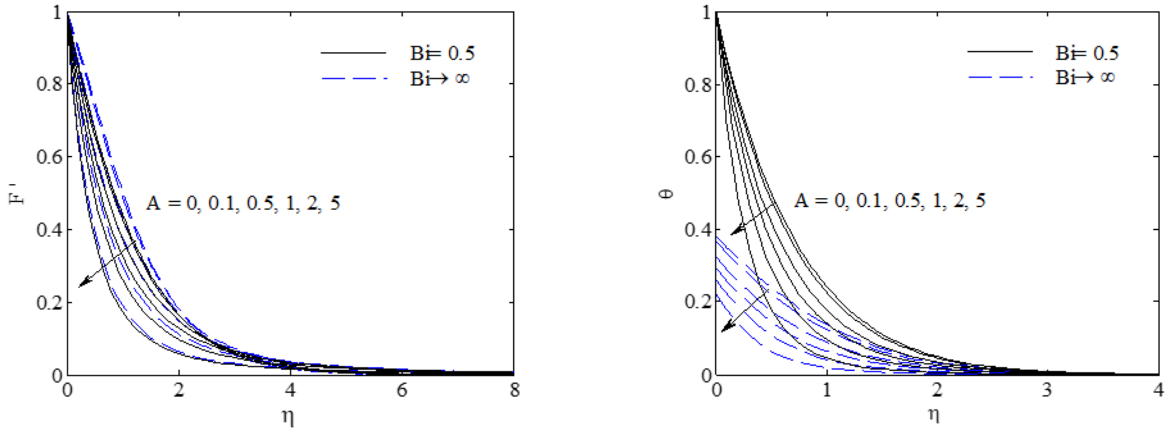


**Fig. 2.** Velocity & temperature profiles versus unsteadiness parameter $A$

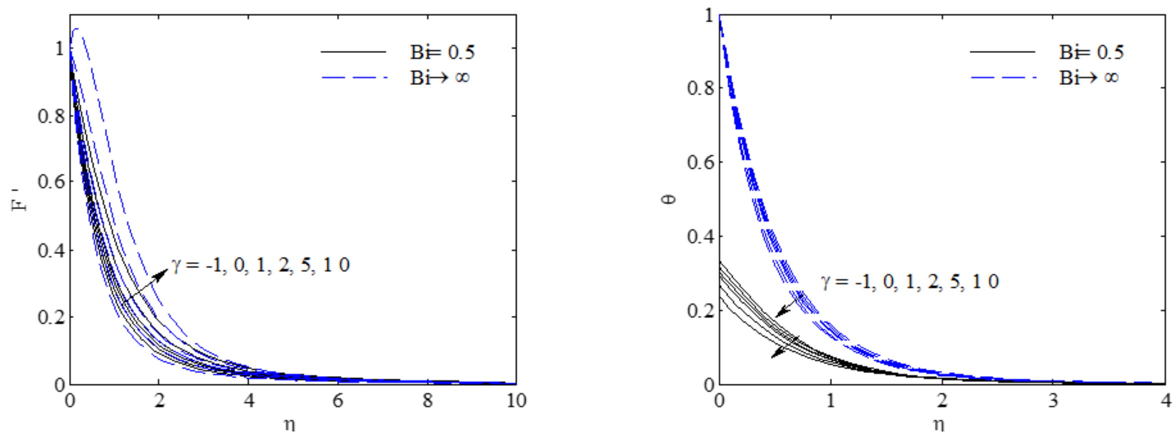


**Fig. 3.** Velocity & temperature profiles versus mixed convection parameter

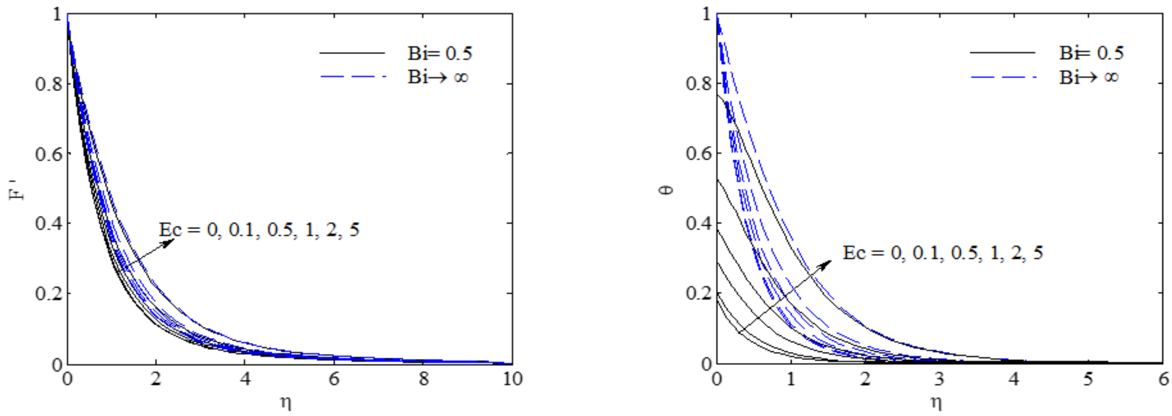


**Fig. 4.** Velocity & temperature profiles versus Eckert number

Figure 5 reveals the influence of radiative factor $Rd$ and Biot number $Bi$ respectively, on hybrid nanofluid temperature profile $\theta(\eta)$. We observe that an upgrade in radiation parameter and Biot number lead to boosts in thermal boundary layer thickness. An upgrade in thermal radiative provides energy to the particles of the hybrid nanofluid which results in an augment in temperature and thermal boundary layer thickness. Essentially, this parameter become clear at the boundary of the system and is so helpful to boost the temperature. Further, hybrid nanofluid temperature transfers from the boundaries to inner system. As clear, temperature is strongly strengthened with rising in Biot number $Bi$ and therefore improvements the thermal boundary layer. When $Bi < 1$, i.e., low impact of Biot number, the domain is usually pointed out as thermally simple and more constant temperature profiles are introduced within the boundary layer and the solid sheet plate. As $Bi > 1$, thermal distributions were postulated to be variant. Essentially, Biot number gives a mechanism for the comparison of heat transfer of the convective reluctance outer to body to the conduction reluctance inside a solid body. Of course, $Bi > 0.1$, refers to thermally thick material, whilst $Bi < 0.1$, denotes a material of thermally delicacy. As $Bi$ is proportional to the thermal conductivity ($k_f$) inversely, hence as $Bi$ strengthens, thermal conductivity of the hybrid nanofluid is weakened at the plate, and therefore, the rate of heat transmission is weakened from the boundary layer to the sheet surface. Therewith, also Biot number is proportional to the Grashof number $Gr_x$ inversely. Hence, as Biot number boosts, the local Grashof number reduces and generates an acceleration in the boundary layer region. Of course, when $Gr_x$ boosts, the change in buoyancy leads to decelerates the boundary layer movement. The skin friction factor and the local Nusselt number as a function of the nanoparticles fraction factor $\varphi$ for variant values of unsteadiness factor $A$ are portrayed in Figure 6. It can be illustrated that for higher value of $A$ the skin friction factor is weakened and the local Nusselt number is strengthened. The drag friction factor and the local Nusselt number variations expressed mathematically as $F''(0)$ and $-\theta'(0)$ with $\gamma$ were plotted in Figure 7. It is exposed from this figure that an assisting buoyant movement $\gamma > 0$ leads to strengthen the drag friction factor, whereas an opposing buoyant motion gives an opposite behavior. That is according to the fluid speed boosts as force of buoyancy enlarges and thus improves the surface shear stress. These yields strengthen the drag friction factor, and also improves the heat transmission rate around the wall. The impact of Eckret number $Ec$ and radiation factor $Rd$ on the local Nusselt number were given in Figure 8. There is an inverse relationship between Eckert number and Nusselt number $-\theta'(0)$ which refers to reduce in Nusselt number when Eckert number is improved. Finally, the relation between the local Nusselt number and the radiative factor is plotted in Figure 8, an upgrade values of $Rd$ results in decline the Nusselt number. Figure 9 highlights the impact of Cu-$H_2O$ with $\varphi = 0.05$, $Al_2O_3$-water with $\varphi = 0.05$ nanofluid and Cu-$Al_2O_3$-water with $\varphi_{\mathrm{Cu}} = \varphi_{\mathrm{Al_2O_3}} = 0.025$ hybrid nanofluid on the drag friction factor and the local Nusselt number respectively.

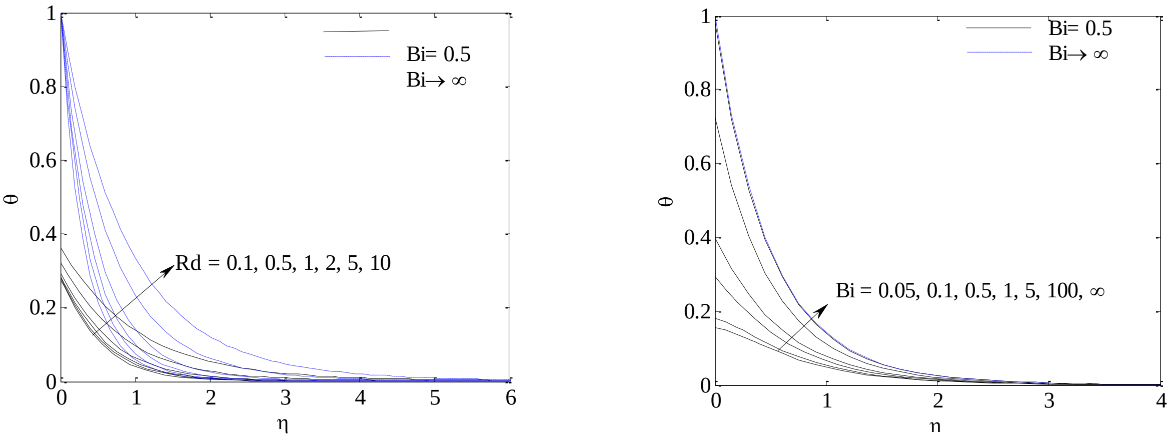


**Fig. 5.** Temperature curves versus radiation parameter and Biot number

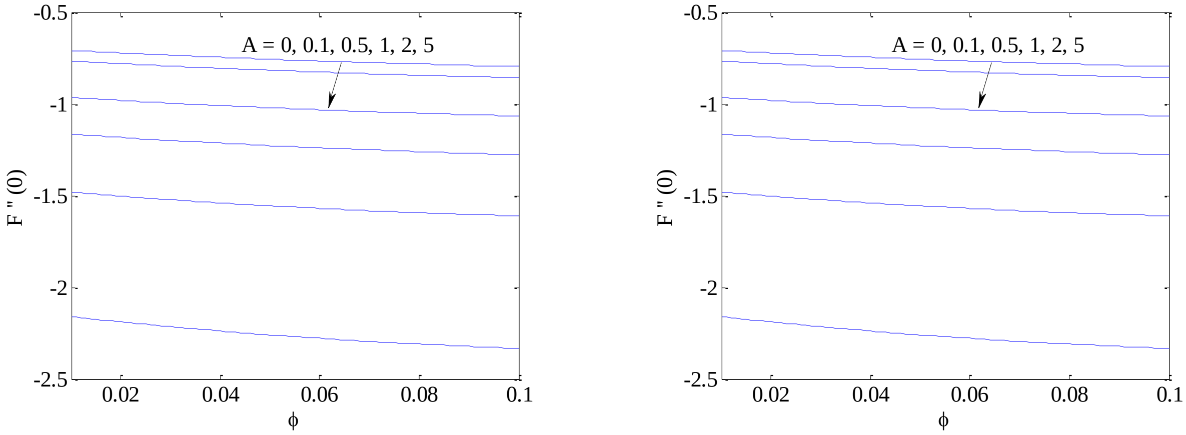


**Fig. 6**. Skin friction & Nusselt number fluctuations versus unsteadiness parameter

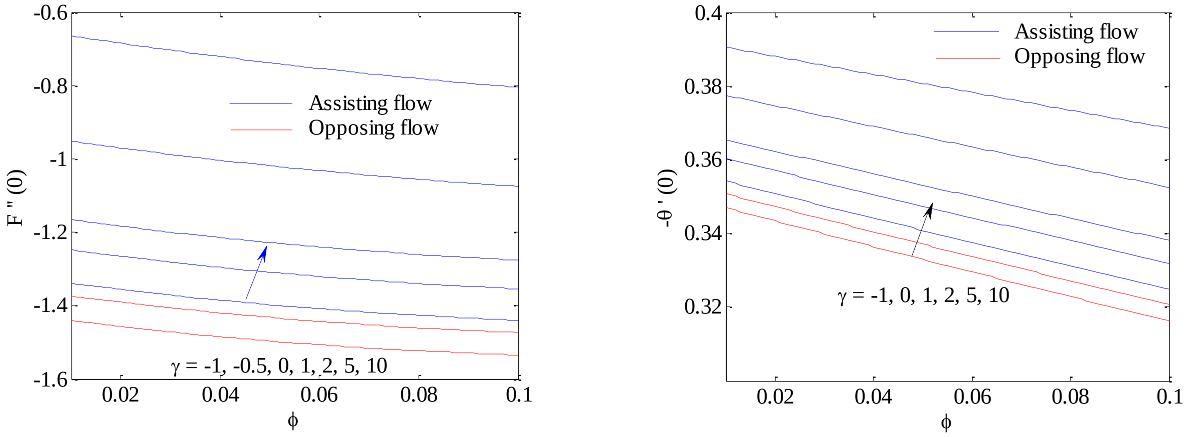


**Fig. 7.** Skin friction & Nusselt number fluctuations versus mixed convection parameter

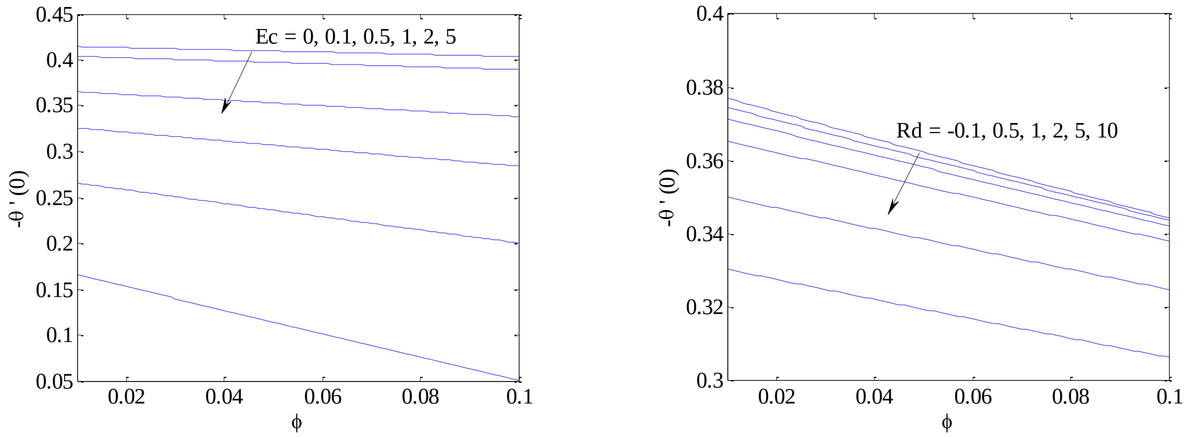


**Fig. 8.** Nusselt number & Nusselt number fluctuations versus Eckert number

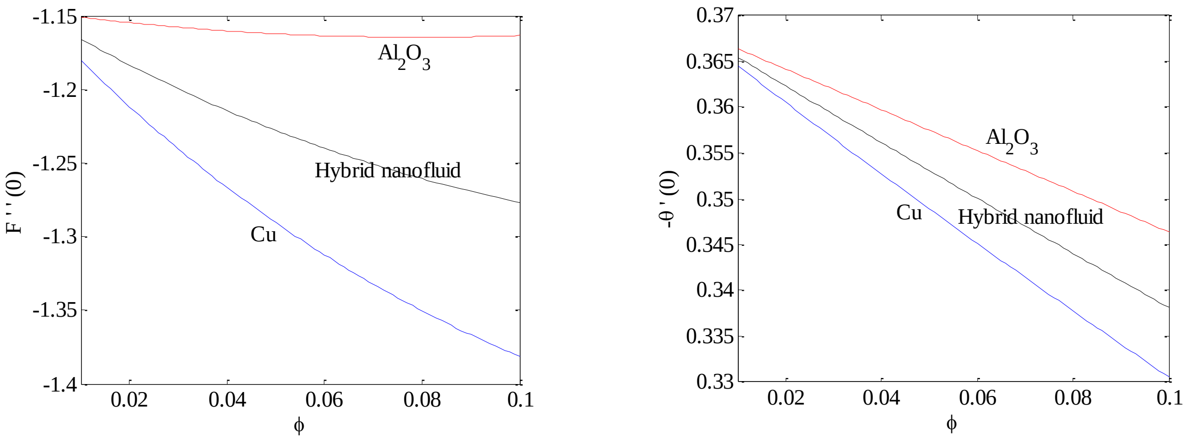


**Fig. 9.** Skin friction & Nusselt number for nanoliquid and hybrid nanoliquid

**Table 2**
Values of $-\theta'(0)$ for various values of Pr, $A$ and $\gamma$ with $Rd = Ec = 0$ of pure fluid

| $A$ | $\gamma$ | Pr | Ishak *et al.,* [45] | Present |
|---|---|---|---|---|
| 0.0 | 0 | 0.01 | 0.0197 | 0.01999 |
| | | 0.72 | 0.8086 | 0.80868 |
| | | 1.0 | 1.0000 | 1.0000 |
| | | 3.0 | 1.9237 | 1.92368 |
| | | 7.0 | 3.0723 | 3.07224 |
| | | 10.0 | 3.7207 | 3.72067 |
| | | 100.0 | 12.294 | 12.2940 |
| | 1.0 | 1.0 | 1.0873 | 1.08727 |
| | 2.0 | | 1.1423 | 1.14233 |
| | 3.0 | | 1.1853 | 1.18528 |
| 1.0 | 0.0 | | 1.6820 | 1.68197 |
| | 1.0 | | 1.7039 | 1.70390 |
| | -0.5 | 10.0 | 5.5585 | 5.55847 |
| | 0.5 | | 5.5690 | 5.56895 |

## 4. Conclusions

Computational investigation of unsteady mixed convection flow of hybrid nanoliquid past a vertical radiative stretchable sheet with convective condition is accomplished. In this study, thermal radiation, viscous dissipation, unsteadiness as well as Newtonian heating aspects are considered. However, the outcomes are summed up depend on the variation in entropic production, hybrid nanofluids motion, and the distribution of temperature. Following are some of the critical findings in this investigation:

i. An assisting buoyant movement leads to strengthen the drag friction factor, whereas an opposing buoyant motion gives an opposite behavior.
ii. As Biot number strengthens, thermal conductivity of the hybrid nanofluid is weakened which results in the rate of heat transmission is weakened from the boundary layer to the sheet surface.
iii. Higher value of unsteadiness factors the skin friction factor is weakened and the local Nusselt number is strengthened.

iv. An increment of Ecert number leads to stock the heat energy in the hybrid nanomaterial due to the forces of the friction which strengthen the hybrid nanofluid temperature profile.
v. An upgrade in Eckert number and radiation factor result in decline the Nusselt number.

## 5. Future Work

The outcomes obtained in this investigation can be fruitful as an advice for upcoming works wherever the thermal efficacy of thermal system is deliberated for several hybrid nanolquids. The computations can be simplified to integrate the impressions of thermal radiation, viscous dissipation and penetrability depending on temperatures with multi-dimensional magneto hydrodynamic slippery flowing.

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