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# Ternary Hybrid Nanofluids Containing Gyrotactic Microorganisms with Magnetohydrodynamics Effect over a Shrinking/Stretching of the Horizontal Plate

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### ABSTRACT

Nowadays, ternary hybrid nanofluids have been discovered by the researchers to play an important role in heat transfer systems. This study focuses on the study of heat and mass transfer of ternary hybrid nanofluids over a shrinking/stretching horizontal plate induced by gyrotactic microorganisms with the effects of magnetohydrodynamics (MHD). Motile microorganisms were added to a suspension of nanoparticles in a base fluid and mixed in most microsystems to enhance the convective properties of the nanofluids. This research uses water as the base fluid to investigate the effects of silver (Ag), Aluminium Oxide ( $Al_2O_3$ ) and Copper (Cu) nanoparticles. The similarity transformation was used to reduce the partial differential governing equations into ordinary differential equations. The transformed nonlinear ordinary differential equations with appropriate transformed boundary conditions were solved numerically using the bvp4c procedure in the MATLAB software. The combination of the Tiwari and Das with Buongiorno model used in this mathematical equation. The results of the research are illustrated graphically and presented via tables to show the behaviour of velocity, temperature, concentration of nanoparticles, and microorganism density as well as the coefficients of skin friction, Nusselt number, Sherwood numbers, and motile microorganism density for the nanoparticles of the THNF. The results show that, for the shrinking case, the magnetic field parameter increases for skin friction and Sherwood number, while it decreases for the Nusselt number and density of motile microorganisms. The Brownian motion parameter and bioconvection Péclet number are shown to significantly increase the Sherwood Number and density motile microorganisms. Thermophoresis, however, has been proven to lower the Sherwood number and density of motile microorganisms. Then, THNF (water/Ag- $Al_2O_3$ -Cu) is shown to better compared to nanofluids (water/ $Al_2O_3$ ) and hybrid nanofluids (water/Ag- $Al_2O_3$ ) in both cases.

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## 1. Introduction

In recent times, researchers have extensively studied various aspects of bioconvective problems in dilute suspensions of solid particles (nanoparticles) with motile gyrotactic microorganisms. The movement of microorganisms is self-driven, giving rise to bioconvection. Unlike microorganisms, nanoparticles motion is not self-driven and their motion is due to Brownian motion and thermophoresis effects. Brownian diffusion, depending on the concentration gradient, and thermophoresis, depending on temperature gradient, can be the dominant physical mechanisms. The principle of nanofluids bioconvection describes the spontaneous pattern forming and density stratification induced by the simultaneous interaction of denser self-propelled microorganisms, nanoparticles, and buoyancy powers. Motile microorganisms are added to a suspension of nanoparticles in a base fluid and mixed in most microsystems to enhance the convective properties of the nanofluid. The interaction of motile microorganisms and nanoparticles along with buoyancy forces will produce nanofluid bioconvection. Generally, single cell microorganisms (algae, bacteria etc.) are involved in bioconvection flow. The swimming direction of motile microorganisms is determined by the balance between two torques that occurs due to the viscous drag force that is created by the shear flow and gravity acting on the cell at the time when these microorganisms are in a flow field [1]. Motile microorganisms are added with the dilute suspension of nanoparticles in the base fluid to enhance mass transfer, microscale mixing, to improve nanofluids stability, and to prevent nanoparticle agglomeration in nanofluids [2]. It has been discovered that the nanofluids and hybrid nanofluids have a substantial influence on fluid velocity, temperature, skin friction, and heat transfer rates under different models and situations [3-6].

A bioconvection phenomenon has many applications in modelling oil and microbial enhanced oil recovery. Hill and Bees [7] analysed the theory of generalized Taylor dispersion for suspensions of Brownian particles to investigate the dispersion of gyrotactic microorganisms in a linear shear flow. They concluded that the dispersion occurs even when Brownian translational motion is negligible because of the size of the cells. Kuznetsov and Avramenko [8] mentioned the pharmaceutical application of nanofluid bioconvection by enhancing the mixing and slowing down the settling of the nanoparticles. They also analysed that average number density of small particles increases with an increment of critical Rayleigh number and they concluded that in the presence of a small particle, the suspension becomes more stable. Hill and Pedley [9] reviewed previous work on bioconvection, focusing on the mechanism of up swimming motion of different kinds of microorganisms and new theoretical and experimental developments, including nonlinear analysis of the patterns, and dispersion in shear flows. Kuznetsov and Geng [10] distinguished between the studies of Kuznetsov and Geng [11] and Kuznetsov [12], analysing a suspension of gyrotactic microorganisms and continuing the study on the stability of nanofluid containing oxytactic microorganisms. It was found that the destabilizing effect of microorganisms is larger if their concentration in the nanofluid suspension is higher. Bég *et al.*, [13] investigated the non-Newtonian power-law nanofluid containing oxytactic microorganisms over an isothermal horizontal flat plate. It was found that the bioconvection parameters have strong effects on the flow, heat and mass transfer, and motile microorganism numbers.

The incompressible flow of viscous magnetic fluid across a boundary layer is important in numerous industrial manufacturing processes. Magnetohydrodynamics has a lot of applications in space sciences, generators, various machines, drugs etc [14,15]. The influence of the MHD with stretching or shrinking sheet has developed quickly by considering the different problems in the bioconvection flow. Uddin *et al.*, [16] studied the two-dimensional boundary layer slip flow of Newtonian bio nanofluids flow over a stretching/shrinking sheet magnetic field. They found that the

effect of the Lewis number on the temperature distribution is minimal, and the local concentration of nanoparticles decreases as the Lewis number increases. Meanwhile, for fixed Prandtl number and Lewis number, the reduced Nusselt number decreases, but the reduced Sherwood number increases as the Brownian motion and thermophoresis effects become stronger. Akbar and Khan [17] analysed the effect of magnetic field in a suspension of gyrotactic microorganisms and nanoparticles past a stretching surface. They found that the cell swimming speed increases by increasing the values of the Peclet number, which reduces the density of motile microorganisms. Aman *et al.*, [18] investigated the mixed convection flow of a nanofluids combined with gyrotactic microorganisms over a stretching/shrinking sheet under the influence of magnetic field. They found that magnetic field decreases all the parameter, where it affects the velocity profiles significantly. Additionally, the skin friction coefficient and the local Nusselt number increase, while the local Sherwood number and the local density of the motile microorganisms decrease for the shrinking case. For the stretching case, the skin friction coefficient and the local Nusselt number decrease, while the local Sherwood number and the local density of the motile microorganisms increase. Zuhra *et al.*, [19] studied the flow of a second-grade nanofluids containing nanoparticles and gyrotactic with influence of magnetic field and found that the motile microorganism density function enhances with an increase in momentum slip. Additionally, the velocity depreciates with the bioconvection Rayleigh number and magnetic field parameter, while it elevates with the second-grade nanofluids parameter, buoyancy parameter and buoyancy ratio parameter. Amirsom *et al.*, [20] investigated the three-dimensional flow of bioconvection nanofluids containing gyrotactic micro-organisms over a bi-axial stretching sheet. It was shown in their study that an increase in Brownian motion parameter corresponds to a stronger thermophoretic force which encourages the transport of nanoparticles from the hot bi-axial sheet to the quiescent fluid. The fluid temperature and thermal boundary layer thickness decrease with an increasing stretching rate ratio of the bi-axial sheet, while the nanoparticle volume fraction boundary layer increases. Batool and Ashraf [21] analysed the flow with gyrotactic microorganisms due to a stretching sheet with presence of MHD. It shows that the applied magnetic field tends to rise the shear stress while reducing the rates of heat transfer, nanoparticle volume fraction, and density of microorganisms. Alharbi *et al.*, [22] investigated a hybrid nanofluid flow over a horizontal porous stretching sheet along with an induced magnetic field with comprised of gyrotactic microorganisms. The result shows that increasing the value of the magnetic field parameter decreases the velocity profiles. The fluid moves through the exponential stretching sheet of the bioconvection in the presence of a magnetic field as studied by Asjad *et al.*, [23] and it was found that the velocity profile and temperature increase with the magnetic parameter. Soid *et al.*, [24] studied the gyrotactic microorganisms in the presence of magnetic field, which contained Copper (Cu) and Aluminium Oxide ( $\text{Al}_2\text{O}_3$ ) immersed in water. It was found that the local density motile microorganism bioconvection Peclet number and bioconvection Lewis number increased. Studies on MHD effect were conducted in different situations by some researchers and they found that with the effect of magnetic field increases the heat transfer as well as the velocity of fluid [25-34]. Other researchers who studied the effect of magnetic field were discussed in Bosli *et al.*, [34], Wale *et al.*, [35], Akindele and Ogunsola [36], Ishak *et al.*, [37], Zukri *et al.*, [38], and Ab Raji *et al.*, [39].

As an observation, the literature has mentioned the issue of the problem involving nanofluids or hybrid nanofluids containing microorganisms for enhancing the thermal conductivity of the base fluid. This can be helpful in the thermal management of industrial and technological processes, as well as for heat exchangers and in microelectronics. To ensure that the research is more helpful, the THNF is developed. A new theoretical THNF model was developed by Manjunatha *et al.*, [40] that defines mathematically the properties such as density, viscosity, thermal conductivity, electrical conductivity, and specific heat capacitance. The nanoparticles  $\text{TiO}_2$ ,  $\text{Al}_2\text{O}_3$  and  $\text{SiO}_2$  are suspended

into water used and it was found that the ternary hybrid nanofluids has better thermal conductivity than the hybrid nanofluids. The bioconvection of a ternary hybrid nanofluids ( $\text{Al}_2\text{O}_3\text{-Cu-CNT/water}$ ) flow containing motile gyrotactic microorganisms was studied by Yaseen *et al.*, [41] and fit was found that an application of suction reduces the thickness of the momentum boundary layer. Additionally, the microorganism concentration profile decreases as a bioconvection Lewis number increases. Algehyne *et al.*, [42] analysed mass and energy transmission through trihybrid nanofluids flow across a stretched permeable surface using Buongiorno model and without containing a microorganism and concluded that the trihybrid nanofluids has significantly boosted the energy propagation rate compared to simple and hybrid nanofluids. Alanazi *et al.*, [43] found that with the influence of the magnetic field, fluid velocity decreased in opposition to the increasing magnetic parameter. However, fluid temperature shows an increasing trend in their study to improve the transmission of thermal energy above a stretching inclined surface over an upper surface. The effect of magnetic field containing gyrotactic microorganisms and bioconvection problems can be seen in many research studies, such as those done by Ali *et al.*, [44], Ali *et al.*, [45], Waqas *et al.*, [46], and Habib *et al.*, [47]. It has been found that magnetic field can be utilized as a good controller of the nanofluids flow field incorporating bioconvection, Brownian motion and motile microorganisms.

In view of the above, the present study is initiated based on past investigations in the literature. The study of nanofluids combined with gyrotactic microorganisms over a stretching/shrinking sheet under the influence of magnetic field by Aman *et al.*, [18] revealed that the volume fraction of nanoparticles is not considered. Therefore, the aim of the present study is to analyse the behaviour of some governing parameters on THNF containing gyrotactic microorganisms on stretching/shrinking sheet in the presence of magnetic field with considering the volume fraction of nanoparticles. The interaction of nanoparticles, concentration, and microorganisms with the magnetic field will be presented in this study. Since the study involves the volume fraction of nanoparticles in THNF, the mathematical model used is a combination of the Tiwari and Das model as well as the Buongiorno model [48,49].

## 2. Mathematical Formulation

Consider a two-dimensional incompressible and steady state flow past a horizontal plate filled with nanoparticles containing gyrotactic microorganisms. The presence of nanoparticles was assumed to have no effect on the direction in which microorganisms, the THNF is in thermal equilibrium and no slip conditions exist. The physical model and coordinate system of this problem are shown in Figure 1.

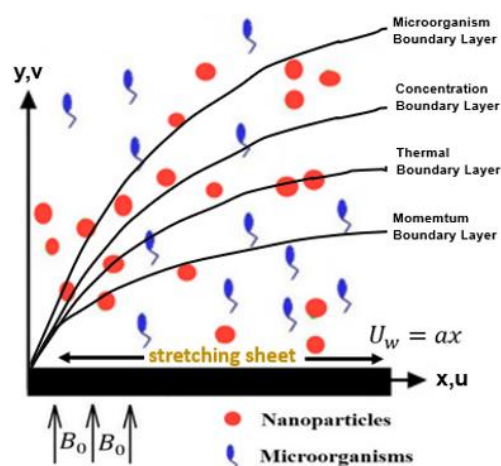


Fig. 1. Physical Model for Horizontal Plate

Assuming that the wall is subjected to a constant wall temperature boundary condition, by using scale analysis and applying the boundary layer approximations, the governing equations can be written as [18,40,43];

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{Thnf}}{\rho_{Thnf}} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(x)}{\rho_{Thnf}} u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{Thnf} \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_B \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right] \tag{3}$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{4}$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_n \frac{\partial^2 N}{\partial y^2} - \frac{1}{C_w - C_\infty} \frac{\partial}{\partial y} \left[ N \frac{\partial C}{\partial y} \right] b W_c \tag{5}$$

While the boundary conditions used in this study are as follows [18]:

$$\begin{aligned} u = \lambda U_w, \quad v = v_0, \quad T = T_w, \quad C = C_w, \quad N = N_w, \quad & \text{on } y = 0 \\ u \rightarrow 0, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty, \quad N \rightarrow N_\infty & \text{as } y \rightarrow \infty \end{aligned} \tag{6}$$

where  $u$  is the fluid velocity and  $v$  is the normal velocity components along the  $x$ -axis and  $y$ -axis.  $T$  is the temperature of the fluids,  $T_f$  is the nanofluids temperature,  $T_\infty$  is the free stream temperature,  $U_w$  is the free stream velocity. The velocity of the surface is linear and is taken as  $U_w = ax$  where  $a$  is a positive constant, and  $x$  is the coordinate component measured along the stretching/shrinking surface. The stretching/shrinking surface is denoted as a  $\lambda$ .  $C$  is the concentration,  $N$  is motile microorganisms,  $b$  is chemotaxis constant,  $W_c$  is the maximum cell swimming speed (assume as constant),  $D_B$  is the Brownian diffusion coefficient,  $D_n$  is the diffusivity of microorganisms,  $D_T$  is the thermophoresis diffusion coefficient,  $\alpha$  is the thermal diffusivity,  $\sigma$  is the electrical conductivity and  $\tau = \frac{(\rho c)_p}{(\rho c)_f}$  which is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the base fluid with  $\rho_f$  being the density of the base fluid. The flow is subjected to the transverse magnetic field of strength  $B_0$ , which is assumed to be applied in the positive  $x$ -direction. Three different nanoparticles are chosen which are  $Al_2O_3$ , Ag and Cu with water as a base fluid. The THNF thermophysical properties attributes are given in Table 1 along with the thermophysical relation of THNF as shown in Table 2 below;

**Table 1**  
 Thermophysical Properties of Base Fluid and Nanoparticles

Properties	Base Fluid (Water)	$Al_2O_3$	Ag	Cu
$\rho(kg/m^3)$	997.1	3970	10500	8933
$C_p(J/kgK)$	4179	765	235	385
$k(W/mk)$	0.613	40	429	400
$Pr$	6.2			

**Table 2**  
 Thermophysical Relation of Ternary Hybrid Nanofluids [40,42,43]

Properties	Ternary Hybrid Nanofluids
Density	$\mu_{Thnf} = \frac{\mu_f}{(1 - \phi_{Al_2O_3})^{2.5} (1 - \phi_{Ag})^{2.5} (1 - \phi_{Cu})^{2.5}}, \quad (7)$
Heat Capacity	$(\rho C_p)_{Thnf} = (1 - \phi_{Al_2O_3}) \left( (1 - \phi_{Ag}) \left[ (1 - \phi_{Cu}) (\rho C_p)_f + \phi_{Cu} (\rho C_p)_{s3} \right] + \phi_{Ag} (\rho C_p)_{s2} + \phi_{Al_2O_3} (\rho C_p)_{s1} \right) \quad (8)$
Viscosity	$\rho_{Thnf} = \left[ (1 - \phi_{Al_2O_3}) \left[ (1 - \phi_{Ag}) \left( (1 - \phi_{Cu}) (\rho_f + \phi_{Cu} \rho_{s3}) \right) + \phi_{Ag} \rho_{s2} \right] + \phi_{Al_2O_3} \rho_{s1} \right], \quad (9)$
Thermal Conductivity	$\frac{k_{nf}}{k_f} = \frac{k_3 + 2k_{nf} - 2\phi_{Cu}(k_{nf} - k_3)}{k_3 + 2k_{nf} + (k_{nf} - k_3)},$ $\frac{k_{hnf}}{k_{nf}} = \frac{k_2 + 2k_{nf} - 2\phi_{Ag}(k_{nf} - k_2)}{k_2 + 2k_{nf} + \phi_{Ag}(k_{nf} - k_2)}, \quad (10)$ $\frac{k_{Thnf}}{k_{hnf}} = \frac{k_1 + 2k_{nf} - 2\phi_{Al_2O_3}(k_{nf} - k_1)}{k_1 + 2k_{nf} + \phi_{Al_2O_3}(k_{nf} - k_1)},$ <p>Then,</p> $\frac{k_{Thnf}}{k_f} = \frac{k_{Thnf}}{k_{hnf}} \times \frac{k_{hnf}}{k_{nf}} \times \frac{k_{nf}}{k_f},$
Thermal Diffusivity	$\alpha_{hnf} = \frac{k_{Thnf}}{(\rho C_p)_{Thnf}} \quad (11)$

Based on formulas above,  $k_{Thnf}$  is the ternary hybrid nanofluids,  $k_{hnf}$  represents the hybrid nanofluids and  $k_f$  denotes as the nanofluids while  $\phi_{Al_2O_3}$ ,  $\phi_{Ag}$ , and  $\phi_{Cu}$  represents the volume fractions of the first, second, and third types of nanoparticles respectively. Where  $\rho_{Thnf}$  is the effective density,  $\mu_{Thnf}$  is the effective dynamic viscosity,  $(\rho C_p)_{Thnf}$  is the heat capacity of the fluid and  $k_{Thnf}$  is the thermal conductivity of the hybrid nanofluids.

The continuity Eq. (1) is satisfied by introducing stream function  $\psi(x, y)$  as shown below,

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (12)$$

The following similarity variables are introduced to solve the governing Eq. (1) to Eq. (5),

$$\eta = y \sqrt{\frac{U_w}{\nu_f x}}, \quad \psi = \sqrt{\nu_f U_w x} f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad \chi = \frac{N - N_\infty}{N - N_\infty} \quad (13)$$

where  $\eta$  is the similarity variable,  $Re_x = \frac{U_w x}{\nu_f}$  refers to Reynolds number,  $\nu_f = \frac{\mu_f}{\rho_f}$  is kinematic viscosity,  $f(\eta)$  and  $\theta(\eta)$  indicate the non-dimensional stream function and temperature, respectively.

By substituting (7), (8) and (9) into (2) to (5), the following nonlinear systems of ordinary differential equations are obtained:

$$\frac{1}{E1} f''' + f f'' - \frac{1}{E2} M(1 - f') = 0 \quad (14)$$

$$\frac{1}{Pr} \theta'' + \frac{E4}{E5} [f\theta' + Nb\phi'\theta' + Nt\theta'^2] = 0 \quad (15)$$

$$\phi'' + Le f \phi' + \frac{Nt}{Nb} \theta'' = 0 \quad (16)$$

$$\chi'' + Sc f \chi' + Pe [\chi' \phi' + \phi''(\sigma + \chi)] = 0 \quad (17)$$

where;

$$E1 = (1 - \phi_{Al_2O_3})^{2.5} (1 - \phi_{Ag})^{2.5} (1 - \phi_{Cu})^{2.5}$$

$$E2 = (1 - \phi_{Al_2O_3}) \left( (1 - \phi_{Ag}) \left[ (1 - \phi_{Cu})(\rho C_p)_f + \phi_{Cu}(\rho C_p)_{s3} \right] + \phi_{Al_2O_3}(\rho C_p)_{s1} + \phi_{Ag}(\rho C_p)_{s2} \right)$$

$$E4 = (1 - \phi_{Al_2O_3}) \left[ (1 - \phi_{Ag}) \left( (1 - \phi_{Cu})(\rho_f + \phi_{Cu}\rho_{s3}) \right) + \phi_{Al_2O_3}\rho_{s1} + \phi_{Ag}\rho_{s2} \right]$$

$$E5 = \frac{k_{Thnf}}{k_{hnf}}$$

where  $Pr$  is the Prandtl number,  $Nb$  is the Brownian motion parameter,  $Nt$  is the thermophoresis parameter,  $Le$  is the Lewis number,  $Sc$  is the Schmidt number,  $Pe$  is the bioconvection Péclet number,  $S$  is the mass flux parameter,  $M$  is the magnetic parameter number, and  $\sigma$  is a dimensionless constant, which are defined as;

$$Pr = \frac{\nu}{\alpha}$$

$$Nb = \frac{\tau D_B (C_w - C_\infty)}{\nu}$$

$$Nt = \frac{\tau D_T (T_w - T_\infty)}{\nu T_\infty}$$

$$Le = \frac{\nu}{D_B}$$

$$Sc = \frac{\nu}{D_n}$$

$$M = \frac{\sigma B_0^2(x)}{\alpha \rho_{hnf}}$$

$$Pe = \frac{b W_c}{D_n}$$

$$\sigma = \frac{N_\infty}{N_w - N_\infty}$$

$$S = \frac{v_0}{\sqrt{\alpha v}}$$

By respecting to (13), the boundary conditions obtained are as follows:

$$\begin{aligned} f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad \phi(0) = 1, \quad \chi(0) = 1 \quad \text{at } y = 0 \\ f'(\eta) = 0, \quad \theta(\eta) = 0, \quad \phi(\eta) = 0, \quad \chi(\eta) = 0 \quad \text{as } y \rightarrow \infty \end{aligned} \quad (18)$$

Where  $\lambda$  is a constant with for stretching  $\lambda > 0$  and for shrinking  $\lambda < 0$ .  $S$  is the mass flux parameter with  $S > 0$  for suction and  $S < 0$  for injection. The discussions of numerical results are based on the skin friction coefficient,  $C_f$  at the surface of the plate, local Nusselt number,  $Nu_x$ , Sherwood number,  $Sh_x$  and Motile Microorganism number,  $Nn_x$ , which are defined as;

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{xq_w}{k_f(T_f - T_\infty)}, \quad Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)}, \quad Nn_x = \frac{xq_n}{D_n(n_w - n_\infty)} \quad (19)$$

where  $\rho_f$  is the density of nanofluids,  $\tau_w$  is the shear stress or wall skin friction,  $q_w$  is the surface heat flux,  $k_f$  is the thermal conductivity of the nanofluids,  $q_m$  is surface mass flux and  $q_n$  is the surface motile microorganisms' flux which defines as;

$$\tau_w = \mu_{hnf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{hnf} \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad q_m = -D_B \left( \frac{\partial C}{\partial y} \right)_{y=0}, \quad q_n = -D_n \left( \frac{\partial N}{\partial y} \right)_{y=0} \quad (20)$$

By substituting (12) and (19) into (20), the solutions obtained are as follows:

$$\frac{C_f}{(Re_x)^{\frac{1}{2}}} = \frac{1}{(1 - \phi_{Al_2O_3})^{2.5} (1 - \phi_{Ag})^{2.5} (1 - \phi_{Cu})^{2.5}} f''(0),$$

$$\frac{Nu_x}{(Re_x)^{\frac{1}{2}}} = -\frac{k_{tnhf}}{k_{hnf}} \theta'(0),$$

$$\frac{Sh_x}{(Re_x)^{\frac{1}{2}}} = -\phi'(0), \quad \frac{Nn_x}{(Re_x)^{\frac{1}{2}}} = -\chi'(0)$$

### 3. Numerical Solution and Validation

The `bvp4c` algorithm is used in MATLAB to solve the problem numerically. MATLAB solver `bvp4c` algorithm which comprises of finite-difference code that executes the three stages of Lobatto IIIa formula was introduced by Shampine *et al.*, [50]. The Lobatto IIIa is a part of the families of Runge-Kutta methods, practices the implicit trapezoidal rule and associated with the collocation method where the `bvp4c` function represents the collocation method. To initiate the `bvp4c` algorithm, the ordinary differential equations must write as systems of first-order ordinary differential equations. This function widely used by researchers to obtain the solutions for the boundary value problems. The (14)-(17) are converted into a set of the ordinary differential equation using a set of newly defined variables as (21).



$$f = y(1), f' = y(2), f'' = y(3), \theta = y(4), \theta' = y(5), \phi = y(6), \phi' = y(7), \chi = y(8), \chi' = y(9) \quad (21)$$

The initial guesses for new assumed variables are taken into consideration. The newly defined set of variables is provided as:

$$\frac{\partial y(1)}{\partial n} = f' = y(2)$$

$$\frac{\partial y(2)}{\partial n} = f'' = y(3)$$

$$\frac{\partial y(3)}{\partial n} = f''' = \left( \left( \frac{E2}{E1} \right) * y(2) * y(2) - y(1) * y(3) + \left( \frac{M}{E2} \right) * y(2) \right)$$

$$\frac{\partial y(4)}{\partial n} = \theta' = y(5)$$

$$\frac{\partial y(5)}{\partial n} = \theta'' = \left( Pr * \left( \frac{E4}{E5} \right) \right) (-y(1) * y(5) - Nb * y(7) * y(5) - Nt * y(5) * y(5))$$

$$\frac{\partial y(6)}{\partial n} = \phi' = y(7)$$

$$\frac{\partial y(7)}{\partial n} = \phi'' = -Le * y(1) * y(7) - \frac{Nb}{Nt} * y(5)$$

$$\frac{\partial y(8)}{\partial n} = \chi' = y(9)$$

$$\frac{\partial y(9)}{\partial n} = \chi'' = -Sc * y(1) * y(9) + Pe[y(9) * y(7) + y(7) * (\sigma + y(8))]$$

and boundary conditions as follows:

$$ya1 = S, ya2 = \lambda, ya4 = 1, ya6 = 1, ya8 = 1$$

$$yb2 = 0, yb4 = 0, yb6 = 0, yb8 = 0$$

The solution of (14) to (17) along with the boundary conditions given in (18) were solved numerically by using function `bvp4c` in MATLAB for the selected value of the effects on the parameters in this study. The comparative results between  $C_f Re_x^{1/2}$ ,  $Nu_x Re_x^{-1/2}$ ,  $Sh_x Re_x^{-1/2}$  and  $Nn_x Re_x^{-1/2}$  with Aman *et al.*, [18] are presented in Table 3. The table displays good comparative results and provides confidence to the method employed in this study.

**Table 3**

Comparison Result of  $C_f Re_x^{1/2}$ ,  $Nu_x Re_x^{-1/2}$ ,  $Sh_x Re_x^{-1/2}$  and  $Nn_x Re_x^{-1/2}$  for Different Values of magnetic parameter ( $M$ ) when  $Pr = 6.2$ ,  $Nb = 0.1$ ,  $Nt = 0.1$ ,  $Le = 10$ ,  $Sc = 1$ ,  $Pe = 0.3$ ,  $\sigma = 0.1$ ,  $S = 1$  and  $\lambda = 0.1$

$M$	Aman et al., [18]				Present result			
	$C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$	$Nn_x Re_x^{-1/2}$	$C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$	$Sh_x Re_x^{-1/2}$	$Nn_x Re_x^{-1/2}$
0	-0.1093	3.4658	6.6933	3.4463	-0.10916	3.46578	6.69335	3.44529
1	-0.1662	3.4630	6.6826	3.4359	-0.16619	3.46299	6.68264	3.43469
1.5	-0.1860	3.4621	6.6793	3.4330	-0.18602	3.46210	6.67933	3.43173
2.0	-0.2033	3.4614	6.6766	3.4307	-0.20329	3.46135	6.67659	3.42937

#### 4. Result and Discussion

In order to study influences of the interaction of magnetic field ( $M$ ), the Brownian motion ( $Nb$ ), thermophoresis parameter ( $Nt$ ) and the bioconvection Péclet number ( $Pe$ ) while Lewis number ( $Le$ ), Schmidt number ( $Sc$ ), mass flux parameter ( $S$ ), and dimensionless constant ( $\sigma$ ) is fixed, which are ( $Pe = 1$ ,  $Sc = 1$ ,  $Le = 2$ ,  $\sigma = 1$ ,  $S = 2.5$ ,  $\lambda = 0.5$  (stretching) and  $\lambda = -0.5$  (shrinking)) are graphically presented. Figure 2 to Figure 16 demonstrate how velocity, temperature, concentration, and motile microorganisms' profiles change when  $\phi_{Al_2O_3}$ ,  $\phi_{Ag}$ ,  $\phi_{Cu}$ ,  $M$ ,  $Nb$ ,  $Nt$  and  $Le$  change at shrinking/stretching for the nanofluids ( $Al_2O_3$ -Water), hybrid nanofluids ( $Al_2O_3/Ag$ -Water) and THNF ( $Al_2O_3/Ag/Cu$ -Water).

The effects of different values of the magnetic strength,  $M = 0, 0.5, 2$  and  $5$  of the the velocity, temperature, concentration, and motile microorganisms are shown in Figure 2 to 5. For the case of shrinking, the velocity, temperature, concentration, and motile microorganisms is increase when  $M$  is increase. Meanwhile, for the case of stretching, the velocity, concentration, and motile microorganisms is decrease while the temperature is increase. This explained with the concurrence with Pal et al., [51] where the increasing strength of  $M$  means that a larger resistive force to the flow. The resistive Lorentz force comes into play with the interaction magnetic and electric fields which causes the decline of velocity profile. The influence of the magnetic field drags the thermal energy as the additional force [52]. This additional force increases the thickness of the thermal boundary layer, so that the temperature profile enhances with rise in  $M$ . It can be concluded that the increasing of magnetic field can be used as a good controller of the THNF flow field incorporating bioconvection, Brownian motion and motile microorganisms. It is shown that increase of  $M$  results is decreasing microorganism's profile for the case of s stretching, whereas a cross-over is noted in microorganism profile at  $\eta \approx 0.5$  as shown in Figure 5. For the dynamic region  $\eta < 0.5$  the growth of  $M$  leads to decline in the microorganism boundary layers, whereas for  $\eta > 0.5$ , the reverse take place as  $M$  is increases. The velocity, temperature, concentration, and microorganism for the THNF ( $Al_2O_3/Ag/Cu$ -water) is better compared to nanofluids ( $Al_2O_3$ -water) and hybrid nanofluids ( $Al_2O_3/Ag$ -water) except for the velocity in stretching case, the velocity of nanofluids ( $Al_2O_3$ -water) is highest compared to others.

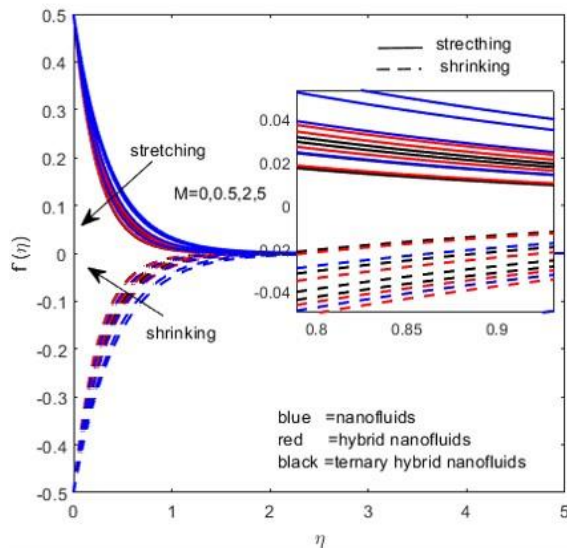


Fig. 2. Effects of  $M$  on velocity profiles

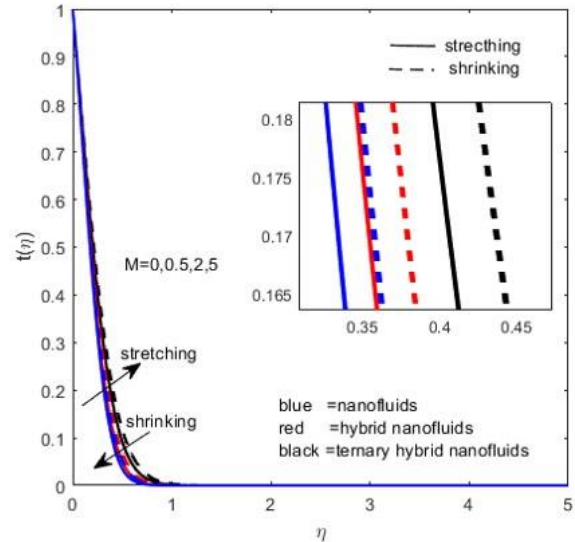


Fig. 3. Effects of  $M$  on temperature profiles

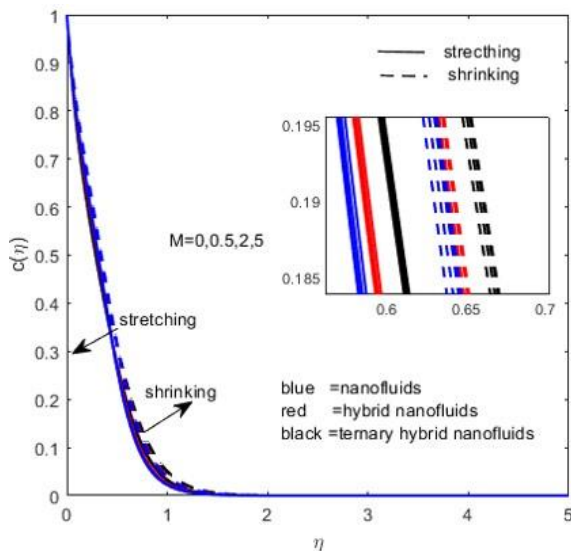


Fig. 4. Effects of  $M$  on concentration profiles

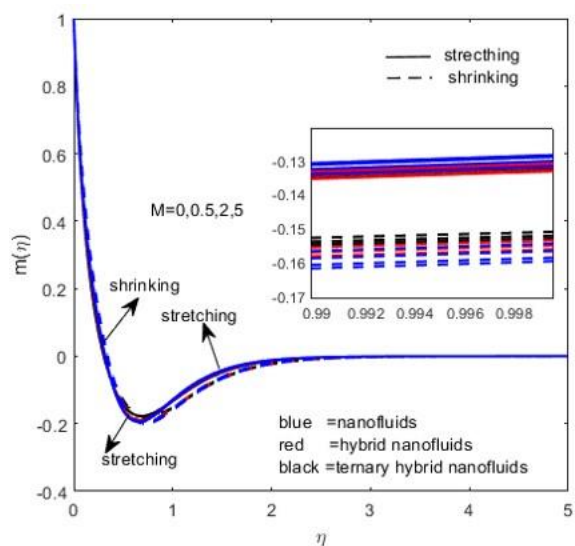


Fig. 5. Effects of  $M$  on microorganism profiles

The variation of velocity, temperature, concentration, and motile microorganisms' profile for different values of thermophoresis parameter  $Nt = 0.1, 0.3, 0.5, 1$  are shown in Figure 6 to Figure 9. It is found that the velocity near the shrinking surface increases with increase in the  $Nt$ . It is observed that the velocity distribution is prominent near the shrinking surface whereas this effect enhances as the value approaches asymptotically to zero as  $\eta \rightarrow \infty$ , whereas there is a decrease in the velocity distribution in the boundary layer with increase in the value of the suction parameter which is because the heated fluid is pushed towards the stretching sheet since the forces due to buoyancy act to decrease the fluid velocity because of greater effect of the Brownian motion [53]. Meanwhile, the effects of the thermophoresis parameter on the temperature, concentration, and motile microorganisms' s profile which gradually diminishes. with the distance from the shrinking and stretching sheet. The nanoparticle volume fraction is affected by the  $Nt$  effect because the fluid's mass transfer rate is increased by the thermophoresis parameter leading to a decreased rate of surface mass transfer. It is shown that increase of  $Nt$  results are decreasing microorganism's profile for the both cases, whereas a cross-over is noted in microorganism profile at  $\eta \approx 0.8$  as shown in Figure 9. For the dynamic region  $\eta < 0.8$  the growth of  $Nt$  leads to decline in the microorganism

boundary layers, whereas for  $\eta > 0.8$ , the reverse take place as  $Nt$  is increases. It noticed that the velocity, temperature, concentration, and microorganism for the THNF ( $Al_2O_3/Ag/Cu$ -water) is better compared to nanofluids ( $Al_2O_3$ -water) and hybrid nanofluids ( $Al_2O_3/Ag$ -water).

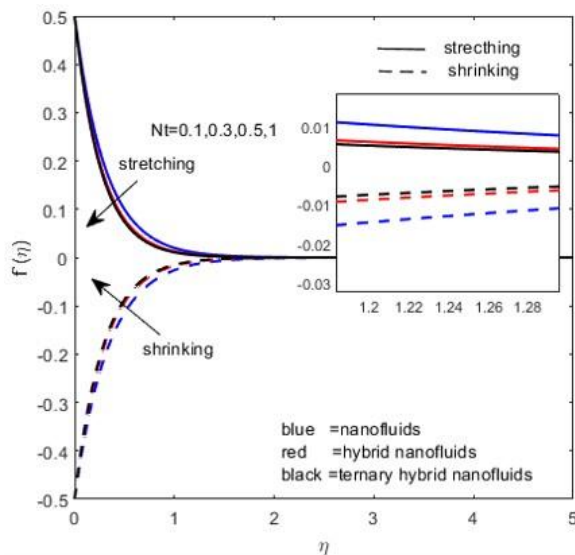


Fig. 6. Effects of  $Nt$  on velocity profiles

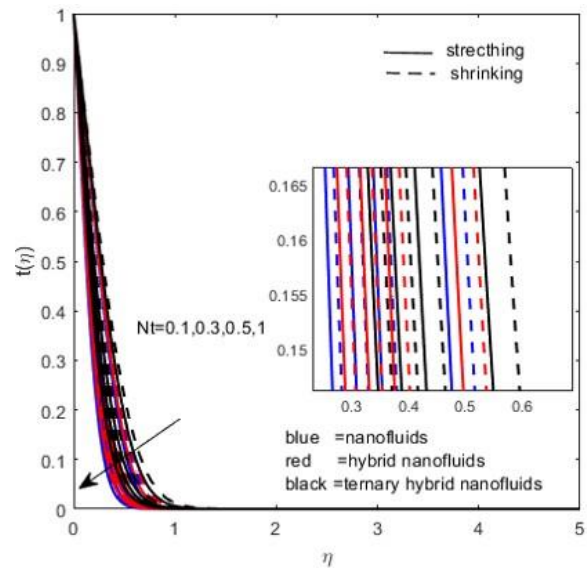


Fig. 7. Effects of  $Nt$  on temperature profiles

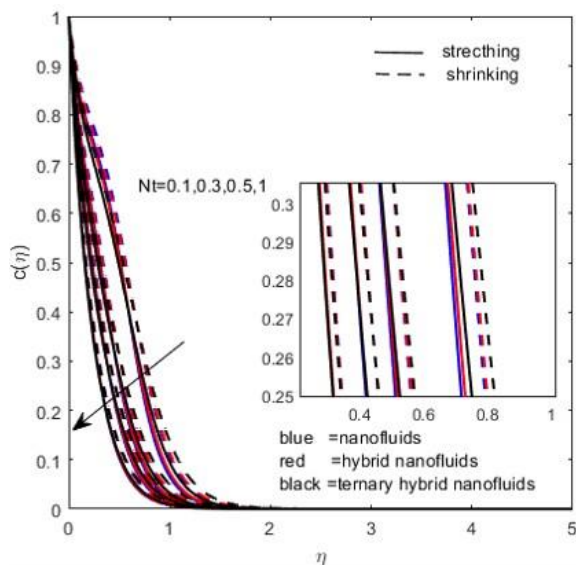


Fig. 8. Effects of  $Nt$  on concentration profiles

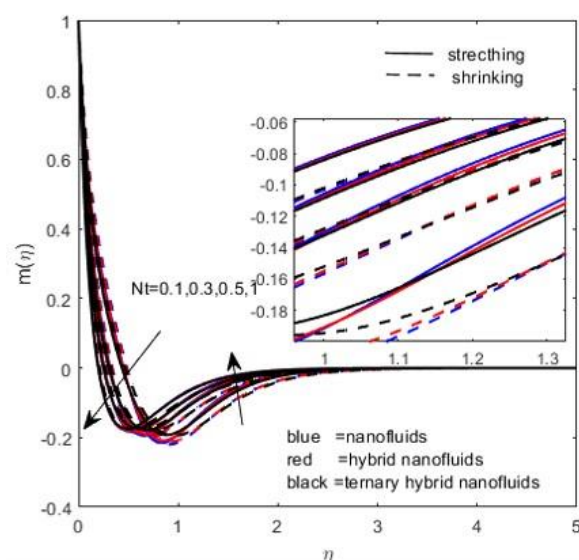


Fig. 9. Effects of  $Nt$  on microorganism profiles

Figure 10 to 13 are shown the effects of different values of the Brownian motion parameter,  $Nb = 0.4, 0.5, 0.6, 0.7$  of the velocity, temperature, concentration, and motile microorganisms' profile. An increment in Brownian motion parameter,  $Nb$  near the shrinking surface the velocity is increase while at stretching surface is gradually decrease. The movement of Brownian motion depends upon the size of the particles and the viscosity of the dispersion medium. When the smaller particles and the less viscous the dispersion medium, the more vigorous is the Brownian motion and vice versa [54]. The temperature at both surface shows gradually diminishes due to the slower movement of particles. The  $Nb$  are seen to be increased near the stretching and shrinking surface on the concentration, and motile microorganisms' s profile. This is due to combined effect of migration of a colloidal particle, buoyancy, magnetic effect and bioconvection plumes that result in a reduced

momentum boundary layer thickness and as such there will be more particles near the boundary layer region hence increased nanoparticle concentration at the plate surface. The reduction in the nanoparticle boundary layer concentration can be due to the decrease in mass diffusivity and the Brownian motion of nanoparticles in the boundary layer region [55]. Figure 13 shows that an increasing of  $Nb$  results in increasing microorganism's profile for the both cases, whereas a cross-over is noted in microorganism profile at  $\eta \approx 0.8$  as shown in Figure 5. For the dynamic region  $\eta < 0.8$  the growth of  $Nt$  leads to upsurge in the microorganism boundary layers, whereas for  $\eta > 0.8$ , the reverse take place as  $Nb$  is decreases. The velocity, temperature, concentration, and microorganism for the THNF ( $Al_2O_3/Ag/Cu$ -water) is better compared to nanofluids ( $Al_2O_3$ -water) and hybrid nanofluids ( $Al_2O_3/Ag$ -water).

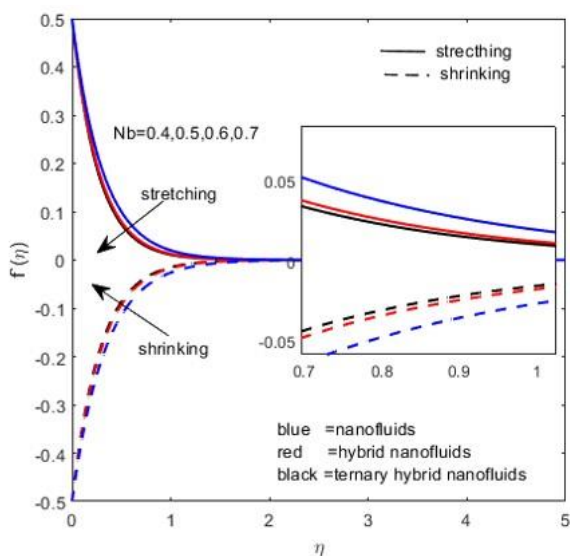


Fig. 10. Effects of  $Nb$  on velocity profiles

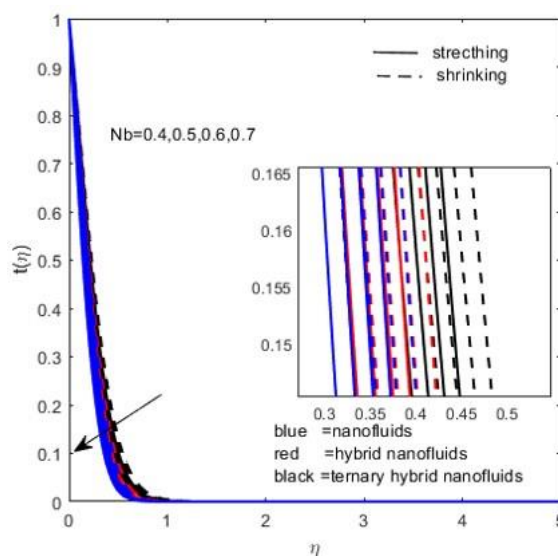


Fig. 11. Effects of  $Nb$  on temperature profiles

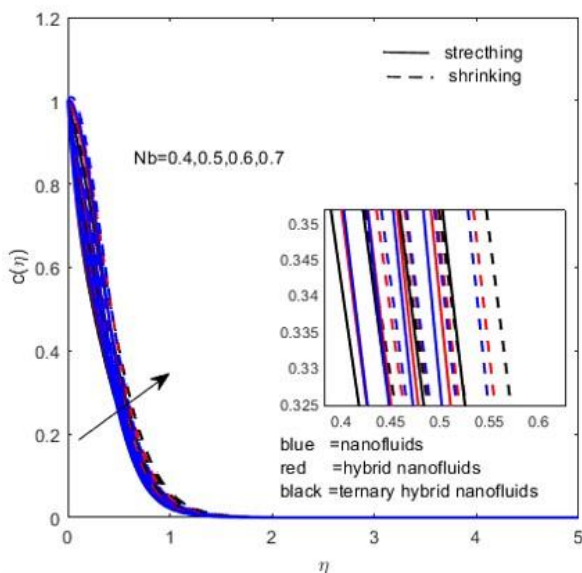


Fig. 12. Effects of  $Nb$  on concentration profiles

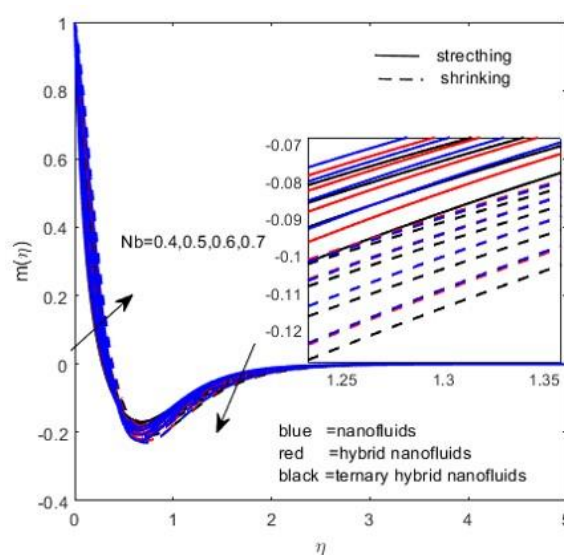


Fig. 13. Effects of  $Nb$  on microorganism profiles

Figure 14 to 17 shows the effect of various values of the bioconvection Péclet number parameter,  $Pe = 0.4, 0.5, 0.6, 0.7$  of the velocity, temperature, concentration, and motile microorganism's profile. It is noted that an increasing of the  $Pe$ , the velocity is decrease at stretching surface while



increase at shrinking surface. On the other hand, the temperature is decrease for both cases. The concentration and motile microorganisms are affected greatly by the  $Pe$ . It is noted that an increase in  $Pe$ , The concentration and motile microorganisms also upsurge due to the bioconvection and magnetic field concentration, drive the fluid towards the plate surface [54]. The velocity, temperature, concentration, and microorganism for the THNF ( $Al_2O_3/Ag/Cu$ -water) is better compared to nanofluids ( $Al_2O_3$ -water) and hybrid nanofluids ( $Al_2O_3/Ag$ -water). The effects of various parameters on the the skin friction, Nusselt number, Sherwood number and density motile microorganisms at shrinking/stretching for the nanofluids ( $Al_2O_3$ -Water), hybrid nanofluids ( $Al_2O_3/Ag$ -Water) and THNF ( $Al_2O_3/Ag/Cu$ -Water) are shown in Table 4 to Table 6.

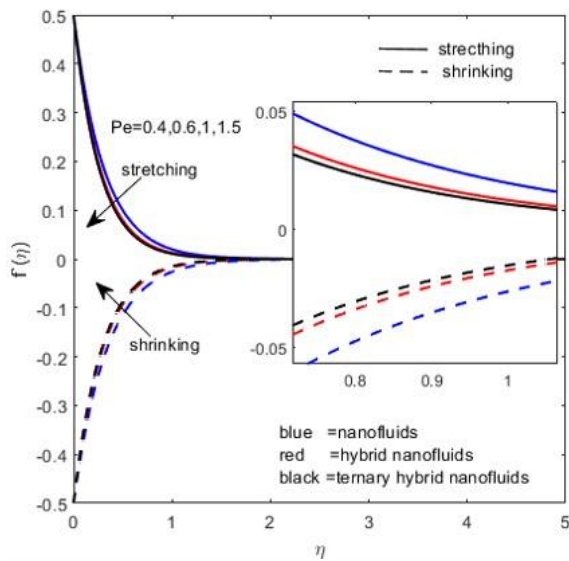


Fig. 14. Effects of  $Pe$  on velocity profiles

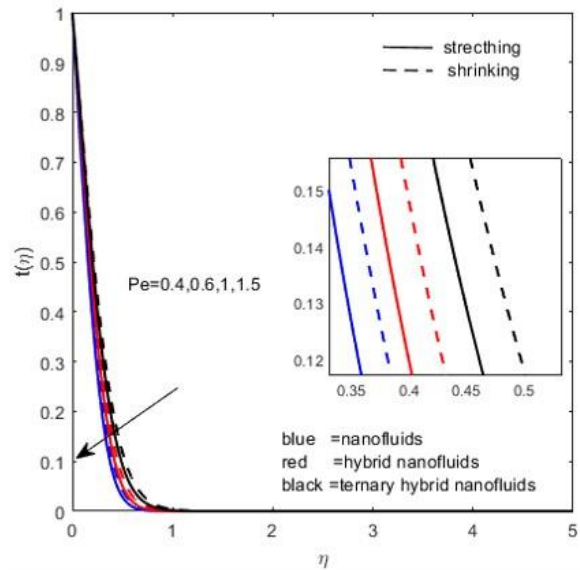


Fig. 15. Effects of  $Pe$  on temperature profiles

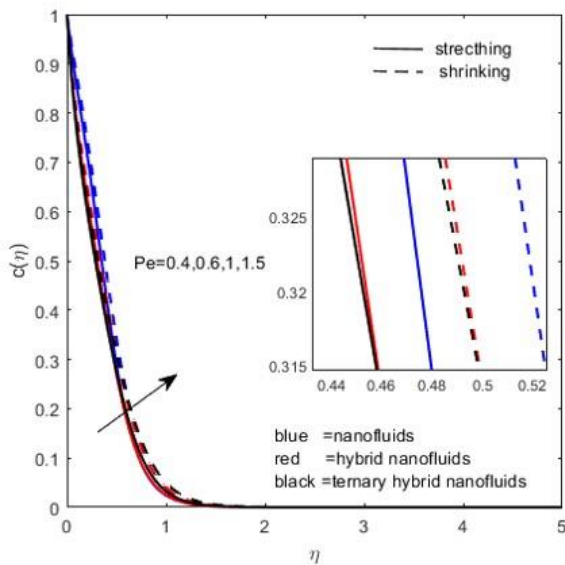


Fig. 16. Effects of  $Pe$  on concentration profiles

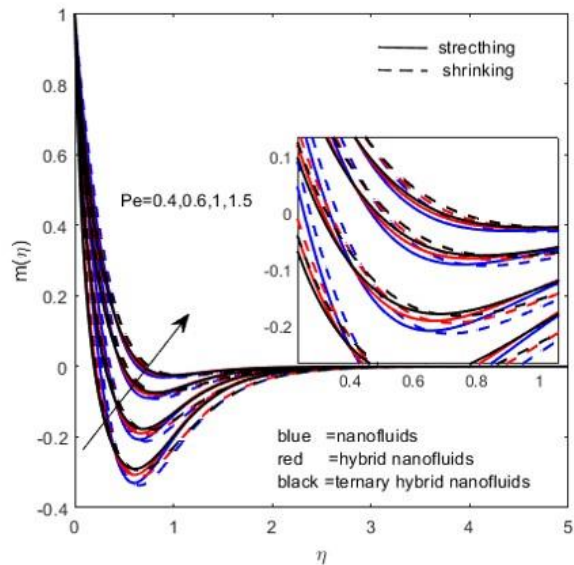


Fig. 17. Effects of  $Pe$  on microorganism profiles

From Table 6, an increment value in  $M, Nb, Nt$  and  $Pe$ , it shows that the skin friction, Nusselt Number, Sherwood Number and Density Motile Microorganisms at shrinking/stretching is observed to growth in THNF ( $Al_2O_3/Ag/Cu$ -Water) compared to nanofluids ( $Al_2O_3$ -Water) in Table 4 and hybrid nanofluids ( $Al_2O_3/Ag$ -Water) in Table 5. It shows that an increase in the value of  $M$ , the skin friction is increase at shrinking surface but reverse trend is seen at stretching surface. On the other hand, it remains unchanged for both surfaces. It is found that the Nusselt number decreases with an increase in the value of the  $M, Nt$  and  $Nb$  at shrinking surface but at the stretching an increase in  $M$  enhances the value of Nusselt number. It is observed that there is an increase in the Sherwood number with increase in the value of the  $M$  at shrinking surface and  $Nt$  for both cases, but reverse trend is seen by increasing the value of the  $Nb$  at both cases. A rise in  $Nb$  and  $Pe$  enhance the motile microorganism's local density number for both cases. This may be attributed to the fact that at high  $Pe$ , there is low concentration of nanoparticles, due to the higher plate surface concentration than in the fluid causes mass transfer to the fluid from the plate [53].

**Table 4**

Variation in Skin Friction, Nusselt Number, Sherwood Number and Density Motile Microorganisms at Different Dimensionless Parameters for nanofluids ( $\phi_{Al_2O_3} = 0.05, \phi_{Ag} = 0, \phi_{Cu} = 0$ )

$M$	$Nt$	$Nb$	$Pe$	shrinking				stretching			
				Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms	Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms
0	0.5	0.6	1	1.0151	3.19743	2.46325	12.3623	-1.1933	3.08784	2.95547	14.3167
0.5				1.0998	3.19143	2.47614	12.4101	-1.2569	3.09070	2.94877	14.2919
2				1.2983	3.17943	2.50258	12.5087	-1.4199	3.09738	2.93289	14.2331
5				1.5862	3.16592	2.53361	12.6259	-1.6771	3.10622	2.91123	14.1519
2	0.1	0.6	1	1.2983	4.76568	4.16588	19.1864	-1.4199	4.73914	4.44569	20.2634
	0.3			1.2983	3.86934	3.14747	15.1002	-1.4199	3.80688	3.50156	16.4978
	0.5			1.2983	3.17943	2.50258	12.5087	-1.4199	3.09738	2.93289	14.2331
	1			1.2983	2.07477	1.75594	9.49826	-1.4199	1.98060	2.34471	11.8997
2	0.5	0.4	1	1.2983	5.26356	-0.95474	-1.35722	-1.4199	5.17675	-0.41122	0.88823
		0.5		1.2983	4.12607	1.20260	7.29578	-1.4199	4.03891	1.68054	9.23493
		0.6		1.2983	3.17943	2.50258	12.5087	-1.4199	3.09738	2.93289	14.2331
		0.7		1.2983	2.40763	3.32256	15.7966	-1.4199	2.33423	3.71672	17.3615
2	0.5	0.6	0.4	1.2983	3.17943	2.50258	4.43296	-1.4199	3.09738	2.93289	4.90960
			0.6	1.2983	3.17943	2.50258	5.44487	-1.4199	3.09738	2.93289	6.07287
			1	1.2983	3.17943	2.50258	7.46622	-1.4199	3.09738	2.93289	8.40158
			1.5	1.2983	3.17943	2.50258	9.98912	-1.4199	3.09738	2.93289	11.3159

**Table 5**

Variation in Skin Friction, Nusselt Number, Sherwood Number and Density Motile Microorganisms at Different Dimensionless Parameters for Hybrid Nanofluids ( $\phi_{Al_2O_3} = 0.05, \phi_{Ag} = 0.05, \phi_{Cu} = 0$ )

M	Nt	Nb	Pe	shrinking				stretching			
				Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms	Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms
0	0.5	0.6	1	1.11566	3.56775	2.56900	12.7765	-1.2717	3.51624	2.97036	14.3827
0.5				1.16899	3.56477	2.57585	12.8020	-1.3135	3.51782	2.96643	14.3681
2				1.30521	3.55800	2.59176	12.8618	-1.4263	3.52175	2.95644	14.3309
5				1.51899	3.54939	2.61294	12.9419	-1.6149	3.52741	2.94156	14.2749
2	0.1	0.6	1	1.30521	5.17619	4.21630	19.3823	-1.4263	5.20040	4.47296	20.3776
	0.3			1.30521	4.27052	3.23585	15.4489	-1.4263	4.25718	3.54498	16.6757
	0.5			1.30521	3.55800	2.59176	12.8618	-1.4263	3.52175	2.95644	14.3309
	1			1.30521	2.37711	1.80524	9.69426	-1.4263	2.31938	2.28845	11.6767
2	0.5	0.4	1	1.30521	5.63224	-0.50895	0.42695	-1.4263	5.60907	-0.07608	2.22860
		0.5		1.30521	4.50773	1.41720	8.15211	-1.4263	4.47550	1.81102	9.75913
		0.6		1.30521	3.55800	2.59176	12.8618	-1.4263	3.52175	2.95644	14.3309
		0.7		1.30521	2.76931	3.34420	15.8786	-1.4263	2.73277	3.68617	17.2435
2	0.5	0.6	0.4	1.30521	3.55800	2.59176	4.50827	-1.4263	3.52175	2.95644	4.92507
			0.6	1.30521	3.55800	2.59176	5.55453	-1.4263	3.52175	2.95644	6.09893
			1	1.30521	3.55800	2.59176	7.64499	-1.4263	3.52175	2.95644	8.44850
			1.5	1.30521	3.55800	2.59176	10.2548	-1.4263	3.52175	2.95644	11.3884

**Table 6**

Variation in Skin Friction, Nusselt Number, Sherwood Number and Density Motile Microorganisms at Different Dimensionless Parameters for Ternary Hybrid Nanofluids ( $\phi_{Al_2O_3} = 0.05, \phi_{Ag} = 0.05, \phi_{Cu} = 0.05$ )

M	Nt	Nb	Pe	shrinking				stretching			
				Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms	Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms
0	0.5	0.6	1	1.05112	4.56861	2.67049	13.1812	-1.1883	4.56554	3.02492	14.6019
0.5				1.08987	4.56649	2.67530	13.1991	-1.2189	4.56668	3.02207	14.5913
2				1.19199	4.56148	2.68703	13.2429	-1.3035	4.56959	3.01461	14.5636
5				1.35785	4.55484	2.70354	13.3053	-1.4488	4.57391	3.00303	14.5202
2	0.1	0.6	1	1.19199	6.41819	4.26563	19.5783	-1.3035	6.50434	4.51191	20.5345
	0.3			1.19199	5.38961	3.32878	15.8201	-1.3035	5.42905	3.61356	16.9506
	0.5			1.19199	4.56148	2.68703	13.2429	-1.3035	4.56959	3.01461	14.5636
	1			1.19199	3.13652	1.84924	9.8713	-1.3035	3.10712	2.27336	11.6158
2	0.5	0.4	1	1.19199	6.88518	-0.05180	2.2592	-1.3035	6.92520	0.31566	3.79269
		0.5		1.19199	5.63452	1.64151	9.0506	-1.3035	5.65586	1.98653	10.4602
		0.6		1.19199	4.56148	2.68703	13.2429	-1.3035	4.56959	3.01461	14.5636
		0.7		1.19199	3.65306	3.36760	15.9717	-1.3035	3.65234	3.68105	17.2236
2	0.5	0.6	0.4	1.19199	4.56148	2.68703	4.5859	-1.3035	4.56959	3.01461	4.97034
			0.6	1.19199	4.56148	2.68703	5.6701	-1.3035	4.56959	3.01461	6.16768
			1	1.19199	4.56148	2.68703	7.8364	-1.3035	4.56959	3.01461	8.56416
			1.5	1.19199	4.56148	2.68703	10.5411	-1.3035	4.56959	3.01461	11.5626



Table 7 and 8 show the comparison of skin friction, Nusselt number, Sherwood number and density motile microorganisms for water, Al<sub>2</sub>O<sub>3</sub>-Water, Al<sub>2</sub>O<sub>3</sub>/Ag-Water and Al<sub>2</sub>O<sub>3</sub>/Ag/Cu-Water for shrinking/ stretching. It shown the THNF (Al<sub>2</sub>O<sub>3</sub>/Ag/Cu-Water) is higher compared to others for both cases and it can be concluded that the THNF (Al<sub>2</sub>O<sub>3</sub>/Ag/Cu-Water) have a better rate of heat transfer.

**Table 7**

Comparison Result of Skin Friction and Nusselt Number for Nanofluids, Hybrid Nanofluids and THNF at Shrinking when  $M = 2, Nb = 0.6, Nt = 0.5, and Pe = 1$

$\phi_{Al_2O_3}$	$\phi_{Ag}$	$\phi_{Cu}$	$M$	$Nt$	$Nb$	$Pe$	Shrinking				
							Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms	
0	0	0	2	0.5	0.6	1	1.5000	2.7695	2.4992	7.4594	Water
0.05	0	0					1.2983	2.5026	3.1794	12.5087	Water- Al <sub>2</sub> O <sub>3</sub>
0.05	0.05	0					1.3052	2.5918	3.5580	12.8618	Water- Al <sub>2</sub> O <sub>3</sub> /Ag
0.05	0.05	0.05					1.4920	2.6870	4.5615	13.2429	Water- Al <sub>2</sub> O <sub>3</sub> /Ag/Cu

**Table 8**

Comparison Result of Sherwood Number and Density Motile Microorganisms for Nanofluids, Hybrid Nanofluids and THNF at Stretching when  $M = 2, Nb = 0.6, Nt = 0.5, and Pe = 1$

$\phi_{Al_2O_3}$	$\phi_{Ag}$	$\phi_{Cu}$	$M$	$Nt$	$Nb$	$Pe$	stretching				
							Skin Friction	Nusselt number	Sherwood number	Motile Microorganisms	
0	0	0	2	0.5	0.6	1	-1.6328	2.6965	3.9284	8.3927	Water
0.05	0	0					-1.4199	3.0974	2.9329	14.2331	Water- Al <sub>2</sub> O <sub>3</sub>
0.05	0.05	0					-1.4263	3.5218	2.9564	14.3309	Water- Al <sub>2</sub> O <sub>3</sub> /Ag
0.05	0.05	0.05					-1.4435	4.5696	3.0146	14.5636	Water- Al <sub>2</sub> O <sub>3</sub> /Ag/Cu

In percentage-wise:

for the case of shrinking;

- i. By comparing water and Al<sub>2</sub>O<sub>3</sub>, the skin friction and Nusselt number factor is decreased by 13.45% and 9.64% respectively while for Sherwood number and Motile microorganisms is increased by 27.22% and 67.69% respectively.
- ii. By comparing water and Al<sub>2</sub>O<sub>3</sub>/Ag-water, the skin friction, Nusselt number, Sherwood number and Motile microorganisms' factor is increased by 0.53%, 3.56%, 11.9% and 2.82% respectively.
- iii. By comparing water and Al<sub>2</sub>O<sub>3</sub>/Ag/Cu-water, the skin friction, Nusselt number, Sherwood number and Motile microorganisms' factor is increased by 14.3%, 3.67%, 28.2% and 2.96% respectively.

for the case of stretching;

- i. By comparing water and Al<sub>2</sub>O<sub>3</sub>, the skin friction and Sherwood number is decreased by 13.04% and 25.34% respectively while for Nusselt number and Motile microorganisms is increased by 14.87% and 69.59% respectively.
- ii. By comparing water and Al<sub>2</sub>O<sub>3</sub>/Ag-water, the skin friction, Nusselt number, Sherwood number and Motile microorganisms' factor is increased by 0.45%, 13.7%, 0.8% and 0.69% respectively.

- iii. By comparing water and  $\text{Al}_2\text{O}_3/\text{Ag}/\text{Cu}$ -water, the skin friction, Nusselt number, Sherwood number and Motile microorganisms' factor is increased by 1.21%, 29.8%, 1.97% and 1.62% respectively.

## 5. Conclusions

This paper study the THNF flow containing gyrotactic microorganisms with MHD effects over the shrinking/stretching. The influences of the interaction of magnetic field,  $M$ , volume fraction of nanoparticles,  $\phi$ ,  $Nb$  is the Brownian motion parameter,  $Nt$  is the thermophoresis parameter and  $Pe$  is the bioconvection Péclet number on the effect of THNF and base fluid results obtained:

- i. An increasing of  $M$  lead to growth the velocity while the temperature, concentration, and motile microorganisms' s profile of the THNF which gradually diminishes.
- ii. An increment of  $Nt$  for both cases decrease the temperature, concentration, and motile microorganism.
- iii. For the case shrinking,  $M$  is increase for skin friction and Sherwood number and show opposite result in Nusselt number and density of motile microorganisms.
- iv.  $Nb$  and  $Pe$  is increase for both cases in skin friction, Nusselt number, Sherwood number and density of motile microorganism.
- v.  $Nt$  is decrease in skin friction, Nusselt number, Sherwood number and density of motile microorganism for both cases.
- vi. THNF ( $\text{Al}_2\text{O}_3/\text{Ag}/\text{Cu}$ -Water) is better compared with nanofluids ( $\text{Al}_2\text{O}_3$ -Water) and hybrid nanofluids ( $\text{Al}_2\text{O}_3/\text{Ag}$ -Water) for all cases.

These results have revealed that there is an influence of microorganisms to THNF in comparison to nanofluids and hybrid nanofluids. This means that THNF in the presence of gyrotactic microorganisms are important in the cooling and heating industrial processes. Although the results obtained with this method are satisfactory, there are some parameters that show no effect, but in the future, additional investigations should be done to show a better effect. In this study, the significance of THNF have been highlighted which makes them have numerous heat transfer applications as well as other applications in areas such as delivery of bio-industries.

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