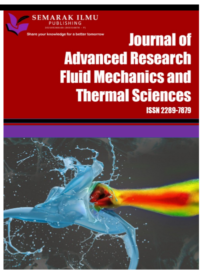
Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879



Flow and Heat Transfer of Williamson Hybrid Ferrofluid with Combined Convective Transport

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ARTICLE INFO

Article history:
Received 17 June 2023
Received in revised form 29 August 2023
Accepted 8 September 2023
Available online 27 September 2023

Keywords:
Williamson; combined convective transport; heat transfer; hybrid ferrofluid; shrinking; bvp4c

ABSTRACT

The present research investigated the characteristics of combined convective blood flow carrying copper and magnetite nanoparticles over a shrinking sheet, modelled as Williamson hybrid ferrofluid. The mathematical model is converted to similarity equations by suitable transformations. The similarity equations are then solved using the bvp4c function. The characteristics and effects of Williamson and combined convective transport parameter, as well as the nanoparticle volume fraction in the Williamson hybrid ferrofluid towards the temperature profiles, velocity profiles as well as the Nusselt number and the skin friction coefficient are analysed and discussed. Outcomes reveal the increment in Williamson parameter suppress the heat transfer performance of the fluid but not for the combined convective transport and suction parameter. Meanwhile the heat transfer performance is also improved as concentration of copper is increased.

1. Introduction

Heat transfer is the process of exchanging thermal energy between physical systems through the dissipation of heat. The process of transferring heat is induced by pressure and temperature difference that occurs within the physical systems. Energy can be saved and kept at maximum level by using devices which have maximum rate of heating and cooling. Thus, in industries applications, it is crucial to have the best heat transfer material.

Hybrid nanofluid (HNF) is a new potential of nanofluid (NF) that has great thermo-physical properties and thermal performance compared to mono-nanofluid. HNF consists of two elements of nanoparticle immersed in a base fluid. The frequently used nanoparticles are classified into (i) metals (copper/Cu, silver/Ag, Nickel/Ni), (ii) metal oxides (aluminum oxide/Al_2O_3, ferric oxide/Fe_2O_3, cupric

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https://doi.org/10.37934/arfmts.109.2.95112

oxide/CuO, silicon dioxide/SiO_2), (iii) carbon materials (carbon nanotubes/CNTs, multi-walled carbon nanotubes/MWCNTs, diamond, graphite), (iv) metal nitride (aluminum nitride/AlN) and (v) metal carbide (silicon carbide/SiC). On the other hand, water, ethylene glycol and oil are commonly used as the base fluid in the formation of nanofluids. Through this technology, the heat transfer performance has been improved and their advantages have led to numerous research to further investigate the unknown behavior of HNF over different body geometries and physical parameters [1-11]. Based on recent study, not only the role of type and concentration of nanoparticle contributes to heat transfer enhancement but also its base fluid [12].

Referring to Huminic and Huminic [13], heat transfer characteristics of HNF were investigated through numerous experimental and numerical studies. Although investigation on flow and heat transfer of HNF via numerical approach is hard to get accurate results, it is economical compared to experimental approaches which are costly and even sometimes give bad effects to humans and environment [14-16]. Several existing numerical approaches for solving fluid flow problems which have been found in the literatures include method of lines, bvp4c & Keller-box and Runge-kutta method, may be used to overcome this limitation [17-22].

Ferrofluid (FF) is a special nanofluid (NF) which consist of magnetic nanoparticles known as ferro particles from oxide nanoparticle family, such as ferric oxide/maghemite (Fe_2O_3), magnetite (Fe_3O_4), cobalt ferrite ($CoFe_2O_4$) or some other compound containing iron which immersed in a base fluid such as water, oil, ethylene, etc. [23]. It was first developed in 1963 at the National Aeronautics and Space Administration (NASA) Research Centre to overcome zero gravity in space. The presence of magnetic force through the combination of ferro particles with fuel which act as the carrier fluid, the fuel can be injected into the combustion chamber [24]. Since FF exhibits superparamagnetic behaviors, this fluid is designed to be directed in a specific direction using magnetic field strength. This fluid is then known as one of the magnetically controllable fluids and has been studied by many researchers since the past decade until recent such as Blennerhassett *et al.,* [25], Barclay [26], Jamaludin *et al.,* [27], Mohamed *et al.,* [28], and Yasin *et al.,* [29].

Further then, current studies found the heat transfer performance of FF is enhanced through the combination with another nano particle which then be one type of hybrid nanofluid (HNF) specifically called hybrid ferrofluid (HFF) [30,31]. Recently, FF has been widely used not only in the industrial sector but also in the biomedical sector. In the industrial field, FF has been used related to heat transfer, such as coolant in thermal management devices, heat exchangers, processes that include boiling, and even to improve the cooling function of loudspeakers [32,33]. Meanwhile, in biomedical applications, FF is used in medical treatment for cancer therapy, bleeding, stopping agents, magnetic resonance imaging, and other diagnostic tests [34-36]. In medical treatment, the medication and proteins transportation must be delivered to only the targeted tissue. Considering human blood as the carrier fluid, the specific nano-size and magnetic behaviors of particles in the carrier fluid, can deliver high doses of therapeutic factors into tumor cells without contaminating normal cells.

The study of non-Newtonian fluids has piqued the interest of many academics due to its wide range of applications in the industrial and technical fields, particularly in manufacturing and processing. Non-Newtonian fluids are more complicated compared to Newtonian fluids and Navier-Stokes equations alone are insufficient to describe the rheological properties of these fluids. Several existing models which improved the Navier-Stokes equations in describing the non-Newtonian fluid have been adopted in recent literature. Among them are Casson, Cross, Maxwell, Reiner-Philippoff, Viscoelastic, and Williamson model [37-45]. These models are represented in the form of differential equations. For differential fluids, stress is expressed by an explicit velocity gradient. Differential models describe the fluids in terms of their shear thickening and thinning effects, normal stress effects, thixotropy exhibition, and nonlinear behavior of creeping and yielding.

In non-Newtonian fluids, pseudoplastic fluids are the most frequently encountered fluids. A pseudoplastic is a shear-thinning fluid and has less resistance at high strain rates like polymer solution, paint, blood, plasma. Williamson model have been commonly used to represent the rheological properties of the pseudoplastic fluid [46]. Subbarayudu *et al.,* [47] study the assessment of time dependent flow of Williamson fluid (WF) with radiative blood flow against a wedge. Later, Shateyi and Muzara [46] study on the numerical analysis of unsteady MHD boundary layer flow of WF over a stretching sheet and heat transfers. Both studies found the Williamson parameter decreases the velocity and temperature profile. Next, the WF studies are extended with the involvement of nano particles which modelled as Williamson Nanofluid (WNF). Among WNF studies includes Khan *et al.,* [48], and Bouslimi *et al.,* [49] which both found the velocity decreased and the temperature profile increased as the Williamson parameter decreased. Since NF is upgraded to HNF, several studies on HNF have been embedded with WF model known as Williamson hybrid nanofluid (WHNF). This model have been explored by Jamshed *et al.,* [50,51] and Kavya *et al.,* [52] which involved different types of nano particles and bases. All of them found the same findings as the previous WNF studies. Moreover, most stated previous studies, concluded that heat transfer performance is diminished as the Williamson parameter increases.

The numerical studies on fluid flow and heat transfer due to stretching/shrinking surfaces were also being subject of interest for current researchers due to its numerous and significant applications in technological and industrial queries such as wire drawing, aerodynamic extrusion of plastic sheets, hot rolling, metal spinning and so on [53]. The flow on shrinking surface is important because during this flow, the separation process occurs where the flow tends to change from laminar to turbulent. Previous literature on HNF flow has shown that the solutions for the shrinking surface cases were not unique [54-58]. Hence, many researchers reported the duality of solutions with stability analysis for most problems regarding HNF flow due to shrinking surfaces. Hamid *et al.,* [59] study on WF over a shrinking sheet, obtained multiple solutions for the flow fields and found that the range of dual solutions exist expands with unsteadiness parameter. There were also several literatures on shrinking sheet involving WF which comprising nanoparticles includes Khan *et al.,* [48], and Khan *et al.,* [53] which both found dual solution. Khan *et al.,* [48] studied stagnation point flow of WNF towards a permeable stretching/shrinking sheet with a partial slip. While Khan *et al.,* [53] studied computational simulation of crossflow of WHNF fluid over a porous shrinking/stretching surface and thermal radiation.

Apart from the flow on shrinking surfaces, the separation process also can happen at buoyancy opposing flow in combined convective transport problems. These problems have more interest due to their various applications in the science and industrial sector. Some of such applications are mentioned as electronic receivers, drying technologies, and many others. From literature, various combined convective transport of HNF over different geometries and effects have been found [60-67]. WNF fluid flow problems with combined convective also have been identified with different effects. Hamid and Khan [68] studied unsteady combined convective flow of WNF with heat transfer in the presence of variable thermal conductivity and magnetic field. Velocity profile was found to be increased and temperature profile was found to be decreased as combined convective parameter is increased. This finding is also in line with Khan *et al.,* [69] while studying the effects of thermal radiation and slip mechanism on combined convective flow of WNF over an inclined stretching cylinder. While Eswaramoorthi *et al.,* [70] found different findings with the previous studies for velocity profile where fluid velocity decelerates when combined convective parameter increases. Eswaramoorthi *et al.,* [70] focused on combined convective and thermally radiative flow of MHD WNF with arrhenius activation energy and Cattaneo–Christov heat-mass flux. Among others combined convective WNF studies include Hayat *et al.,* [71] and Ahmad *et al.,* [72]. Hayat *et al.,* [71] studied the combined convective three-dimensional flow of WNF subject to chemical reaction, while Ahmad *et al.,* [72] research on numerical investigation for combined convective 3D radiative flow of chemically reactive WNF with power law heat/mass fluxes. The literature observed that velocity of the fluid and momentum boundary layer thickness depicts significant enhancement with elevation in combined convective parameters.

Considering HFF with blood as its base, the Williamson model has been embedded in the existing HFF flow formulation, which is known as Williamson hybrid ferrofluid (WHFF) and be one type of WHNF [73-75]. Bhatti and Abdelsalam [73] involved ferroparticle of copper oxide (CuO) while Rosli *et al.,* [74,75] involved ferroparticle of magnetite (Fe_3O_4). Both Rosli *et al.,* [74] and Rosli *et al.,* [75] combined the ferroparticle with copper (Cu) and study on WHFF over a stretching sheet but with different effects. Rosli *et al.,* [74] found the non-Newtonian WHFF potentially provides better performance in heat transfer capability compared to FF with the same volume of nanoparticle volume fraction. Meanwhile, Rosli *et al.,* [75] found the HFF has the same performance of heat transfer as FF with the same volume of nanoparticle volume fraction. Since the WHFF studies are found to be very limited in the literature, it is crucial for conducting this study for further understanding of fluid flow and heat transfer characteristics of this upgraded fluid. To the best of the author’s knowledge, there is no WHFF flow study which involved the combined convective transport and shrinking sheet problem. Thus, the objective of this research is to further study the fluid flow and heat transfer characteristics of combined convective transport of WHFF flow over a shrinking sheet which consists of magnetite (Fe_3O_4) and copper (Cu) with blood as its base.

2. Methodology

2.1 Mathematical Formulation

A steady two-dimensional flow of Williamson hybrid ferrofluid (WHFF) under combined convective transport over a shrinking sheet is considered with the surface velocity, $u_w(x) = ax$ where a is a positive constant and the combined convective, $\frac{(\rho\beta_T)_{hff}}{\rho_{hff}} g(T - T_\infty)$ with the thermal expansion β and gravitational acceleration g. Figure 1 demonstrates the geometric structure of the study case.

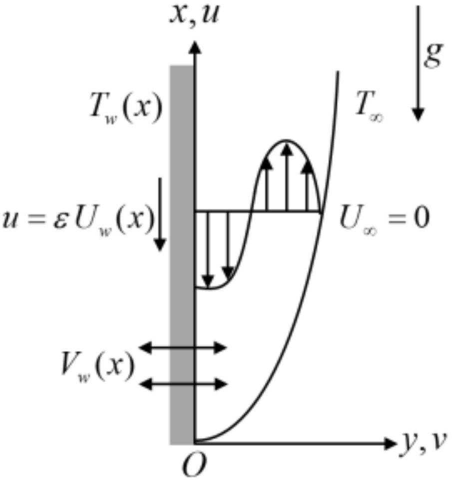

Fig. 1. Geometry of the physical problem

Additionally, the surface temperature of the sheet is considered as, $T_w = T_\infty + T_0 x$, where T_0 is a constant and the constant ambient temperature is signified as T_∞. The derivation of the WHFF model is based on a graphical method for separating the viscous and plastic resistances of pseudoplastic dispersions into separate measurable quantities. This theory was proposed and explained by Williamson [76]. Meanwhile, because the hybrid ferrofluid (HFF) is created as a stable compound, the size of nanoparticles in the HFF is uniform, and the agglomeration effect is ignored.

In Cartesian coordinate system, the governing boundary layer equations of the WHFF can be stated as using the conventional boundary layer approximations for the continuity, momentum, and energy equations under the presumptions outlined above and depicting the fluid model can be conveyed as [61,67,75];

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{hff}}{\rho_{hff}}\left(\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\Gamma\frac{\partial^2 u}{\partial y^2}\frac{\partial u}{\partial y}\right) + \frac{(\rho\beta_T)_{hff}}{\rho_{hff}} g\left(T - T_\infty\right) \tag{2}$$

$$\left(\rho c_p\right)_{hff}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k_{hff}\left(\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

with the boundary conditions:

$$\begin{aligned} &u = \varepsilon u_w(x), \quad v = v_w, \quad T = T_w \quad \text{at } y = 0, \\ &u \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty \end{aligned} \tag{4}$$

where (u, v) are the velocity components in (x, y) directions respectively, ρ_{hff} is the density, whereas the μ_{hff} defined as dynamic viscosity, c_p is the specific heat at constant pressure, $\left(\rho c_p\right)_{hff}$ is the heat capacitance, k_{hff} is thermal conductivity, while $\Gamma = \Gamma_0 x^{-1}$ is fluid parameter of the Williamson model with constant Γ_0. Besides, the parameter ε is for the deformable sheet such that $\varepsilon > 0$ stands for a stretching sheet, $\varepsilon < 0$ indicates a shrinking sheet and $\varepsilon = 0$ represents a static sheet. Moreover, $v_w = -S\left(a v_f\right)^{1/2}$ denotes the constant mass velocity for the surface and S is the suction/injection parameter such that $S > 0$ corresponds to the suction effect, and $S < 0$ refers to the injection (fluid removal) effect. While v_f is fluid kinematic viscosity.

Further, Table 1 gives the characteristics of the blood, Cu, and Fe_3O_4. Note that, ϕ_1 and ϕ_2 denote Fe_3O_4 and Cu nanoparticles respectively, where $\phi_{hff} = \phi_1 + \phi_2$. Meanwhile, Table 2 provides the HFF correlation. The governing Eq. (1), Eq. (2) and Eq. (3) are a system of nonlinear partial differential equations (PDEs) and due to its complexity, a practical transformation method namely similarity transformation is introduced to convert the PDEs into a simplified set of a nonlinear ordinary differential equation (ODEs). Thus, the relevant similarity transformations which applicable to simplify on Eq. (1), Eq. (2) and Eq. (3) inclusive with the boundary conditions (4) are,

$$\eta = \left(\frac{a}{\nu_f}\right)^{1/2} y, \quad \psi = \left(a\nu_f\right)^{1/2} xf(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

Where η defined as dimensionless similarity variable, and ψ is the stream function. The velocity component becomes,

$$u = \frac{\partial \psi}{\partial y} = axf'(\eta), v = -\frac{\partial \psi}{\partial x} = -\left(a\nu_f\right)^{\frac{1}{2}} f(\eta) \tag{6}$$

which satisfied the continuity Eq. (1). Next, the momentum (2) and energy (3) equations become,

$$\frac{\mu_{hff}/\mu_f}{\rho_{hff}/\rho_f}\left(1+\gamma f''\right)f''' + ff'' - f'^2 + \frac{\left(\rho\beta_T\right)_{hff}/\left(\rho\beta_T\right)_f}{\rho_{hff}/\rho_f}\lambda\theta(\eta) = 0 \tag{7}$$

$$\theta'' + \Pr\frac{\left(\rho c_p\right)_{hff}/\left(\rho c_p\right)_f}{k_{hff}/k_f}\left(f\theta' - f'\theta\right) = 0 \tag{8}$$

with the boundary conditions,

$$\begin{aligned} &f(0) = S,\ f'(0) = \varepsilon,\ \theta(0) = 1 \\ &f'(\eta) \to 0,\ \theta(\eta) \to 0 \ \ as\, \eta \to \infty \end{aligned} \tag{9}$$

where a notation prime (′) is derivative with respect to η and

$$\gamma = \Gamma_o a\left(\frac{2a}{\nu_f}\right)^{\frac{1}{2}},\ \lambda = \frac{Gr}{\left(\mathrm{Re}_x\right)^2},\ Pr = \frac{\left(\rho c_p \nu\right)_f}{k_f}, \tag{10}$$

represent Williamson fluid parameter, combined convective parameter, and Prandtl number respectively. The combined convective parameter with $\lambda > 0$ corresponds to the aiding or assisting flow, $\lambda < 0$ corresponds to the opposing flow and $\lambda = 0$ denotes the pure forced convective flow. Further, $Gr = \frac{g\left(\beta_T\right)_f\left(T_w - T_\infty\right)x^3}{{\nu_f}^2}$ is the Grashof number and $Re_x = \frac{ax^2}{\nu_f}$ is the local Reynolds number.

2.2 The Skin Friction Coefficient and Local Nusselt Number

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are given by,

$$C_f = \frac{\mu_{hff}}{\rho_f u_w^2}\left[\frac{\partial u}{\partial y} + \frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right]_{y=0},\ Nu_x = -\frac{x k_{hff}}{k_f\left(T_w - T_\infty\right)}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{11}$$

Using Eq. (5) and Eq. (11) one obtains,

$$\mathrm{Re}_x^{\frac{1}{2}} C_f = \frac{\mu_{hff}}{\mu_f} f''(0)\left(1 + \frac{\gamma}{2} f''(0)\right),\ \mathrm{Re}_x^{-\frac{1}{2}} Nu_x = -\frac{k_{hff}}{k_f}\theta'(0) \tag{12}$$

where $Re_x = \frac{ax^2}{\nu_f}$ is the Reynolds number and $\gamma = \Gamma_o a\left(\frac{2a}{\nu_f}\right)^{\frac{1}{2}}$ is the Williamson fluid parameter.

2.3 Model and Thermo-Physical Properties

The thermo-physical properties are presented to elucidate the flow of HFF. In this model $\phi_1 = \phi_2 = 0.01$ to yield Cu-Fe_3O_4/blood throughout the problem. To make it clear, the valuable thermo-physical characteristics of ferrofluid (FF) and hybrid ferrofluid (HFF) are presented in Table 1 [77]. From the above literature review the basic thermo-physical characteristics of nanofluids are taken. The thermo-physical characteristics of blood taken as base fluid nanoparticle involved are given in Table 2 [78,79].

Table 1
Thermo-physical properties for FF and HFF

Element	FF	HFF
Viscosity	$\frac{\mu_{ff}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}$	$\frac{\mu_{hff}}{\mu_f} = \frac{1}{\left(1-\phi_{hff}\right)^{2.5}}$
Density	$\rho_{ff} = (1-\phi)\rho_f + \phi\rho_s$	$\rho_{hff} = \left(1-\phi_{hff}\right)\rho_f + \phi_1\rho_{s1} + \phi_2\rho_{s2}$
Heat Capacity	$\left(\rho C_p\right)_{ff} = (1-\phi)\left(\rho C_p\right)_f + \phi\left(\rho C_p\right)_s$	$\left(\rho C_p\right)_{hff} = \left(1-\phi_{hff}\right)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_{s1} + \phi_2\left(\rho C_p\right)_{s2}$
Thermal Conductivity	$\frac{k_{ff}}{k_f} = \frac{k_s + 2k_f - 2\phi\left(k_f - k_s\right)}{k_s + 2k_f + \phi\left(k_f - k_s\right)}$	$\frac{k_{hff}}{k_f} = \frac{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_{hff}}\right) + 2k_{bf} + 2\left(\phi_1 k_{s1} + \phi_2 k_{s2}\right) - 2\phi_{hff}k_{bf}}{\left(\frac{\phi_1 k_{s1} + \phi_2 k_{s2}}{\phi_{hff}}\right) + 2k_{bf} - \left(\phi_1 k_{s1} + \phi_2 k_{s2}\right) + \phi_{hff}k_{bf}}$
Thermal expansion	$\left(\rho\beta_T\right)_{ff} = (1-\phi)\left(\rho\beta_T\right)_f + \phi\left(\rho\beta_T\right)_s$	$\left(\rho\beta_T\right)_{hff} = \left(1-\phi_{hff}\right)\left(\rho\beta_T\right)_f + \phi_1\left(\rho\beta_T\right)_{s1} + \phi_2\left(\rho\beta_T\right)_{s2}$ where $\phi_{hff} = \phi_1 + \phi_2$

Table 2
Thermo-physical properties for the base fluid and nanoparticles

Thermo-physical properties	Base fluid	Nano particle	
	Blood	Fe_3O_4	Cu
Density, $\rho(kg/m^3)$	1,053	5200	8933
Heat capacitance, $C_p(J/kgK)$	3,594	670	385
Thermal conductivity, $k(W/mK)$	0.492	6	401
Thermal expansion, $\beta_T \times 10^{-5}\left(K^{-1}\right)$	0.18	1.35	1.67

3. Results and Discussion

Numerical solutions of physical quantities of interest Cu-Fe_3O_4/blood fluid flow is computed by using the bvp4c function in MATLAB software. Verification of above results is assured when these hold reasonable comparison with previous results for Nusselt Number under different values of Pr in limiting cases for stretching sheet as illustrated in Table 3. Table 4 also validated the previous results for skin friction and Nusselt Number in limiting cases for shrinking sheet problem. By utilizing the above numerical scheme, the physical nature of flow and heat transfer for WHFF is revealed through a prolonged computational process involving suitable variations of the controlling parameters. It is noted that, in this study, Pr=21 is used which is suitable enough for blood-based non-Newtonian HFF [74,75,80].

Table 3
Comparative values of $-\theta'(0)$ under different values of Pr when $\phi_{hff} = S = \lambda = \gamma = 0,$ and $\varepsilon = 1$ (stretching sheet)

Pr	Grubka and Bobba [81]	Ishak *et al.*, [82]	Waini *et al.*, [83]	Present Results
0.72	0.8086	0.8086	0.8086	0.8086
1	1.0000	1.0000	1.0000	1.0000
3	1.9237	1.9237	1.9237	1.9237
10	3.7207	3.7207	3.7207	3.7207

Table 4
Comparative model and values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ and $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ when $\phi_{hff} = 2\%,\ S = 2,$ and $\varepsilon = -1$ (shrinking sheet)

Author	Model	Limiting cases	$\mathrm{Re}_x^{\frac{1}{2}} C_f$		$\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$	
			First Solution	Second Solution	First Solution	Second Solution
Current	Momentum Equation $\frac{\mu_{hff}/\mu_f}{\rho_{hff}/\rho_f}\left(1+\gamma f''\right)f''' + ff'' - f'^2 + \frac{(\rho\beta_T)_{hff}/(\rho\beta_T)_f}{\rho_{hff}/\rho_f}\lambda\theta(\eta) = 0$ Energy Equation $\theta'' + \Pr\frac{(\rho c_p)_{hff}/(\rho c_p)_f}{k_{hff}/k_f}\left(f\theta' - f'\theta\right) = 0$	$\lambda = 0$, $\gamma = 0$	1.36	0.8566	11.2525	11.1872

Waini *et al.*, [83]	Momentum Equation $\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + ff'' - f'^2 + \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} Mf' = 0$ Energy Equation $\frac{1}{\Pr\left(\rho c_p\right)_{hnf}/\left(\rho c_p\right)_f}\left(\frac{k_{hnf}}{k_f}+\frac{4}{3}R\right)\theta''$ $+f\theta' - mf'\theta = 0$	$M = 0$, $m = 1$, $R = 0$	1.3622	0.8566	11.2748	11.2126

In Figure 2 to Figure 9, graphs of velocity and temperature profile for WHFF (Cu- Fe_3O_4/blood) flow under different variations of parameters, are presented. The influences of Williamson number, γ on velocity $f'(\eta)$ and temperature $\theta(\eta)$ are highlighted in Figure 2 and Figure 3 respectively. It seems that the velocity of the fluid recedes down whereas the temperature recedes up with the increasing values of the parameter, γ. Since the Williamson number is the ratio of relaxation time to specific process time, a decline in the specific time of the process will enhance the Williamson number, indicating a reduction in the velocity field. So that velocity goes down. Physically, the Williamson variable strengthens the non-Newtonian nature of the ephemeral fluid through the frictional manipulations which make the fluidity tougher. Since slower fluidity allows for more time for thermal absorption from the surface, thus the thermal distribution rises.

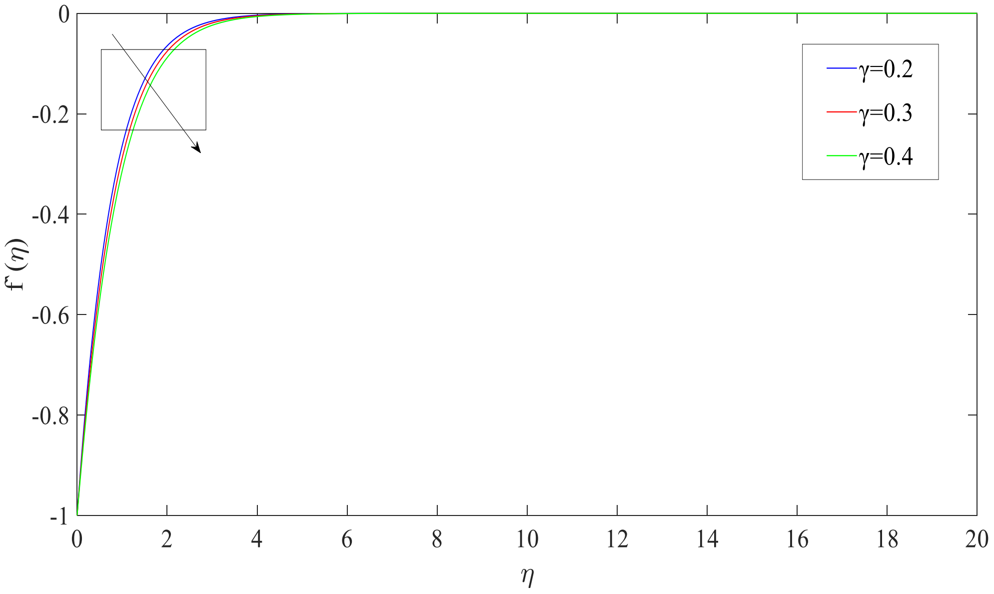

Fig. 2. The velocity profile for different values of γ

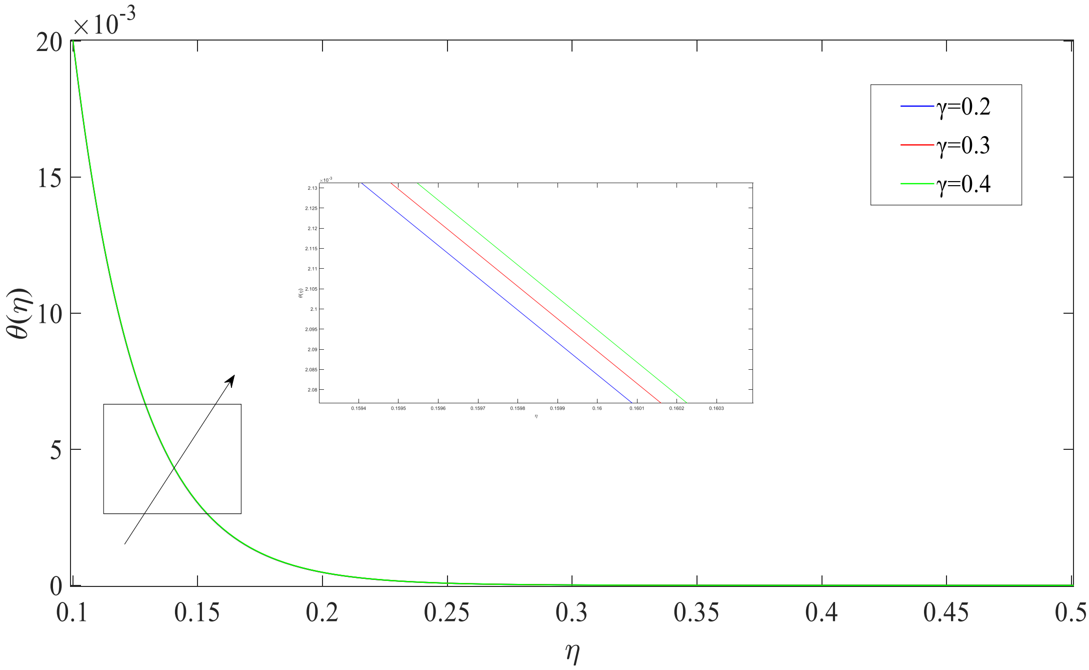

Fig. 3. The temperature profile for different values of γ

Figure 4 and Figure 5 describe the impact of combined convective parameter, λ on velocity $f'(\eta)$ and temperature $\theta(\eta)$ respectively. The speed of the flow increases whereas the temperature decrease, against a greater combined convective parameter, λ as value $(\lambda < 0)$. This enhancement in the velocity is because of higher thermal buoyancy force. Additionally, since the buoyancy force tends to upgrade the temperature gradient hence the temperature diminishes by higher values of combined convective parameter.

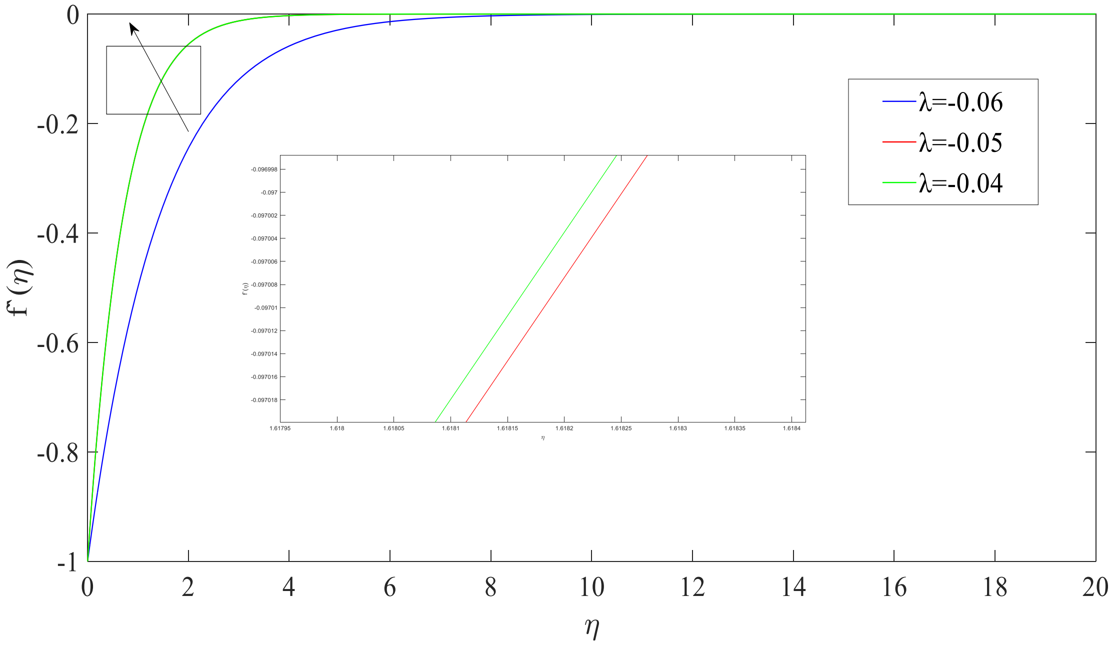

Fig. 4. The velocity profile for different values of λ

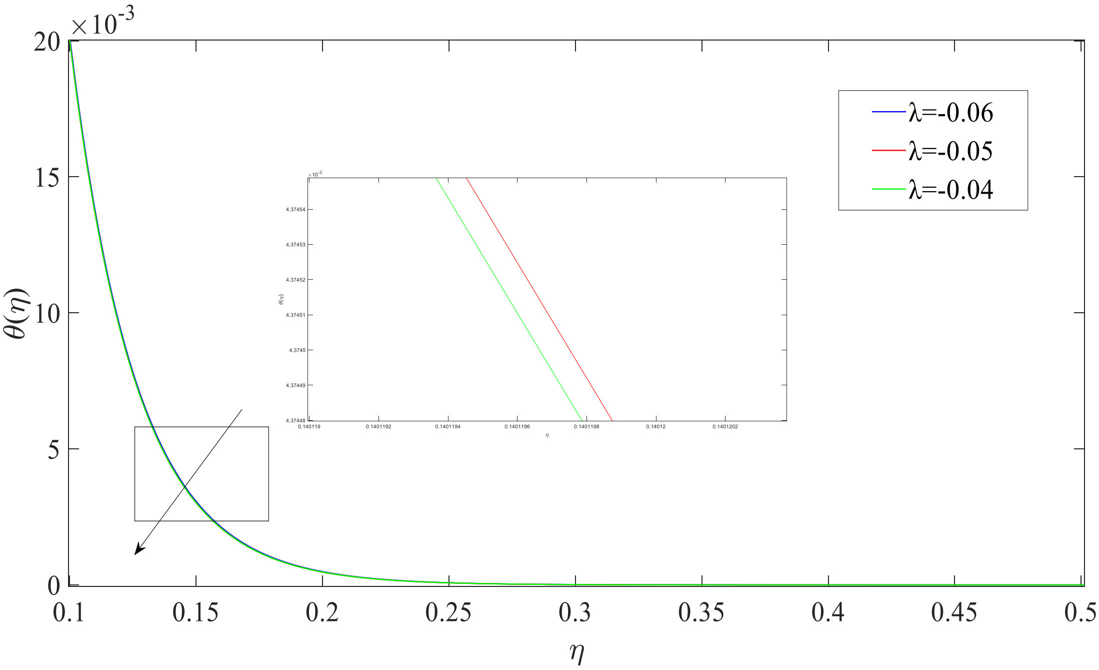

Fig. 5. The temperature profile for different values of λ

Figure 6 and Figure 7 explain the influences of suction parameter, S as value $(S>0)$ on velocity and temperature respectively. It can be found that the velocity is increasing due to mass transfer at the suction of the wall while temperature is decreasing. Physically, with the increasing suction strength in flow, the heat velocity increases because of the removal of the decelerated fluid particles through the surface. On the other hand, when the heat is dispersed faster around it, the temperature of the fluid reduces.

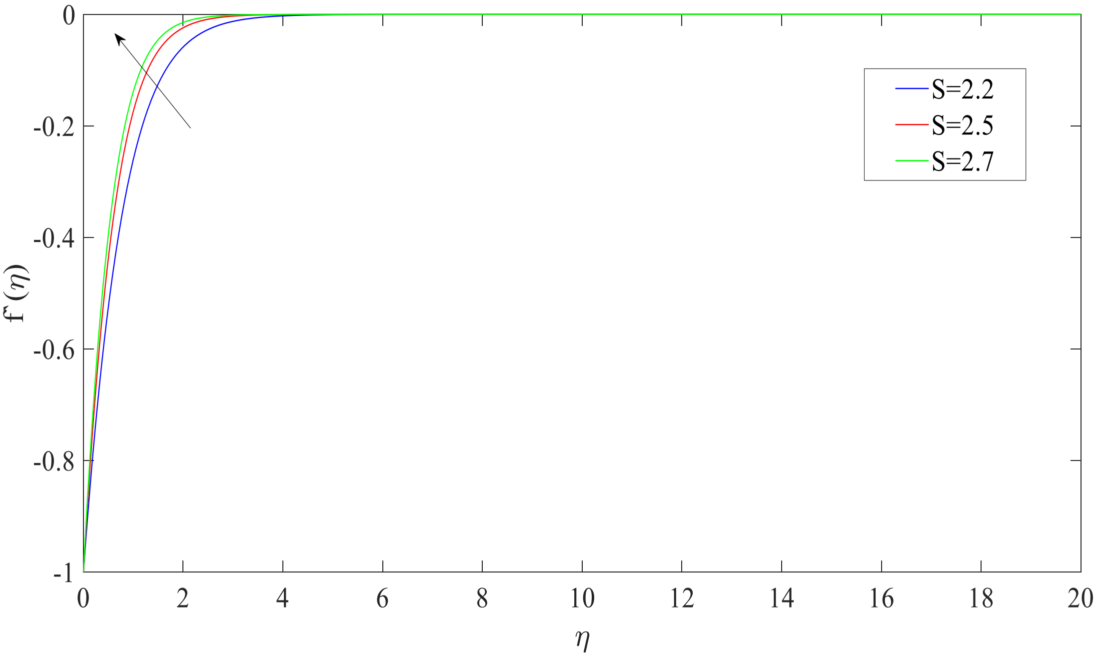

Fig. 6. The velocity profile for different values of S

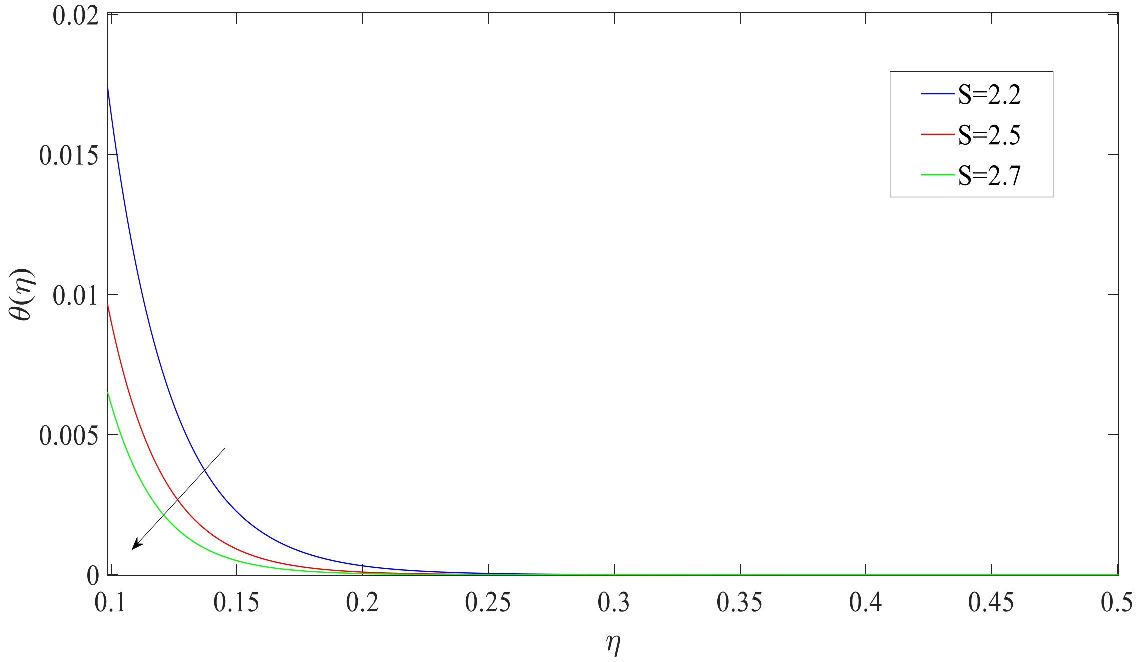

Fig. 7. The temperature profile for different values of S

The impact of all parameters of Williamson, combined convective, and suction on skin friction and Nusselt Number are recorded in Table 5. While role of volume fraction ϕ_2 of Cu on skin friction and Nusselt number is detected from data recorded in Table 6 and Table 7 respectively. It is seen that the skin friction and Nusselt Number of WHFF are decreased slowly with the increments in the parameter of Williamson.

Table 5

Values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ and $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ for different physical parameters when $\phi_1 = \phi_2 = 0.01$, $\varepsilon = -1$ (base fluid: blood)

λ	γ	S	$\mathrm{Re}_x^{\frac{1}{2}} C_f$	$\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$
-0.09	0.1	2.1	0.739533441	43.284761008
-0.06			0.741158414	43.284780650
-0.05			1.548663649	43.304040309
-0.04			1.549231651	43.304047069
	0.2		1.488276238	43.300804664
	0.3		1.426245016	43.298054795
	0.4		1.360569169	43.295602146
		2.2	1.599703609	45.459936639
		2.5	2.113459266	51.921124234
		2.7	2.405122695	56.210710110

Table 6

Values of $\mathrm{Re}_x^{\frac{1}{2}} C_f$ when $\phi_1 = 0.01$, $\mathrm{Pr} = 21$, $S = 2.1$, $\varepsilon = -1$ and $\lambda = -0.04$

γ	$\phi_2 = 0.007$	$\phi_2 = 0.01$	$\phi_2 = 0.02$
0.1	1.499446369	1.549231651	1.709061213
0.2	1.437185407	1.488276238	1.650317651
0.3	1.372840602	1.426245016	1.592351236
0.4	1.302888985	1.360569169	1.533696965

Table 7

Values of $\mathrm{Re}_x^{-\frac{1}{2}} Nu_x$ when $\phi_1 = 0.01$, $\mathrm{Pr} = 21$, $S = 2.1$, $\varepsilon = -1$ and $\lambda = -0.04$

γ	$\phi_2 = 0.007$	$\phi_2 = 0.01$	$\phi_2 = 0.02$
0.1	43.281242508	43.304047069	43.379030005
0.2	43.278109204	43.300804664	43.375362840
0.3	43.275422735	43.298054795	43.372321634
0.4	43.272981756	43.295602146	43.369693249

Figure 8 and Figure 9 show the impact of ϕ_2 on skin friction and Nusselt Number respectively. It is clear that ϕ_2 will increase the skin friction and Nusselt Number starting from concentration of 0.007 and above. It comes to know that the velocity $f'(\eta)$ becomes slower with augmentation of ϕ_2 but the temperature rises. The reason for the slowing of flow speed is that the viscosity increases with ϕ_2 and hence the viscosity effects decelerate the flow. In the meanwhile, the thermal conductivity increases and hence the rise in temperature has resulted.

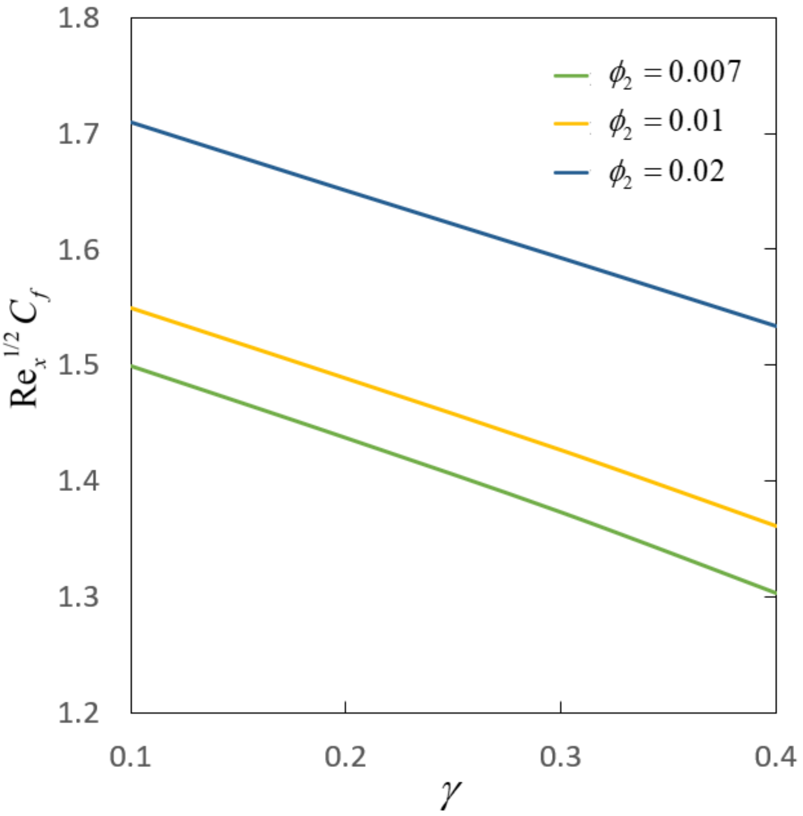

Fig. 8. Variation of skin friction against Williamson parameter, γ for

different values of ϕ_2

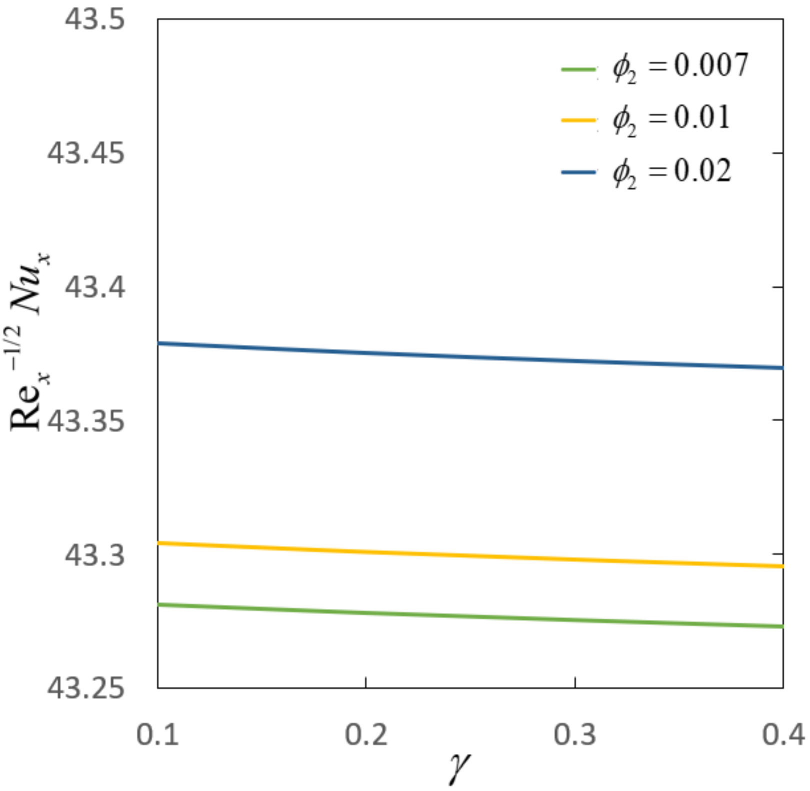

Fig. 9. Variation of Nusselt Number against Williamson parameter, γ for different values of ϕ_2

4. Conclusion

In this study, the impact of several parameters including Williamson, combined convective, and suction parameter on velocity and temperature have been investigated. It is mentionable that the increase in Williamson parameters will decelerate the velocity and increase the temperature. On the other hand, the increase combined convective and suction parameter will increase the velocity and thus diminishes the temperature. The increase in Williamson parameter suppresses the thermal performance of the fluid but not for the combined convective and suction parameter. The nanoparticle concentration also shows a role in heat transfer performance where the Nusselt number is increased as concentration of copper is increased.

Acknowledgement

The authors gratefully acknowledge Universiti Malaysia Pahang Al-Sultan Abdullah for providing the financial support under grant no. UIC221518.

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