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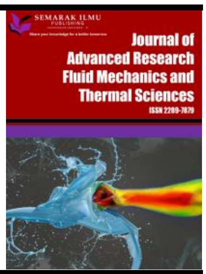

Unsteady Free Convection Dusty MHD Flow with Dissipation Effect Over Non-Isothermal Vertical Cone

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ABSTRACT

The implications of suspended particles on MHD flow in a viscous fluid over a non-isothermal vertical cone with dissipation effect was investigated in this recent research. To maximise process efficiency and avoid equipment failure, it is important to thoroughly investigate the complex and dynamic behaviours that might result from the interaction between the fluid and the dust particles inside the cone. The Crank-Nicolson method will be used in the study to analyse the mathematical model quantitatively. MATLAB software was used to compute the discretization equations and depict the numerical outcomes. The velocity distribution, temperature distribution, skin friction and Nusselt number affected by Prandtl number, magnetic, mass concentration of particle phase, viscous dissipation and fluid-particle interaction for particle and fluid phases are presented graphically as well as in a table and are thoroughly analysed. The outcome indicated that the fluid and dust phase velocity decreases when magnetic, mass concentration of particle phase and fluid-particle interaction parameters are escalated while viscous dissipation parameter shows an opposite behaviour. Moreover, the temperature and velocity profiles for the fluid phase are consistently greater than those for the dust phase in every result that was examined.

1. Introduction

The advantages of Magnetohydrodynamics (MHD) on natural convection flow are significant in the development of technology for industry and engineering applications. MHD is defined as the study of the dynamics of electrically conducting fluids. Essentially, magnetic fields in conductive fluids generate currents, which create a drag force on the fluid flow called the Lorentz force. This force consequently affects the fluid flow. Investigation of the effects of this buoyancy force together with MHD is essential in various fields. Previously, Nayan *et al.,* [1] investigated aligned MHD flow with a constant wall temperature. They discovered that when the interaction of the magnetic parameter

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and the angle of aligned magnetic field enhances, the velocity profiles increase, and the temperature profiles decrease. Mahat *et al.,* [2] conducted theoretical research on the impacts of magnetic fields and how they affect a system's thermal property. They found that an increase in magnetic properties causes velocity to slow down and rise. Meanwhile, as the magnetic parameter is increased, skin friction rises, but the heat transfer coefficient falls. Reyaz *et al.,* [3] analyzed the impacts of Riga plates MHD flow with fractional derivatives. They discovered that fluid velocity increases while the skin friction coefficient decreases with the presence of Riga plates. Saranya and Al-Mdallal [4] investigated a unique type of ferrofluid flowing around an unstable cylinder impacted by a magnetic field. They found that the fluid may behave in unusual ways, including reversing its flow direction due to magnetic field. Al-Mdallal *et al.,* [5] released research in which they investigated what occurs when heat is generated while a liquid flows through a shrinking cylinder via a porous medium. They observed that the heat generation rate and the magnetic field strength may have a significant influence on how the liquid moves. Saranya *et al.,* [6] investigated how two distinct types of fluids containing microscopic particles and microbes react on both fixed and moving surfaces. They revealed that the presence of small particles and microbes can significantly influence how fluids move. Finally, some recent intriguing MHD research is presented in previous studies [7-13].

Studies have been done on the impact of dissipation on natural convection flow in various ways. It has been discovered that substantial dissipation from viscous fluid may occur in free convection as a result of significant deceleration or in equipment that rotates at high speeds. For instance, the fundamentals of thermophysical features of viscous dissipation are examined by Gopal *et al.,* [14]. In addition, variable temperature and mass diffusion are studied by Shukla and Dube [15] on unsteady MHD flow for their combined effects of radiation and Soret-Dufour. When the parameter of viscous dissipation is increased, the distribution of velocity, temperature, and concentration all increase significantly. The concentration profile initially drops before increasing. Computational analysis was performed by Hanif *et al.,* [16] on a viscous MHD fluid caused by an inclined magnetic field with pressure gradient. It was noted that viscous dissipation impacted the production of heat. Increased values of the magnetic field and inclination angle are found to cause a decrease in fluid velocity. Moreover, the thermal boundary layer thickness falls and the temperature declines as the Prandtl number increases. Ashraf *et al.,* [17] investigate the periodic heat transfer along a cone affected by MHD and viscous dissipation. The results show that the presence of a magnetic field causes an increase in the skin friction coefficient and heat transfer rates, while viscous dissipation leads to a decrease in these parameters. More noteworthy and interesting research with viscous dissipation can be found in previous studies [18-20].

The investigation of dusty fluid is relevant to MHD because it reflects the impact of corrosion and wear procedures that create solid dispersion in a conducting fluid, in the form of soot or ash. Dusty fluid can be found in various applications, and due to its significance, the basic characteristics of dusty fluid flow have been described in numerous published research works. Previously, Ali *et al.,* [21] observed the resistive porous medium of dusty flow via permeable media when Lorentz force is present. They revealed that skin friction tends to rise, while magnetic and porosity parameters have the tendency to reduce the heat transport rate. Goh and Shab [22] investigated how various parameters affected the dusty MHD fluid flow. The momentum boundary layer was found to decrease and the thermal boundary layer to grow as the magnetic, thermal conductivity, and slip variation parameters increased. The porosity parameter, however, displayed the opposite tendency. The experimental study published by Ali *et al.,* [23] studied the impact of Newtonian heating on the MHD flow of dusty fluid. They found that as the dusty fluid values increase, the velocity of the fluid and particle phases falls. Other significant studies on dusty fluid flow can be explored in previous researches [24-26].

The model proposed by Saffman [27] will be used to include the particle phase in this research. Most of these fluid issues are linear and were designed to tackle steady-state fluid problems. However, in this study, the magnetic field and viscous dissipation implications on the unsteady dusty vertical cone with variable wall temperature will be considered. Neglecting the variable wall temperature boundary condition results in significant deviations in the predicted heat transfer characteristics compared to experimental data. Therefore, it is important to include this boundary condition in numerical simulations for accurate predictions of convective heat transfer in such systems [28]. The non-dimensional, non-linear, coupled partial differential equations (PDE) are numerically solved using the Crank-Nicolson method in conjunction with the Thomas algorithm [29]. To obtain numerical results, an algorithm is created in MATLAB software and explained graphically and in a table.

2. Methodology

Under the impact of magnetic and viscous dissipation, an unsteady flow of an incompressible dusty viscous fluid is examined as it passes by a vertical cone with a half-angle α and radius $r(r = x \sin \alpha)$. The cone's surface is represented by the x −axis, while the cone's normal is represented by the y-axis. At rest, the cone surface and the surrounding dusty fluid temperature are assumed to be equal to T_∞. The number density throughout the flow, and the dust particles are assumed to be constant, conducting, and homogeneous in size. It is believed that the external electrical field caused by polarization is insignificant. The drag force for the fluid-dust particle interaction is taken into consideration. Figure 1 depicts a homogeneous magnetic field parallel to the y-axis as follows:

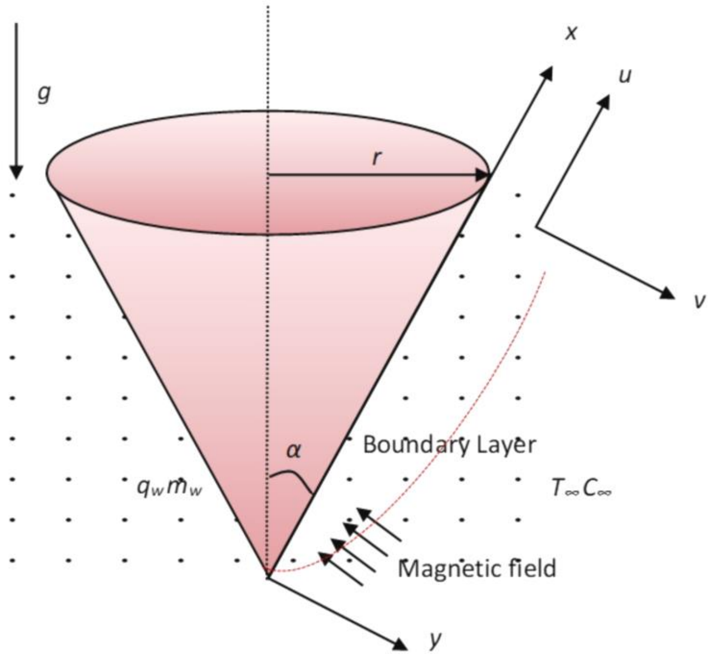

Fig. 1. Geometric coordinates system and problem layouts

The governing equations for heat transfer and dusty flow subject to the above assumptions [30]:

For fluid phase equation:

Continuity:

$$\frac{\partial}{\partial x^*}(ru^*) + \frac{\partial}{\partial y^*}(rv^*) = 0, \tag{1}$$

Momentum:

$$\rho\left(\frac{\partial u^*}{\partial t^*} + u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = \mu\frac{\partial^2 u^*}{\partial {y^*}^2} + g(\rho\beta_T)\cos\alpha\,(T^* - T_\infty^*) - \sigma B^2 u^* + \frac{\rho_p}{\tau_m}\left(u_p^* - u^*\right), \tag{2}$$

Energy:

$$\rho C_p\left(\frac{\partial T^*}{\partial t^*} + u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*}\right) = k\frac{\partial^2 T^*}{\partial {y^*}^2} + \mu\left(\frac{\partial u^*}{\partial y^*}\right)^2 + \frac{\rho_p C_s}{\tau_T}\left(T_p - T^*\right), \tag{3}$$

For particle phase equation:

Continuity:

$$\frac{\partial}{\partial x^*}\left(ru_p^*\right) + \frac{\partial}{\partial y^*}\left(rv_p^*\right) = 0, \tag{4}$$

Momentum:

$$\rho_p\left(\frac{\partial u_p^*}{\partial t^*} + u_p^*\frac{\partial {u^*_p}_p}{\partial x^*} + v_p^*\frac{\partial u_p^*}{\partial y^*}\right) = -\frac{\rho_p}{\tau_m}\left(u_p^* - u^*\right), \tag{5}$$

Energy:

$$\rho_p C_s\left(\frac{\partial T_p^*}{\partial t^*} + u_p^*\frac{\partial T_p^*}{\partial x^*} + v_p^*\frac{\partial T_p^*}{\partial y^*}\right) = -\frac{\rho_p C_s}{\tau_T}\left(T_p^* - T^*\right), \tag{6}$$

Given the dimensional initial boundary conditions:

$$\begin{aligned}
&u^*(x^*,y^*,0) = 0, u_p^*(x^*,y^*,0) = 0, v^*(x^*,y^*,0) = 0, v_p^*(x^*,y^*,0) = 0,\\
&T^*(x^*,y^*,0) = T_\infty^*, T_p^*(x^*,y^*,0) = T_\infty^*,\\
&u^*(x^*,0,t^*) = 0, u_p^*(x^*,0,t^*) = 0, v^*(x^*,0,t^*) = 0, v_p^*(x^*,0,t^*) = 0,\\
&T^*(x^*,0,t^*) = T_\infty^* + a{x^*}^m, T_p^*(x^*,0,t^*) = T_\infty^* + a{x^*}^m,\\
&u^*(0,y^*,t^*) = 0, u_p^*(0,y^*,t^*) = 0, T^*(0,y^*,t^*) = T_\infty^*, T_p^*(0,y^*,t^*) = T_\infty^*,\\
&u^*(x^*,y^*,t^*) \to 0, u_p^*(x^*,y^*,t^*) \to 0, T^*(x^*,y^*,t^*) \to T_\infty^*, T_p^*(x^*,y^*,t^*) \to T_\infty^*, \text{ as } y^* \to \infty.
\end{aligned} \tag{7}$$

Following are the corresponding physical values for local shear stress and local Nusselt number:

$$\tau_x = \mu\left(-\frac{\partial u^*}{\partial y^*}\right)_{y^*=0}, \tag{8}$$

$$Nu_x = \frac{-x^* k \left(\frac{\partial T^*}{\partial y^*}\right)_{y^*=0}}{k(T_w^* - T_\infty^*,)}. \tag{9}$$

By using dimensionless parameter [31]:

$$x = \frac{x^*}{L}, y = \frac{y^*}{L}(Gr_L)^{\frac{1}{4}}, R = \frac{r}{L}, t = \frac{\nu_f t^*}{L^2}(Gr_L)^{\frac{1}{2}}, (v, v_p) = \frac{L}{\nu_f}(Gr_L)^{-\frac{1}{4}}(v^*, v_p^*),$$
$$(u, u_p) = \frac{L}{\nu_f}(Gr_L)^{-\frac{1}{2}}(u^*, u_p^*),\ (T, T_p) = \frac{(T^*, T_p^*) - T_\infty^*}{T_w^* - T_\infty^*}, Gr_L = \frac{g\beta_T (T_w^* - T_\infty^*) L^3}{\nu_f^2}. \tag{10}$$

Eq. (1) to Eq. (6) are then simplified to the dimensionless form:

For fluid phase equation:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{u}{x} = 0, \tag{11}$$

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + T\cos\alpha - Mu + D_p \alpha_d (u_p - u), \tag{12}$$

Energy:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} + \varepsilon\left(\frac{\partial u}{\partial y}\right)^2 + \frac{2}{3Pr} D_p \alpha_d (T_p - T), \tag{13}$$

For particle phase equation:

Continuity:

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} + \frac{u_p}{x} = 0, \tag{14}$$

Momentum:

$$\frac{\partial u_p}{\partial t} + u_p\frac{\partial u_p}{\partial x} + v_p\frac{\partial u_p}{\partial y} = -\alpha_d (u_p - u), \tag{15}$$

Energy:

$$\frac{\partial T_p}{\partial t} + u_p\frac{\partial T_p}{\partial X} + v_p\frac{\partial T_p}{\partial y} = -\frac{2}{3\gamma Pr}\alpha_d (T_p - T), \tag{16}$$

where, $Pr = \frac{\nu_f}{\alpha}$, $\gamma = \frac{C_s}{C_p}$, $D_p = \frac{\rho_p}{\rho}$, $\tau_T = \frac{3}{2}\gamma\tau_m\, Pr$, $\alpha_d = \frac{L^2}{\nu_f \tau_m}(Gr_L)^{-\frac{1}{2}}$, $M = \frac{\sigma B^2 L^2}{\mu}(Gr_L)^{-\frac{1}{2}}$

and $\varepsilon = \frac{g\beta_T L}{C_p}$.

The dimensionless initial and boundary conditions:

$$\begin{aligned}
&u(x,y,0) = 0, u_p(x,y,0) = 0, v(x,y,0) = 0, v_p(x,y,0) = 0,\\
&T(x,y,0) = T_\infty, T_p(x,y,0) = T_\infty,\\
&u(x,0,t) = 0, u_p(x,0,t) = 0, v(x,0,t) = 0, v_p(x,0,t) = 0,\\
&T(x,0,t) = x^m, T_p(x,0,t) = x^m,\\
&u(0,y,t) = 0, u_p(0,y,t) = 0, T(0,y,t) = 0, T_p(0,y,t) = 0,\\
&u(x,y,t) \to 0, u_p(x,y,t) \to 0, T(x,y,t) \to 0, T_p(x,y,t) \to 0, \text{ as } y \to \infty.
\end{aligned} \tag{17}$$

The dimensionless local shear stress and local Nusselt number:

$$\tau_x = Gr_L^{\frac{3}{4}} \left(\frac{\partial u}{\partial y}\right)_{y=0}, \tag{18}$$

$$Nu_x = Gr_L^{\frac{1}{4}} \frac{x}{T_{y=0}} \left(-\frac{\partial T}{\partial y}\right)_{y=0}. \tag{19}$$

2.1 The Implicit Crank Nicolson Method

The Crank Nicolson method is an implicit finite difference scheme which is not only unconditionally stable, but also offers promising characteristics such as quick convergence, precision, and reliability [32,33]. To specify difficulties in programming, the equations must be converted from continuous to discrete ones. It is converted to a tridiagonal system when the dimensionless equations are discretized and iterations i and j are carried out. Analogous to Eq. (11) to Eq. (16), following are the finite difference equation [34]:

For fluid phase equation:

Continuity:

$$\frac{u_{i,j}^{k+1} - u_{i-1,j}^{k+1} + u_{i,j}^{k} - u_{i-1,j}^{k}}{2\Delta x} + \frac{v_{i,j}^{k+1} - v_{i,j-1}^{k+1} + v_{i,j}^{k} - v_{i,j-1}^{k}}{2\Delta y} + \frac{1}{x_i}\frac{u_{i,j}^{k+1} + u_{i,j}^{k}}{2} = 0, \tag{20}$$

Momentum:

$$\begin{aligned}
&\left(\frac{u_{i,j}^{k+1} - u_{i,j}^{k}}{\Delta t}\right) + u_{i,j}^{k}\left(\frac{u_{i,j}^{k+1} - u_{i-1,j}^{k+1} + u_{i,j}^{k} - u_{i-1,j}^{k}}{2\Delta x}\right) + v_{i,j}^{k}\left(\frac{u_{i,j+1}^{k+1} - u_{i,j-1}^{k+1} + u_{i,j+1}^{k} - u_{i,j-1}^{k}}{4\Delta y}\right)\\
&= \frac{u_{i,j+1}^{k+1} - 2u_{i,j}^{k+1} + u_{i,j-1}^{k+1} + u_{i,j+1}^{k} - 2u_{i,j}^{k} + u_{i,j-1}^{k}}{2(\Delta y)^2} + \frac{T_{i,j}^{k+1} + T_{i,j}^{k}}{2}\cos\alpha - M\frac{u_{i,j}^{k+1} + u_{i,j}^{k}}{2}\\
&+ D_p\alpha_d\left(\frac{{u_p}_{i,j}^{k+1} + {u_p}_{i,j}^{k}}{2} - \frac{u_{i,j}^{k+1} + u_{i,j}^{k}}{2}\right),
\end{aligned} \tag{21}$$

Energy:

$$\left(\frac{T_{i,j}^{k+1}-T_{i,j}^{k}}{\Delta t}\right)+u_{i,j}^{k}\left(\frac{T_{i,j}^{k+1}-T_{i-1,j}^{k+1}+T_{i,j}^{k}-T_{i-1,j}^{k}}{2\Delta x}\right)+v_{i,j}^{k}\left(\frac{T_{i,j+1}^{k+1}-T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-T_{i,j-1}^{k}}{4\Delta y}\right)$$
$$=\frac{1}{Pr}\left(\frac{T_{i,j+1}^{k+1}-2T_{i,j}^{k+1}+T_{i,j-1}^{k+1}+T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{2(\Delta y)^2}\right)+\varepsilon\left(\frac{u_{i,j+1}^{k+1}-u_{i,j-1}^{k+1}+u_{i,j+1}^{k}-u_{i,j-1}^{k}}{4\Delta y}\right)^2 \quad (22)$$
$$+\frac{2}{3Pr}D_p\alpha_d\left(\frac{T_{p_{i,j}}^{k+1}+T_{p_{i,j}}^{k}}{2}-\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}\right),$$

For particle phase equation:

Equation on continuity:

$$\frac{u_{p_{i,j}}^{k+1}-u_{p_{i-1,j}}^{k+1}+u_{p_{i,j}}^{k}-u_{p_{i-1,j}}^{k}}{2\Delta x}+\frac{v_{p_{i,j}}^{k+1}-v_{p_{i,j-1}}^{k+1}+v_{p_{i,j}}^{k}-v_{p_{i,j-1}}^{k}}{2\Delta y}+\frac{1}{x_i}\frac{u_{p_{i,j}}^{k+1}+u_{p_{i,j}}^{k}}{2}=0, \quad (23)$$

Equation on momentum:

$$\left(\frac{u_{p_{i,j}}^{k+1}-u_{p_{i,j}}^{k}}{\Delta t}\right)+u_{p_{i,j}}^{k}\left(\frac{u_{p_{i,j}}^{k+1}-u_{p_{i-1,j}}^{k+1}+u_{p_{i,j}}^{k}-u_{p_{i-1,j}}^{k}}{2\Delta x}\right)+v_{p_{i,j}}^{k}\left(\frac{u_{p_{i,j+1}}^{k+1}-u_{p_{i,j-1}}^{k+1}+u_{p_{i,j+1}}^{k}-u_{p_{i,j-1}}^{k}}{4\Delta y}\right)=$$
$$-\alpha_d\left(\frac{u_{p_{i,j}}^{k+1}+u_{p_{i,j}}^{k}}{2}-\frac{u_{i,j}^{k+1}+u_{i,j}^{k}}{2}\right), \quad (24)$$

Equation on energy:

$$\left(\frac{T_{p_{i,j}}^{k+1}-T_{p_{i,j}}^{k}}{\Delta t}\right)+u_{p_{i,j}}^{k}\left(\frac{T_{p_{i,j}}^{k+1}-T_{p_{i-1,j}}^{k+1}+T_{p_{i,j}}^{k}-T_{p_{i-1,j}}^{k}}{2\Delta x}\right)+v_{p_{i,j}}^{k}\left(\frac{T_{p_{i,j+1}}^{k+1}-T_{p_{i,j-1}}^{k+1}+T_{p_{i,j+1}}^{k}-T_{p_{i,j-1}}^{k}}{4\Delta y}\right)=$$
$$-\frac{2}{3\gamma Pr}\alpha_d\left(\frac{T_{p_{i,j}}^{k+1}+T_{p_{i,j}}^{k}}{2}-\frac{T_{i,j}^{k+1}+T_{i,j}^{k}}{2}\right) \quad (25)$$

3. Results

The value of y_{max} and y_{∞} are compatible, constant and the location is outside both the boundary layers of velocity and temperature. The meshing size have been set as $[\Delta y, \Delta x, \Delta t] = [0.05, 0.05, 0.01]$, with the shortness ignorance $O(\Delta y^2 + \Delta t^2 + \Delta x)$ when $\Delta y, \Delta x$ and Δt approach the value of null. Moreover, $[x_{max}, y_{max}] = [1,10]$ is the integral area. It is feasible to deduce from the computations, predictions, and calculations made before that a complex and systematic plan of action reveals that a solution may appear and function in agreement as explained in previous studies [35-37]. Lastly, the numerical algorithms are developed using MATLAB to find the solution and results will be presented in a form of graph. The modifying parameters outcome on the distribution are explored. The results of numerical assessment for $M, \varepsilon, Pr, \alpha_d$ D_p and γ are visually depicted. Except where noted, the numerical calculations are performed using the following fixed parameters: $t = 10, m = 0.5, \alpha = 20^o, M = 2, \varepsilon = 0.1, Pr = 0.71, \alpha_d = 0.5, D_p = 10$ and $\gamma = 0.1$ [38,39].

The consequences of Pr on temperature and velocity distribution are shown in Figure 2, respectively. This plot shows that both velocities and temperature diminish as Pr climbs. A lower Pr has a higher thermal diffusivity, which improves conduction and the thermal boundary layer thickness while hindering the heat transfer rate. Figure 3 depicts how M in the fluid and particle phases altered the velocity and temperature distributions. According to this graph, the velocity of fluid and particle phase fall while both temperatures rise as M rises.

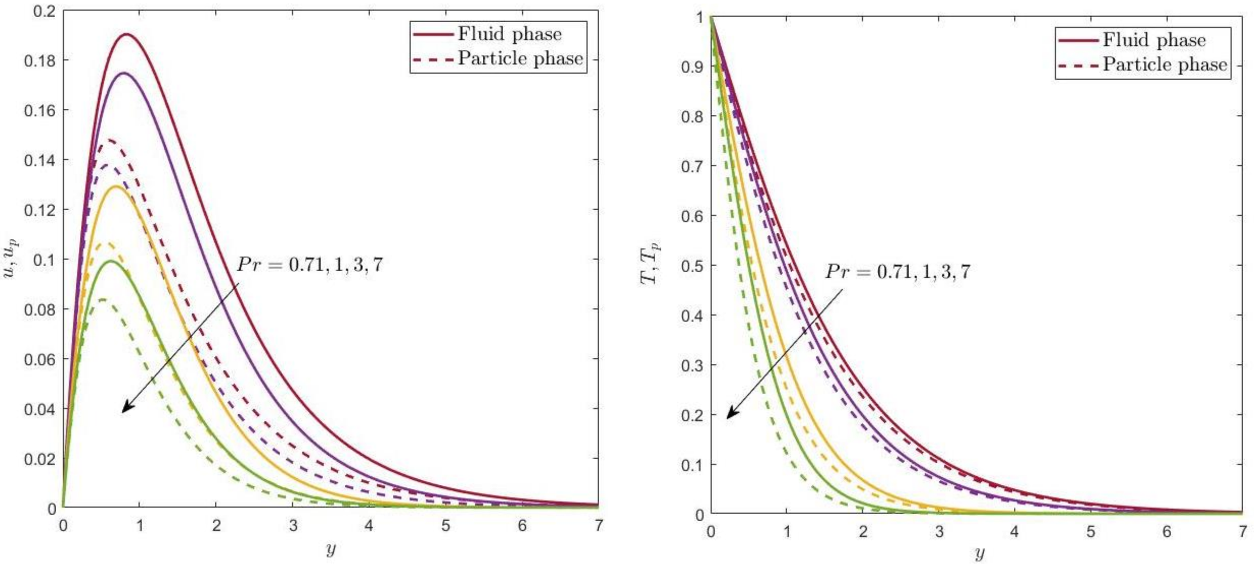

Fig. 2. Impact of Prandtl number on the profiles of velocity and temperature

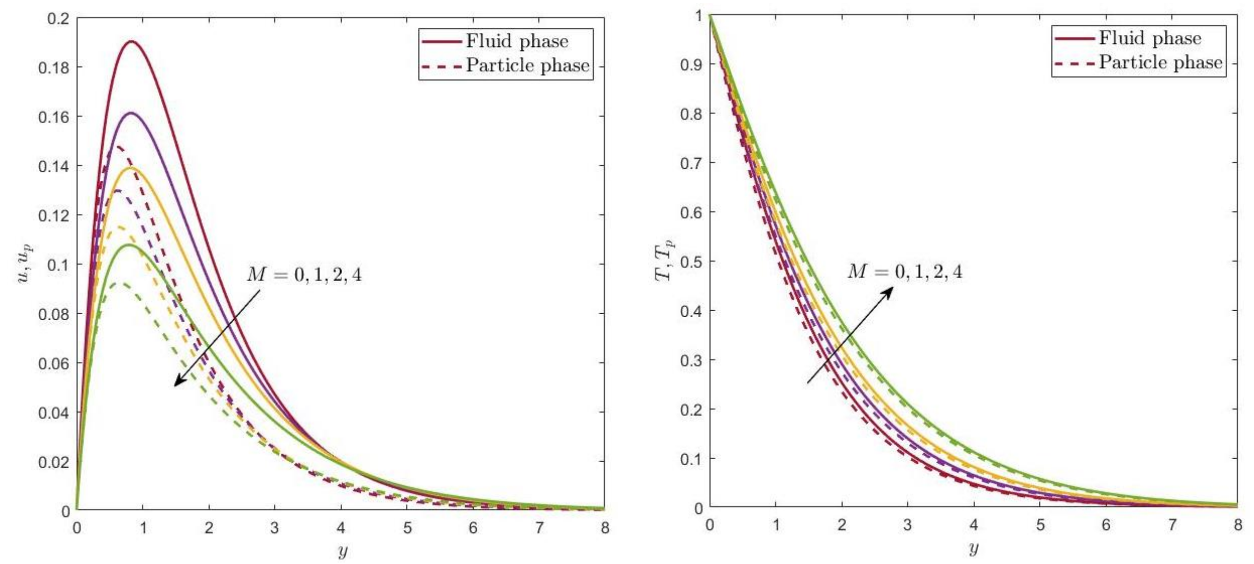

Fig. 3. Impact of magnetic field on the profiles of velocity and temperature

Figure 4 shows the ε impacts on temperature and velocity distribution. Both phases velocity and temperature are noted to enhance as ε rises. The temperature profile rises because of the energy source that is the temperature distribution on fluid flow caused by viscous dissipation. The velocity and temperature profile rises as a result of the energy source caused by viscous dissipation. Magnetic and viscous dissipations are considered negligible when the magnetic field and viscosity effects are relatively weak. The magnetic and viscous forces are proportional to the magnetic permeability and viscosity, respectively. When the magnetic permeability or viscosity is low, the magnetic and viscous forces become weak, and the resulting dissipations become negligible. Figure 5 depicts the impacts of α_d on velocity and temperature distribution. The rise in α_d enhances the velocity and temperature of particle phase but reduces velocity and temperature of fluid phase. The temperature distribution lowers because the particle phase generates a drag force in contrast to the fluid phase, where there is high connection between the fluid and particle phases.

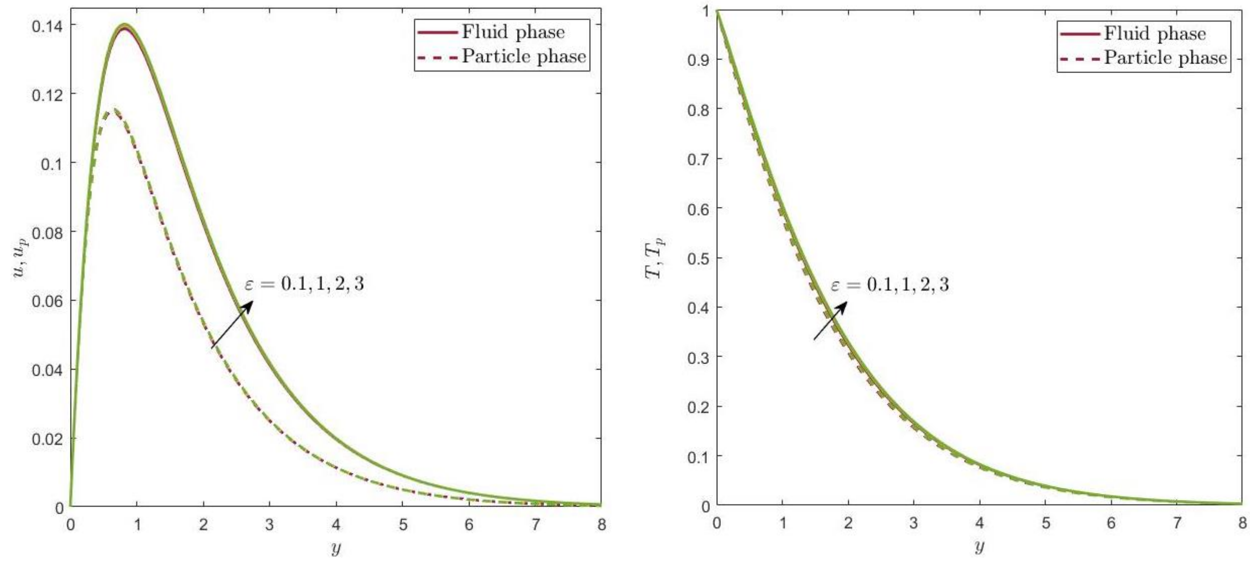

Fig. 4. Impact of viscous dissipation on the profiles of velocity and temperature

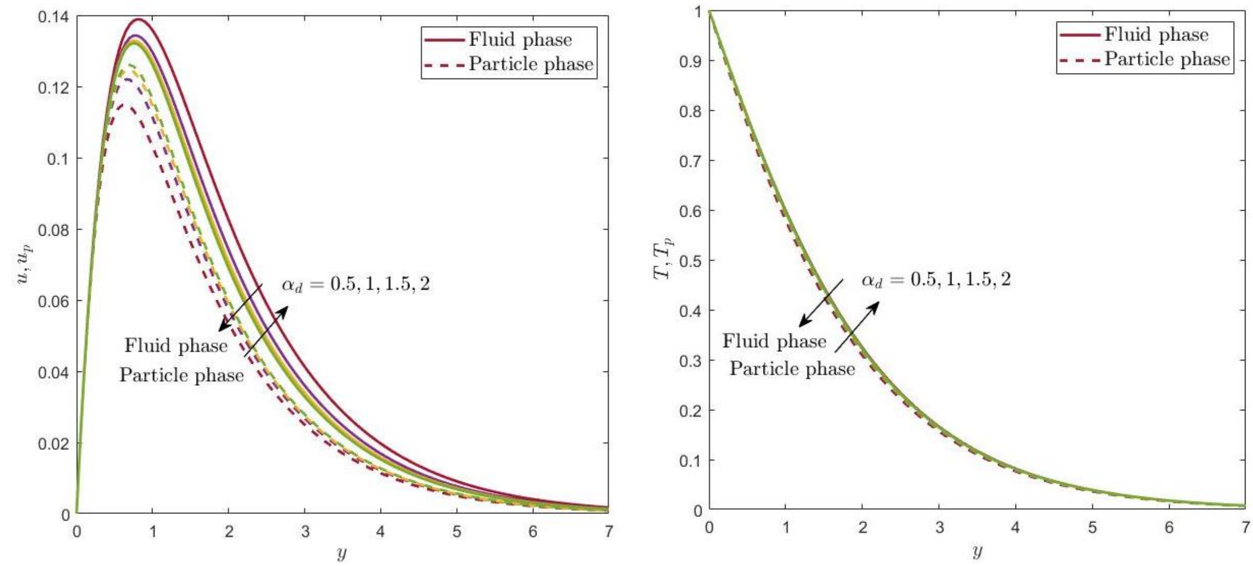

Fig. 5. Impact of fluid-particle interaction on the profiles of velocity and temperature

Figure 6 illustrates how D_p affects the velocity and temperature distribution of both phases. It is noted that the temperature and velocity distribution for both phases are dropping as D_p values rise. This happens since the presence of particles makes the fluid internal friction rise, which slows the fluid flow. Intriguing to notice is also that the flow system is set up so that the value $D_p = 0$ represents a typical viscous fluid. Local $Nu_x/Gr_L^{1/4}$ and $\tau_x/Gr_L^{3/4}$ affected by $Pr, M, \varepsilon, \alpha_d$ and D_p are depicted in Table 1. It is obvious that as the number of Pr, D_p and α_d rises, so does the local Nu_x. Meanwhile, as the number of ε and M enhances, the local Nu_x drops. Furthermore, the value of local τ_x goes down as the number of Pr, M, D_p and α_d rises. In the meantime, when the local τ_x value goes up, so does ε.

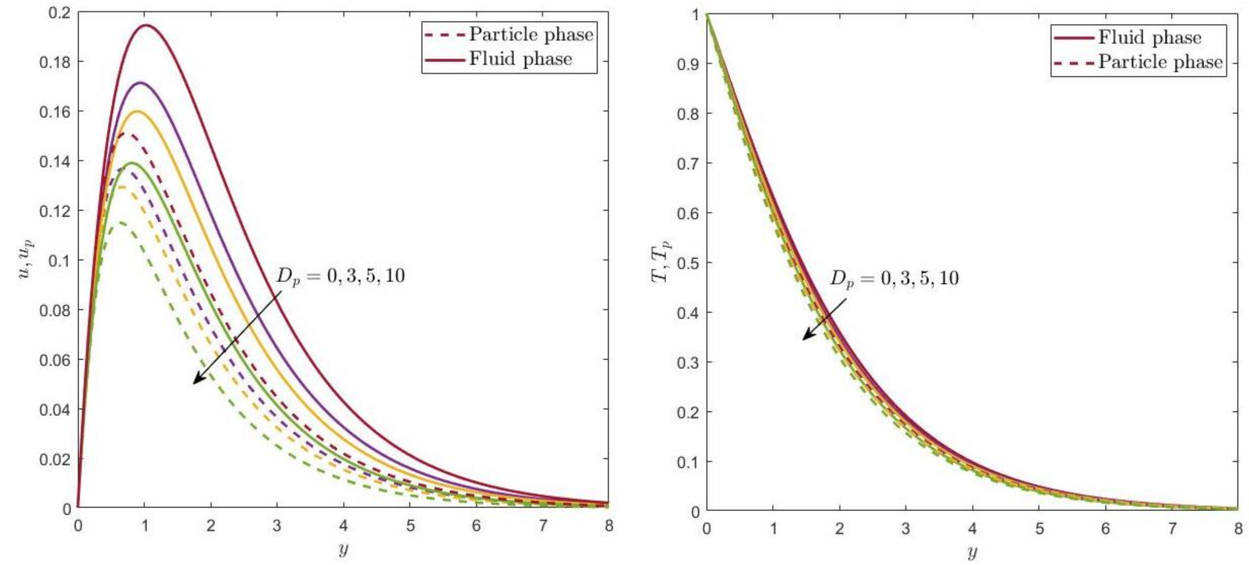

Fig. 6. Impact of mass concentration of particle phase on the profiles of velocity and temperature

Table 1
$Nu_x/Gr_L^{1/4}$ and $\tau_x/Gr_L^{3/4}$ affected by $Pr, M, \varepsilon, \alpha_d$ and D_p

γ	ε	Pr	D_p	M	α_d	$Nu_x/Gr_L^{1/4}$	$\tau_x/Gr_L^{3/4}$
0.1	1	0.71	10	2	0.5	0.4474	0.4197
	2					0.4418	0.4200
	3					0.4381	0.4202
	0.1					0.4456	0.4198
		1				0.5114	0.4042
		3				0.7757	0.3497
		7				1.0591	0.3048
		0.71	0			0.3964	0.4835
			3			0.4130	0.4596
			5			0.4234	0.4465
			10	0		0.5191	0.5303
				1		0.4786	0.4677
				4		0.3968	0.3518
				2	1	0.4479	0.4171
					1.5	0.4485	0.4161
					2	0.4488	0.4156

4. Conclusions

The problem of free convective dusty flow with magnetic field and dissipation impacts across a non-isothermal vertical cone has been explored and taken into consideration. This interaction can help the fluid flow more effectively and uniformly throughout the surface of the cone. The MHD effect can also produce electrical currents inside the fluid, which can be useful in a variety of areas. For example, MHD generators may transform fluid flow energy into electrical energy. This conversion process is made more efficient by using a cone geometry. Overall, fluid flow over a vertical cone with MHD effect offers practical applications in power production, space technology, and material processing. The results indicate that the fluid and particle phases velocity decreases when the magnetic field strength, mass concentration of particle phase, and fluid-particle interaction parameters are enhanced. The opposite reaction is observed for the viscous dissipation parameter. Moreover, the temperature distribution increases as the magnetic and viscous dissipation parameters are increased, while the opposite tendency is seen for the fluid-particle interaction parameter and the mass concentration of particle phase parameter. It is also observed that the fluid and particle phases velocity and temperature show a dual nature against the fluid-particle interaction parameter. Furthermore, the temperature and velocity profiles for the fluid phase consistently exceed those for the dust phase in all examined results.

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References

[1] Nayan, Asmahani, Nur Izzatie Farhana Ahmad Fauzan, Mohd Rijal Ilias, Shahida Farhan Zakaria, and Noor Hafizah Zainal Aznam. "Aligned Magnetohydrodynamics (MHD) Flow of Hybrid Nanofluid Over a Vertical Plate Through Porous Medium." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 92, no. 1 (2022): 51-64. https://doi.org/10.37934/arfmts.92.1.5164

[2] Mahat, Rahimah, Muhammad Saqib, Imran Ulah, Sharidan Shafie, and Sharena Mohamad Isa. "MHD Mixed Convection of Viscoelastic Nanofluid Flow due to Constant Heat Flux." *Journal of Advanced Research in Numerical Heat Transfer* 9, no. 1 (2022): 19-25.

[3] Reyaz, Ridhwan, Ahmad Qushairi Mohamad, Lim Yeou Jiann, Muhammad Saqib, and Sharidan Shafie. " Presence of Riga Plate on MHD Caputo Casson Fluid: An Analytical Study." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 93, no. 2 (2022): 86-99. https://doi.org/10.37934/arfmts.93.2.8699

[4] Saranya, S., and Qasem M. Al-Mdallal. "Non-Newtonian ferrofluid flow over an unsteady contracting cylinder under the influence of aligned magnetic field." *Case Studies in Thermal Engineering* 21 (2020): 100679. https://doi.org/10.1016/j.csite.2020.100679

[5] Al-Mdallal, Qasem M., Sidra Aman, Khalil Ur Rehman, Laila Y. Al Sakkaf, and S. Saranya. "Numerical Investigation of Heat Generation and Magnetohydrodynamic Flow of Fluid Over a Shrinking Infinite Long Cylinder Through Porous Medium." *Journal of Nanofluids* 11, no. 2 (2022): 285-295. https://doi.org/10.1166/jon.2022.1830

[6] Saranya, S., Qasem M. Al-Mdallal, and I. L. Animasaun. "Shifted Legendre collocation analysis of time-dependent Casson fluids and Carreau fluids conveying tiny particles and gyrotactic microorganisms: Dynamics on static and moving surfaces." *Arabian Journal for Science and Engineering* 48, no. 3 (2023): 3133-3155. https://doi.org/10.1007/s13369-022-07087-8

[7] Ali, Md Yousuf, Sk Reza-E-Rabbi, Md Mehedi Hasan Rasel, and Sarder Firoz Ahmmed. "Combined impacts of thermoelectric and radiation on hydromagnetic nanofluid flow over a nonlinear stretching sheet." *Partial Differential Equations in Applied Mathematics* 7 (2023): 100500. https://doi.org/10.1016/j.padiff.2023.100500

[8] Dar, Ajaz Ahmad. "Perturbation analysis on simultaneously occurring of Mhd fluid flow and heat transfer with temperature jump and variable viscosity." *Preprint Research Square* (2023). https://doi.org/10.21203/rs.3.rs-2537686/v1

[9] Maiti, Hiranmoy, Amir Yaseen Khan, Sabyasachi Mondal, and Samir Kumar Nandy. "Scrutinization of unsteady MHD fluid flow and entropy generation: Hybrid nanofluid model." *Journal of Computational Mathematics and Data Science* 6 (2023): 100074. https://doi.org/10.1016/j.jcmds.2023.100074

[10] Ab Raji, Nurul Hidayah, Nurul Samiha Mohd Shahabudin, Noorehan Awang, Mohd Rijal Ilias, and Siti Shuhada Ishak. "Aligned Magnetohydrodynamics Mixed Convection on Various Base Fluids with Carbon Nanotubes over an Inclined Plate." *CFD Letters* 15, no. 6 (2023): 12-25. https://doi.org/10.37934/cfdl.15.6.1225

[11] Bosli, Fazillah, Mohd Rijal Ilias, Noor Hafizah Zainal Aznam, Siti Shuhada Ishak, Shahida Farhan Zakaria, and Amirah Hazwani Abdul Rahim. "Aligned magnetohydrodynamic effect on magnetic nanoparticle with different base fluids past a moving inclined plate." *International Journal of Advanced and Applied Sciences* 10, no. 3 (2022): 96-107. https://doi.org/10.21833/ijaas.2023.03.013

[12] Bosli, Fazillah, Alia Syafiqa Suhaimi, Siti Shuhada Ishak, Mohd Rijal Ilias, Amirah Hazwani Abdul Rahim, and Anis Mardiana Ahmad. "Investigation of Nanoparticles Shape Effects on Aligned MHD Casson Nanofluid Flow and Heat Transfer with Convective Boundary Condition." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 91, no. 1 (2022): 155-171. https://doi.org/10.37934/arfmts.91.1.155171

[13] Ilias, Mohd Rijal, Noraihan Afiqah Rawi, and Sharidan Shafie. "Steady aligned MHD free convection of Ferrofluids flow over an inclined plate." *Journal of Mechanical Engineering (JMechE)* 14, no. 2 (2017): 1-15.

[14] Gopal, D., S. Saleem, S. Jagadha, Farooq Ahmad, A. Othman Almatroud, and N. Kishan. "Numerical analysis of higher

order chemical reaction on electrically MHD nanofluid under influence of viscous dissipation." *Alexandria Engineering Journal* 60, no. 1 (2021): 1861-1871. https://doi.org/10.1016/j.aej.2020.11.034

[15] Shukla, A. K., and Shubham Kumar Dube. "Numerical simulation of changes in Soret-Dufour number, Radiation, chemical reaction and viscous dissipation on unsteady MHD flow past an inclined porous plate embedded in porous medium with heat generation or absorption." *International Journal of Advances in Applied Mathematics and Mechanics* 10, no. 2 (2022): 15-26.

[16] Hanif, Hanifa, Sharidan Shafie, and Ali Chamkha. "Effect of Ohmic heating on magnetohydrodynamic flow with variable pressure gradient: a computational approach." *Waves in Random and Complex Media* (2022): 1-16. https://doi.org/10.1080/17455030.2022.2141916

[17] Ashraf, Muhammad, Asifa Ilyas, Zia Ullah, and Aamir Ali. "Combined effects of viscous dissipation and magnetohydrodynamic on periodic heat transfer along a cone embedded in porous medium." *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering* 236, no. 6 (2022): 2325-2335. https://doi.org/10.1177/09544089221089135

[18] Gamsha, Abdullahi Madaki, A. A. Hussaini, S. K. Alaramma, A. M. Musa, A. A. Tata, and A. Barde. "Electron Magnetohydrodynamic (EMHD) Nanofluid flow with triple effects of Chemical reaction, Dufour diffusivity and impermeability of the surface along a Slandering Stretching Sheet." *The Sciencetech* 4, no. 1 (2023).

[19] Sadighi, Sina, Hossein Afshar, Hossein Ahmadi Danesh Ashtiani, and Mohsen Jabbari. "MHD flow and conductive heat transfer on a permeable stretching cylinder: Benchmark solutions." *Case Studies in Thermal Engineering* 44 (2023): 102886. https://doi.org/10.1016/j.csite.2023.102886

[20] Gangadhar, Kotha, M. Rupa Lavanya, and Ali J. Chamkha. "EMHD flow of second-grade fluid through a heated permeable disk with space dependent heat source." *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering* (2023): 09544089231159203. https://doi.org/10.1177/09544089231159203

[21] Ali, Kashif, Sohail Ahmad, Muhammad Aamir, Wasim Jamshed, Amjad Ali Pasha, and Syed M. Hussain. "Application of the successive over relaxation method for analyzing the dusty flow over a surface subject to convective boundary condition." *Ain Shams Engineering Journal* 14, no. 8 (2023): 102044. https://doi.org/10.1016/j.asej.2022.102044

[22] Goh, Jun Shen, and Noorzehan Fazahiyah Bt Md Shab. "Numerical Analysis of Mhd Dusty Fluid Flow Over Stretching Sheet in A Porous Medium." *Enhanced Knowledge in Sciences and Technology* 2, no. 1 (2022): 402-412.

[23] Ali, Farhad, Gohar Ali, Arshad Khan, Ilyas Khan, Elsayed Tag Eldin, and Matin Ahmad. "Effects of Newtonian heating and heat generation on magnetohydrodynamics dusty fluid flow between two parallel plates." *Frontiers in Materials* 10 (2023): 1120963. https://doi.org/10.3389/fmats.2023.1120963

[24] Kalpana, Gajendran, and Salman Saleem. "Heat Transfer of Magnetohydrodynamic Stratified Dusty Fluid Flow through an Inclined Irregular Porous Channel." *Nanomaterials* 12, no. 19 (2022): 3309. https://doi.org/10.3390/nano12193309

[25] Vijayalakshmi, R., A. Govindarajan, and B. Thiripura Sundari. "Impact of Hall Current and Rotation on Unsteady MHD Oscillatory Dusty Fluid Flow Through Horizontal Porous Plates." In *Advances in Mathematical Modelling, Applied Analysis and Computation: Proceedings of ICMMAAC 2021*, pp. 527-547. Singapore: Springer Nature Singapore, 2022. https://doi.org/10.1007/978-981-19-0179-9_31

[26] Abbas, Wael, Osama Khaled, Salah Beshir, Mostafa Abdeen, and Mohamed Elshabrawy. "Analysis of chemical, ion slip, and thermal radiation effects on an unsteady magnetohydrodynamic dusty fluid flow with heat and mass transfer through a porous media between parallel plates." *Bulletin of the National Research Centre* 47, no. 1 (2023): 1-10. https://doi.org/10.1186/s42269-023-01024-x

[27] Saffman, P. G. "On the stability of laminar flow of a dusty gas." *Journal of Fluid Mechanics* 13, no. 1 (1962): 120-128. https://doi.org/10.1017/S0022112062000555

[28] Nabwey, Hossam A., and A. Mahdy. "Numerical approach of micropolar dust-particles natural convection fluid flow due to a permeable cone with nonlinear temperature." *Alexandria Engineering Journal* 60, no. 1 (2021): 1739-1749. https://doi.org/10.1016/j.aej.2020.11.023

[29] Hanif, Hanifa, Ilyas Khan, and Sharidan Shafie. "Heat transfer exaggeration and entropy analysis in magneto-hybrid nanofluid flow over a vertical cone: a numerical study." *Journal of Thermal Analysis and Calorimetry* 141, no. 5 (2020): 2001-2017. https://doi.org/10.1007/s10973-020-09256-z

[30] Sambath, P. "A Study on Unsteady Natural Convective Flow past a Vertical Cone with Heat and Mass Transfer Effects." *PhD diss., SRM University*, 2017.

[31] Blottner, F. G. "Finite difference methods of solution of the boundary-layer equations." *AIAA Journal* 8, no. 2 (1970): 193-205. https://doi.org/10.2514/3.5642

[32] Luskin, Mitchell, Rolf Rannacher, and Wolfgang Wendland. "On the smoothing property of the Crank-Nicolson scheme." *Applicable Analysis* 14, no. 2 (1982): 117-135. https://doi.org/10.1080/00036818208839415

[33] Sun, Guilin, and Christopher W. Trueman. "Efficient implementations of the Crank-Nicolson scheme for the finite-

difference time-domain method." *IEEE Transactions on Microwave Theory and Techniques* 54, no. 5 (2006): 2275-2284. https://doi.org/10.1109/TMTT.2006.873639

[34] Anderson, Dale, John C. Tannehill, and Richard H. Pletcher. *Computational fluid mechanics and heat transfer*. CRC Press, 2012.

[35] Pullepu, Bapuji, E. Ekambavanan, and A. J. Chamkha. "Unsteady laminar natural convection from a non-isothermal vertical cone." *Nonlinear Analysis: Modelling and Control* 12, no. 4 (2007): 525-540. https://doi.org/10.15388/NA.2007.12.4.14684

[36] Pullepu, Bapuji, Ali J. Chamkha, and I. Pop. "Unsteady laminar free convection flow past a non-isothermal vertical cone in the presence of a magnetic field." *Chemical Engineering Communications* 199, no. 3 (2012): 354-367. https://doi.org/10.1080/00986445.2011.592443

[37] Thandapani, E., A. R. Ragavan, and G. Palani. "Finite-difference solution of unsteady natural convection flow past a nonisothermal vertical cone under the influence of a magnetic field and thermal radiation." *Journal of Applied Mechanics and Technical Physics* 53 (2012): 408-421. https://doi.org/10.1134/S0021894412030133

[38] Janke, V. Ramana Reddy, Sandeep Naramgari, and Sugunamma Vangala. "MHD flow of a nanofluid embedded with dust particles due to cone with volume fraction of dust and nano particles." *Procedia Engineering* 127 (2015): 1026-1033. https://doi.org/10.1016/j.proeng.2015.11.452

[39] Nabwey, Hossam A., and A. Mahdy. "Numerical approach of micropolar dust-particles natural convection fluid flow due to a permeable cone with nonlinear temperature." *Alexandria Engineering Journal* 60, no. 1 (2021): 1739-1749. https://doi.org/10.1016/j.aej.2020.11.023