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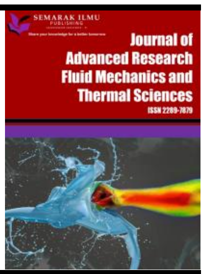

Numerical Computation of Stagnation Point Flow and Heat Transfer over a Nonlinear Stretching/Shrinking Sheet in Hybrid Nanofluid with Suction/Injection Effects

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ABSTRACT

The steady, laminar, stagnation point flow of hybrid nanofluid past a nonlinearly stretching and shrinking sheet is studied. Hybrid nanofluid is regarded by disseminated two distinct nano-sized particles, silver (Ag) and copper oxide (CuO) in pure water. Similarity technique was used for the transformation of partial differential equations (PDEs) into an ordinary differential equation (ODEs). Obtained ODEs were solved using Matlab's built-in function (bvp4c). The results of important governing parameters which are nonlinear parameter, suction/injection parameter, stretching/shrinking parameter and nanoparticle volume fraction are evaluated and discussed in graphical and tabular form for the velocity and temperature profiles, along with local skin friction, local Nusselt number. Nonunique solutions (first and second branch) are visible for some limit of shrinking parameter. It is noticed that suction parameter delays the boundary layer hastens flow separations.

1. Introduction

In various applications, including wire drawing, extrusion, metal spinning, and hot rolling, boundary layer flow over a stretching sheet is essential in the applications. Other than that, as well as various investigations on flow and heat transfer generated by a nonlinearly stretching/shrinking surface have been studied. Vajravelu [1] developed the study on the flow of nonlinear stretching sheet in viscous fluid. Only a few researchers did research on viscous fluid for nonlinear stretching/shrinking sheet due to viscous fluid having a large amount of viscosity. As the result, they discovered that the heat flow from the stretched sheet to the fluid is consistent. The nonlinear vertical stretching surface with varying fluid properties has been investigated by Prasad *et al.,* [2]. Meanwhile, the case of nonlinear shrinking surface in nanofluid was studied by Rana and Bhargava [3]. The stagnation point flow towards a nonlinearly stretching/shrinking sheet in nanofluid has been

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discussed by Mat *et al.,* [4] using a single-phase nanofluid model. They noticed that two solutions exist for some range of the shrinking parameter when the value of nonlinear parameter is more than 1/3. In addition, they noticed that both the skin friction and local Nusselt number increase as the value of the nanoparticle volume fraction increases. Then, Anwar *et al.,* [5] considered the stagnation point flow over a nonlinear stretching sheet using the Buongiorno model. Their study took into consideration the effects of Brownian motion and the thermophoresis parameter. In their study, they found that the concentration diminishes with higher Brownian motion, but it is enhanced with a higher thermophoresis parameter. This problem was then extended by Anwar *et al.,* [6] by considering the MHD and radiation effects. Their study showed that the reduced Nusselt number and Sherwood number decrease for higher values of the magnetic parameter and nonlinear parameter. Later, Zaimi *et al.,* [7] analysed the problem of stretching and shrinking surface respectively. The nonlinear shrinking sheet with slip impact in the stagnation region was studied by Fauzi *et al.,* [8] and Rana *et al.,* [9] explored the radiative nanofluid's flow over a nonlinear surface with suction and slip.

Hybrid nanofluid is considered as new fluid since its first development by researchers. Several reviews in hybrid nanofluid on boundary layer flow are available in this literature for further reading [10-13]. However, the literature on hybrid nanofluid over nonlinear stretching/shrinking is very limited. Anuar *et al.,* [14] analysed the numerical solution of stagnation point flow and heat transfer over a nonlinear stretching/shrinking sheet in hybrid nanofluid with stability analysis. The first solution denotes stable flow, while the second solution denotes unstable flow, and the nonlinear parameter delays the boundary layer separation. It was found that dual solutions exist for a certain range of the stretching/shrinking parameter.

In 1977, the problem of Crane [15] was extended to heat and mass transfer with the effect of suction or blowing Gupta and Gupta [16]. Heat transfer analysis of Jeffery fluid flow over a stretching sheet with suction/injection and magnetic dipole effect was proposed by Zeeshan and Majeed [17]. Raju *et al.,* [18] analysed the effects of nonlinear thermal radiation on 3D Jeffrey fluid flow over a stretching/shrinking surface in the presence of homogeneous-heterogeneous reactions, non-uniform heat source/sink, and suction/injection. Effect of suction/injection on stagnation point flow of hybrid open nanofluid over an exponentially shrinking sheet with stability analysis [19]. Stagnation-point flow and heat transfer over a permeable stretching/shrinking sheet with heat source effect analysed by Kamal *et al.,* [20]. Waini *et al.,* [21] interested to do research on hybrid nanofluid flow and heat transfer over a nonlinear permeable stretching/shrinking surface. In this paper, it was found that still dual solutions exist with parameter of shrinking/stretching but additional suction parameter. Anuar *et al.,* [22] solved for the problem of hybrid nanofluid flow over a permeable moving surface in presence of hydromagnetic and suction effects. The skin friction coefficient and the local Nusselt number were found to increase as the suction/injection increases. When heat source rises, the local Nusselt number was shown to be reduced. According to the literature, it becomes clear that a good deal of the studies on the stagnation-point flow of hybrid nanofluid is constructed based on two-phase modelling framework with the effects of suction or injection in the boundary layer of a steady, laminar, and incompressible hybrid nanoparticle. Moreover, the deployed model from Anuar *et al.,* [14] provides us with information regarding the impact of combining two nanoparticles size and the heat transfer direction in addition to the volume concentration of nanoparticles on the evolution of velocity and temperature fields as well as the main quantities of engineering interest. Therefore, this paper is extended work from Anuar *et al.,* [14] with the effects of suction or injection.

2. Methodology

The basic governing equations for time-independent $\partial/\partial t = 0$ and two-dimensional flow can be written as (Bachok *et al.,* [23])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{\partial x}+\frac{\mu_{hnf}}{\rho_{hnf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{hnf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

the corresponding boundary conditions are

$$\begin{aligned} &v=V_w(x), \quad u=U_w(x), \quad T=T_w \quad at \quad y=0 \\ &u\rightarrow U_w(x), \quad T\rightarrow T_w \quad as \quad y\rightarrow\infty \end{aligned} \tag{4}$$

where u and v are corresponding velocity components in x and y directions, T is the temperature, μ_{hnf} , ρ_{hnf} and α_{hnf} are the hybrid nanofluid dynamic viscosity, density of hybrid nanofluid and thermal diffusivity of hybrid nanofluid, respectively.

$$\begin{aligned} &\alpha_{nf}=\frac{k_{nf}}{(\rho C_p)_{nf}}, \mu_{hnf}=\frac{\mu_f}{(1-\varphi_1)^{2.5}(1-\varphi_2)^{2.5}}, \ \rho_{hnf}=\varphi_2\,\rho_{s2}+(1-\varphi_2)\big[(1-\varphi_1)\rho_f+\varphi_1\rho_{s1}\big], \\ &\frac{k_{hnf}}{k_f}=\frac{k_{s2}+2k_{nf}-2\varphi_2(k_{nf}-k_{s2})}{k_{s2}+2k_{nf}-\varphi_2(k_{nf}-k_{s2})}\ where\ \frac{k_{nf}}{k_f}=\frac{k_{s1}+2k_f-2\varphi_1(k_f-k_{s1})}{k_{s1}+2k_f-\varphi_1(k_f-k_{s1})} \\ &(\rho C_p)_{hnf}=\varphi_2(\rho C_p)_{s2}+(1-\varphi_2)\left[(1-\varphi_1)(\rho C_p)_f+\varphi_1(\rho C_p)_{s1}\right], \end{aligned} \tag{5}$$

Here, $\varphi 1$ is the CuO (copper oxide) nanoparticle while $\varphi 2$ is the addition of nanoparticles which is Ag (silver) nanoparticles. When CuO disseminated in basic fluid it will form nanofluid while hybrid nanofluid is formed by disseminated two distinct nano-sized particles which are CuO and Ag in the basic fluid such as water. The subscript of f , nf , hnf, $s1$ and $s2$ signify fluid, nanofluid, hybrid nanofluid, CuO nanoparticles and Ag nanoparticles respectively.

The following similarities variables are introduced to solve Eq. (1) to Eq. (3) along with the boundary conditions (4)

$$\eta=\left(\frac{(n+1)b}{2v_f}\right)^{1/2} yx^{\frac{n-1}{2}},\ \psi=\left(\frac{2bv_f}{n+1}\right)^{\frac{1}{2}} x^{\frac{n+1}{2}}f(\eta),\ T=(T_w-T_\infty)\theta(\eta)+T_\infty, \tag{6}$$

where η denotes as similarity variable, v_f is the kinematic viscosity of the fluid and ψ represents the stream function which identically satisfy Eq. (1) and defined as $v=-\frac{\partial\psi}{\partial x}$ and $u=\frac{\partial\psi}{\partial y}$.

Using (6), Eq. (2) and Eq. (3) take the following form

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f}\,f'''+ff''-\left(\frac{2n}{n+1}\right)(f'^2+1)=0, \tag{7}$$

$$\frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)} \frac{1}{\mathrm{Pr}} \theta'' + f\theta' = 0 \tag{8}$$

The transformed boundary conditions are

$$\begin{aligned} &f(0) = S, \quad \theta(0) = 1, \quad f'(0) = \varepsilon, \\ &f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad as \quad \eta \to \infty \end{aligned} \tag{9}$$

where f' denotes the differentiation with respect to η . While the stretching ($\varepsilon > 0$) and shrinking ($\varepsilon < 0$) parameter denoted by $\varepsilon = \frac{a}{b}$ and $\mathrm{Pr} = \frac{v}{\alpha}$ is the Prandtl number.

The local skin friction and the local Nusselt number are

$$Re_x^{\frac{1}{2}} C_f = \frac{\mu_{hnf}}{\mu_f} \sqrt{\frac{n+1}{2}} f''(0), \; Re_x^{\frac{1}{2}} Nu_x = -\frac{k_{hnf}}{k_f} \sqrt{\frac{n+1}{2}} \theta'(0), \tag{10}$$

Here, Re_x is the local Reynold number given by $U_\infty x/v_f$.

3. Result and Discussions

Shampine *et al.,* [24] claimed that the effective techniques to solve boundary value problems for ordinary differential equations (ODEs) is by using bvp4c. Hence, the bvp4c solver, MATLAB software is used to solve the individual equations, Eq. (7) and Eq. (8) as well as the boundary conditions, Eq. (9). The thermophysical characteristics of Ag-CuO and base fluid are referred from Table 1 in Hayat and Nadeem [25]. The range of parameters CuO–water nanofluid is originally created by disseminating 0.1 volume percent of CuO nanoparticle into the water, $\varphi 1 = 0.1$ and $\varphi 2 = 0$ [14]. Therefore, the nanoparticles volume fraction φ values are in the range of $0 \le \varphi \le 0.2$ while the nonlinear parameters, n are taken from 1 to 2. For the suction value is S = 0.1 and injection value is $S = -0.1$. Prandtl number, Pr is fixed to 6.2, which represent water as stated in the work of Oztop and Abu-Nada [26]. According to Anuar *et al.,* [14], the existence of a dual solution is validated from $\varepsilon = -1$ until its critical value, ε_c.

Table 1
Thermophysical properties of nanoparticles

Properties	Ag	CuO	Base fluid (water)
$-\rho(kgm^{-3})$	10500	6320	997.1
$k(Wm^{-1}K^{-1})$	429	76.50	0.613
$C_p(Jkg^{-1}K^{-1})$	235	531.80	4179

Figure 1 and Figure 2 show the effect of suction/injection S on the variations of velocity profiles $f'(\eta)$ and temperature profile $\theta(\eta)$ in the (Ag–CuO/water) hybrid nanofluid with nanoparticle volume fractions of $\varphi 1 = \varphi 2 = 0.1$ for shrinking case ($\varepsilon = -1.2$). This signifies that the presence of suction $S = 0.1$ affects the range of duality to be widened. Figure 3 and Figure 4 present the graphical results of reduced skin friction coefficient $f''(0)$ and reduced heat transfer rate at the surface $-\theta'(0)$ with ε for different S. It shows that the existence of unique solution when $\varepsilon > -1$ while the dual solution exists when $\varepsilon \le -1$. The figures of $f''(0)$ and $-\theta'(0)$ are observed to be increasing when the values of S get higher for both solutions. The increasing value of S pushes the fluid into an empty space affecting the surface limit. As a result, more force is used to fluid flow, and more temperature will eventually rise.

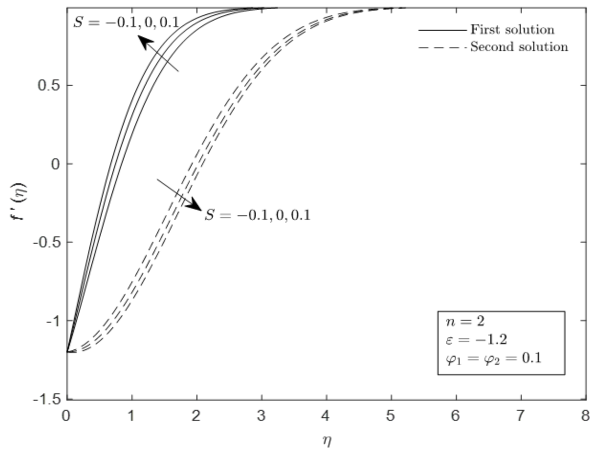

Fig. 1. Velocity profile for different S

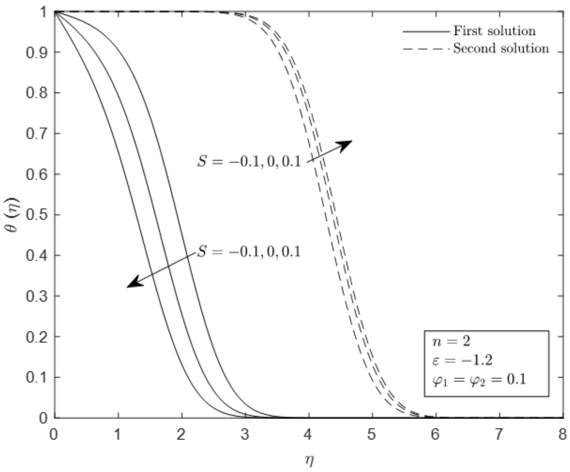

Fig. 2. Temperature profile for different S

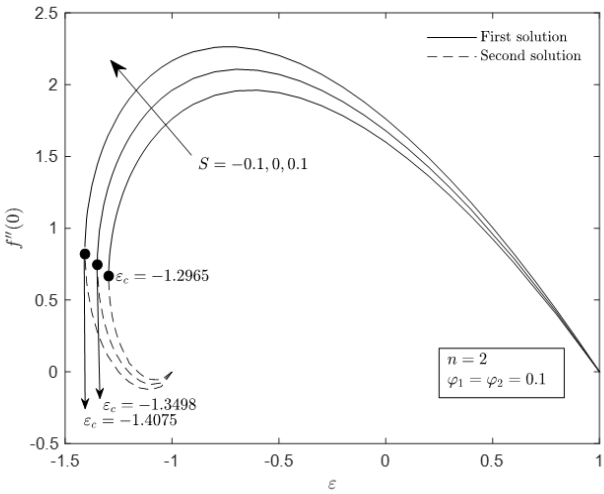

Fig. 3. Variation of $f''(0)$ with ε for different S

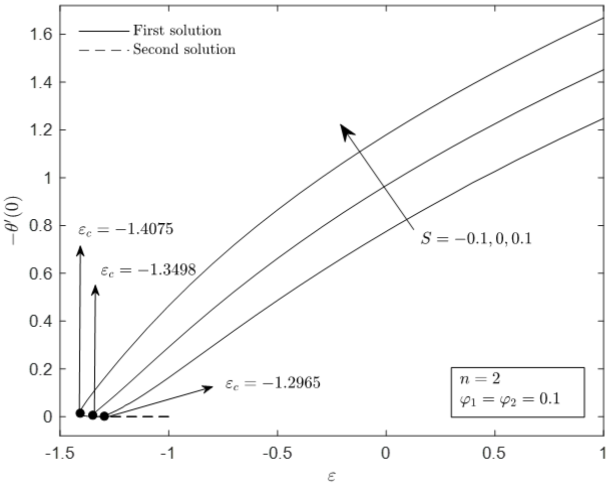

Fig. 4. Variation of $-\theta'(0)$ with ε for different S

4. Conclusions

The study of stagnation point flow in hybrid nanofluid past a nonlinear stretching/shrinking sheet with the effect of suction/injection effect is discussed. Hence, the findings of this study can be summarized as follows

i. For nonlinear stretching/shrinking surface, there exists non-unique solutions at certain values of shrinking case ($\varepsilon \leq -1$) while unique solution exists at streching case $\varepsilon > -1$.
ii. The presence of nonlinear suction parameter widens the range of solutions and delay the boundary layer separation.

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