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Solitary Wave Solutions with Compact Support for The Nonlinear Dispersive K(m,n) Equations by Using Approximate Analytical Method

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ABSTRACT

The study of solitons and compactons is important in nonlinear physics. In this paper we combined the Adomian polynomials with the multi-step approach to present a new technique called Multi-step Modified Reduced Differential Transform Method (MMRDTM). The proposed technique has the advantage of producing an analytical approximation in a fast converging sequence with a reduced number of calculated terms. The MMRDTM is presented with some modification of the Reduced Differential Transformation Method (RDTM) with multi-step approach and its nonlinear term is replaced by the Adomian polynomials. Therefore, the nonlinear initial value problem can easily be solved with less computational effort. Besides that, the multi-step approach produces a solution in fast converging series that converges the solution in a wide time area. Two examples are provided to demonstrate the capability and benefits of the proposed method for approximating the solution of NKdVEs with compactons. Graphical inputs are used to represent the solution and to demonstrate the precision and validity of the MMRDTM in graphic illustration. From the results, it was found that it is possible to obtain highly accurate results or exact solutions by using the MMRDTM.

1. Introduction

The Korteweg–de Vries (KdV) equation is a seminal model in fluid mechanics [1]. The KdV equation is used to simulate a variety of nonlinear phenomena, including ion acoustic waves in plasmas [2], pattern formation in liquid drops [3] and shallow water waves [4]. Shallow water wave equations commonly used in oceanography and atmospheric science [5]. The KdV equation was originally derived to describe shallow water waves of long wavelength and small amplitude. The KdV equation is introduced as follows [4],

$$u_t + 6uu_x + u_{xxx} = 0. \tag{1}$$

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The derivative u_t describes the time evolution of a wave propagating in one direction, the nonlinear term uu_x describes the wave's steepening, and the linear term u_{xxx} accounts for the wave's spreading or dispersion. The nonlinear steepening of the water wave can be balanced by dispersion.

Due to the balance between the nonlinear convection uu_x and the linear dispersion u_{xxx} the KdV equation (1) leads towards solitons. Soliton is a localised wave with an infinite support, or an exponentially winged localised wave. After mutual collision, solitons maintain their identities. This means that a soliton has particle-like properties. Solitary wave solutions were obtained for hyperbolic partial differential equations in [6-9] while exact solution for wave equations were obtained in literatures [10-15]**.** Solitary-wave special solutions with compact support for the nonlinear dispersive were discussed in [16-20]. Solitary waves emerge at the variable topographic effects on the evolution of the internal undular bores of depression [21]. Mun Hoe *et al.,* [22] also investigated the effect of rapidly varying topography by using mathematical model of the variable-coefficient extended KdV. In addition to low waves along the beaches, the KdV equation and its solitary wave solution also apply to internal ocean waves. Internal waves are slower waves with great amplitude in the open ocean that travel through the interface of layers of various densities [5].

Contrasting to solitons, a compacton is a special solitary traveling wave that does not have exponential tails. In studying the role of nonlinear dispersion in liquid drops pattern formation, $K(n,n)$ has been found as a simplified model and also proposed in the analysis of liquid surface patterns. Compact solutions have also been identified in applications such as the long nonlinear surface waves in a rotating ocean when the high-frequency dispersion is null, the pulse propagation in the ventricle aorta system, dispersive models for magma dynamics, or even particle wave functions in nonlinear quantum mechanics. The propagation of compacton-like kinks in nonlinear lattices has been found using mechanical, electrical, and magnetic analogues.

Compactons, also known as solitons with compact support or strict localization of solitary waves, was recently introduced by Wazwaz [16]. He [16] investigated the role of nonlinear dispersion in the formation of patterns in liquid drops and developed a genuinely nonlinear dispersive equation $K(m,n)$, a special type of the KdV equation. The proposed $K(m,n)$ equation as stated by Rosenau and Hyman [3], which is a generalisation of the KdV equation has the form $u_t + (u^m)_x + (u^n)_{xxx} = 0$, $m > 0$,$1 \leq n \leq 3$, $t > 0$. Compactons are solitary waves with exact compact support that are referred as compactons. Unlike the soliton, which narrows as the amplitude increases, the width of the compacton is independent of the amplitude.

There are many effective and powerful methods for approximate analytical solution that have been developed and improved. For instance, the Adomian Decomposition Method (ADM), Homotopy Perturbation Method (HPM), Homotopy Analysis Method (HAM), Variation Iteration Method (VIM), Hirota's Bilinear Method, Balance Method, Inverse Scattering Method, and Differential Transform Method (DTM). However, Ray [23] developed and introduced a modification to the fractional RDTM in order to solve fractional KdV equation. The modification in this method includes the replacement of the nonlinear term by relating Adomian polynomials. As a consequence, the solutions to the nonlinear problem can be obtained in a more straightforward manner with less computed terms. El-Zahar [24] later implemented an adaptive multi-step DTM to solve singular perturbation initial-value problems. It generates a solution in a rapidly convergent sequence that is converging over a large time span. This paper proposed these two methods in solving KdV equations in application of pattern formation liquid drop.

Multistep Modified Reduced Differential Transform Method (MMRDTM) for solving NLSEs is proposed by Che Hussin *et al.,* [25]. In addition, the MMRDTM also was experimentally tested to approximate the Klein-Gordon equations[26]. Later, Che Hussin *et al.,* [27] applied the MMRDTM to obtain the approximate solution of fractional NLSEs. Che Hussin *et al.,* [28] also solved the nonlinear KdV equation by using MMRDTM. Recently, MMRDTM was applied to obtain NLSE with power law nonlinearity by Che Hussin *et al.*[29]. The results of the approximation are obtained with a smaller number of calculated terms and with high precision. Furthermore, the findings converge over a large time frame in a shorter period of time.

The modification by implementing Adomian polynomials and the multistep method are combined in this paper to perform the MMRDTM for solving KdV equations with compact support in application of pattern formation liquid drop. Furthermore, we use parametrization methods to generate Adomian polynomials without the need for time-consuming high-derivative calculations [30]. The proposed technique will quickly produce a fast-convergent sequence of analytical approximations. As a consequence, the solutions converge over a wide time period. Simultaneously, the number of computed terms is greatly decreased.

2. The Development of Multistep Modified Reduced Differential Transform Method

Original functions are denoted by lowercase letters, such as the letter u in the function $u(x,t)$, while transformed functions are denoted by uppercase letters, such as the letter U in the function $U_k(x)$. The differential transformation of the function $u(x,t) = f(x)g(t)$ is obtained as follows [31],

$$u(x,t) = \sum_{i=0}^{\infty} F(i)x^i \sum_{j=0}^{\infty} G(j)t^j = \sum_{k=0}^{\infty} U_k\,(x)t^k,$$

where $U_k(x)$ is known as the function of $u(x,t)$. Some basic properties of RDTM are defined in the following descriptions.

Definition 1: For an analytically and continuously differential function $u(x,t)$ with respect to time t and space variable x, the differential transformation of $u(x,t)$ is defined by,

$$U_k(x) = \left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0}, \tag{2}$$

where $U_k(x)$ is the transformed function.

Definition 2. The inverse transform of $U_k(x)$ is given by,

$$u(x,t) = \sum_{k=0}^{\infty} U_k(x)t^k. \tag{3}$$

By combining equations (2) and (3), the following equation is obtained,

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{\partial^k}{\partial t^k} u(x,t)\right]_{t=0} t^k. \tag{4}$$

Consider the following nonlinear PDE to represent the RDTM's core features,

$$Du(x,t) + Pu(x,t) + Qu(x,t) = h(x,t),$$

where $u(x,\ 0) = f(x)$ is the initial condition. Note that, $D = \frac{\partial}{\partial t}$ and P is the remaining part of a linear operator. The nonlinear and inhomogeneous terms are represented as $Nu(x,t)$ and $h(x,t)$ respectively.

The iteration formula can be derived from the MMRDTM as follows:

$$(k+1)U_{k+1}(x) = H_k(x) - PU_k(x) - NU_k(x). \tag{5}$$

The functions $Du(x,t), Pu(x,t), Nu(x,t)$ and $h(x,t)$ are transformed and then represented as $U_k(x), PU_k(x), NU_k(x)$ and $H_k(x)$ respectively. We have

$$U_0(x) = f(x). \tag{6}$$

From the initial condition. Referring to Ray, the nonlinear term is denoted as follows [23],

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n\big(U_0(x), U_1(x), \dots, U_n(x)\big).$$

Recently, Kataria and Vellaisamy proposed a novel method for calculating the Adomian polynomials [30],

$$A_0 = N\big(U_0(x)\big),$$

$$A_n(U_0(x), U_1(x), \dots, U_n(x)) = \frac{1}{2\pi}\int_{-\pi}^{\pi} N\left(\sum_{k=0}^{n} U_k(x)e^{ikx}\right)e^{-in\lambda}\, d\lambda, \quad n \geq 1.$$

It can be observed that the algorithm does not involve tedious calculations with high derivatives. By combining equations (5) and (6), the $U_k(x)$ values can be obtained through iterative calculation. Furthermore, the set of values $\{U_k(x)\}_{k=0}^{n}$ of the inverse transformation produces the approximate solution as follows,

$$u(x,t) = \sum_{k=0}^{K} U_k(x)t^k, \qquad t \in [0,T].$$

For $m = 1,2,\dots,M$, divide the interval $[0,T]$ is into M subintervals $[t_{m-1}, t_m]$ by equal step size $s = \frac{T}{M}$ and nodes $t_m = ms$. The following steps are used to calculate MMRDTM. Firstly, apply modified RDTM to the initial value problem of interval $[0, t_1]$. Then by using the initial conditions

$$u(x,0) = f_0(x), \qquad u_1(x,0) = f_1(x),$$

The approximate result

$$u_1(x,t) = \sum_{k=0}^{K} U_{k,1}(x)t^k, \qquad t \in [0, t_1]$$

is obtained. At each subinterval $[t_{m-1}, t_m]$, the initial conditions

$u_m(x, t_{m-1}) = u_{m-1}(x, t_{m-1}),$

$(\partial/\partial t)u_m(x, t_{m-1}) = (\partial/\partial t)u_{m-1}(x, t_{m-1}),$

are used for $m \geq 2$ and the multistep RDTM is implemented to the initial value problem on $[t_{m-1}, t_m]$, where t_0 is replaced with t_{m-1}. To produce a sequence of approximate solutions $u_m(x,t)$ the step is performed and carried out repeatedly for $m = 1,2, \dots, M$, such as,

$$u_m(x,t) = \sum_{k=0}^{K} U_{k,m}(x)(t - t_{m-1})^k, \qquad t \in [t_{m-1}, t_m].$$

Finally, MMRDTM proposes the following solutions

$$u(x,t) = \begin{cases} u_1(x,t), \text{for } t \in [0, t_1] \\ u_2(x,t), \text{for } t \in [t_1, t_2] \\ \vdots \\ u_M(x,t), \text{for } t \in [t_{M-1}, t_M]. \end{cases}$$

With better computing performance, the new algorithm MMRDTM is straightforward for all values of s. Note that, the MMRDTM reduces to the modified RDTM once the step size $s = T$.

3. Results

3.1 Numerical Example 1

Consider nonlinear KdV equations as follows [23]

$$u_t + u^2{}_x + u^2{}_{xxx} = 0, \tag{7}$$

Subject to initial conditions

$$u(x,0) = \frac{4}{3} c \cos^2\left(\frac{x}{4}\right).$$

The exact solution is $\frac{4}{3} c \cos^2\left(\frac{x-t}{4}\right)$ where c is a constant.

Using basic properties of MMRDTM and then applying MMRDTM to Equation (7), we can obtain

$$U_{k+1,i}(x) = \left(\frac{1}{(k+1)}\right)\left(-\frac{\partial}{\partial x}\left(A_{k,i}(x)\right) - \frac{\partial^3}{\partial x^3}\left(A_{k,i}(x)\right)\right). \tag{8}$$

From initial condition, we write when $c = 1$,

$$U_0(x) = \frac{4}{3} \cos^2\left(\frac{x}{4}\right). \tag{9}$$

Now write first four examples of the nonlinear term as

$A_0 = U_0^2(x),$
$A_1 = 2U_0(x)U_1(x),$
$A_2 = 2U_0(x)U_2(x) + U_1^2(x),$
$A_3 = 2U_0(x)U_3(x) + 2U_1(x)U_2(x).$

For these nonlinear terms, we use general form of formula A_n Adomian polynomials. We calculate these Adomian polynomials formula by using Maple 13.

Replacing Equation (9) into Equation (8) and through iterative calculation, the $U_k(x)$ values can be obtained. Next, set of values $\{U_6(x)\}_{k=0}^{6}$ of the inverse transformations gives the 6-terms approximate solution as follows,

$$u_1(x,t) = \frac{4}{3}\cos\left(\frac{1}{4}x\right)^2 + \frac{2}{3}\cos\left(\frac{1}{4}x\right)\sin\left(\frac{1}{4}x\right)t + \left(\frac{1}{12} - \frac{1}{6}\cos\left(\frac{1}{4}x\right)^2\right)t^2 - \left(\frac{1}{36}\cos\left(\frac{1}{4}x\right)\sin\left(\frac{1}{4}x\right)\right)t^3 \ldots, \quad t \in [0,0.1].$$

$$u_2(x,t) = 1.3317\cos(0.25)^2 + 0.0666\cos(0.25\,x)\sin(0.25x) + 0.0008 + (-6\times10^{-10}\cos(0.25)^3 sin(0.25x) - 0.0333\cos(0.25)^2 + 2.5328\times10^{-11}\cos(0.25)^4 + 0.6658\cos(0.25\,x)\sin(0.25x) + 0.0166)(t - 0.1) \ldots, \quad t \in [0.1,0.2].$$

$$u_3(x,t) = 1.3267\cos(0.25)^2 + 0.1331\cos(0.25\,x)\sin(0.25x) + 0.0033 + (-0.0666\cos(0.25)^2 + 1.99\times10^{-11}\cos(0.25)^4 + 0.6633\cos(0.25\,x)\sin(0.25x) + 0.0333)(t - 0.2) \ldots \quad t \in [0.2,0.3].$$

By using the nodes $t_i = mh$, divide the interval $[\,0,2\,]$ into 20 subintervals $[t_{m-1}, t_m], m = 1,2,\ldots,20$, equally sized with $h = 0.1$. Then, follow the multi-step scheme for approximate solution.

Figure 1 shows the exact solution, Figure 2 shows graph of approximate solution MMRDTM for $t \in [0,2]$ and $x \in [0,1]$ while Figure 3 shows graph of approximate solution MRDTM for $t \in [0,2]$ and $x \in [0,1]$. Therefore, obviously the multi-step approximate solutions for this type of nonlinear KdV equations obtained the exact solutions. The performance error analyses obtained by MMRDTM are summarized in Table 1 for $x = 1$.

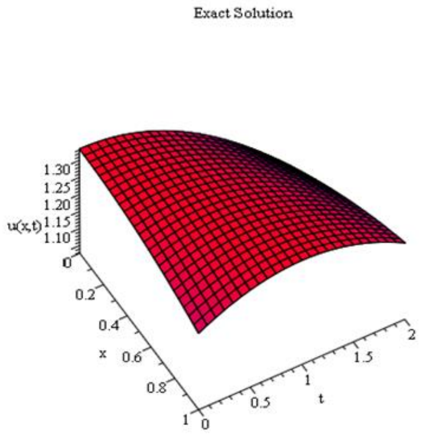

Fig. 1. Exact solution for Example 1

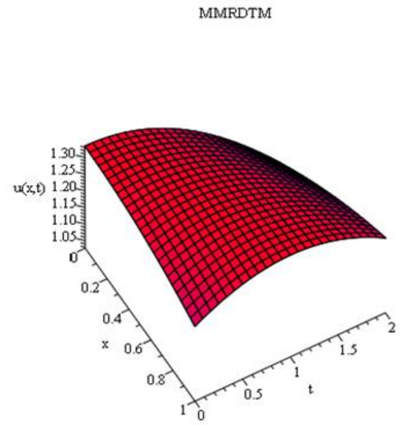

Fig. 2. MMRDTM for Example 1

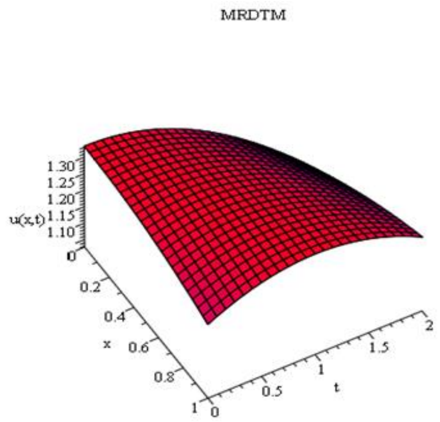

Fig. 3. MRDTM for Example 1

Table 1
Comparison error results of MMRDTM and MRDTM for Example 1

t	Exact Solution	Absolute Error MMRDTM	Absolute Error MRDTM
0.1	1.266964735	0.000	1.000×10^{-9}
0.2	1.280707329	0.000	1.000×10^{-9}
0.3	1.292915142	1.000×10^{-9}	0.000
0.4	1.303557659	0.000	1.000×10^{-9}
0.5	1.312608281	1.000×10^{-9}	4.000×10^{-9}
0.6	1.320044385	2.000×10^{-9}	1.300×10^{-8}
0.7	1.325847385	1.000×10^{-9}	3.800×10^{-8}
0.8	1.330002777	1.000×10^{-9}	9.500×10^{-8}
0.9	1.332500174	2.000×10^{-9}	2.120×10^{-7}
1.0	1.333333333	0.000	4.380×10^{-7}
1.1	1.332500174	1.000×10^{-9}	8.400×10^{-7}
1.2	1.330002777	0.000	1.524×10^{-6}
1.3	1.325847385	2.000×10^{-9}	2.631×10^{-6}
1.4	1.320044385	3.000×10^{-9}	4.355×10^{-6}
1.5	1.312608281	3.000×10^{-9}	6.956×10^{-6}
1.6	1.303557659	3.000×10^{-9}	1.077×10^{-5}
1.7	1.292915142	2.000×10^{-9}	1.621×10^{-5}
1.8	1.280707329	3.000×10^{-9}	2.380×10^{-5}
1.9	1.266964735	3.000×10^{-9}	3.421×10^{-5}
2.0	1.251721708	3.000×10^{-9}	4.819×10^{-5}

3.2 Numerical Example 2

Next, consider nonlinear KdV equations as follows [23]:

$$u_t - 3(\,u^2)_x + \,u^2{}_{xxx} = 0, \tag{10}$$

Subject to initial conditions

$u(x,0) = 6x.$

The exact solution is $\frac{6x}{1-36t}$.

Using basic properties of MMRDTM and then applying MMRDTM to Equation (10), will obtain

$$U_{k+1,i}(x) = \left(\frac{1}{(k+1)}\right)\left(3\frac{\partial}{\partial x}\left(A_{k,i}(x)\right) - \frac{\partial^3}{\partial x^3}\left(A_{k,i}(x)\right)\right), \tag{11}$$

with transformed initial condition

$$U_0(x) = 6x. \tag{12}$$

The solution in Equation (10), becomes the exact Solitary Wave solution. Next, set of values $\{U_6(x)\}_{k=0}^{6}$ of the inverse transformations gives the 6-terms approximate solution as follows,

$$u_1(x,t) = 6x + 216xt + 7776xt^2 + 279936xt^3 + 10077696xt^4 + 362797056xt^5 + 13060694016xt^6, \quad t \in [0,0.1].$$

$$u_2(x,t) = -2.308x + 31.953x(t-0.1) - 442.421x(t-0.1)^2 + 6125.84x(t-0.1)^3 - 84819x(t-0.1)^4 + 1.17\times 10^6 x(t-0.1)^5 - 1.626\times 10^7 x(t-0.1)^6, \quad t \in [0.1,0.2]$$

$$u_3(x,t) = -0.967x + 5.619x(t-0.2) - 32.627x(t-0.2)^2 + 189.448x(t-0.2)^3 - 1100x(t-0.2)^4 + 6387x(t-0.2)^5 - 37087x(t-0.2)^6, \quad t \in [0.2,0.3].$$

Figure 4 shows the exact solution, Figure 5 shows graph of approximate solution MMRDTM for $t \in [0,2]$ and $x \in [0,1]$ while Figure 6 shows graph of approximate solution MRDTM for $t \in [0,2]$ and $x \in [0,1]$. Therefore, it shows that the multi-step approximate solutions of MMRDTM and MRDTM for this type of nonlinear KdV equations have good agreement with the exact solution.

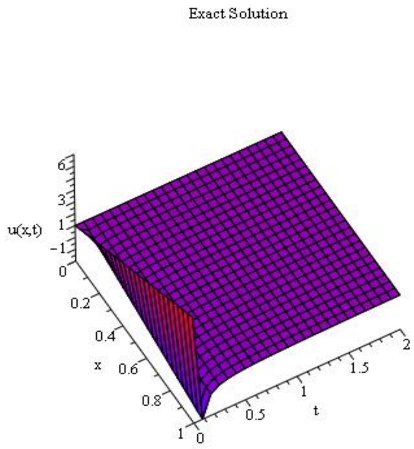

Fig. 4. Exact solution for Example 2

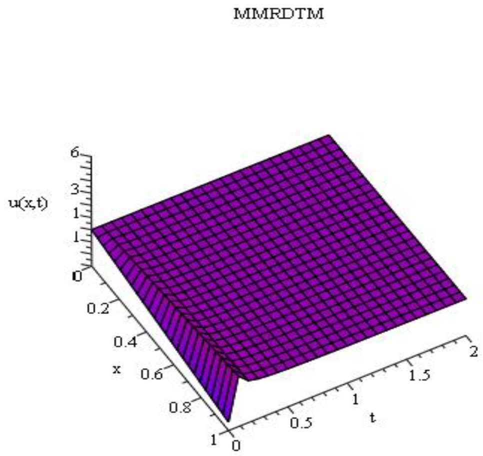

Fig. 5. MMRDTM for Example 2

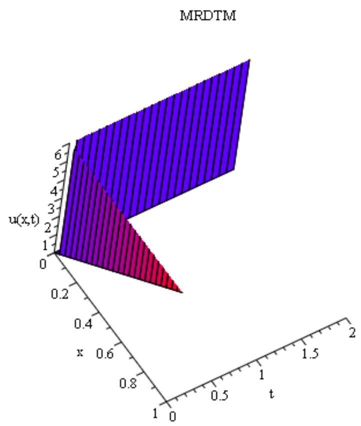

Fig. 6. MRDTM for Example 2

4. Conclusions

Compactons are used in many fields of physics and scientific applications. In order to examine the role of nonlinear dispersion in pattern formation in the liquid drops, $K(n,n)$ was developed as a simplified model [3]. Equations with Compact Solutions also found applications such as fluid mechanics, thin viscous film lubrication approximation [3], Bose Einstein condensates, long non-linear surface waves in rotating ocean when the high-frequency dispersion is null, ventricle aorta pulse propagation, magma dynamic dispersive models or, particle wave functions in nonlinear quantum mechanics [3]. The series of solutions of NKdV equations of compacton using MMRDTM is successfully applied in this paper. We compared the obtained solutions with exact solutions and MRDTM. The improvement is made by substituting the nonlinear term for its Adomian polynomials and adapting a multi-step approach. The obtained results verified that the approximate solutions of NkdV equations with compact support are obtained with high accuracy. As a conclusion, the MMRDTM is more effective, consistent, and precise than the MRDTM in obtaining an analytic approximate solution for these types of equations. All computations in this paper had been carried out by using Maple 13.

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