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Slip Effects on MHD Boundary Layer Flow over a Flat Plate in Casson Ferrofluid

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ABSTRACT

Present study investigated the slip effects on boundary layer flow and heat transfer on a horizontal flat surface immersed in blood-based Casson ferrofluid. The magnetite and cobalt ferrite ferroparticles suspended into Casson fluid represent by human blood to form Casson ferrofluid are numerically analysed. The study began with the transforming the governing equations which in dimensional partial differential equations to non-dimensional ordinary differential equations by using the similarity variable. Resulting similarity equations then solved by Runge-Kutta-Fehlberg (RKF45) method. The characteristics and effects of the slip parameter, the magnetic parameter and the ferroparticles volume fraction for magnetite and cobalt ferrite on the variation of Nusselt number and the skin friction coefficient are analyzed and discussed.

1. Introduction

Nowadays, ferrofluid is employed in plenty of applications. The importance of this kind of magnetic fluid attracted the researchers to explore its potential, especially in convective flow and heat transfer process. Recent studies on ferrofluid included the study of heat, thermal radiation and slip flow of ferrofluid towards various geometry like stagnation point, stretching/shrinking surface as well as a flat surface with heat flux and Newtonian heating boundary conditions by Khan *et al.,* [1], Zeeshan *et al.,* [2], Ramli *et al.,* [3], Jusoh *et al.,* [4], Mohamed *et al.,* [5,6], Yasin *et al.,* [7], Jamaludin *et al.,* [8], Gangadhar *et al.,* [9] and Kumar *et al.,* [10].

In the industrial segment, the ferrofluid is applied in electronic devices cooling system for example in hi-fi speakers and computer hard-disc. Ferrofluid is found in transportation segment as heat controlling agents in electric motor [2]. Last but not least, ferrofluid also employed in medical treatment segment. Ferrofluid play role in cancer therapy, bleeding stopping agent, magnetic resonance imaging and other diagnostic tests [11].

Studying the ferrofluid potential in medicine, specifically blood as the based-fluid is challenging. The blood is a non-Newtonian fluid, thus it shows different characteristics compared to a Newtonian

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fluid like water. The ordinary Navier-Stokes equations did not fulfil the characteristics to present the fluid flow model. Therefore, some modifications to the Navier-Stokes equations are proposed in many previous studies [12]. Casson model is introduced as one of the non-Newtonian models to characterize the fluid elastic solid behaviour. This model is identified as the most preferred rheological model for characterizing the human blood flow [13].

Present studies investigate the convective heat transfer of blood-based Casson ferrofluid on a flat surface. The Casson non-Newtonian fluid model combines with Tiwari and Das nanofluid model are solve numerically considering the pertinent fluid parameters [14]. Further, these configurations against the no slip conditions. It is known that in no slip conditions, the fluid particle at the surface body is static thus have no relative velocity [15]. Recent studies consider the slip flow in Maxwell nanofluid, viscoelastic nanofluid, Williamson nanofluid, hybrid nanofluid and Maxwell nanofluid includes the works by Ishak *et al.,* [16], Kho *et al.,* [17], Ibrahim and Negera [18], Zemedu and Ibrahim [19] and Muhammad *et al.,* [20]. To the best of the author's knowledge, this study had never been discussed before, thus the results published here is new.

2. Mathematical Formulations

Consider the convective heat transfer of blood-based Casson ferrofluid along a stationary flat surface with the constant magnetic field and uniform wall temperature T_w. Assuming the flow is in steady-state, incompressible and laminar. The physical model is shown in Figure 1 where T_∞ and U_∞ is an ambient temperature and the free stream velocity, respectively. The slip flow is considered at the surface while the viscous dissipation effects is neglected. In two-dimensional coordinates, the equations that governed this flow can be represented as [2,6]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2(x)}{\rho_{nf}}(u-U_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

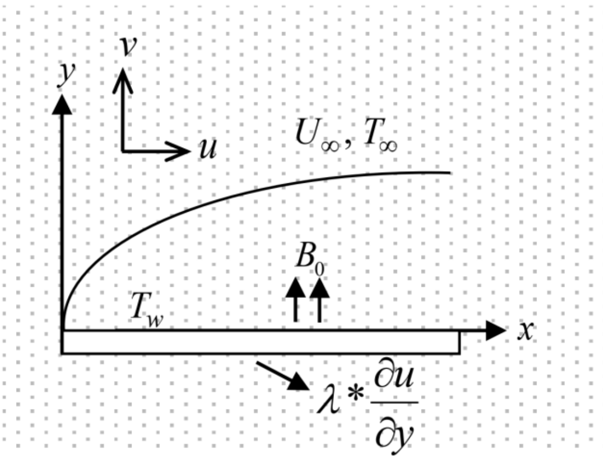

Fig. 1. Physical model and the coordinate system

subject to the boundary conditions

$$u=\lambda^{*}\frac{\partial u}{\partial y},\ \ v=0,\ \ T=T_w \text{ at } y=0,$$

$$u=U_\infty,\ \ T\to T_\infty,\ \text{as}\ y\to\infty, \tag{4}$$

where u and v are the velocity components along the x and y axes, respectively. T is the temperature inside the boundary layer, σ is the electrical conductivity, ν_{nf}, ρ_{nf}, $\left(\rho C_p\right)_{nf}$ and k_{nf} is the Casson ferrofluid kinematic viscosity, the Casson ferrofluid density, the Casson ferrofluid heat capacity and the Casson ferrofluid thermal conductivity, respectively. Further, $B_o(x)=Bx^{-1}$ is the magnetic field strength and $\lambda^{*}=Cx^{1/2}$ is the velocity slip factor where *B* and *C* is a constant [1]. Other properties of Casson ferrofluid can be expressed in terms of the properties of based-fluid $_f$ and the solid ferroparticles $_s$ as follows [3]

$$\begin{gathered}\nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\quad \rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\quad \alpha_{nf}=\frac{k_{nf}}{\rho_{nf}\left(C_p\right)_{nf}},\quad \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}},\\ \left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_f+\phi\left(\rho C_p\right)_s,\quad \frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+2(k_f-k_s)}.\end{gathered} \tag{5}$$

Note that μ_{nf} is the ferrofluid dynamic viscosity and ϕ is ferroparticles volume fraction. The partial differential Eq. (1)-(3) are dimensional and contain many dependent variables, thus the similarity solution which in non-dimensional form is convenience to consider [2,4]

$$\eta=\left(\frac{U_\infty}{\nu x}\right)^{1/2} y,\ \ \psi=\left(U_\infty \nu x\right)^{1/2} f(\eta),\ \ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

where η is a similarity variable, ψ is a non-dimensional stream function defined as $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$ while θ as a non-dimensional temperature. The Eq. (1) was satisfied by similarity in Eq. (6). Next, substitute Eq. (5) and Eq. (6) into governing Eq. (2) and Eq. (3) with boundary conditions (4) gives the following transformed ordinary differential equations

$$\frac{1}{(1-\phi)^{2.5}\left[1-\phi+(\phi\rho_s)/(\rho_f)\right]}\left(1+\frac{1}{\beta}\right)f'''+\frac{1}{2}ff''-M(f'-1)=0 \tag{7}$$

$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{(1-\phi)+\phi(\rho C_p)_s/(\rho C_p)_f}\theta''+\frac{1}{2}f\theta'=0 \tag{8}$$

subject to

$$f(0)=0,\ \ f'(0)=\lambda f''(0),\ \ \theta(0)=1,$$

$$f'(\eta) \to 1,\ \theta(\eta) \to 0,\ \text{as } y \to \infty. \tag{9}$$

Noticed that, $M = \dfrac{\sigma B^2(x)}{U_\infty \rho_{nf}}$ is the magnetic parameter, $\Pr = \dfrac{\nu_f \left(\rho C_p\right)_f}{k_f}$ is the Prandtl number which set as Pr=21 in calculation with respect to blood-based fluid while $\lambda = C\left(\dfrac{U_\infty}{\nu_f}\right)^{1/2}$ is the velocity slip parameter. The physical quantities interested are the skin friction coefficient C_f and the local Nusselt number Nu_x which defined as

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \qquad Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)}, \tag{10}$$

with the surface shear stress along *x*-direction τ_w and the surface heat flux q_w are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}. \tag{11}$$

The physical quantities in (10) can be reduced to a non-dimensional form as

$$C_f \operatorname{Re}_x^{1/2} = \frac{f''(0)}{(1-\phi)^{2.5}} \text{ and } Nu_x \operatorname{Re}_x^{-1/2} = -\frac{k_{nf}}{k_f}\theta'(0). \tag{12}$$

3. Results and Discussions

The transformed ordinary differential Eq. (7) and Eq. (8) with Eq. (9) were solved numerically using the RKF45 technique in Maple software. The solution in form of numerical and graphical variations are obtained for the temperature profiles, velocity profiles as well as the reduced Nusselt number $Nu_x \operatorname{Re}_x^{-1/2}$ and the reduced skin friction coefficient $C_f \operatorname{Re}_x^{1/2}$.

For numerical computing purpose, the boundary layer thickness from 6 to 12 is sufficient to provide the accurate numerical results for blood-based Casson ferrofluid with magnetite Fe_3O_4 and Cobalt Ferrite $CoFe_2O_4$ particles. The thermophysical properties of a ferroparticles and fluid considered are tabulated in Table 1.

The efficiency of the method used in this study have been validate with the previous studied done by Khan *et al.,* [1] and Ramli *et al.,* [3]. Table 2 shows the comparison values of $-\theta'(0)$ in various values of Prandtl number Pr. It is concluded that the presence numerical results are in good agreement with previous studies, thus validate the efficiency of the method used.

Table 1
Thermophysical properties of water and ferroparticles

Physical Properties	Blood (*f*)	Magnetite, Fe_3O_4	Cobalt ferrite, $CoFe_2O_4$
ρ (kg/m^3)	1053	5180	4907
C_p (J/kg·K)	3594	670	700
k(W/m·K)	0.492	9.7	3.7

Table 2
Comparison values of $f''(0)$ in viscous fluid for various values of λ and M when $\Pr = 21, \phi = 0$ and $\beta \to \infty$

λ	M	Khan *et al.,* [1]	Ramli *et al.,* [3]	Present
0	0	0.33206	0.33206	0.332057
0	1	1.04400	1.04400	1.044009
0.5	1	0.69872	0.69872	0.698724

Figure 2 to Figure 5 illustrate the effects of slip parameter λ on the temperature profiles $\theta(\eta)$, velocity profiles $f'(\eta)$, variation of the Nusselt number $Nu_x \mathrm{Re}_x^{-1/2}$ as well as variation of the reduced skin friction coefficient $C_f \mathrm{Re}_x^{1/2}$. From Figure 2 and Figure 3, it is found that the increase of λ has thinning the thermal and velocity boundary layer thicknesses. This situation physically enhanced the temperature gradient as represent by the reduced Nusselt number in Figure 4. The presence of λ lead to a fluid velocity at the surface. As we know, at no-slip conditions, the particle of fluid at the surface has zero velocity. The increase of λ directly proportional to the increase of velocity at the surface as describe clearly in boundary conditions Eq. (10).

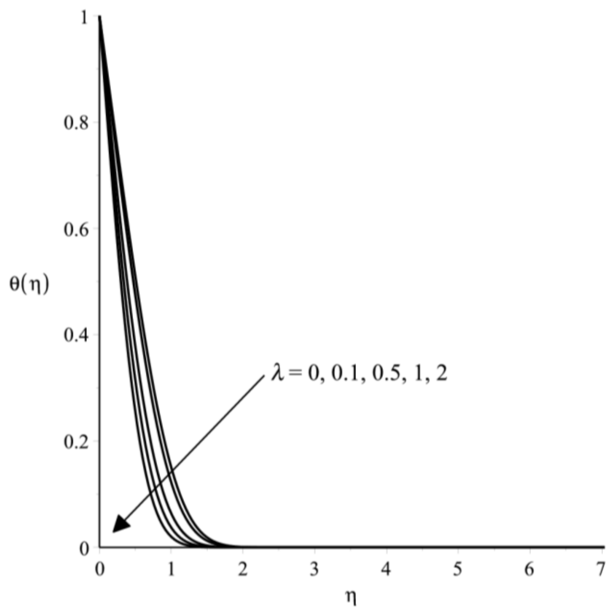

Fig. 2. Temperature profiles $\theta(\eta)$ with various values of λ when $\Pr = 21, M = 1, \beta = 0.1$ and $\phi = 0.1$

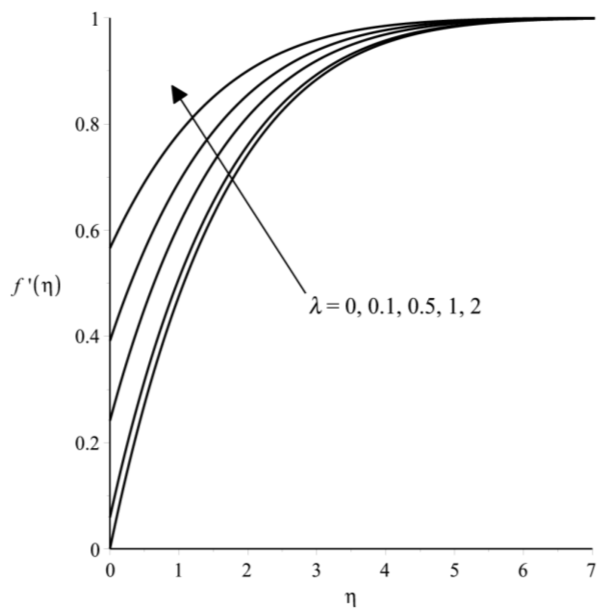

Fig. 3. Velocity profiles $f'(\eta)$ with various values of λ when $\Pr = 21, M = 1, \beta = 0.1$ and $\phi = 0.1$

Next, the variation of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ with various values of λ are shown in Figure 4 and Figure 5, respectively. From Figure 4, the increase of λ results to the increase of $Nu_x \mathrm{Re}_x^{-1/2}$ which physically represent the fluid convection capability. The increase in λ lead to increase the fluid velocity at the surface, thus promote the more fluid particle movement and convective heat transfer in the boundary layer.

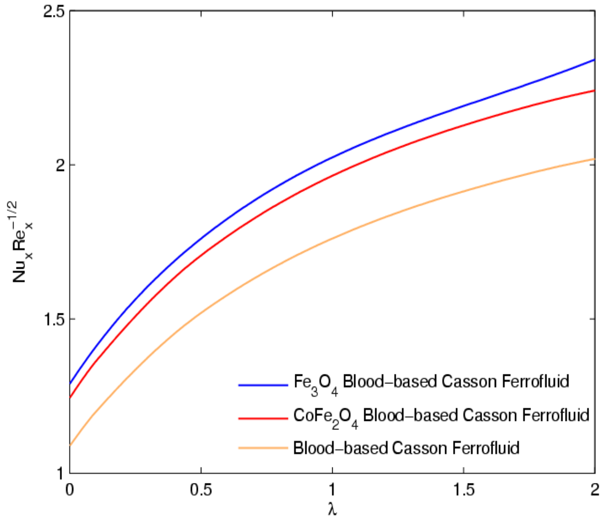

Fig. 4. Variation values of $Nu_x \operatorname{Re}_x^{-1/2}$ with various values of λ when $\Pr = 21, M = 1, \beta = 0.1$ and $\phi = 0.1$

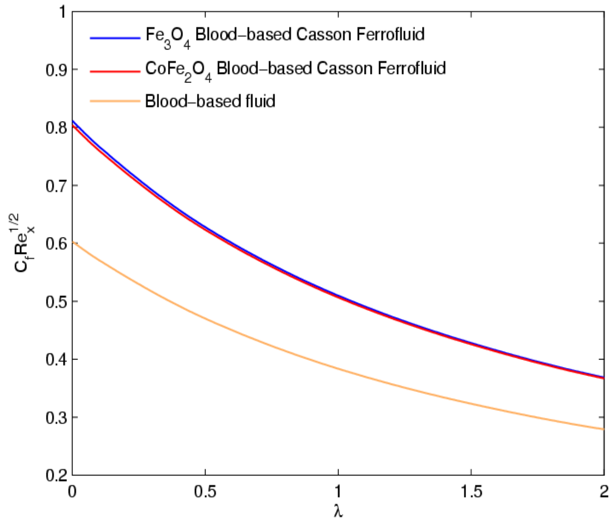

Fig. 5. Variation values of $C_f \operatorname{Re}_x^{1/2}$ with various values of λ when $\Pr = 21, M = 1, \beta = 0.1$ and $\phi = 0.1$

On the other hand, the increase in velocity at the surface has shortened the velocity difference thus reduced the fluid and surface friction as shown in Figure 5. Further, the blood-based Casson ferrofluid with Fe_3O_4 and $CoFe_2O_4$ produced high in $Nu_x \operatorname{Re}_x^{-1/2}$ and $C_f \operatorname{Re}_x^{1/2}$ compared to blood-based fluid. In comparing both Fe_3O_4 and $CoFe_2O_4$ ferroparticles, the highly in thermal conductivity of Fe_3O_4 influenced the fluid heat transfer capability thus scored better in $Nu_x \operatorname{Re}_x^{-1/2}$. Fe_3O_4 are also denser than $CoFe_2O_4$ hence, contribute to a more friction with surface.

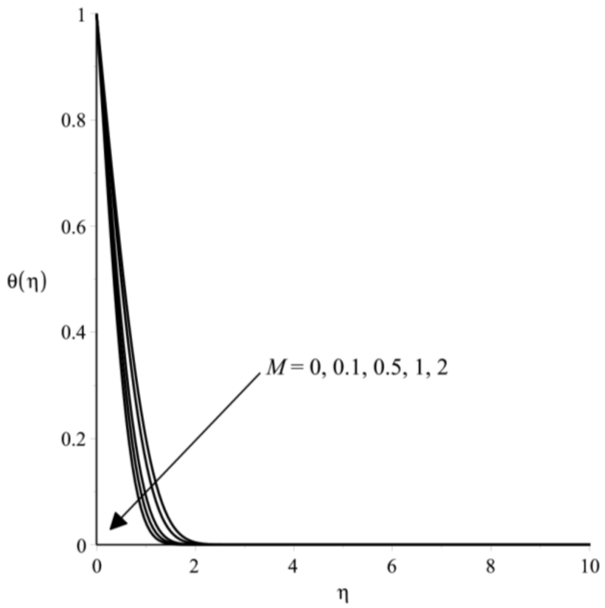

Fig. 6. Temperature profiles $\theta(\eta)$ with various values of M when $\Pr = 21, \beta = \phi = 0.1$ and $\lambda = 0.5$

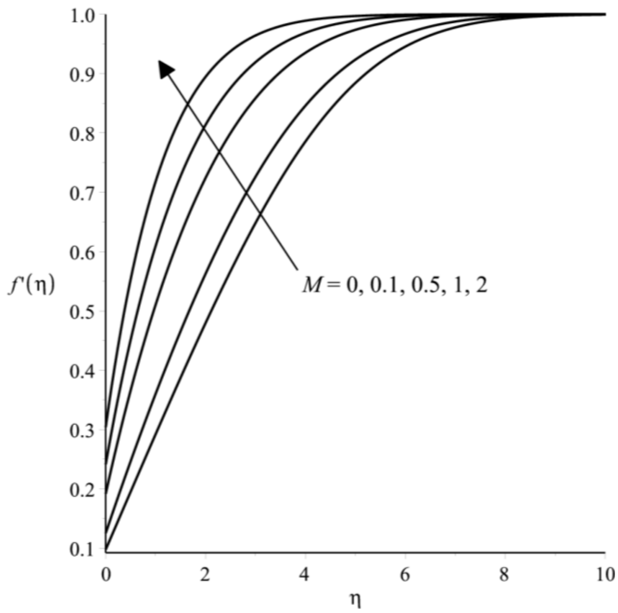

Fig. 7. Velocity profiles $f'(\eta)$ with various values of M when $\Pr = 21, \beta = \phi = 0.1$ and $\lambda = 0.5$

In investigating the magnetic effects towards the flow and heat transfer characteristics, the Figure 6 until Figure 9 are illustrated. Figure 6 and Figure 7 show the temperature profiles $\theta(\eta)$ and velocity profiles $f'(\eta)$ with various values of magnetic parameter M. From both figures, it is observed that the increase of M reduced both thermal and velocity boundary layer thicknesses. The presence of

Lorentz force has accelerated the fluid particle, increase the fluid velocity at the surface thus thinning the velocity boundary layer thickness.

Thinning the thermal and velocity boundary thicknesses, with the increasing both temperature and velocity gradient has results to the increase in reduced Nusselt number $Nu_x \mathrm{Re}_x^{-1/2}$ and reduced skin friction coefficient $C_f \mathrm{Re}_x^{1/2}$ as M increases. This observation has been illustrated in Figure 8 and Figure 9, respectively. Further, the increase in ferroparticles volume fraction ϕ in blood-based fluid shows the increment in $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$. The increment is more significant as M increases. From the numerical calculation, the 10% Fe_3O_4 blood-based Casson ferrofluid enhanced 15% in $Nu_x \mathrm{Re}_x^{-1/2}$ and 32% in $C_f \mathrm{Re}_x^{1/2}$ compared to blood-based fluid. By increase the Fe_3O_4 ferroparticles to 15%, the $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ increased significantly to 23% and 53%, respectively compared to its based fluid.

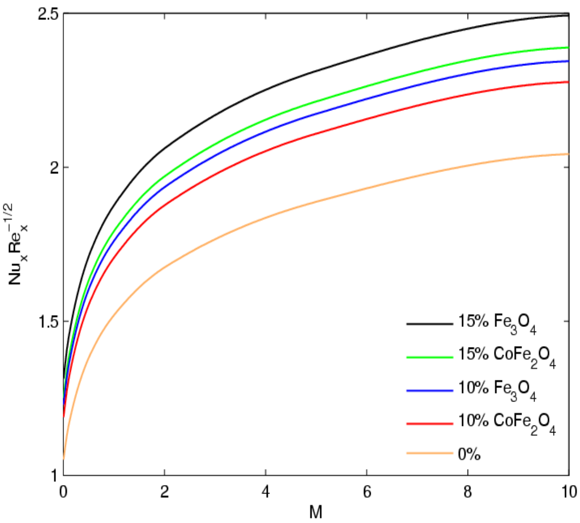

Fig. 8. Variation values of $Nu_x \mathrm{Re}_x^{-1/2}$ with various values of M when $\mathrm{Pr}=21, \beta=0.1$ and $\lambda=0.5$

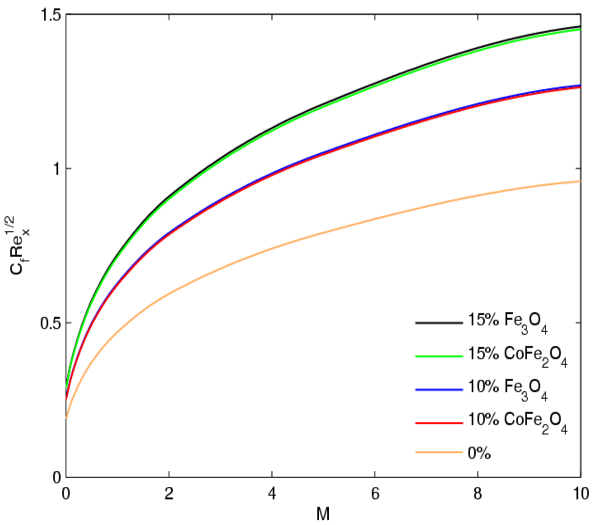

Fig. 9. Variation values of $C_f \mathrm{Re}_x^{1/2}$ with various values of M when $\mathrm{Pr}=21, \beta=0.1$ and $\lambda=0.5$

4. Conclusions

As a conclusion, from the numerical computation, it is observed that by adding 10% of magnetite in blood-based fluid to form the blood-based Casson ferrofluid will enhanced 15% in Nusselt number which represent the fluid convective heat transfer capability. The skin friction also increases 32% than the based fluid. Next, the increase of slip and magnetic parameter have reduced the boundary layer thicknesses. This results to an increase in temperature gradient thus promote the Nusselt number. Lastly, it is found that the increase in slip parameter cause the fluid velocity at the surface thus disobey the no-slip condition. This situation reduced the velocity differences between stream velocity and the fluid at the surface, therefore decrease a skin friction coefficient.

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