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The Mixed of Hybrid Nanofluid GO-MoS₂/Engine Oil Over a Shrinking Sheet with Mass Flux Effect: Reiner-Philippoff Model

Nur Syahidah Nordin^{1,2}, Abdul Rahman Mohd Kasim^{1,4,*}, Masyfu'ah Mokhtar^{1,2}, Iskandar Waini³

¹ Centre for Mathematical Sciences, Universiti Malaysia Pahang, Gambang, 26300 Kuantan, Pahang, Malaysia

² Mathematical Sciences Studies, College of Computing, Informatics and Media, Universiti Teknologi MARA (UiTM) Johor Branch, Segamat Campus, 85000 Segamat, Johor, Malaysia

³ Fakulti Teknologi Kejuruteraan Mekanikal dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, Durian Tunggal 76100, Melaka, Malaysia

⁴ Center for Research in Advanced Fluid and Process, University Malaysia Pahang, Lebuhraya Tun Razak, Gambang, Kuantan, 26300, Pahang, Malaysia

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ABSTRACT

The mixed convection flow and heat transfer of the hybrid nanofluid over a shrinking sheet are investigated. Molybdenum disulphide (MoS₂) and graphene oxide (GO) are employed as two hybrid nanoparticles while engine oil (EO) as the base fluid is considered. In this study, the Reiner-Philippoff model as one of non-Newtonian types is deliberated since it has the ability to function on three distinct types of fluids: viscous, shear thickening and shear thinning. The Reiner-Philippoff relation, the momentum and energy equations under Tiwari and Das model are all employed in the study. Influences from mass flux are also considered in the flow. Before computation using the *bvp4c* function in MATLAB, the respected equations are first converted into ordinary differential equation form using the similarity transformation. When the established and current models are discovered to be identical in a specific case, a direct comparative investigation is conducted to confirm the correctness of the current model. In addition, the present results are shown graphically and in tabular form. It is hypothesized that the presence of a hybrid nanofluid significantly affects the fluid characteristic and gives more satisfactory results than a single nanofluid. The skin friction coefficient and heat transfer rate of hybrid nanofluids are greater than the nanofluids. In terms of velocity and temperature profile, the reduction in velocity and the enhancement in temperature profile are caused by a rise in the Reiner-Philippoff parameter. The same outcome is also seen when the volume fraction of hybrid nanofluids increases.

1. Introduction

An efficient working fluid is required for industrial and technological applications to regulate processes and produce superior final products. Although many processes use non-Newtonian types of fluid to accelerate advancements, pure water (Newtonian) is still used as a cooling agent in many of them. There are numerous non-Newtonian fluid varieties that each have unique characteristics. In

* Corresponding author.

E-mail address: rahmanmohd@ump.edu.my

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contrast to the Newtonian type of fluid, whose strain is in line with the stress tensor, the non-Newtonian type of fluid is classified by either shear-thickening or shear-thinning. Shear-thinning fluid shows the behavior of Newtonian fluids at extreme shear rates, whereas shear-thickening fluids show the development in viscosity proportional to shear rate. As indicated in Deshpande *et al.*, [1] the models that represent shear-thickening and shear-thinning characteristics include the Powell-Eyring, Sisko, Carreau-Yasuda, Carreau viscosity, and as well as Reiner-Philippoff models. The Reiner-Philippoff model, which belongs to the non-Newtonian group, is the most fascinating to study because it exhibits Newtonian fluid behavior at low or high values of shear stress and non-Newtonian behavior at other values. Many researchers paid attention to the Reiner-Philippoff model investigation because of its enormous significance in engineering applications [2-10]. Furthermore, the Reiner-Philippoff model is essential and unique in representing natural fluid in industrial applications. In certain situations, it can exhibit three fluid characteristics where it can behave like Newtonian, dilatant, and pseudo-plastics. This is significant in manufacturing procedures since the applied fluid might vary in a specific process to achieve the best production. In addition, numerous research examined the flow's movement over various geometries and its effects on the flow field [11-23].

The process by which heat is transferred from one medium to another by the movement of fluids is known as convection. There are two types of convection: 1) forced convection - the process by which fluid motion is generated from an external source, and 2) natural or free convection - the phenomenon in which buoyant forces which result from changes in density are the only source of fluid motion. When forced and natural convection systems merge, mixed convection is formed. Due to the significance of mixed convection flow in industrial systems including nuclear reactors, solar collectors, heat exchangers, and electronic devices, academics are particularly fascinated with the topic. For instance, Merkin [24] investigated the mixed convection flow towards a vertical plate in a porous substance. He discovered that dual solutions are possible for particular values of the mixed convection parameter, for which Merkin [25] had previously shown that these solutions were stable. Ingham [26] also discovered the mixed convection flow over a moving vertical flat plate. Ramachandran *et al.*, [27] then applied this work to the stagnation flow problem and discovered that the opposing flow area was where the solution's non-uniqueness occurred. Similar behavior was seen in the mixed convection stagnation flow of a micropolar fluid by Lok *et al.*, [28]. Dual solutions for the assisting flow concurrently with the opposing flow were also discovered by Ishak *et al.*, [29]. In addition, Harris *et al.*, [30], Zokri *et al.*, [31], Khashi'ie *et al.*, [32,33], Ghalambaz *et al.*, [34], Waini *et al.*, [35,36], have also considered the work on the mixed convection flow.

In recent years, hybrid nanofluids have taken the place of nanofluids in several technologies to enhance thermal performance. In their experimental work, Turcu *et al.*, [37] and Jana *et al.*, [38] appear to be among the first researchers to integrate hybrid nano-composite particles into consideration. A hybrid nanofluid is an innovative fluid that contains several nanoparticles and can increase the rate of heat transfer due to its synergistic effects [39]. Additionally, the appropriate nanoparticles can be combined or hybridized to achieve the optimum heat transfer [40]. A fundamental requirement of the modern world is the rate of heat transfer. Many scientists and researchers are working on methods for expediting heat transfer rates. Engine oil is one of the base fluids that many researchers have recently employed. It is treated as blood for automobiles, plays a crucial part in how the machine functions, and is most vitally used to lubricate and clean the engine [41]. Asadi *et al.*, [42] examined thermophysical properties and heat transmission performance using MWCNT + ZnO in engine oil hybrid nanofluid. In their experimental study using WO₃+MWCNTs in regular engine oil, Aghahadi *et al.*, [43] conducted research on the rheological behavior of a hybrid nanofluid, while Soltani *et al.*, [44] discovered that the engine oil's thermal conductivity increased by

up to 19.85%. Omrani *et al.*, [45] recently investigated the impact of adding some micro- and nanosized particles to engine oil to speed up heat transfer. Huang *et al.*, [46] discussed how graphene oil nanofluid performs tribologically in rotation phenomena. While Arif *et al.*, [47] studied engine oil as a base fluid and considered using nanoparticles to expedite heat transfer. They discussed some crucial applications of engines that make use of nanoparticles in base fluid engine oil. Additionally, in another research, Arif *et al.*, [48] employed engine oil as the base fluid and GO+MoS₂ as the Maxwell hybrid nanofluid (MHNF). They discovered that MHNF speeds up heat transport by up to 23.17%.

The character of the flow across a stretching and shrinking surface in a hybrid nanofluid with temporal stability analysis was described by Waini *et al.*, [49]. They discovered that one of the solutions was shown to be unstable over time. The problem of a hybrid nanofluid flow over a stretching/shrinking sheet was then extended to encompass a variety of facets, as discussed by Waini *et al.*, [50-52]. The boundary layer flow of a hybrid nanofluid has also been addressed in several articles, with the influence of different physical parameters past a stretching and shrinking sheet/surface [53-58].

In the present study, the nanoparticles of graphene oxide (GO) and molybdenum disulfide (MoS₂) were considered with engine oil (EO) as a base fluid. GO is a unique material that can be viewed as a single monomolecular layer of graphite with various oxygen-containing functionalities such as epoxide, carbonyl, carboxyl, and hydroxyl groups. The membranes with GO incorporation show excellent mechanical and thermal stability. As a result, GO has seen significant use in various sectors, including catalysis, energy storage, biomedical applications, nanocomposite materials, polymer composite materials, and as a surfactant. Molybdenum disulfide (MoS₂), also known as moly, is an inorganic metallic compound composed of molybdenum and sulfur. This substance has a crystal lattice-layered structure, and it occurs in natural conditions as the mineral molybdenite (the principal ore of molybdenum). It has numerous industrial and commercial applications, including lubricants and is also the perfect material for low-friction materials because of its low reactivity. This substance's low coefficient of friction and chemical inertness are why it is regarded as a helpful lubricant. Engine oil (EO) represents crucial importance in the automotive sector. To maintain performance and improve the thermal mechanism of engine oil base fluid with applications of hybrid-nanofluid, engine oil thermal capacitance is required. The employment of graphene oxide and molybdenum disulfide nanoparticles supports the thermal mechanism of a hybrid nanofluid in EO. Applications in the auto sector are the driving forces behind the idea of suspending engine oil with graphene oxide and molybdenum nanoparticles.

Motivated by the above literature survey, this study focuses on the mixed convection of Reiner-Philippoff hybrid nanofluid, which was inspired by a study in the literature that used the unique Reiner-Philippoff fluid model. The flow is expected to pass across a shrinking sheet. Also, as engine oil is regarded as a base fluid, GO-MoS₂ nanoparticles are embedded to the engine oil to expedite the heat transfer rate. Influences from mass flux are also considered in the flow. The respected equations are first converted into ordinary differential equation form using the similarity transformation before it solves computationally using the *bvp4c* function in MATLAB. The findings are presented graphically along with a brief discussion of how various physical factors were influenced. To the best knowledge, this problem has not been studied before, so the reported results are new. It is vital to mention that the proposed model sheds light on the complex fluid that predominates in practical applications. Additionally, the proposed model which is examined along with the implication of mixed convection, provides a clearer understanding of what transpired in technologically advanced applications. Another intriguing feature of this model is that in certain circumstances, it can be used to simulate Newtonian fluid where the model's validity may be verified using established output from the literature. Furthermore, since the Reiner-Philippoff fluid can

resemble the shear-thinning fluid which exhibits pseudoplastic behavior, it is frequently used in oil and gas drilling, roller-bearing operations, mixing vessels mostly in food industries, dampers or hydrodynamics bearing, polymer processing, and catalytic chemical reactors. Meanwhile, the shear-thickening fluid which exhibits dilatant behavior is used in many commercial applications like the fabrication of protective gear or equipment such as body armor, vests, army gear and sporting protective clothing as well as other types of materials used for protection.

2. Methodology

Figure 1 depicts the physical configuration of Reiner-Philippoff nanofluid across a shrinking surface where the velocity's surface is $u = ax^{1/3}f'(\eta)$ with $a > 0$. The mass flux velocity $v_w(x)$ represents the surface permeability, while given $T_w = T_\infty + T_0x^{-1/3}$ is the surface temperature where the constant ambient temperature is T_∞ and T_0 is the reference temperature. Additionally, both the fluid and surface were at rest and in steady state with the ambient temperature. Thus, the comprehensive equations for the suggested model are as follows [5,6]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{\tau}{\mu_\infty + \frac{\mu_{hnf} - \mu_\infty}{1 + \left(\frac{\tau}{\tau_s}\right)^2}} \tag{2}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho_{hnf}} \frac{\partial \tau}{\partial y} + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}} g^* (T - T_\infty) \tag{3}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

subject to:

$$\begin{aligned} u &= \varepsilon u_w(x), v = v_w(x), T = T_w \text{ at } y = 0 \\ u &\rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \end{aligned} \tag{5}$$

where (u, v) are the velocity components in the (x, y) directions, respectively. Further, ρ_{hnf} is fluid density, $(\rho\beta)_{hnf}$ is thermal expansion, $(\rho C_p)_{hnf}$ is heat capacity, k_{hnf} is thermal conductivity, μ_{hnf} is dynamic viscosity, μ_∞ is limiting dynamic viscosity, T is temperature, g is acceleration due to gravity, τ is shear stress of Reiner-Philippoff fluid, τ_s is references shear stress and ε is stretching/shrinking parameter. The subscripts of hnf and f stand for hybrid nanofluid and fluid, respectively.

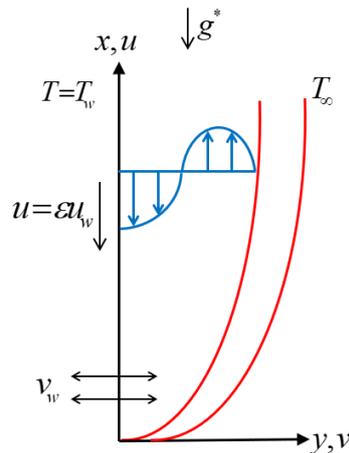


Fig. 1. The physical model

Employing similarity transformation as [59]:

$$\psi = \sqrt{av_f} x^{2/3} f(\eta), \quad \tau = \rho_f \sqrt{a^3 v_f} g(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \frac{y}{x^{1/3}} \sqrt{\frac{a}{v_f}} \quad (6)$$

The term ψ is express by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ yield:

$$u = ax^{1/3} f'(\eta), \quad v = -\sqrt{av_f} x^{-1/3} \left(\frac{2}{3} f(\eta) - \frac{1}{3} \eta f'(\eta) \right) \quad (7)$$

At $\eta = 0$, the wall mass flux velocity obtained as:

$$v_w(x) = -\frac{2}{3} \sqrt{av_{mf}} x^{-1/3} S \quad (8)$$

in which $f(0) = S$ indicate the parameter of constant mass flux. There are three different situations of the value of S , where $S = 0$ denote the impermeable surface, $S < 0$ for injection and $S > 0$ is for suction, while $\nu_f = \mu_\infty/\rho_f$ is the fluid kinematic viscosity. The similarity Eq. (9) to Eq. (12) are obtained after employing Eq. (6) and Eq. (7):

$$g = f'' \left(\frac{g^2 + \left(\frac{\mu_{mf}}{\mu_f} \right) \lambda \gamma^2}{g^2 + \gamma^2} \right) \quad (9)$$

$$g' - \frac{\rho_{mf}}{\rho_f} \left(\frac{1}{3} f'^2 - \frac{2}{3} ff'' \right) + \left[\frac{(\rho\beta)_{mf}}{(\rho\beta)_f} \right] Z\theta = 0 \quad (10)$$

$$\frac{1}{\text{Pr}} \left[\frac{k_{hmf}}{k_f} \right] \theta'' + \left[\frac{(\rho C_p)_{hmf}}{(\rho C_p)_f} \right] \left(\frac{1}{3} f' \theta + \frac{2}{3} f \theta' \right) = 0 \quad (11)$$

subject to:

$$\begin{aligned} f(0) = S, f'(0) = \varepsilon, \theta(0) = 1 \\ f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty \end{aligned} \quad (12)$$

The dimensionless parameter Bingham number γ , Reiner–Philippoff fluid λ , mixed convection Z , and Prandtl number Pr , are defined by:

$$\gamma = \frac{\tau_s}{\rho_f \sqrt{a^3 v_f}}, \lambda = \frac{\mu_f}{\mu_\infty}, Z = \frac{Gr_x}{Re_x^2} = \frac{g^*(\beta)_f T_0}{a^2}, \text{Pr} = \frac{(\mu C_p)_f}{k_f} \quad (13)$$

Note that, $\lambda = 1$ presenting viscous Newtonian type, whereas $\lambda > 1$ signify the shear-thinning and $\lambda < 1$ is shear thickening fluid type respectively. Further, $\varepsilon = 0$ signifies the static sheet, $\varepsilon > 0$ indicate the stretching sheet and $\varepsilon < 0$ is the shrinking sheet. The quantity of physical in term of skin friction and local Nusselt number are given by:

$$C_f = \frac{\tau_w}{\rho_f u_w^2}, Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)} \quad (14)$$

where:

$$\tau_w = \rho_f \sqrt{a^3 v_f} (g(\eta))_{y=0}, q_w = -k_{hmf} \left(\frac{\partial T}{\partial y} \right) \Big|_{y=0} \quad (15)$$

The τ_w symbolizes the quantity of τ on $y = 0$, and q_w presenting the surface heat flux. Then, one gets:

$$\text{Re}_x^{1/2} C_f = g(0), \text{Re}_x^{-1/2} Nu_x = -\frac{k_{hmf}}{k_f} \theta'(0) \quad (16)$$

where $Re_x = u_w(x)x/v_f$ is the local Reynolds number and $Gr_x = (g^*(\beta_T)_f (T_0 x^{-1/3}) x^3)/v_f^2$ is the Grashof number.

The physical properties of GO, MoS₂, and EO are listed in Table 1, whereas the thermophysical characteristics of nanofluid and hybrid nanofluid are listed in Table 2.

Table 1

Thermophysical properties of engine oil and nanoparticles [47,48,60]

Thermophysical properties	EO	GO	MoS ₂
ρ (kg/m ³)	884	1800	5060
C _p (J/kgK)	1910	717	397.21
K (W/mK)	0.144	5000	904.4
Bx10 ⁻⁵ (1 / K)	70	0.284	2.8424

Table 2

Mathematical expression for the thermophysical properties of hnf [48,61]

Thermophysical properties	Hybrid nanofluid
Density	$\rho_{hnf} = (1 - \phi_{hnf})\rho_f + \phi_{GO}\rho_{GO} + \phi_{MoS_2}\rho_{MoS_2}$
Heat capacity	$(\rho C_p)_{hnf} = (\rho C_p)_f (1 - \phi_{hnf}) + \phi_{GO}(\rho C_p)_{GO} + \phi_{MoS_2}(\rho C_p)_{MoS_2}$
Dynamic viscosity	$\mu_{hnf} = \frac{\mu_f}{(1 - (\phi_{GO} + \phi_{MoS_2}))^{2.5}}$
Thermal conductivity	$k_{hnf} = \frac{2k_f + \frac{(\phi_{GO}k_{GO} + \phi_{MoS_2}k_{MoS_2})}{\phi_{hnf}} + 2(\phi_{GO}k_{GO} + \phi_{MoS_2}k_{MoS_2}) - 2k_f\phi_{hnf}}{2k_f + \frac{(\phi_{GO}k_{GO} + \phi_{MoS_2}k_{MoS_2})}{\phi_{hnf}} + (\phi_{GO}k_{GO} + \phi_{MoS_2}k_{MoS_2}) - k_f\phi_{hnf}}$
Thermal expansion	$(\rho\beta_T)_{hnf} = (\rho\beta_T)_f (1 - \phi_{hnf}) + \phi_{GO}(\rho\beta_T)_{GO} + \phi_{MoS_2}(\rho\beta_T)_{MoS_2}$ where $\phi_{hnf} = \phi_{GO} + \phi_{MoS_2}$

3. Results and Discussion

The numerical solutions of Eq. (9) to Eq. (12) are obtained using the boundary value problem solver, bvp4c, a feature of the MATLAB software. It uses the three-stage Lobatta IIIa formula and is a finite difference approach. The choice of the initial guess and the thickness of the boundary layer η_∞ , will rely on the parameters utilised to obtain the required solutions. This solver is also being utilized by various researchers to solve boundary layer flow problems [62,63].

A direct comparison analysis is conducted on the existing value of $f''(0)$ provided by Cortell [64], Ferdows *et al.*, [65], and Waini *et al.*, [18] to vouch for the dependability of the current model. It should be noted that the equations on the current model were the same for the limiting case, making a comparison between the present findings and the current output appropriate. In Table 3 and Table 4, respectively, the validation data on the values of $f''(0)$ are presented. The comparison reveals excellent agreement, which supports the current mathematical formulation and the provided numerical results.

To strengthen the current formulation and the current result, the values of $g(0)$ are also compared with the output reported by Reddy *et al.*, [10] and Waini *et al.*, [66] for various values of the Reiner-Philippoff fluid parameter λ , the Bingham number γ , and the Prandtl number, $Pr = 2$. Strong agreement can be seen in the comparison; hence Table 5 and Table 6 show the corresponding numerical values. For higher values of γ , the values of $g(0)$ significantly increase. With the rise of λ , there is, however, a slight decrease in the values of $g(0)$.

Table 3
 Comparative model in terms of momentum equations

Author	Model (momentum)	Limiting cases
Current	$g' - \frac{\rho_{hmf}}{\rho_f} \left(\frac{1}{3} f'^2 - \frac{2}{3} ff'' \right) + \left[\frac{(\rho\beta)_{hmf}}{(\rho\beta)_f} \right] Z\theta = 0$	$Z = 0$
Cortell [64]	$3f''' + 2ff'' - (f')^2 = 0$	-
Ferdows <i>et al.</i> , [65]	$f''' + \frac{2}{3} ff'' - \frac{1}{3} (f'^2 + Mf'^2) + Gr\theta + Gc\phi = 0$	$M = Gr = Gc = 0$
Waini <i>et al.</i> , [18]	$3 \frac{\mu_{hmf} / \mu_f}{\rho_{hmf} / \rho_f} f''' + 2ff'' - f'^2 = 0$	-

Table 4
 Comparative value of $f''(0)$ at $\varepsilon = \lambda = \gamma = 1$, $Pr = 2$ and $Z = 0$ for different value of S

S	Cortell [64]	Ferdows <i>et al.</i> , [65]	Waini <i>et al.</i> , [18]	Current
-0.75	-0.453521	-0.453523	-0.453523	-0.453523325
-0.5	-0.518869	-0.518869	-0.518869	-0.518869429
0	-0.677647	-0.677648	-0.677648	-0.677647983
0.5	-0.873627	-0.873643	-0.873643	-0.873642863
0.75	-0.984417	-0.984439	-0.984439	-0.984439388

Table 5
 Comparative model in terms of momentum equations

Author	Model (momentum)	Limiting cases
Current	$g' - \frac{\rho_{hmf}}{\rho_f} \left(\frac{1}{3} f'^2 - \frac{2}{3} ff'' \right) + \left[\frac{(\rho\beta)_{hmf}}{(\rho\beta)_f} \right] Z\theta = 0$	$Z = 0$
Sajid <i>et al.</i> , [10]	$g' + \frac{2}{3} ff'' - \frac{1}{3} f'^2 = 0$	-
Waini <i>et al.</i> , [66]	$g' + \frac{2}{3} ff'' - \frac{1}{3} f'^2 - M \sin^2(\beta) f' = 0$	$M = 0$

Table 6
 Comparative value of $g(0)$ for λ and γ when $S = Z = 0$ and $\varepsilon = 1$

γ	λ	Sajid <i>et al.</i> , [10]	Waini <i>et al.</i> , [66]	Current
0.1	0.1	-0.660273	-0.660275	-0.660275189
	0.5	-0.380604	-0.380604	-0.380603983
	1	-0.246415	-0.246415	-0.246414994
0.1	0.3	-0.664497	-0.664498	-0.664497827
	0.5	-0.668484	-0.668486	-0.668486422
	0.7	-0.672282	-0.672277	-0.672276682

Figure 2 and Figure 3 illustrate how λ and γ affect variations in $Re_x^{1/2} C_f$ and $Re_x^{-1/2} Nu_x$ when $\varepsilon = 1$, $Pr = 21$ and $S = Z = 0$. The increase in $Re_x^{-1/2} Nu_x$ (except at $\gamma = 0.1$) and the decrease in $Re_x^{1/2} C_f$ were both influenced by an increase in λ . When $\gamma = 0.1, 0.3, 0.5$ and $\lambda = 1$ (Newtonian fluid), the values of $Re_x^{1/2} C_f = -0.67764798$ and $Re_x^{-1/2} Nu_x = 3.75348202$ remain unchanged (see Table 7).

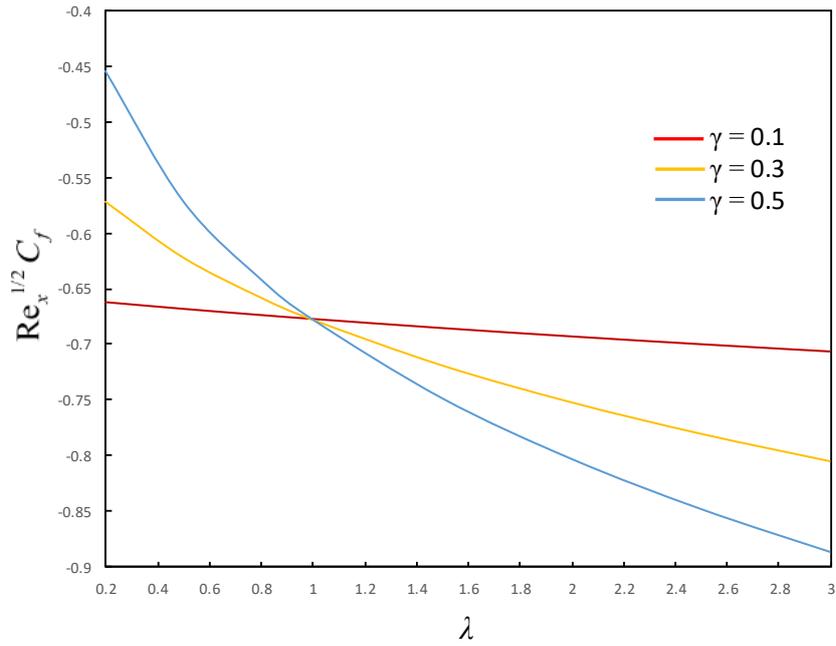


Fig. 2. $Re_x^{1/2} C_f$ vs λ for various values of γ

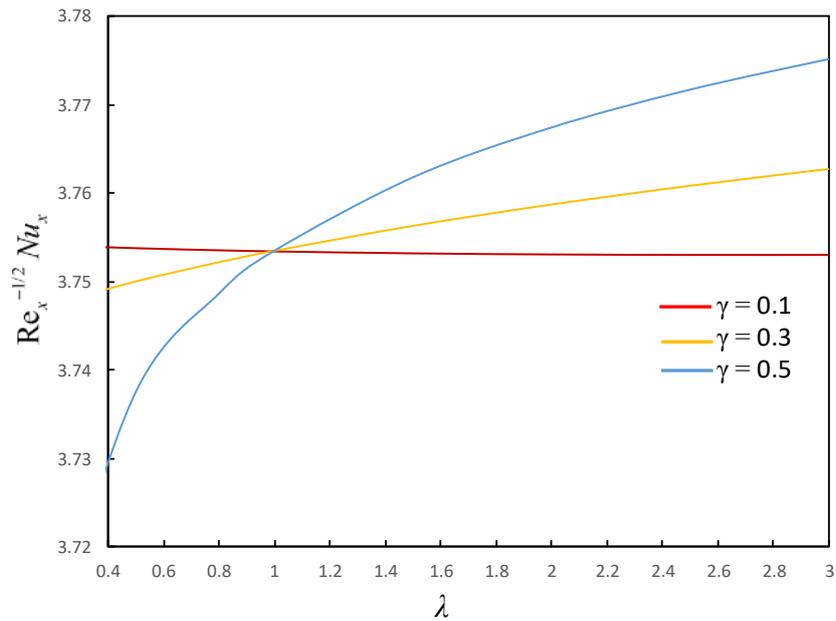


Fig. 3. $Re_x^{-1/2} Nu_x$ vs λ for various values of γ

Further, as γ increases, it becomes clear that the quantity of $Re_x^{1/2} C_f$ rises when $\lambda < 1$ (shear-thickening fluid) and decreases when $\lambda > 1$ (shear-thinning fluid), while the thermal rate yields opposite outcomes. Additionally, for future references, Table 7 tabulates the computed values of $Re_x^{1/2} C_f$ and $Re_x^{-1/2} Nu_x$ with various values of λ and γ .

This work has examined the thermal characteristics of the hybrid nanofluid and nanofluid to assess the thermal behaviour of a hybrid nanofluid (GO-MoS₂/EO) and a nanofluid (GO/EO and MoS₂/EO). The quantities of $Re_x^{1/2} C_f$ and $Re_x^{-1/2} Nu_x$ against Z for different values of fluids (regular engine oil, nanofluids and hybrid nanofluid) are tabulated in Table 8. The comparison of a (GO-MoS₂/EO) with (MoS₂/EO), (GO/EO) and regular EO are also plotted in Figure 4 and Figure 5. This

study's primary goal is to assess the heat transfer rate of hybrid nanofluid in EO and evaluate the outcomes with a nanofluid. As can be seen from the comparison, the (GO-MoS₂/EO) has a higher rate of heat transfer than (MoS₂ + EO), (GO + EO), and regular EO. The (GO-MoS₂/EO) and (MoS₂ + EO) yield the greatest results for the rate of heat transfer. Physically, hybrid nanoparticles increased the thermal conductivity, which increased the rate of heat transfer in both nanofluids.

Table 7

Values of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ for λ and γ when $\varepsilon = 1$, $Pr = 21$ and $S = Z = 0$

λ	$Re_x^{1/2}C_f$			$Re_x^{-1/2}Nu_x$		
	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$	$\gamma = 0.1$	$\gamma = 0.3$	$\gamma = 0.5$
0.5	-0.66848642	-0.62158953	-0.571307799	3.75383530	3.750043514	3.737631739
1.0	-0.677647983	-0.677647981	-0.677647983	3.75348202	3.753482015	3.753482015
1.5	-0.68591905	-0.718769932	-0.748419859	3.75326718	3.756288435	3.761767024

Table 8

Values of $Re_x^{1/2}C_f$ and $Re_x^{-1/2}Nu_x$ for selected values of regular engine oil, nanofluids, hybrid nanofluid and Z when $\varepsilon = -1$, $Pr = 21$, $\lambda = 1.5$, $\gamma = 0.1$ and $S = 2.4$

Z	$Re_x^{1/2}C_f$			
	Regular EO	GO + EO	MoS ₂ + EO	GO + MoS ₂ +EO
-1	1.070639148	1.091231998	1.173068981	1.192058246
-0.5	1.086811156	1.107956731	1.189619042	1.208949090
0	1.102979760	1.124678016	1.206167067	1.225838040
0.5	1.119144981	1.141395883	1.222713059	1.242725103
1	1.135306870	1.158110367	1.239257033	1.259610293

Z	$Re_x^{-1/2}Nu_x$			
	Regular EO	GO + EO	MoS ₂ + EO	GO + MoS ₂ + EO
-1	32.97603626	32.87190642	33.01678707	33.36337800
-0.5	32.97620496	32.87209622	33.01697204	33.36344330
0	32.97637356	32.87228593	33.01715694	33.36350857
0.5	32.97654208	32.87247554	33.01734178	33.36357383
1	32.97671052	32.87266505	33.01752656	33.36363906

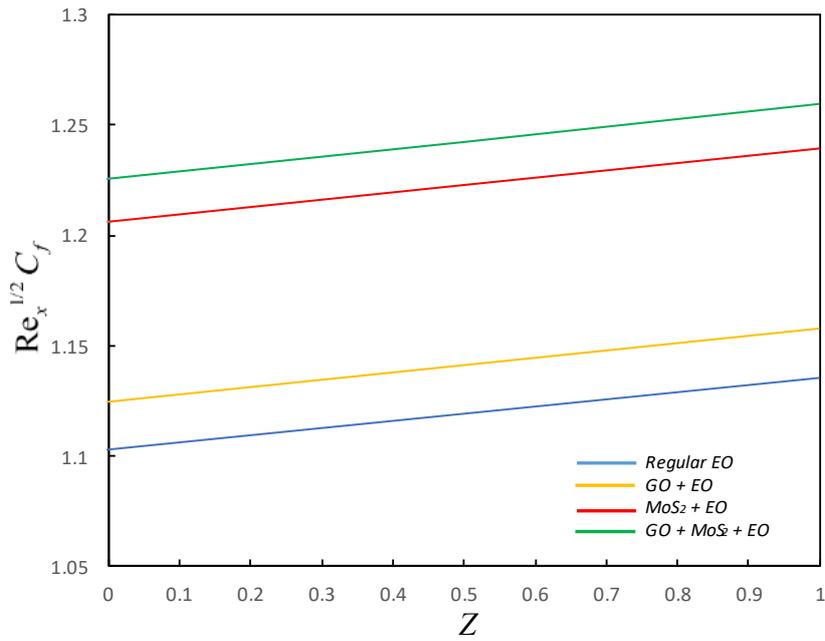


Fig. 4. $Re_x^{1/2} C_f$ vs Z for selected values of engine oil, nanofluids, and hybrid nanofluid

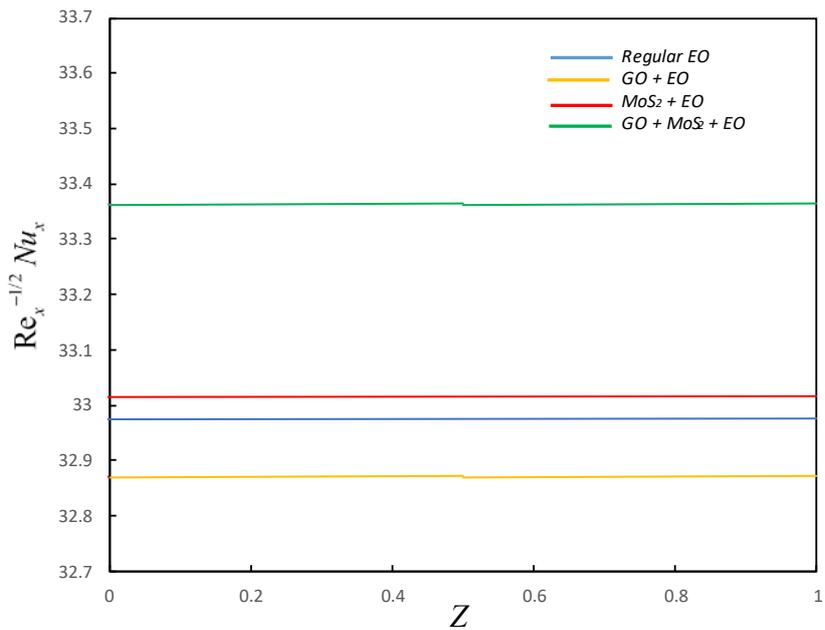


Fig. 5. $Re_x^{-1/2} Nu_x$ vs Z for selected values of engine oil, nanofluids, and hybrid nanofluid

The quantities of $Re_x^{-1/2} Nu_x$ against Z for the different volume fractions of MoS_2 (ϕ_{MoS_2}) are tabulated in Table 9. Results show that the values of $Re_x^{-1/2} Nu_x$ increase with the rising of Z and the volume fraction, ϕ_{MoS_2} . The assisting flow ($Z > 0$) has a greater heat transfer than the opposing flow ($Z < 0$). A higher wall temperature than the fluid temperature indicates heat is transferred from the wall to the fluid in the assisting flow. Yet, the opposing flow has the opposite effect. Hence, the heat transfer process for an assisting flow is always greater than an opposing flow. This heat transmission

rate is quite efficient and advantageous for the thermal performance of engine oil utilised in various machines.

Table 9

Values of $Re_x^{-1/2} Nu_x$ for selected values of ϕ_{MoS_2} and Z when $\epsilon = -1$, $Pr=21$, $\lambda = 1.5$, $\gamma = 0.1$, $S = 2.4$ and $\phi_{GO} = 0.005$

Z	$Re_x^{-1/2} Nu_x$		
	$\phi_{MoS_2} = 0.015$	$\phi_{MoS_2} = 0.025$	$\phi_{MoS_2} = 0.035$
-1	32.964406627	33.023130232	33.061146140
-0.5	32.976900377	33.023359340	33.061400883
0	32.984620580	33.023588381	33.061655553
0.5	32.984826969	33.023817355	33.061910149
1	32.985033293	33.024046262	33.062164674

The velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles for selected parameters are provided in Figure 6 to Figure 13. These profiles asymptotically satisfy the boundary condition, thus supports the validity of the numerical results. The analysis of the temperature and velocity profiles employing Reiner-Philippoff parameters is shown in Figure 6 and Figure 7, respectively. The profiles demonstrate that the fluid's velocity decreases as λ increases, while temperature profiles exhibit the opposite behaviour. Figure 8 and Figure 9, and Figure 10 and Figure 11 showed the analysis of velocity and temperature profile with the variation of S and Z , respectively. The increasing behaviour is observed with the increasing of S and Z as portrayed in Figure 8 and Figure 10. However, the temperature profiles as displayed in Figure 9 and Figure 11 shows a contradictory behaviour. The volume fraction for the hybrid nanofluid GO-MoS₂/EO was highlighted in Figure 12. As in the figure, the velocity decreases as the volume fraction rises. It is a fact that increasing the values of the volume fractional parameter can cause resistance in the flow and slow down the fluid motion, which lowers the velocity of the hybrid nanofluid. While as shown in Figure 13, the temperature profile rises as the volume fraction of hybrid nanoparticles increase.

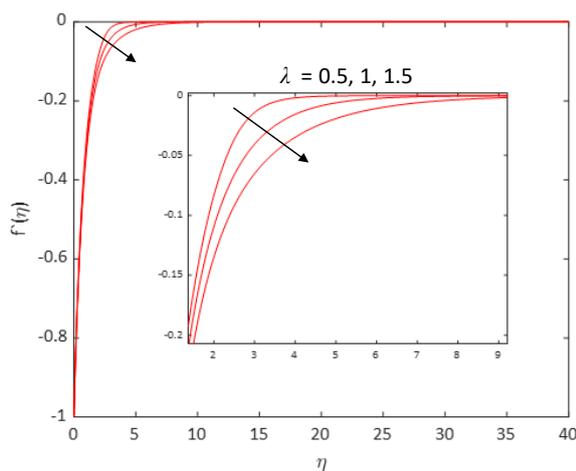


Fig. 6. $f'(\eta)$ for several values of λ and $\phi_{hnf} = 0.02$

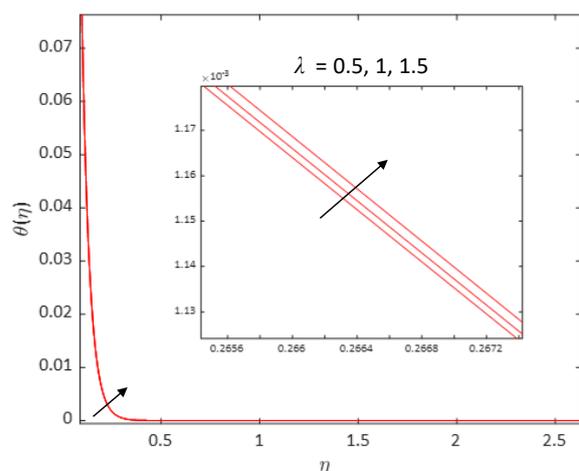


Fig. 7. $\theta(\eta)$ for several values of λ and $\phi_{hnf} = 0.02$

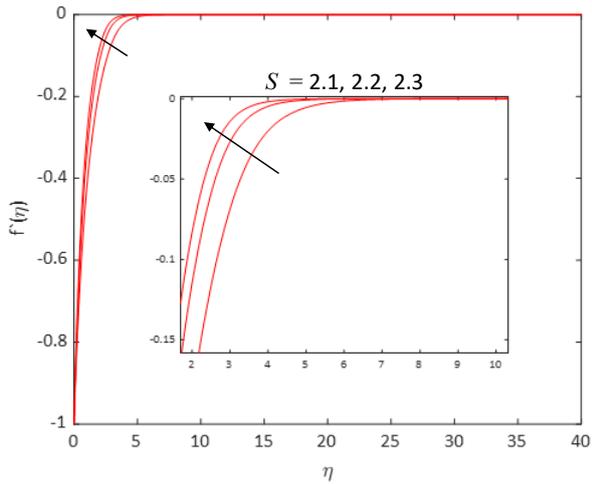


Fig. 8. $f'(\eta)$ for several values of S and $\phi_{hnf} = 0.02$

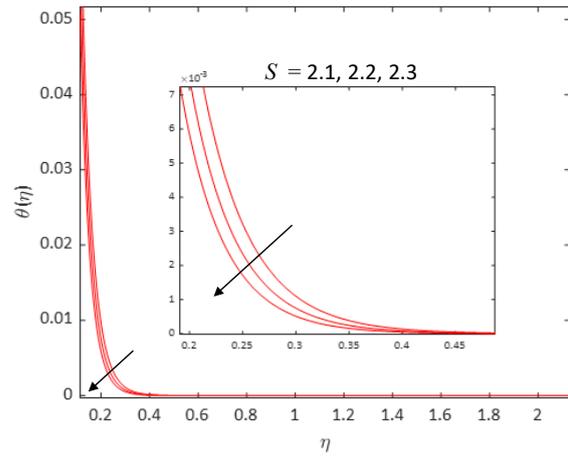


Fig. 9. $\theta(\eta)$ for several values of S and $\phi_{hnf} = 0.02$

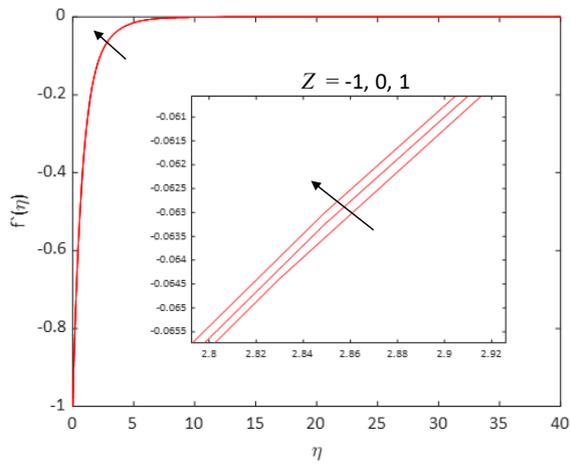


Fig. 10. $f'(\eta)$ for various values of Z and $\phi_{hnf} = 0.02$

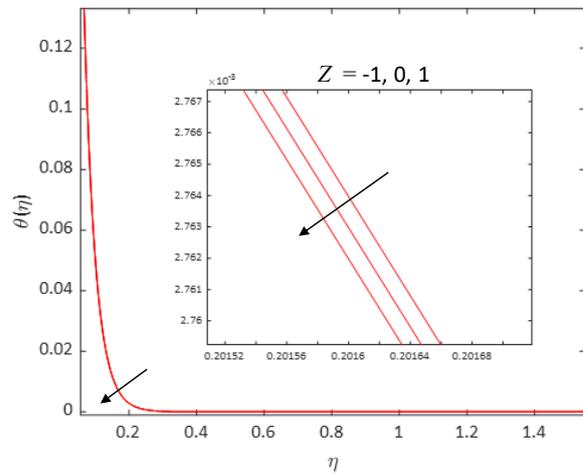


Fig. 11. $\theta(\eta)$ for various values of Z and $\phi_{hnf} = 0.02$

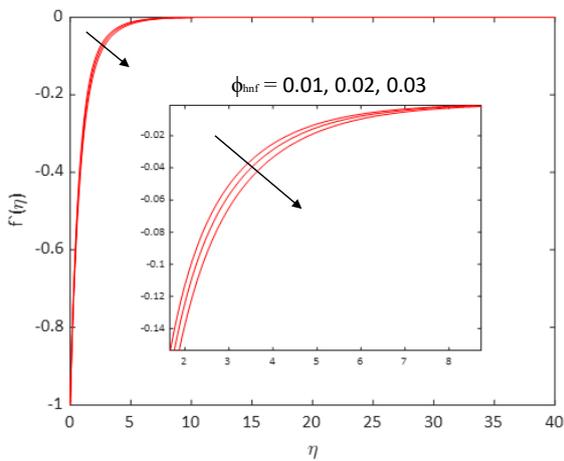


Fig. 12. $f'(\eta)$ for various values of ϕ_{hnf} and $Z = -1$

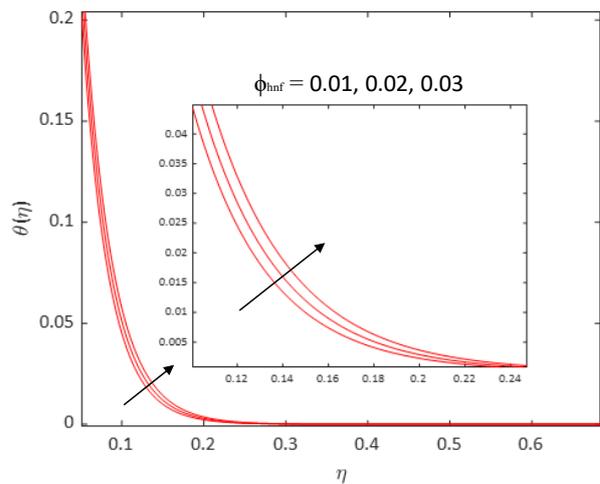


Fig. 13. $\theta(\eta)$ for various values of ϕ_{hnf} and $Z = -1$

4. Conclusion

In the present study, the Reiner-Philippoff model with mixed convection of hybrid GO-MoS₂/EO nanofluid flow past a shrinking sheet is established. The `bvp4c` function in the MATLAB programme was used to numerically solve the nonlinear ordinary differential equations (ODE) with the transformed boundary conditions. The results validation was carried out in a few specified cases when the comparison between the current results and the previous results was excellent. The findings for the present problem are as follows:

- i. Hybrid nanofluid GO-MoS₂/EO has greater skin friction coefficient and heat transfer rate compared to nanofluids (MoS₂+EO) and (GO+EO), and regular EO.
- ii. The increase in local Nusselt number and the decrease in skin friction coefficient were both influenced by an increasing in λ (Reiner-Philippoff parameter) and remain unchanged at $\lambda=1$ (Newtonian fluid).
- iii. In term of velocity and temperature profile, the rising in λ reduces the velocity of the fluid and enhance the temperature profiles.
- iv. The increasing behavior in velocity profile is also observed with the increasing of S (mass flux parameter) and Z (mixed convection parameter), but the temperature profiles show a contradictory behavior.
- v. The enhance of volume fraction of hybrid nanofluid (ϕ_{hnf}) will decrease the velocity and increase the temperature profiles.

In addition, the current model can be expanded to include other geometries for a wide range of future applications, including engineering processes like cooling and heating in electrical devices, vehicle radiators (engine cooling systems), nuclear reactors, solar collectors, and heat exchangers. Furthermore, this problem can be simulated in a channel via a vertical plate with various boundary conditions, such as Newtonian heating. Various other techniques can also be used to answer the aforementioned issues accurately.

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References

- [1] Krishnan, J. Murali, Abhijit P. Deshpande, and PB Sunil Kumar, eds. *Rheology of complex fluids*. New York: Springer, 2010. <https://doi:10.1007/978-1-4419-6494-6>
- [2] Kapur, J. N., and R. C. Gupta. "Two dimensional flow of Reiner-Philippoff fluids in the inlet length of a straight channel." *Applied Scientific Research, Section A* 14 (1965): 13-24. <https://doi:10.1007/BF00382227>
- [3] Na, Tsung-Yen. "Boundary layer flow of Reiner-Philippoff fluids." *International journal of non-linear mechanics* 29, no. 6 (1994): 871-877. [https://doi:10.1016/0020-7462\(94\)90059-0](https://doi:10.1016/0020-7462(94)90059-0)
- [4] Yam, K. S., S. D. Harris, D. B. Ingham, and I. Pop. "Boundary-layer flow of Reiner-Philippoff fluids past a stretching wedge." *International Journal of Non-Linear Mechanics* 44, no. 10 (2009): 1056-1062. <https://doi:10.1016/j.ijnonlinmec.2009.08.006>
- [5] Ahmad, A., M. Qasim, and S. Ahmed. "Flow of Reiner-Philippoff fluid over a stretching sheet with variable thickness." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 39 (2017): 4469-4473. <https://doi:10.1007/s40430-017-0840-7>
- [6] Gnanaswara Reddy, M., M. V. V. N. L. Sudharani, K. Ganesh Kumar, Ali J. Chamkha, and G. Lorenzini. "Physical aspects of Darcy-Forchheimer flow and dissipative heat transfer of Reiner-Philippoff fluid." *Journal of Thermal Analysis and Calorimetry* 141 (2020): 829-838. <https://doi:10.1007/s10973-019-09072-0>

- [7] Khashi'ie, Najiyah Safwa, Iskandar Waini, Abdul Rahman Mohd Kasim, Nurul Amira Zainal, Anuar Ishak, and Ioan Pop. "Magnetohydrodynamic and viscous dissipation effects on radiative heat transfer of non-Newtonian fluid flow past a nonlinearly shrinking sheet: Reiner–Philippoff model." *Alexandria Engineering Journal* 61, no. 10 (2022): 7605-7617. <https://doi.org/10.1016/j.aej.2022.01.014>
- [8] Kumar, K. Ganesh, M. Gnanaswara Reddy, M. V. V. N. L. Sudharani, S. A. Shehzad, and Ali J. Chamkha. "Cattaneo–Christov heat diffusion phenomenon in Reiner–Philippoff fluid through a transverse magnetic field." *Physica A: Statistical Mechanics and its Applications* 541 (2020): 123330. <https://doi:10.1016/j.physa.2019.123330>
- [9] Sajid, T., M. Sagheer, and S. Hussain. "Impact of temperature-dependent heat source/sink and variable species diffusivity on radiative Reiner–Philippoff fluid." *Mathematical Problems in Engineering* 2020 (2020). <https://doi:10.1155/2020/9701860>
- [10] Reddy, M. Gnanaswara, Sudha Rani, K. Ganesh Kumar, Asiful H. Seikh, Mohammad Rahimi-Gorji, and El-Sayed Mohamed Sherif. "Transverse magnetic flow over a Reiner–Philippoff nanofluid by considering solar radiation." *Modern Physics Letters B* 33, no. 36 (2019): 1950449. <https://doi:10.1142/S0217984919504499>
- [11] Crane, Lawrence J. "Flow past a stretching plate." *Zeitschrift für angewandte Mathematik und Physik ZAMP* 21 (1970): 645-647. <https://doi:10.1007/BF01587695>
- [12] Goldstein, Sydney. "On backward boundary layers and flow in converging passages." *Journal of Fluid Mechanics* 21, no. 1 (1965): 33-45. <https://doi:10.1017/S0022112065000034>
- [13] Shahrim, Muhammad Nazirul, Ahmad Qushairi Mohamad, Lim Yeou Jiann, Muhamad Najib Zakaria, Sharidan Shafie, Zulkhibri Ismail, and Abdul Rahman Mohd Kasim. "Exact solution of fractional convective Casson fluid through an accelerated plate." *CFD Letters* 13, no. 6 (2021): 15-25. <https://doi.org/10.37934/cfdl.13.6.1525>
- [14] Lund, Liaquat Ali, Zurni Omar, Sumera Dero, Dumitru Baleanu, and Ilyas Khan. "Rotating 3D flow of hybrid nanofluid on exponentially shrinking sheet: Symmetrical solution and duality." *Symmetry* 12, no. 10 (2020): 1637. <https://doi:10.3390/sym12101637>
- [15] Zainal, Nurul Amira, Roslinda Nazar, Kohilavani Naganthran, and Ioan Pop. "Stability analysis of MHD hybrid nanofluid flow over a stretching/shrinking sheet with quadratic velocity." *Alexandria Engineering Journal* 60, no. 1 (2021): 915-926. <https://doi:10.1016/j.aej.2020.10.020>
- [16] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, Natalia C. Rosca, Alin V. Rosca, and Ioan Pop. "Three-dimensional flow of radiative hybrid nanofluid past a permeable stretching/shrinking sheet with homogeneous-heterogeneous reaction." *International Journal of Numerical Methods for Heat & Fluid Flow* 32, no. 2 (2022): 568-588. <https://doi:10.1108/HFF-01-2021-0017>
- [17] Jamaludin, Anuar, Kohilavani Naganthran, Roslinda Nazar, and Ioan Pop. "MHD mixed convection stagnation-point flow of Cu-Al₂O₃/water hybrid nanofluid over a permeable stretching/shrinking surface with heat source/sink." *European Journal of Mechanics-B/Fluids* 84 (2020): 71-80. <https://doi:10.1016/j.euromechflu.2020.05.017>
- [18] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow and heat transfer over a nonlinear permeable stretching/shrinking surface." *International Journal of Numerical Methods for Heat & Fluid Flow* 29, no. 9 (2019): 3110-3127. <https://doi:10.1108/HFF-01-2019-0057>
- [19] Yashkun, Ubaidullah, Khairy Zaimi, Nor Ashikin Abu Bakar, Anuar Ishak, and Ioan Pop. "MHD hybrid nanofluid flow over a permeable stretching/shrinking sheet with thermal radiation effect." *International Journal of Numerical Methods for Heat & Fluid Flow* 31, no. 3 (2020): 1014-1031. <https://doi:10.1108/HFF-02-2020-0083>
- [20] Wahid, Nur Syahirah, Norihan Md Arifin, Mustafa Turkyilmazoglu, Mohd Ezad Hafidz Hafidzuddin, and Nor Aliza Abd Rahmin. "MHD hybrid Cu-Al₂O₃/water nanofluid flow with thermal radiation and partial slip past a permeable stretching surface: analytical solution." *Journal of Nano Research* 64 (2020): 75-91. <https://doi:10.4028/www.scientific.net/jnanor.64.75>
- [21] Abu Bakar, Shahirah, Norihan Md Arifin, Najiyah Safwa Khashi'ie, and Norfifah Bachok. "Hybrid nanofluid flow over a permeable shrinking sheet embedded in a porous medium with radiation and slip impacts." *Mathematics* 9, no. 8 (2021): 878. <https://doi:10.3390/math9080878>
- [22] Waini, I., A. Ishak, and I. Pop. "Magnetohydrodynamic flow past a shrinking vertical sheet in a dusty hybrid nanofluid with thermal radiation." *Applied Mathematics and Mechanics* (2022): 1-14. <https://doi:10.1007/s10483-022-2807-8>
- [23] Zainal, N. A., R. Nazar, K. Naganthran, and I. Pop. "Impact of anisotropic slip on the stagnation-point flow past a stretching/shrinking surface of the Al₂O₃-Cu/H₂O hybrid nanofluid." *Applied Mathematics and Mechanics* 41 (2020): 1401-1416. <https://doi:10.1007/s10483-020-2642-6>
- [24] Merkin, J. H. "Mixed convection boundary layer flow on a vertical surface in a saturated porous medium." *Journal of Engineering Mathematics* 14, no. 4 (1980): 301-313. <https://doi:10.1007/BF00052913>
- [25] Merkin, J. H. "On dual solutions occurring in mixed convection in a porous medium." *Journal of engineering Mathematics* 20, no. 2 (1986): 171-179. <https://doi:10.1007/BF00042775>

- [26] Ingham, D. B. "Singular and non-unique solutions of the boundary-layer equations for the flow due to free convection near a continuously moving vertical plate." *Zeitschrift für angewandte Mathematik und Physik ZAMP* 37 (1986): 559-572. <https://doi:10.1007/BF00945430>
- [27] Ramachandran, N., T. S. Chen, and Bassem F. Armaly. "Mixed convection in stagnation flows adjacent to vertical surfaces." (1988): 373-377. <https://doi:10.1115/1.3250494>
- [28] Lok, Y. Y., Norsarahaida Amin, D. Campean, and I. Pop. "Steady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface." *International Journal of Numerical Methods for Heat & Fluid Flow* 15, no. 7 (2005): 654-670. <https://doi:10.1108/09615530510613861>
- [29] Ishak, Anuar, Roslinda Nazar, Norihan M. Arifin, and Ioan Pop. "Dual solutions in mixed convection flow near a stagnation point on a vertical porous plate." *International Journal of Thermal Sciences* 47, no. 4 (2008): 417-422. <https://doi:10.1016/j.ijthermalsci.2007.03.005>
- [30] Harris, S. D., D. B. Ingham, and I. Pop. "Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip." *Transport in Porous Media* 77 (2009): 267-285. <https://doi:10.1007/s11242-008-9309-6>
- [31] Zokri, Syazwani Mohd, Nur Syamilah Arifin, Muhammad Khairul Anuar Mohamed, Abdul Rahman Mohd Kasim, Nurul Farahain Mohammad, and Mohd Zuki Salleh. "Mathematical model of mixed convection boundary layer flow over a horizontal circular cylinder filled in a Jeffrey fluid with viscous dissipation effect." *Sains Malaysiana* 47, no. 7 (2018): 1607-1615. <https://doi:10.17576/jsm-2018-4707-32>
- [32] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, Mohammad Mehdi Rashidi, Ezad Hafidz Hafidzuddin, and Nadiyah Wahid. "Magnetohydrodynamics (MHD) stagnation point flow past a shrinking/stretching surface with double stratification effect in a porous medium." *Journal of Thermal Analysis and Calorimetry* 139 (2020): 3635-3648. <https://doi:10.1007/s10973-019-08713-8>
- [33] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, Roslinda Nazar, Ezad Hafidz Hafidzuddin, Nadiyah Wahid, and Ioan Pop. "Mixed convective flow and heat transfer of a dual stratified micropolar fluid induced by a permeable stretching/shrinking sheet." *Entropy* 21, no. 12 (2019): 1162. <https://doi:10.3390/e21121162>
- [34] Ghalambaz, Mohammad, Natalia C. Roşca, Alin V. Roşca, and Ioan Pop. "Mixed convection and stability analysis of stagnation-point boundary layer flow and heat transfer of hybrid nanofluids over a vertical plate." *International Journal of Numerical Methods for Heat & Fluid Flow* 30, no. 7 (2020): 3737-3754. <https://doi:10.1108/HFF-08-2019-0661>
- [35] Waini, Iskandar, Anuar Ishak, Teodor Groşan, and Ioan Pop. "Mixed convection of a hybrid nanofluid flow along a vertical surface embedded in a porous medium." *International Communications in Heat and Mass Transfer* 114 (2020): 104565. <https://doi:10.1016/j.icheatmasstransfer.2020.104565>
- [36] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Mixed convection flow over an exponentially stretching/shrinking vertical surface in a hybrid nanofluid." *Alexandria Engineering Journal* 59, no. 3 (2020): 1881-1891. <https://doi:10.1016/j.aej.2020.05.030>
- [37] Turcu, R., A. L. Darabont, A. Nan, N. Aldea, D. Macovei, D. Bica, L. Vekas et al. "New polypyrrole-multiwall carbon nanotubes hybrid materials." *Journal of optoelectronics and advanced materials* 8, no. 2 (2006): 643-647.
- [38] Jana, Soumen, Amin Salehi-Khojin, and Wei-Hong Zhong. "Enhancement of fluid thermal conductivity by the addition of single and hybrid nano-additives." *Thermochimica acta* 462, no. 1-2 (2007): 45-55. <https://doi:10.1016/j.tca.2007.06.009>
- [39] Sarkar, Jahar, Pradyumna Ghosh, and Arjumand Adil. "A review on hybrid nanofluids: recent research, development and applications." *Renewable and Sustainable Energy Reviews* 43 (2015): 164-177. <https://doi:10.1016/j.rser.2014.11.023>
- [40] Esfe, Mohammad Hemmat, Ali Alirezaie, and Mousa Rejvani. "An applicable study on the thermal conductivity of SWCNT-MgO hybrid nanofluid and price-performance analysis for energy management." *Applied Thermal Engineering* 111 (2017): 1202-1210. <https://doi:10.1016/j.applthermaleng.2016.09.091>
- [41] Dinesh, R., MJ Giri Prasad, R. Rishi Kumar, N. Jerome Santharaj, J. Santhip, and AS Abhishek Raaj. "Investigation of tribological and thermophysical properties of engine oil containing nano additives." *Materials Today: Proceedings* 3, no. 1 (2016): 45-53. <https://doi.org/10.1016/j.matpr.2016.01.120>
- [42] Asadi, Amin, Ali Naderi Bakhtiyari, and Ibrahim M. Alarifi. "Predictability evaluation of support vector regression methods for thermophysical properties, heat transfer performance, and pumping power estimation of MWCNT/ZnO-engine oil hybrid nanofluid." *Engineering with Computers* 37 (2021): 3813-3823. <https://doi.org/10.1007/s00366-020-01038-3>
- [43] Aghahadi, Mohammad Hasan, Mohammadreza Niknejadi, and Davood Toghraie. "An experimental study on the rheological behavior of hybrid Tungsten oxide (WO₃)-MWCNTs/engine oil Newtonian nanofluids." *Journal of Molecular Structure* 1197 (2019): 497-507. <http://dx.doi.org/10.1016/j.molstruc.2019.07.080>

- [44] Soltani, Farid, Davood Toghraie, and Arash Karimipour. "Experimental measurements of thermal conductivity of engine oil-based hybrid and mono nanofluids with tungsten oxide (WO₃) and MWCNTs inclusions." *Powder technology* 371 (2020): 37-44. <http://dx.doi.org/10.1016/j.powtec.2020.05.059>
- [45] Omrani, Emad, Pradeep L. Menezes, and Pradeep K. Rohatgi. "Effect of micro-and nano-sized carbonous solid lubricants as oil additives in nanofluid on tribological properties." *Lubricants* 7, no. 3 (2019): 25. <http://dx.doi.org/10.3390/lubricants7030025>
- [46] Huang, Jigang, Jun Tan, Hui Fang, Feng Gong, and Jie Wang. "Tribological and wear performances of graphene-oil nanofluid under industrial high-speed rotation." *Tribology International* 135 (2019): 112-120. <http://dx.doi.org/10.1016/j.triboint.2019.02.041>
- [47] Arif, Muhammad, Farhad Ali, Nadeem Ahmad Sheikh, and Ilyas Khan. "Enhanced heat transfer in working fluids using nanoparticles with ramped wall temperature: Applications in engine oil." *Advances in Mechanical Engineering* 11, no. 11 (2019): 1687814019880987. <https://doi.org/10.1177/1687814019880987>
- [48] Arif, Muhammad, Poom Kumam, Dolat Khan, and Wiboonsak Watthayu. "Thermal performance of GO-MoS₂/engine oil as Maxwell hybrid nanofluid flow with heat transfer in oscillating vertical cylinder." *Case Studies in Thermal Engineering* 27 (2021): 101290. <http://dx.doi.org/10.1016/j.csite.2021.101290>
- [49] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Unsteady flow and heat transfer past a stretching/shrinking sheet in a hybrid nanofluid." *International Journal of Heat and Mass Transfer* 136 (2019): 288-297. <https://doi:10.1016/j.ijheatmasstransfer.2019.02.101>
- [50] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Flow and heat transfer along a permeable stretching/shrinking curved surface in a hybrid nanofluid." *Physica Scripta* 94, no. 10 (2019): 105219. <https://doi:10.1088/1402-4896/ab0fd5>
- [51] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow and heat transfer over a nonlinear permeable stretching/shrinking surface." *International Journal of Numerical Methods for Heat & Fluid Flow* 29, no. 9 (2019): 3110-3127. <https://doi:10.1108/HFF-01-2019-0057>
- [52] Waini, Iskandar, Anuar Ishak, and Ioan Pop. "Hybrid nanofluid flow over a permeable non-isothermal shrinking surface." *Mathematics* 9, no. 5 (2021): 538. <https://doi:10.3390/math9050538>
- [53] Patil, P. M., and Madhavarao Kulkarni. "Analysis of MHD mixed convection in a Ag-TiO₂ hybrid nanofluid flow past a slender cylinder." *Chinese Journal of Physics* 73 (2021): 406-419. <https://doi:10.1016/j.cjph.2021.07.030>
- [54] Khan, Umair, Anum Shafiq, A. Zaib, and Dumitru Baleanu. "Hybrid nanofluid on mixed convective radiative flow from an irregular variably thick moving surface with convex and concave effects." *Case Studies in Thermal Engineering* 21 (2020): 100660. <https://doi:10.1016/j.csite.2020.100660>
- [55] Wahid, Nur Syahirah, Nor Aliza Abd Rahmin, Norihan Md Arifin, Najiyah Safwa Khashi'ie, Ioan Pop, Norfifah Bachok, and Mohd Ezad Hafidz Hafidzuddin. "Radiative Blasius Hybrid Nanofluid Flow Over a Permeable Moving Surface with Convective Boundary Condition." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 100, no. 3 (2022): 115-132. <https://doi:10.1016/j.aej.2021.08.059>
- [56] Khashi'ie, Najiyah Safwa, Norihan Md Arifin, and Ioan Pop. "Non-Darcy mixed convection of hybrid nanofluid with thermal dispersion along a vertical plate embedded in a porous medium." *International Communications in Heat and Mass Transfer* 118 (2020): 104866. <https://doi:10.1016/j.icheatmasstransfer.2020.104866>
- [57] Khan, Umair, Aurang Zaib, Anuar Ishak, El-Sayed M. Sherif, Iskandar Waini, Yu-Ming Chu, and Ioan Pop. "Radiative mixed convective flow induced by hybrid nanofluid over a porous vertical cylinder in a porous media with irregular heat sink/source." *Case Studies in Thermal Engineering* 30 (2022): 101711. <https://doi:10.1016/j.csite.2021.101711>
- [58] Wahid, Nur Syahirah, Norihan Md Arifin, Najiyah Safwa Khashi'ie, and Ioan Pop. "Mixed convection of a three-dimensional stagnation point flow on a vertical plate with surface slip in a hybrid nanofluid." *Chinese Journal of Physics* 74 (2021): 129-143. <https://doi:10.1016/j.cjph.2021.08.013>
- [59] Ahmad, Adeel. "Flow of Reiner-Philippoff based nano-fluid past a stretching sheet." *Journal of Molecular Liquids* 219 (2016): 643-646. <https://doi:10.1016/j.molliq.2016.03.068>
- [60] He, Jun-Zhe, Xi-Xi Wang, Yan-Lan Zhang, and Mao-Sheng Cao. "Small magnetic nanoparticles decorating reduced graphene oxides to tune the electromagnetic attenuation capacity." *Journal of Materials Chemistry C* 4, no. 29 (2016): 7130-7140. <https://doi.org/10.1039/C6TC02020H>
- [61] Saqib, Muhammad, Ilyas Khan, Sharidan Shafie, and Ahmad Qushairi. "Recent advancement in thermophysical properties of nanofluids and hybrid nanofluids: an overview." *City Univ. Int. J. Comput. Anal* 3, no. 2 (2019): 16-25. <http://dx.doi.org/10.33959/cuijca.v3i2.27>
- [62] Khan, Ansab Azam, Khairy Zaimi, Suliadi Firdaus Sufahani, and Mohammad Ferdows. "MHD Mixed Convection Flow and Heat Transfer of a Dual Stratified Micropolar Fluid Over a Vertical Stretching/Shrinking Sheet With Suction, Chemical Reaction and Heat Source." *CFD Letters* 12, no. 11 (2020): 106-120. <https://doi.org/10.37934/araset.21.1.114>

- [63] Bakar, Fairul Naim Abu, and Siti Khuzaimah Soid. "MHD stagnation-point flow and heat transfer over an exponentially stretching/shrinking vertical sheet in a micropolar fluid with a Buoyancy effect." *Journal of Advanced Research in Numerical Heat Transfer* 8, no. 1 (2022): 50-55.
- [64] Cortell, Rafael. "Heat and fluid flow due to non-linearly stretching surfaces." *Applied Mathematics and Computation* 217, no. 19 (2011): 7564-7572. <https://doi:10.1016/j.amc.2011.02.029>
- [65] Ferdows, M., Md Jashim Uddin, and A. A. Afify. "Scaling group transformation for MHD boundary layer free convective heat and mass transfer flow past a convectively heated nonlinear radiating stretching sheet." *International Journal of Heat and Mass Transfer* 56, no. 1-2 (2013): 181-187. <https://doi:10.1016/j.ijheatmasstransfer.2012.09.020>
- [66] Waini, Iskandar, Abdul Rahman Mohd Kasim, Najiyah Safwa Khashi'ie, Nurul Amira Zainal, Anuar Ishak, and Ioan Pop. "Insight into Stability Analysis on Modified Magnetic Field of Radiative Non-Newtonian Reiner–Philippoff Fluid Model." *Journal of Applied and Computational Mechanics* 8, no. 2 (2022): 745-753.