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Various Speed Ratios of Two-Sided Lid-Driven Cavity Flow using Lattice Boltzmann Method

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Abstract – *In the present study, the flow configuration of two-sided lid-driven cavity has been investigated using the Lattice Boltzmann method. First, the code was validated against the numerical results taken from previous study of fluid flow in a single-lid driven cavity. The influence of various speed ratios which vary from 0 to 1 and several Reynolds number (100, 400, and 1,000) on the flow configuration of the cavity were analyzed. The results show that the increase in both speed ratio and Reynolds number gives an effect on flow configuration of the cavity.*

Keywords: Lattice Boltzmann method, Two-sided, Lid-driven cavity, Parallel wall motion

1. INTRODUCTION

The problem of cavity flow of moving boundary has been a major topic for research studies due to its simplicity in geometry in the last four decades. It has also been widely used frequently in industrial and technological applications which include coating system [1-2], mixing [3], drying technologies [4], polymer processing [5] and ceramic tape casting [6].

Numerous investigations of flow field in a single-sided lid-driven cavity flow have been conducted either by experimental or numerical studies. A great number of papers on lid-driven cavity flow can be found in available literature [7-12]. An extended study of two-sided lid-driven cavity from single-sided lid-driven cavity flow problem was done by Kuhlmann and other investigators [13], where their investigation specified that the cavity aspect ratio and the Reynolds number will lead to the existence of non-unique two-dimensional steady flow, which is determined by the wall velocities. At low Reynolds number, the flow consists of separate co-rotating vortices next to each of the moving walls. As the velocities of the wall increase, a jump transition occurs, and the two vortices partially merge to generate a flow pattern which resembles cat's eyes. At high Reynolds number, the flow of the cat's eye becomes unstable and transforms into a steady three-dimensional cellular flow.

One of the alternative methods in CFD is the lattice Boltzmann method (LBM). LBM is a relatively new simulation technique for complex fluid systems and has attracted interest from researchers in many fields. Unlike traditional CFD methods, which solve conservation equations of macroscopic properties (i.e. mass, momentum, and energy) numerically, LBM models fluid that contains fictive particles, and such particles perform successive propagation and collision processes over a discrete lattice mesh. Due to its particulate nature and local dynamics, LBM has several advantages over other conventional CFD methods, especially in dealing with complex boundaries, incorporation of microscopic interactions, and parallelization of the algorithm. The lid-driven cavity flow is also the subject of numerous lattice Boltzmann studies. For example, Nor Azwadi et al. [14] performed a numerical investigation of lid-driven cavity flow based on two different methods: lattice Boltzmann method and splitting method. In their study, the results from uniform and stretched form of splitting method were compared with the results from lattice Boltzmann method. Lid-driven cavity problem at various Reynolds numbers was used as a numerical test case.

Meanwhile, Predrag et al. [15] used lattice Boltzmann method in order to explore incompressible fluid flow inside a two-sided lid-driven staggered cavity. They also presented the characteristics of flow pattern for a variety of Reynolds numbers (50–3,200) for parallel motion of lids. The authors had also obtained an asymmetric steady-state flow pattern for parallel motion of lids.

Recently, Perumal and Dass [16] presented a result of a numerical study in a two-sided lid-driven square cavity by the LBM. They found that for parallel motion of the walls, there was a pair of counter-rotating secondary vortices of equal size near the center of the wall. The main aim of the present study is to investigate the effects of speed ratio and Reynolds number on the development of vortex in cavity using LBM.

2.0 MATHEMATICAL FORMULATION

2.1 Lattice Boltzmann Method

The results presented in this paper are obtained using D2Q9 lattice Boltzmann model (where the numbers following the letters D and Q refer to the model's dimensionality and number of lattice speeds, respectively).

The starting point for lattice Boltzmann simulation is the evolution equation for a set of distribution functions f_i which is discrete in both space and time:

$$f_i(\mathbf{x}+\mathbf{e}_i, t+1) - f_i(\mathbf{x}, t) = \frac{1}{\tau_f}\left[f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)\right] + F \quad (1)$$

Where **e** is the particle's velocity, τ is the relaxation time for the collision, f_i^{eq} is an equilibrium distribution function and $i = 0, 1, \ldots, 8$ for two-dimension nine-velocity model (D2Q9). Noted that the right hand side of Eq. 1 is the collision term where the Bhatnagar-Gross-Krook (BGK) approximation has been applied [17]. The discrete velocity is expressed as $\mathbf{e}_i = (0, 0)$ for $i = 0$, $\mathbf{e}_i = (\cos(i-1)\pi/4, \sin(i-1)\pi/4)$ for i = 1, 3, 5, 7 and $\mathbf{e}_i = 2^{1/2}(\cos(i-1)\pi/4, \sin(i-1)\pi/4)$ for i = 2, 4, 6, and 8. Macroscopic density ρ and velocity $\boldsymbol{u}$ of the fluid are determined by the following velocity moments of the distribution function:

$$\sum_i f_i^{eq} = \rho \quad (2)$$

$$\sum_i e_{i,\alpha} f_i^{eq} = \rho u_\alpha \quad (3)$$

The equilibrium distribution function, f_i^{eq} is chosen such that the continuum macroscopic equations approximated by the evolution equation correctly describe the hydrodynamics of the fluid. For D2Q9 model, f_i^{eq} is defined as:

$$f_i^{eq} = \rho\omega_i \left[1 + 3\frac{\mathbf{e}_i \cdot \mathbf{u}}{c^2} + 9\frac{(\mathbf{e}_i \cdot \mathbf{u})^2}{2c^4} - \frac{3\mathbf{u}^2}{2c^2} \right] \tag{4}$$

where c = $(3RT)^{1/2}$ and the weights are ω_0 = 4/9, $\omega_{1,3,5,7}$ = 1/9 and $\omega_{2,4,6,8}$ = 1/36. Through multiscaling expansion, the mass and momentum equations can be derived from D2Q9 model as follows:

$$\nabla \cdot \mathbf{u} = 0 \tag{5}$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\nabla \cdot \mathbf{u} = \frac{1}{\rho}\nabla p + \upsilon\nabla^2\mathbf{u} \tag{6}$$

The viscosity, υ can be related to the time relation in lattice Boltzmann equation as follows:

$$\tau = 3\upsilon + \frac{1}{2} \tag{7}$$

2.2 Code Validation

To validate the present numerical method, the LBM code was used to compute the single lid-driven flow for Re = 1,000 on a 251 × 251 lattice size. A lid velocity of U = 0.1 was considered in this work. Fig. 1 shows the comparison of steady-state u-velocity profile along a vertical line and v-velocity profile along a horizontal line passing through the geometric center of the cavity at Re = 1,000 with the benchmark solutions of Ghia et al. [9]. The excellent match of the present results with the present study demonstrates its validity for the simulation.

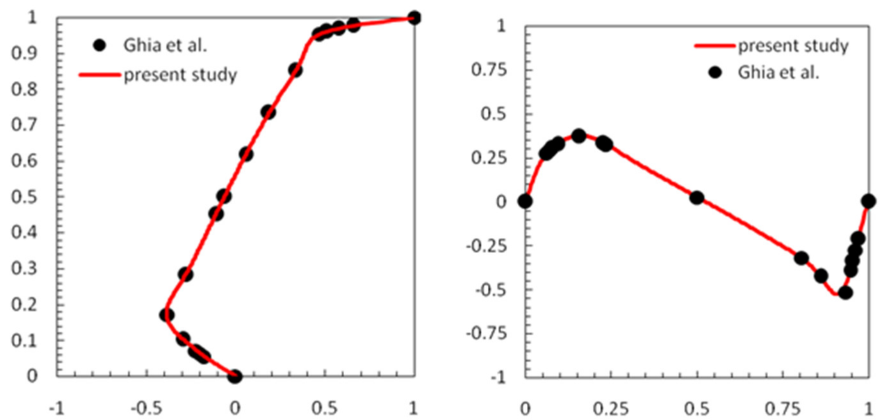

Figure 1: u-velocity profile (left) and v-velocity profile (right) at Re = 1,000

To ensure grid dependence, LBM code was simulated using three different grids; 151 × 151, 201 × 201 and 251× 251. All reported results in the present study converged to a maximum residual of 10^{-8}.

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3.0 RESULTS AND DISCUSSION

A comprehensive analysis has been conducted for a square cavity with L = H. Most of the previous investigations were carried out by considering equal magnitude of the velocity of the opposite walls irrespective of the direction of motion.

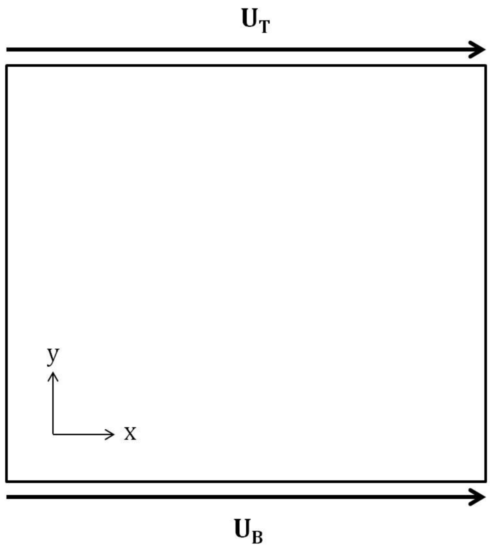

Figure 2: Geometry of two-sided parallel wall motion

Therefore, this present study attempts to examine the change in the fluid motion. The magnitude of the lid velocities and Reynolds number were varied (Re = 100, 400, 1,000) for particular parallel wall motion where the top and bottom lid moved in the same direction but different velocities as shown in Fig.2.

To track various flow transformations arising in a cavity, speed ratio ($S = U_B / U_T$) was varied from 0 to 1. Fig. 3 signifies classical cavity flow driven by the uniform motion of one of the lids while all other lids are stationary. In this situation, one primary vortex and one secondary corner vortices were observed in the right bottom cavity. As S increased from 0, the secondary corner vortices started to increase. The vortices combined and formed another primary vortex, counter rotating with respect to the already existed vortex at the bottom of the cavity. With the increase of S, the size of the bottom vortex increased and finally became equal to the top vortex with a free shear layer in between at S = 1. Meanwhile, it was observed that the location of the center of the primary vortex at the top remains more or less fixed in spite of the change in S. However, the center of the vortex at the bottom side slowly moved up as the speed ratio increased.

Figs. 4 and 5 also show the similar streamline pattern where the secondary vortices or bottom vortex combined and increased until the vortices became symmetrical with te top primary vortex. Nevertheless, it can also be seen that the location of the top and bottom vortices was influenced by the variation of Reynolds number. Increased Reynolds number made the primary vortex cores moved towards the centers of the top and bottom halves of the cavity. It can also be observed that with the increase of Reynolds number, the secondary vortex pair grew in size at the right side of the cavity.

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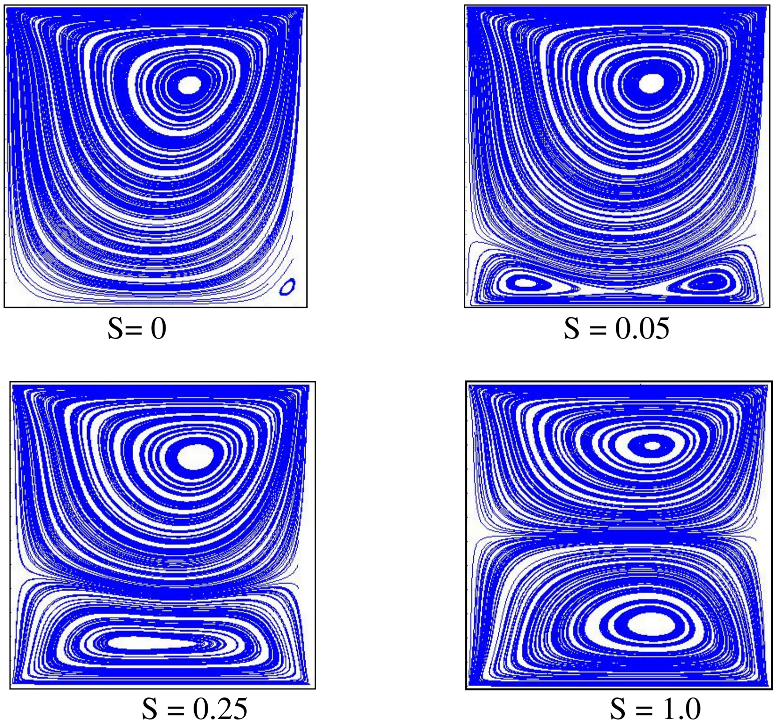

Figure 3: Streamline plots for various speed ratios of the lid for parallel motion at Re = 100.

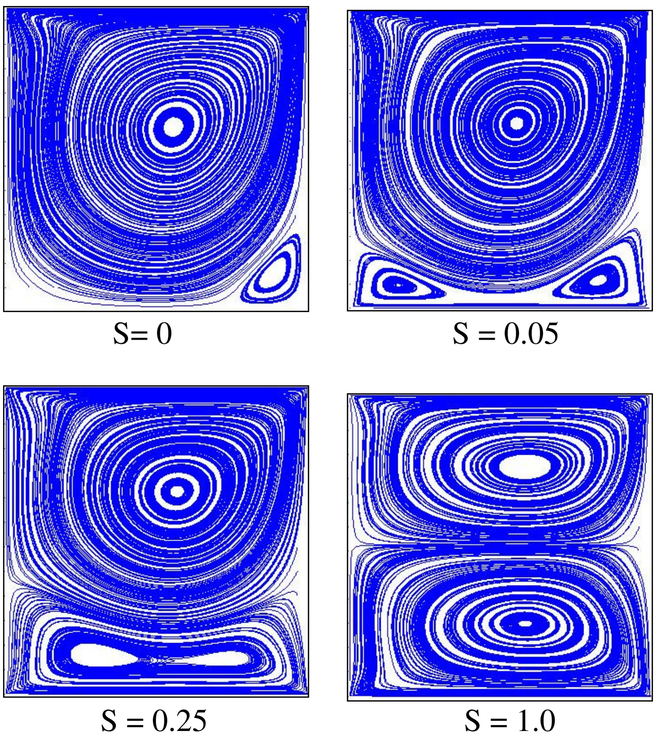

Figure 4: Streamline plots for various speed ratios of the lid for parallel motion at Re = 400.

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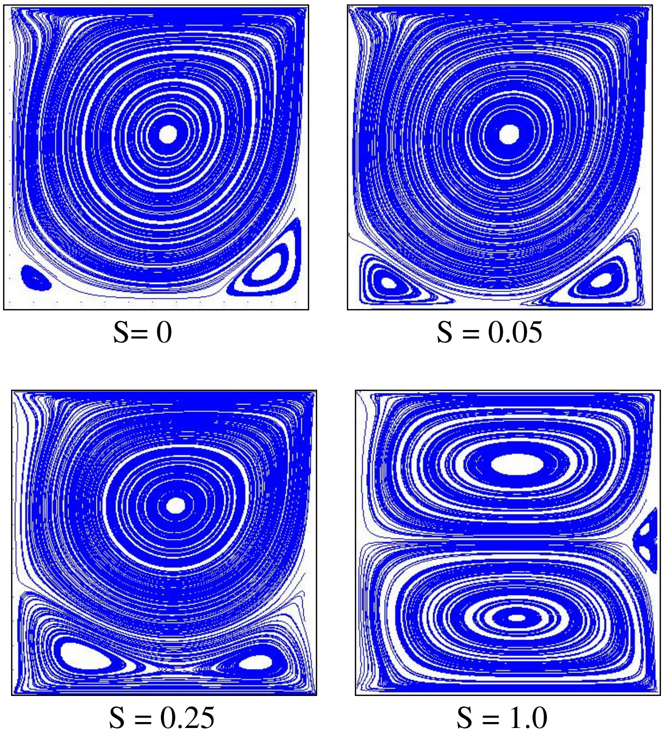

Figure 5: Streamline plots for various speed ratios of the lid for parallel motion at Re = 1,000.

Both Tables 1 and Table 2 show the location of the center of vortices for two-sided lid-driven cavity flow at Reynolds numbers of 100, 400 and 1,000. 5 speed ratios (S) were considered in this study (i.e. 0, 0.05, 0.25, 0.5, and 1.0). S is the ratio between the velocities of the bottom lid to the velocity of the top lid.. From both tables, it can be determined that at Reynolds number 100, 400, 1,000, and S=1.0, the primary vortex center and the secondary vortex center in the x/H direction had similar values, which indicated these vortices became symmetrical.

Table 1: Locations of the primary vortex center for parallel wall motion for Reynolds number of 100, 400, and 1,000.

Re		Primary Vortex Center				
		S=0	S=0.05	S=0.25	S=0.5	S=1.0
100	x/H	0.6176	0.6111	0.6172	0.6142	0.6166
	y/H	0.7398	0.7405	0.7604	0.7751	0.7982
400	x/H	0.5565	0.5577	0.5668	0.5705	0.5839
	y/H	0.6078	0.6183	0.6581	0.6961	0.7675
1,000	x/H	0.5341	0.5351	0.5367	0.5317	0.5381
	y/H	0.5694	0.5757	0.6171	0.6519	0.7558

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Table 2: Locations of the secondary vortex center for parallel wall motion for Reynolds number of 100, 400, and 1,000.

Re		Secondary Vortex Center							
		S=0		S=0.05		S=0.25		S=0.5	S=1.0
		Left	Right	Left	Right	Left	Right		
100	x/H	0.9471		0.1967	0.8126	0.3887		0.546	0.6119
	y/H	0.0614		0.0795	0.0872	0.1361		0.1716	0.2077
400	x/H	0.8869		0.1635	0.8331	0.2908	0.7459	0.6731	0.5853
	y/H	0.1216		0.0799	0.0976	0.1323	0.1191	0.1563	0.2409
1,000	x/H	0.0888	0.9008	0.1056	0.8592	0.2368	0.7982	0.7534	0.5325
	y/H	0.0767	0.1127	0.0786	0.1115	0.1206	0.1076	0.1416	0.2469

4.0 CONCLUSION

The present study numerically simulated an incompressible two-dimensional laminar flow inside a square cavity. Two-sided and parallel motions were considered. In this investigation, the aim was to examine the effects of different speed ratios and Reynolds numbers on the development of vortex in cavity using lattice Boltzmann method. For various speed ratios, the bottom vortex increased with the increased of speed ratio and became symmetrical between the top and bottom vortices. Meanwhile, the variation of Reynolds number also gave an effect on flow configuration, which produced primary vortex at different location. A pair of counter-rotating secondary vortices symmetrically placed about the centerline parallel to the motion of the walls was also observed.

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