SEMARAK ILMU
PUBLISHING
202103268166(003316878-P)

Journal of Advanced Research in Fluid
Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Analytical Study of MHD Mixed Convection Flow for Maxwell Nanofluid Through a Vertical Cone with Porous Material in the Presence of Variable Thermal Conductivity and Soret, Dufour Effects

Indira Sundaram[1,*], Srinivasa Raju Rallabandi[2]

[1] Department of Mathematics, CVR College of Engineering, Hyderabad, 501506, Telangana State, India
[2] Department of Mathematics, GITAM (Deemed to be university), Hyderabad campus, Rudraram, Sanga Reddy (Dt), 502329, Telangana State, India

ARTICLE INFO

Article history:
Received 28 January 2023
Received in revised form 12 May 2023
Accepted 20 May 2023
Available online 5 June 2023

Keywords:
Mixed convection; Maxwell nanofluid; MHD; chemical reaction; diffusion thermo and thermal radiation

ABSTRACT

The key purpose of the existing article is to discuss the magnetohydrodynamic (MHD) mixed convection flow for Maxwell nanofluid is deliberated a vertical cone with porous material. A variable thermal conductivity and Dufour, Soret effects are also taken into consideration. The modeled equations are transformed into a set of non-linear ODEs by employing similar transformable variables. These equations are then solved numerically using the shooting method, through the fourth-order Runge–Kutta integration procedure. Effects of some prominent physical parameters, such as diffusion thermo, Prandtl number, thermophoresis parameter, and magnetic parameter on the velocity, temperature, and concentration profiles are discussed graphically and numerically. The main outcomes of this investigation are that Velocity slows down with augmentation in Maxwell and magnetic parameters. Temperature increases with radiation and thermophoresis parameters and reduces with growing values of Prandtl number and Brownian motion parameters. The values of skin-friction coefficient, Nusselt number coefficient and Sherwood number coefficient are presented in table. A comparison with previously reported data is made and an excellent agreement is noted.

1. Introduction

Non-Newtonian flows have attained considerable significance due to its applications in the fields of applied science and engineering. Viscoelastic fluid is a subclass of non-Newtonian fluid that exhibit both viscous and memory effect after the removal of applied stress. Some common viscoelastic fluids are flour dough, egg white, polymers, bitumen, blood and paints. Viscoelastic impacts are primarily essential when there are abrupt changes in the strain rate as during contractions/expansions, pulsating flows and during start-up flow or stoppage. Maxwell model designates the viscoelastic effects in terms of stress relaxation time that is the time required for the elastic effects to decay. Viscoelastic materials are used in automobile bumpers, on computer drives to protect from

---

[*] *Corresponding author.*
E-mail address: indirakannan75@gmail.com

https://doi.org/10.37934/arfmts.106.2.129142

mechanical shock, in helmets (the foam padding inside), in wrestling mats, in shoe insoles to reduce impact transmitted to a person's skeleton. Synthetic viscoelastic materials can be injected directly into an osteoarthritic knee, enveloping cartilage-deficient joints and acting as a lubricant and shock absorber. Abel *et al.,* [1] performed numerical computations for steady flow of Maxwell fluid in view of isothermal stretched surface. Hayat *et al.,* [2] analyzed the problem of Maxwell fluid with effects of melting heat transfer via homotopic technique. Thermally stratified flow of Maxwell fluid with radiation effect is explored by Hayat *et al.,* [3]. Mustafa *et al.,* [4] presented numerical analysis of Maxwell nanofluid over an exponentially stretched plate. Radiative flow of Maxwell fluid in addition to non-uniform heat source and slip effects is inspected by Zheng *et al.,* [5]. Hayat *et al.,* [6] investigated nonlinear convective flow of Maxwell fluid with Cattaneo Christov double diffusion by variable thickness sheet.

Magnetohadrodynamics (MHD) is the study of those liquids which are electrically conducted such as salty water, electrolytes, plasma and liquid metals etc. The term MHD was first introduced by Alfen [7]. This type of fluid has a number of engineering and industrial applications such as growth of crystals, reactors cooling, magnetic drug targeting, MHD sensors and power generation. MHD is dependent on intensity of magnetic induction. Gul *et al.,* [8] discussed hybrid nanofluid effect upon MHD boundary layer flow for viscous fluid. K. Raghunath [9] has studied Study of Heat and Mass Transfer of an Unsteady Magnetohydrodynamic Nanofluid Flow Past a Vertical Porous Plate in the Presence of Chemical Reaction, Radiation and soret Effects. Raghunath *et al.,* [10] has analyzed Diffusion Thermo and Chemical Reaction Effects on Magnetohydrodynamic Jeffrey Nanofluid over an Inclined Vertical Plate in the Presence of Radiation Absorption and Constant Heat Source. Maatoug *et al.,* [11] have expressed Variable chemical species and thermo-diffusion Darcy–Forchheimer squeezed flow of Jeffrey nanofluid in horizontal channel with viscous dissipation effects. Omar *et al.,* [12] have possessed Hall Current and Soret Effects on Unsteady MHD Rotating Flow of Second-Grade Fluid through Porous Media under the Influences of Thermal Radiation and Chemical Reactions. Deepthi *et al.,* [13] have discussed Recent Development of Heat and Mass Transport in the Presence of Hall, Ion Slip and Thermo Diffusion in Radiative Second Grade Material: Application of Micromachines.

Magnetic field including thermal radiation effects for nanofluid flow has been analyzed along stretching surface through Khan *et al.,* [14]. The flow field was discussed by them with dissimilar time steps and reported that average shear stress reductions with the development of magnetic field are observed. MHD boundary layer nanofluid flow regarding heat with mass transfer has been stated through porous media by Haile and Shankar [15] with considering thermal radiation including viscous dissipation with chemical reaction effects. They were considered copper (Cu)-water and Al2O3-water nanofluids and noted out that velocity field decreases with increase of magnetic field. Raghunath *et al.,* [16] have studied processing to pass unsteady MHD flow of a second-grade fluid through a porous medium in the presence of radiation absorption exhibits Diffusion thermo, hall and ion slip effects. Raghunath *et al.,* [17] have studied Influence of MHD mixed convection flow for maxwell nanofluid through a vertical cone with porous material in the existence of variable heat conductivity and diffusion. Raghunath *et al.,* [18] Radiation absorption on MHD Free Conduction flow through porous medium over an unbounded vertical plate with heat source. Li *et al.,* [19] have studied Effects of activation energy and chemical reaction on unsteady MHD dissipative Darcy–Forchheimer squeezed flow of Casson fluid over horizontal channel**.** Suresh Kumar [20] have expressed Numerical analysis of magneto hydrodynamics Casson nanofluid flow with activation energy, Hall current and thermal radiation.

For considering the steady case, MHD mixed convective nanofluid flow through porous medium which has been deliberated past along a stretching sheet by Ferdows *et al.,* [21]. They concluded that velocity together with temperature increases while concentration decreases gradually with increase of Eckert number. Heat transfer physical characteristics of flow field with three dissimilar categories of nanofluid pass through permeable stretching/shrinking surface has been considered and observed through porous medium via Pal *et al.,* [22]. They found with the increment of suction/injection parameters as local Nusselt number rises for stretching sheet while decreasing for shrinking sheet.

In nature and innovation, various transport procedures can be discovered in various manners in which heat and mass transfer happen because of the buoyancy forces produced by alterations in warmth and concentration. At the point when the exchange of warmth and mass in a moving liquid happens immediately, the connections among the motions and the driving potentials are highly unpredictable. Fluxes of energy are created by the gradients of temperature and concentration. A composition gradient called the Dufour effect produces the energy circulation, while mass circulations produced by the temperature gradient is called the Soret effect. These outcomes play a vital role when there are differences in density in the stream regime. For instance, when species in a liquid domain enter a surface, the Soret and Dufour impacts may become huge significant with an alternate (lower) thickness than the encompassing fluid. For heat and mass exchange issues, Soret and Dufour impacts are exceptionally significant for medium atomic weight gasses in double liquid frameworks, which are frequently found in fast optimal design and synthetic cycle building. Chamkha and Rashad [23] have studied Unsteady Heat and Mass Transfer by MHD Mixed Convection Flow from a Rotating Vertical Cone with Chemical Reaction and Soret and Dufour Effects. Rashid and Chamkha [24] have reviewed Heat and Mass Transfer by Natural Convection Flow about a Truncated Cone in Porous Media with Soret and Dufour Effects, International Journal of Numerical Methods for Heat and Fluid Flow. Kabeir *et al.,* [25] have expressed Heat And Mass Transfer By Unsteady Natural Convection Over A Moving Vertical Plate Embedded In A Saturated Porous Medium With Chemical Reaction, Soret And Dufour Effects. Al-Mudhaf, *et al.,* [26] has possessed Soret and Dufour effects on unsteady double diffusive natural convection in porous trapezoidal enclosures. Very recently Raghunath and Mohanaramana [27] have studied Hall, Soret, And Rotational Effects On Unsteady MHD Rotating flow of A Second-Grade Fluid Through a Porous Medium in the Presence of Chemical Reaction and aligned magnetic field. Raghunath *et al.,* [28] have discussed Hall and Ion Slip Radiative Flow of Chemically Reactive Second grade through Porous Saturated Space via Perturbation approach. Raghunath *et al.,* [29] studied Effects of Soret, Rotation, Hall, and Ion Slip on unsteady MHD flow of a Jeffrey Fluid through a Porous Medium in the Presence of Heat absorption and chemical reaction. Jawad *et al.,* [30] have studied analytical study of MHD mixed convection flow for Maxwell nanofluid with variable thermal conductivity and Soret and Dufour effects.

The principal aim of the present work is to study the MHD Mixed Convection Flow for Maxwell Nanofluid through a vertical cone through a porous material in the presence of Variable Thermal Conductivity and Soret, Dufour Effects has been studied. Lie's scaling` group transformations (also known as Lie group analysis or as symmetry analysis) can be used to obtain similarity transformations that can reduce a system of governing partial differential equations and associated boundary conditions to a system of ordinary differential equations. With this transformation, a third order and a second order ordinary differential equations corresponding to momentum, energy and concentration equations are derived. These equations are solved with the help of Runge Kutta fourth order along with shooting technique. The effects of different flow parameters on velocity, temperature and concentration profiles are investigated and analyzed with the help of graphical representation.

2. Flow Governing Equations

Two-dimensional electrically conducting, viscous, non-compressible, boundary-layer fluid flow containing Maxwell fluid and nanofluid particles approaching a vertical cone in the presence of a magnetic field and porous media with the effect of Soret and Dufour effects will be studied in this present study effort. Figure 1 depicts the physical geometry of this study at y = 0. Brownian motion and thermophoresis effects are considered using the Buongiorno model [31] for the nanofluid. Further, the effects of viscous dissipation and Joule heating are ignored.

i. The following hypotheses underlie this investigation.
ii. The coordinate system is preferred as the x-axis is equivalent with the flow direction over the cone surface.
iii. The temperature and nanoparticle volume fraction of the ambient fluid are $T_\infty$and C_∞.
iv. An external magnetic field of strength B_o is applied in the direction of the y-axis.
v. It is assumed that T_w, to be determined, is the result of convective heating process which is characterized by a temperature T_f and a heat transfer coefficient h_f , and C_w is the nanoparticle volume fraction at the surface of the cone (y = 0).
vi. The rheological equation for a non-Newtonian fluid is defined as,$\tau_1 = \tau_o + \mu_1 \alpha^*$ (1)

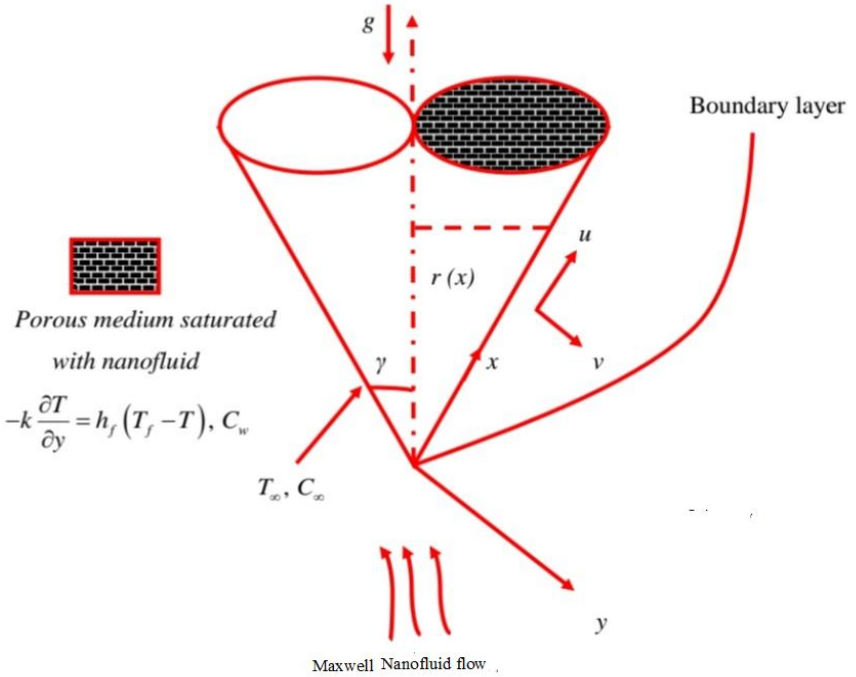

Fig. 1. Physical configuration of the problem

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial x^2} - \lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\,\partial y}\right) + g\begin{bmatrix}(1 - C_\infty)_{\rho f\infty}\beta(T - T_\infty) - \\ \left(\rho_p - \rho_{f_\infty}\right)(C - C_\infty)\end{bmatrix} Cos\gamma - \left(\frac{\sigma B_0^2}{\rho_f}\right)u - \left(\frac{\mu}{k^*}\right)u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T^2}{\partial y^2}\right) \tag{4}$$

For this flow, corresponding boundary conditions are

$$\left.\begin{array}{c} u=0, v=0, -k\left(\frac{\partial T}{\partial y}\right)=h_f\left(T_f-T\right), C=C_w aty=0 \\ u\rightarrow 0, T\rightarrow T_\infty, C\rightarrow C_\infty asy\rightarrow\infty \end{array}\right\} \tag{5}$$

The radiative heat fluxq_r (using Roseland approximation followed [32]) is defined as

$$q_r=-\frac{4\sigma^*}{3K^*}\left(\frac{\partial T^4}{\partial y}\right) \tag{6}$$

We assume that the temperature variances inside the flow are such that the termT^4can be represented as linear function of temperature. This is accomplished by expandingT^4in a Taylor series about a free stream temperatureT_∞ as follows

$$T^4=T_\infty^4+4T_\infty^3(T-T_\infty)+6T_\infty^2(T-T_\infty)^2+\ldots\ldots\ldots.. \tag{7}$$

After neglecting higher-order terms in the above equation beyond the first degree term in$(T-T_\infty)$, we get

$$T^4\cong 4T_\infty^3 T-3T_\infty^4 \tag{8}$$

Thus substituting Eq. (8) in Eq. (6), we get

$$q_r=-\frac{16T_\infty^3\sigma^*}{3K^*}\left(\frac{\partial T}{\partial y}\right) \tag{9}$$

Using (9), Eq. (3) can be written as

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\alpha\left(\frac{\partial^2 T}{\partial y^2}\right)+\tau\left(D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right)+ \\ \frac{1}{\rho C_p}\frac{16T_\infty^3\sigma^*}{\partial K^*}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2} \tag{10}$$

The following similarity variables are introduced for solving governing equations (2), (6) and (4) as

$$\eta=\left(\frac{y}{x}\right)Ra_x^{\frac{1}{4}}, \psi=\alpha Ra_x^{\frac{1}{4}}f(\eta), \theta=\frac{T-T_\infty}{T_w-T_\infty}, \phi=\frac{C-C_\infty}{C_w-C_\infty} \tag{11}$$

Where$Ra_x=\frac{g\beta\rho_{f\infty}(T-T_\infty)(1-C_\infty)\cos(\gamma)x^3}{\mu\alpha}$is the Rayleigh number.Here, *'r'* can be approximated by the local radius of the cone, if the thermal boundary layer is thin, and is related to the *x* coordinate by$r=x\sin(\gamma)$. Substituting Eq. (11) into Eqs. (2), (3) and (4), we get the following system of non-linear ordinary differential equations

$$f^{'''} - f^{'2} + ff^{''} + \Lambda(2ff^{'}f^{''} - f^2 f^{'''}) + (\theta - N_r\varphi) - (M+K)f^{'} = 0 \tag{12}$$

$$\theta^{''}(1+R_d) + Pr\, f\, \theta^{'} + Pr\, N_b\left(\theta^{'}\varphi^{'} + \frac{N_t}{N_b}\theta^{'2}\right) + PrD_u \quad \varphi^{'} = 0 \tag{13}$$

$$\varphi^{''} + Pr \quad L_e f\varphi^{'} + \frac{N_t}{N_b}\theta^{''} = 0 \tag{14}$$

The corresponding boundary conditions (5) become

$$\left.\begin{array}{c} f(\eta) = 0, f^{'}(\eta) = 1, \theta^{'}(\eta) = -Bi[1-\theta(\eta)], \phi(\eta) = 1 at\eta = 0 \\ f^{'}(\eta) \to 0, \theta(\eta) \to 0, \phi(\eta) \to 0 as\eta \to \infty \end{array}\right\} \tag{15}$$

where prime denotes differentiation with respect to η, and the significant thermophysical parameters indicating the flow dynamics are defined by

$$\begin{cases} M = \frac{\sigma B_0^2 x}{\rho Ra_x^{1/2}}, Pr = \frac{\nu}{\alpha} = \frac{\nu\rho C_p}{k}, N_b = \frac{\tau D_B(C_w - C_\infty)}{\alpha}, N_t = \frac{\tau D_T(T_w - T_\infty)}{\alpha}, \\ Du = \frac{D_M k_T (C_w - C_\infty)}{C_S C_p v a^2 (T_w - T_\infty)}, N_r = \frac{(\rho_p - \rho_{f\infty})(C_w - C_\infty)}{\rho_{f\infty}\beta(T_w - T_\infty)(1 - C_\infty)}, L_e = \frac{\alpha}{D_B} \\ R_d = \frac{14\sigma^* T_\infty^3}{3kK^*}, B_i = \frac{h_f x}{kRa_x^{1/2}}, \Lambda = \lambda_1 \end{cases} \tag{16}$$

3. Physical Quantities of Interests

The local skin friction coefficient in the direction of x Cf$_x$, and in the direction of z Cf$_z$, the local Nusselt number Nu$_x$, and the local Sherwood number Sh$_x$ are the physical quantities of relevance that influence the flow. These numbers have the following definitions

$$Cf = \frac{2\tau_w}{\rho(ax)^2}, Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, Sh_x = \frac{xj_w}{D_B(C_w - C_\infty)} \tag{17}$$

where τw, qw and jw are the wall skin friction, wall heat flux and wall mass flux respectively given by

$$\tau_w = \mu\left[\frac{\partial u}{\partial y}\right]_{y=0}, q_w = -k\left[\frac{\partial T}{\partial y}\right]_{y=0}, j_w = -D_B\left[\frac{\partial C}{\partial y}\right]_{y=0} \tag{18}$$

The coefficient of skin friction, the Nusselt number, and the Sherwood number are all expressed in their non-dimensional versions in terms of the similarity variable as follows

$$Re_x^{1/2}\, C\, f = 2f^{''}(0), Re_x^{\frac{1}{2}}\, N\, u_x = -\theta^{'}(0), Re_x^{\frac{1}{2}}\, S\, h_x = -\varphi^{'}(0) \tag{19}$$

4. Solution Methodology

The system of non-linear ODEs (12-14) subject to the boundary conditions 15 has been solved by the shooting method for various values of the involved parameters. We observed through graphs that for η > 7, there is no significant variation in the behavior of solutions. Therefore, on the basis of such computational experiments, we are pondering [0, 7] as the domain of the problem instead of

[0, ∞). We denote f by y_1, θ by y_4 and ϕ by y_7 for converting the boundary value problem (12-15) to the following initial value problem consisting of 7 first order differential equations.

$$y'_1 = y_2,$$

$$y'_2 = y_3,$$

$$y'_3 = \frac{1}{(1+y_1^2)}\left(y_2^2 - y_1y_3 - \Lambda(2y_1y_2y_3) - (y_4 - N_r y_6) + (M+K)y_2\right),$$

$$y'_4 = y_5,$$

$$y'_5 = \frac{-1}{1+R}\left(Pry_1\,y_5 + PrN_b\left(y_5y_7 + \frac{Nt}{Nb}(y_5)^2\right) + Pr\quad Duy_5\right)$$

$$y'_6 = y_7,$$

$$y'_7 = -Pr\quad L_e y_1 y_7 - \frac{N_t}{N_b}y'_5$$

To solve the above initial value problem arising in the shooting method, Runge Kutta method of order four is used. Classical Newton method is applied for the refinement of initial guesses Q1, Q2 and Q3 subject to the tolerance of $\varepsilon = 10^{-7}$.

5. Results and Discussion

We have obtained the solution of Eq. (12)- (14) Subject to the Eq. (15) with the help of MATLAB software by its bvp4c methodology. We have sketched graphs to examine the influence of numerous parameters appearing in equations for dimensionless velocity, temperature, and concentration. The parameter's ranges are taken as Le=1.0, Pr=0.2, M=2.0, Rd=1.0, Nb=0.8, Nt=1.2, Λ=0.3, Du=0.5.

For different values of the magnetic parameter M, the velocity and the temperature profiles are plotted in Figure 2 and 3 respectively. From Figure 2, it is clear that an increase in the magnetic parameter M leads to a fall in the velocity. The effects of the magnetic parameter to increase the temperature profiles are noticed from Figure 3. The presence of Lorentz force retards the force on the velocity field and therefore the velocity profiles decreases with the effect of magnetic parameter. This force has the tendency to slow down the fluid motion and the resistance offered to the flow. Therefore, it is possible for the increase in the temperature.

Figure 4 reflect the impact of the Deborah number Λ, which is a ratio of the fluid relaxation time to its characteristic time scale on the velocity profiles. When the shear stress is applied on the fluid, the time in which it gains its equilibrium position is called relaxation time. This time is higher for the fluids having high viscosity. So an increase in the Deborah number may increase the viscosity of the fluid and hence the velocity decreases as shown in Figure 4 - 6, the effect of the variation in the Deborah number on the temperature θ(η) and the nanoparticle concentration ϕ(η) is displayed respectively. It is observed that the temperature and the concentration increase with an increase in the relaxation time.

The variation of Prandtl number on temperature and concentration outlines as shown in Figure 7 and 8. It is conclude that increasing values of Prandtl number result in a thinner temperature boundary layer thickness. Fluids having larger Prandtl number have lower thermal diffusivity, and hence the temperature decreases. The same decreasing trend of Pr number on the mass concentration profile.

The influence of Brownian motion parameter on concentration and temperature profiles is depicted through Figure 9 and 10. From the figures, it can be seen that as the values of Brownian motion parameter rises, the thermal boundary layer thickness increases and at the surface, the temperature gradient demises. But we witnessed an opposite result on the concentration profiles and concentration boundary layer thickness as Brownian motion parameter upsurges. Figure 11 and 12 are devoted to demonstrate the impact of thermophoresis parameter on temperature and concentration profiles. From the figures, it is perceived that when thermophoresis parameter rises, there is an improvement of the thermal and concentration boundary layer thickness.

The temperature and nanoparticle concentration curves for different values of thermal radiation parameter are depicted in Figure 13 and 14. From the graph, it is possible to observe that as the values of thermal radiation parameter upsurge, the temperature graph and the temperature boundary layer thickness are snowballing. Figure 15 depicts the effect of Dufour parameter on temperature profiles. As the Dufour parameter increases, the energy or temperature profiles increases. The Dufour number denotes the contribution of the concentration gradients to the thermal energy flux in the flow. It can be seen that an increase in the Dufour number causes a rise in temperature. Figure 16 and 17 shows the impact of the Lewis number Le on temperature and nanoparticle concentration outlines respectively. It is observed that the temperature increases by increasing Le while concentration decreases with an increase in the Lewis number.

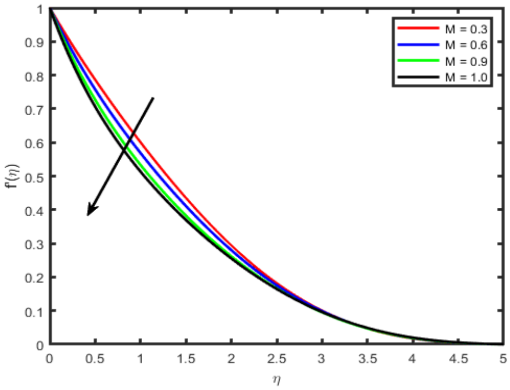

Fig. 2. Influence of M on $f^{I}(\eta)$

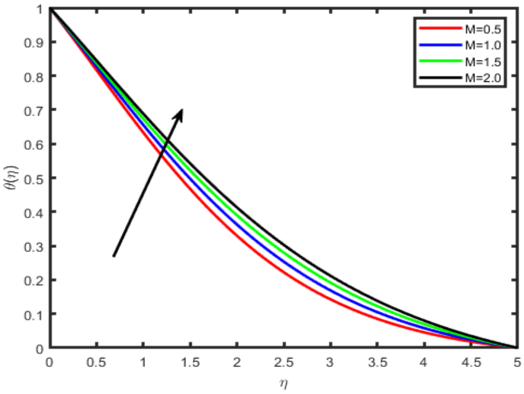

Fig. 3. Influence of M on θ(η)

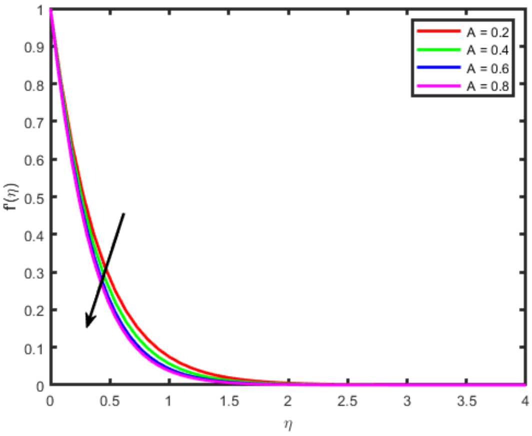

Fig. 4. Influence of Λ on $f^{I}(\eta)$

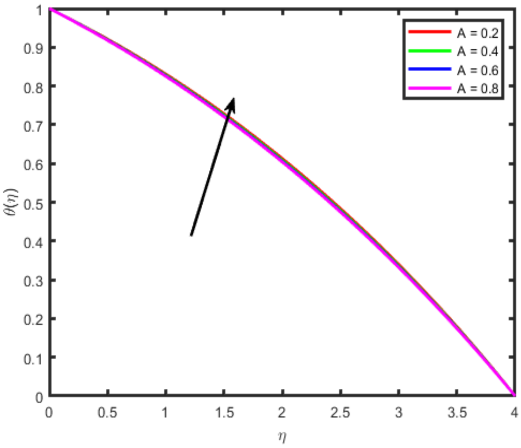

Fig. 5. Influence of Λ on θ(η)

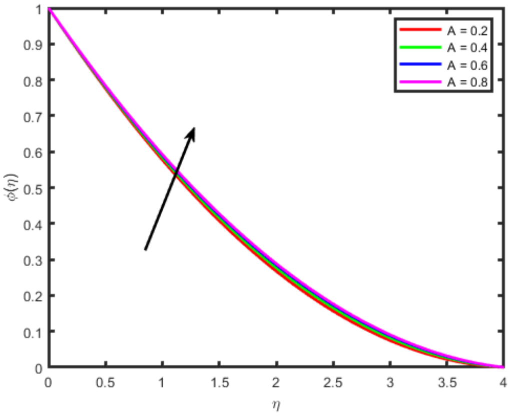

Fig. 6. Influence of Λ on ϕ(η)

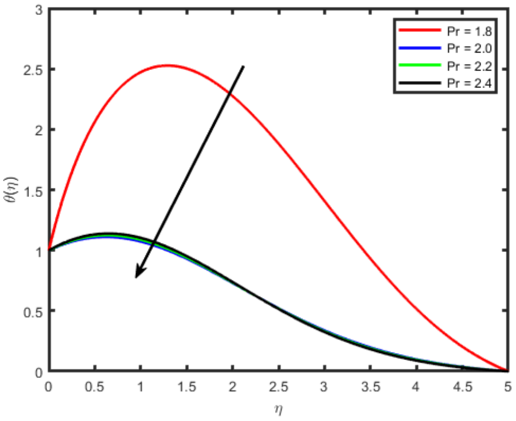

Fig. 7. Influence of Pr on θ(η)

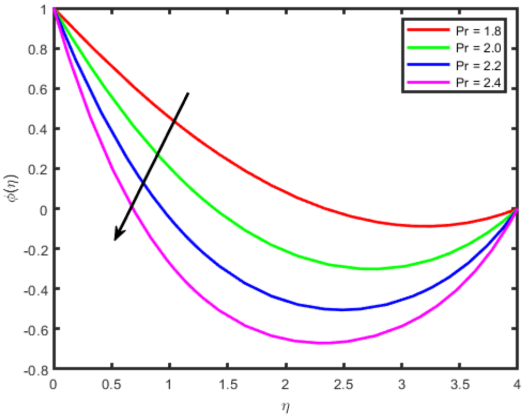

Fig. 8. Influence of Pr on ϕ(η)

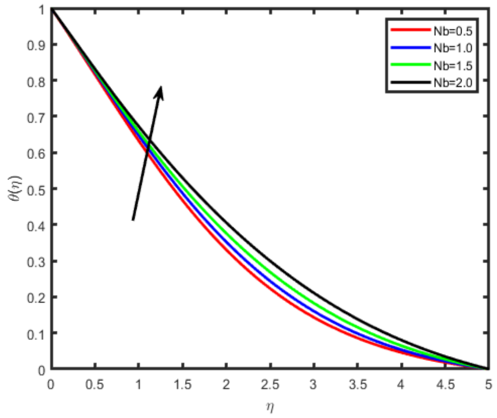

Fig. 9. Influence of Nb on θ(η)

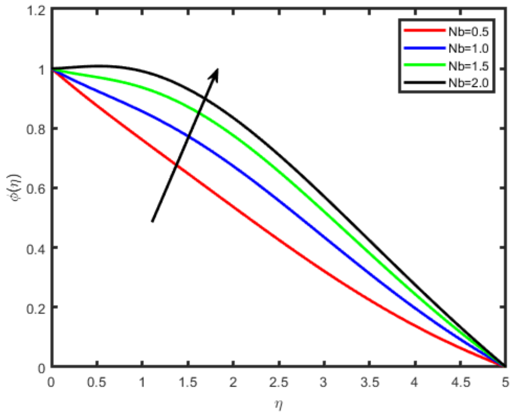

Fig. 10. Influence of Nb on ϕ(η)

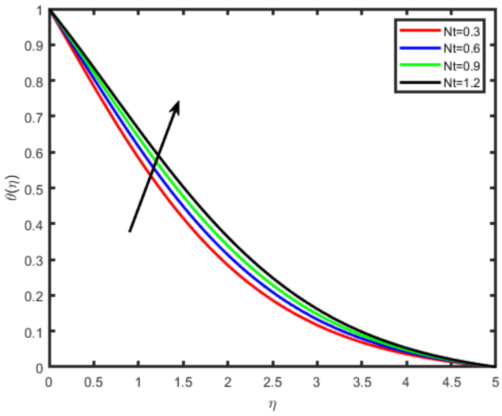

Fig. 11. Influence of Nt on θ(η)

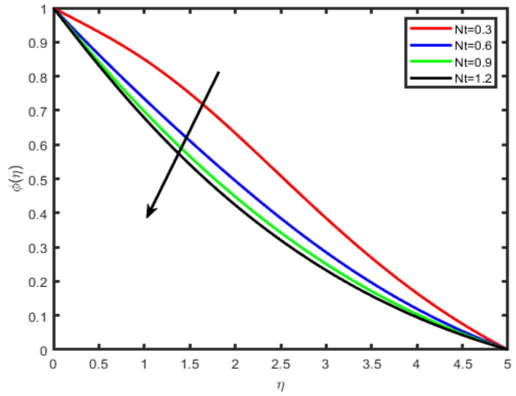

Fig. 12. Influence of Nt on ϕ(η)

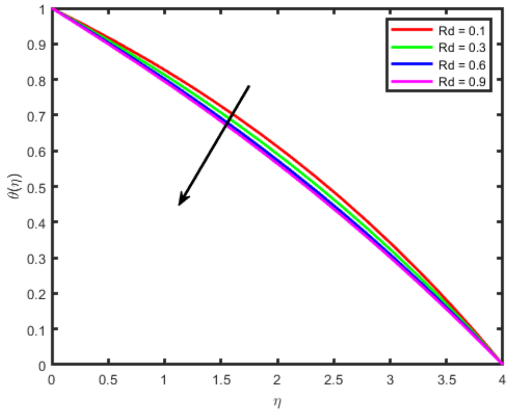

Fig. 13. Influence of Rd on θ(η)

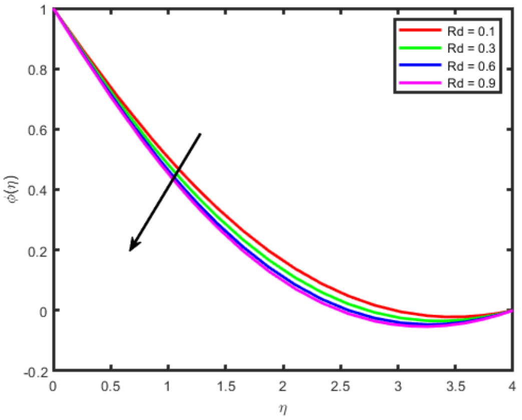

Fig. 14. Influence of Rd on ϕ(η)

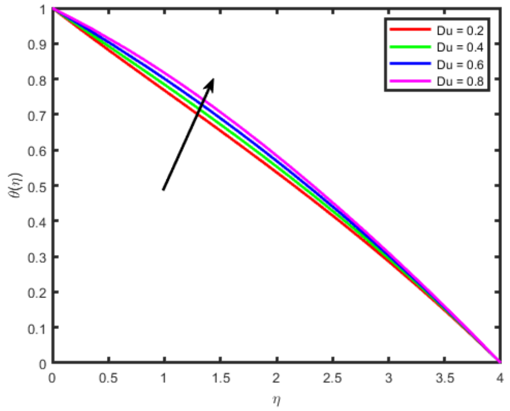

Fig. 15. Influence of Du on θ(η)

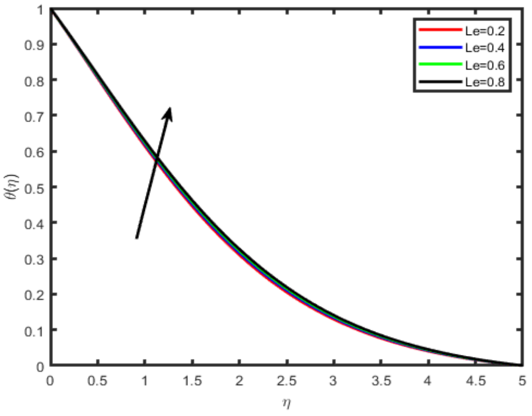

Fig. 16. Effect of Le on θ(n)

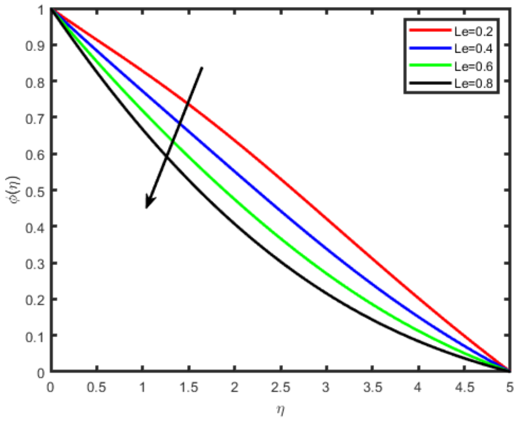

Fig. 17. Effect of Le on φ(n)

The Numerical results of the local Nusselt and Sherwood numbers for various values of different parameters are tabulated in Table 1. From Table 1, it is observed that the rate of heat flux decreases for the increasing values of Deborah number Λ and magnetic parameter M. It happens due to the fact that an increase in the magnetic parameter will enhance the Lorentz force which slows down the motion of the fluid and resultantly the rate of heat flux is reduced. The same phenomenon is observed for the increasing value of Λ and M for the case of Sherwood number ϕl(0). An increment is observed in the rate of the heat flux for the thermal radiation parameter Rd. Similarly for thermal radiation parameter Rd is a decreasing trend is noticed for the mass transfer rate. The influence of the thermophoresis parameter Nt, Brownian motion parameter Nb, Prandtl number Pr, land the Lewis number Le on the rate of heat and mass transfer is also shown in Table 2. From the numerical values, it is noticeable that Nt, Nb and Le have a decreasing effect on Nusselt number while this increases for the increasing values of Prandtl number. On the other hand, Sherwood number decreases for the increasing thermophoresis parameter Nt, however the rate of mass transfer enhances for the increasing values of Nb, Pr and Le.

In order to verify the validity and accuracy of the present analysis, the results for the mass transfer $-\varphi'(0)$ were compared with those reported by jawad *et al.,* (25). The comparison in the above cases is found to be in excellent agreement as shown in Table 2.

Table 1
The Numerical results of the local Nusselt and Sherwood numbers for various values of different parameters

Λ	M	Rd	Bi	Nt	Nb	Pr	Du	Le	Nu_z	Sh_z
1.0	2.0	1.0	0.5	1.2	0.8	0.2	0.5	1.0	1.578521	0.7542141
0.2									1.495214	0.714245
0.4									1.401547	0.648752
0.6									1.364587	0.614578
	0.5								1.245775	0.612478
	1.0								1.002544	0.607247
	1.5								0.895475	0.601247
		0.1							0.787852	0.354785
		0.3							0.894578	0.547851
		0.6							0.942147	0.754782
			0.2						1.078541	0.654785
			0.3						1.987852	0.697852
				0.5					1.345785	0.987852
				1.0					1.297854	0.854578
					0.3				1.457851	0.785452
						0.6			1.120124	0.987852
							0.2		0.785452	0.457852
							0.4		0.794521	0.403214
								1.0	1.245785	0.647851
								1.2	1.124785	0.874512

Table 2
Comparison of present Sherwood number with the published Sherwood number results of Jawad *et al.,* [30] when $\beta i = 0$

Sr	Le	Sherwood number results of jawad *et al.,* [30] results	Present Sherwood number results
0.2	2	0.489 438 9	0.47512484
0.4		0.487 454 0	0.44578541
0.6		0.483 637 2	0.48451235
		1.168 733 1	1.20747852
		1.230 647 3	1.27521452
		1.292 579 7	1.24512450

6. Conclusion

In this work, we have studied Analytical Study of MHD Mixed Convection Flow for Maxwell Nanofluid through a vertical cone with porous material in the presence of Variable Thermal Conductivity and Soret, Dufour Effects. The resulting partial differential equations, which describe the problem, are transformed in to ordinary differential equations solved by numerically by fourth order Runge-Kutta method along with shooting technique. Velocity, temperature and concentration profiles are presented graphically and analyzed. The findings of the numerical results can be summarized as follows

i. An increase in the magnetic parameter leads to a fall in the velocity and rise in the temperature profiles.
ii. It is also found that the temperature profiles increase whereas the concentration profiles decrease with the increase of Brownian motion parameter.
iii. The impact of Du on θ and φ are opposite.
iv. The temperature and concentration profiles tend to fall when the Prandtl number (Pr) is raised.
v. The temperature increases by increasing Le while concentration decreases with an increase in the Lewis number

References

[1] Subhas Abel, M., Jagadish V. Tawade, and Mahantesh M. Nandeppanavar. "MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet." *Meccanica* 47 (2012): 385-393. https://doi.org/10.1007/s11012-011-9448-7

[2] Hayat, T., M. Mustafa, S. A. Shehzad, and S. Obaidat. "Melting heat transfer in the stagnation-point flow of an upper-convected Maxwell (UCM) fluid past a stretching sheet." *International journal for numerical methods in fluids* 68, no. 2 (2012): 233-243. https://doi.org/10.1002/fld.2503

[3] Hayat, T., S. A. Shehzad, Hamed H. Al-Sulami, and S. Asghar. "Influence of thermal stratification on the radiative flow of Maxwell fluid." *Journal of the Brazilian Society of Mechanical Sciences and Engineering* 35 (2013): 381-389. https://doi.org/10.1007/s40430-013-0036-8

[4] Mustafa, Meraj. "Cattaneo-Christov heat flux model for rotating flow and heat transfer of upper-convected Maxwell fluid." *Aip Advances* 5, no. 4 (2015): 047109. https://doi.org/10.1063/1.4917306

[5] Zheng, Liancun, Ning Liu, and Xinxin Zhang. "Maxwell fluids unsteady mixed flow and radiation heat transfer over a stretching permeable plate with boundary slip and nonuniform heat source/sink." *Journal of heat transfer* 135, no. 3 (2013). https://doi.org/10.1115/1.4007891

[6] Hayat, Tasawar, Zeenat Bashir, Sumaira Qayyum, and Ahmed Alsaedi. "Investigation of double diffusion Cattaneo-Christov model in mixed convection flow by variable thickness surface." *Results in physics* 7 (2017): 3873-3881. https://doi.org/10.1016/j.rinp.2017.10.003

[7] Alfvén, Hannes. "Existence of electromagnetic-hydrodynamic waves." *Nature* 150, no. 3805 (1942): 405-406. https://doi.org/10.1038/150405d0

[8] Gul, Taza, Kashifullah, M. Bilal, Wajdi Alghamdi, M. Imran Asjad, and Thabet Abdeljawad. "Hybrid nanofluid flow within the conical gap between the cone and the surface of a rotating disk." *Scientific Reports* 11, no. 1 (2021): 1180. https://doi.org/10.1038/s41598-020-80750-y

[9] Raghunath, Kodi. "Study of Heat and Mass Transfer of an Unsteady Magnetohydrodynamic (MHD) Nanofluid Flow Past a Vertical Porous Plate in the Presence of Chemical Reaction, Radiation and Soret Effects." *Journal of Nanofluids* 12, no. 3 (2023): 767-776. https://doi.org/10.1166/jon.2023.1965

[10] Raghunath, K., R. Mohana Ramana, V. Ramachandra Reddy, and M. Obulesu. "Diffusion Thermo and Chemical Reaction Effects on Magnetohydrodynamic Jeffrey Nanofluid Over an Inclined Vertical Plate in the Presence of Radiation Absorption and Constant Heat Source." *Journal of Nanofluids* 12, no. 1 (2023): 147-156. https://doi.org/10.1166/jon.2023.1923

[11] Maatoug, Samah, K. Hari Babu, V. V. L. Deepthi, Kaouther Ghachem, Kodi Raghunath, Charankumar Ganteda, and Sami Ullah Khan. "Variable chemical species and thermo-diffusion Darcy–Forchheimer squeezed flow of Jeffrey nanofluid in horizontal channel with viscous dissipation effects." *Journal of the Indian Chemical Society* 100, no. 1 (2023): 100831. https://doi.org/10.1016/j.jics.2022.100831

[12] Bafakeeh, Omar T., Kodi Raghunath, Farhan Ali, Muhammad Khalid, El Sayed Mohamed Tag-ElDin, Mowffaq Oreijah, Kamel Guedri, Nidhal Ben Khedher, and Muhammad Ijaz Khan. "Hall current and Soret effects on unsteady MHD rotating flow of second-grade fluid through porous media under the influences of thermal radiation and chemical reactions." *Catalysts* 12, no. 10 (2022): 1233. https://doi.org/10.3390/catal12101233

[13] Deepthi, V. V. L., Maha MA Lashin, N. Ravi Kumar, Kodi Raghunath, Farhan Ali, Mowffaq Oreijah, Kamel Guedri, El Sayed Mohamed Tag-ElDin, M. Ijaz Khan, and Ahmed M. Galal. "Recent development of heat and mass transport in the presence of Hall, ion slip and thermo diffusion in radiative second grade material: application of micromachines." *Micromachines* 13, no. 10 (2022): 1566. https://doi.org/10.3390/mi13101566

[14] Khan, Md Shakhaoath, Md Mahmud Alam, and M. Ferdows. "Effects of magnetic field on radiative flow of a nanofluid past a stretching sheet." *Procedia Engineering* 56 (2013): 316-322. https://doi.org/10.1016/j.proeng.2013.03.125

[15] Haile, Eshetu, and B. Shankar. "Heat and mass transfer through a porous media of MHD flow of nanofluids with thermal radiation, viscous dissipation and chemical reaction effects." *American Chemical Science Journal* 4, no. 6 (2014): 828-846. https://doi.org/10.9734/ACSJ/2014/11082

[16] Raghunath, Kodi, Ravuri Mohana Ramana, Charankumar Ganteda, Prem Kumar Chaurasiya, Damodar Tiwari, Rajan Kumar, Dharam Buddhi, and Kuldeep Kumar Saxena. "Processing to pass unsteady MHD flow of a second-grade fluid through a porous medium in the presence of radiation absorption exhibits Diffusion thermo, hall and ion slip effects." *Advances in Materials and Processing Technologies* (2023): 1-18. https://doi.org/10.1080/2374068X.2023.2191450

[17] Kodi, Raghunath, Charankumar Ganteda, Abhishek Dasore, M. Logesh Kumar, G. Laxmaiah, Mohd Abul Hasan, Saiful Islam, and Abdul Razak. "Influence of MHD mixed convection flow for maxwell nanofluid through a vertical cone with porous material in the existence of variable heat conductivity and diffusion." *Case Studies in Thermal Engineering* 44 (2023): 102875. https://doi.org/10.1016/j.csite.2023.102875

[18] Raghunath, Kodi, Mopuri Obulesu, and Konduru Venkateswara Raju. "Radiation absorption on MHD free conduction flow through porous medium over an unbounded vertical plate with heat source." *International Journal of Ambient Energy* (2023): 1-9. https://doi.org/10.1080/01430750.2023.2181869

[19] Li, Shuguang, Kodi Raghunath, Ayman Alfaleh, Farhan Ali, A. Zaib, M. Ijaz Khan, Sayed M. ElDin, and V. Puneeth. "Effects of activation energy and chemical reaction on unsteady MHD dissipative Darcy–Forchheimer squeezed flow of Casson fluid over horizontal channel." *Scientific Reports* 13, no. 1 (2023): 2666. https://doi.org/10.1038/s41598-023-29702-w

[20] Suresh Kumar, Y., Shaik Hussain, K. Raghunath, Farhan Ali, Kamel Guedri, Sayed M. Eldin, and M. Ijaz Khan. "Numerical analysis of magnetohydrodynamics Casson nanofluid flow with activation energy, Hall current and thermal radiation." *Scientific Reports* 13, no. 1 (2023): 4021. https://doi.org/10.1038/s41598-023-28379-5

[21] Ferdows, M., Md Shakhaoath Khan, Md Mahmud Alam, and Shuyu Sun. "MHD mixed convective boundary layer flow of a nanofluid through a porous medium due to an exponentially stretching sheet." *Mathematical problems in Engineering* 2012 (2012). https://doi.org/10.1155/2012/408528

[22] Pal, Dulal, Gopinath Mandal, and K. Vajravelu. "Flow and heat transfer of nanofluids at a stagnation point flow over a stretching/shrinking surface in a porous medium with thermal radiation." *Applied Mathematics and Computation* 238 (2014): 208-224. https://doi.org/10.1016/j.amc.2014.03.145

[23] Chamkha, Ali J., and A. M. Rashad. "Unsteady heat and mass transfer by MHD mixed convection flow from a rotating vertical cone with chemical reaction and Soret and Dufour effects." *The Canadian Journal of Chemical Engineering* 92, no. 4 (2014): 758-767. https://doi.org/10.1002/cjce.21894

[24] Rashad, A., and Ali Chamkha. "Heat and mass transfer by natural convection flow about a truncated cone in porous media with Soret and Dufour effects." *International Journal of Numerical Methods for Heat & Fluid Flow* 24, no. 3 (2014): 595-612. https://doi.org/10.1108/HFF-05-2012-0105

[25] Rashad, A. M., S. M. M. EL-Kabeir, and M. Modather. "Heat and mass transfer by unsteady natural convection over a moving vertical plate embedded in a saturated porous medium with chemical reaction, Soret and Dufour effects." *Journal of Applied Fluid Mechanics* 8, no. 3 (2015): 453-463. https://doi.org/10.18869/acadpub.jafm.67.222.22864

[26] Al-Mudhaf, Ali F., A. M. Rashad, Sameh E. Ahmed, Ali J. Chamkha, and S. M. M. El-Kabeir. "Soret and Dufour effects on unsteady double diffusive natural convection in porous trapezoidal enclosures." *International Journal of Mechanical Sciences* 140 (2018): 172-178. https://doi.org/10.1016/j.ijmecsci.2018.02.049

[27] Raghunath, Kodi, and Ravuri Mohanaramana. "Hall, Soret, and rotational effects on unsteady MHD rotating flow of a second-grade fluid through a porous medium in the presence of chemical reaction and aligned magnetic field." *International Communications in Heat and Mass Transfer* 137 (2022): 106287. https://doi.org/10.1016/j.icheatmasstransfer.2022.106287

[28] Kodi, Raghunath, Mohanaramana Ravuri, Nagesh Gulle, Charankumar Ganteda, Sami Ullah Khan, and M. Ijaz Khan. "Hall and ion slip radiative flow of chemically reactive second grade through porous saturated space via perturbation approach." *Waves in Random and Complex Media* (2022): 1-17. https://doi.org/10.1080/17455030.2022.2108555

[29] Raghunath, Kodi, G. Charankumar, and L. Giulio. "Effects of Soret, rotation, Hall, and ion slip on unsteady MHD flow of a Jeffrey fluid through a porous medium in the presence of heat absorption and chemical reaction." *J. Mech. Eng. Res. Dev* 45, no. 3 (2022): 80-97.

[30] Jawad, Muhammad, Anwar Saeed, Arshad Khan, Ishtiaq Ali, Hussam Alrabaiah, Taza Gul, Ebenezer Bonyah, and Muhammad Zubair. "Analytical study of MHD mixed convection flow for Maxwell nanofluid with variable thermal conductivity and Soret and Dufour effects." *AIP Advances* 11, no. 3 (2021): 035215. https://doi.org/10.1063/5.0029105

[31] Buongiorno, Jacopo. "Convective transport in nanofluids." (2006): 240-250. https://doi.org/10.1115/1.2150834

[32] Gulle, Nagesh, and Raghunath Kodi. "Soret radiation and chemical reaction effect on MHD Jeffrey fluid flow past an inclined vertical plate embedded in porous medium." *Materials Today: Proceedings* 50 (2022): 2218-2226. https://doi.org/10.1016/j.matpr.2021.09.480