

# Prediction of the Heave Response of a Floating Oscillating Water Column Wave Energy Converter

*B. Stappenbelt*

School of Mechanical, Materials and Mechatronics Engineering  
Faculty of Engineering and Information Sciences  
University of Wollongong, Wollongong, Australia  
*brads@uow.edu.au*

**Abstract** – *The coupled dynamics of the chamber and the water column are central to the successful modelling of the performance of a floating oscillating water column (OWC) wave energy converter. This paper presents a novel system identification method to evaluate the parameters involved in the dynamics of partially submerged bodies in order to predict the motions of the chamber and the water column. The method, based on the reverse SISO approach was employed to allow frequency dependent parameters for both the floating structure and the oscillating water column to be independently determined from several forced vibration experiments. To validate the dynamic model proposed, direct and wave force excitation experiments were conducted using two circular floating OWC wave energy converters. The experimental results obtained agree reasonably well with the dynamic model established. Copyright © 2015 Penerbit Akademia Baru - All rights reserved.*

**Keywords:** Oscillating water column, Wave energy, System identification, Mechanical oscillator model, SISO

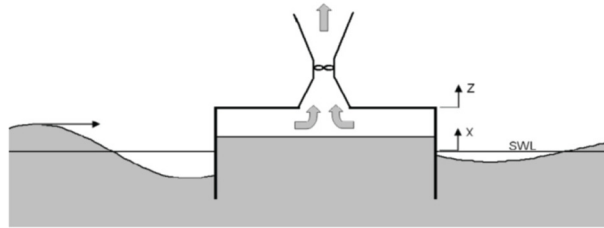
## 1.0 INTRODUCTION

Oscillating water column (OWC) wave energy conversion devices consist of a partially submerged chamber open to wave forces at the base (see figure 1). The wave forces cause the water column within the chamber to rise and fall, driving the air in and out (inhalation and exhalation) of the chamber typically through a Wells or variable pitch type air turbine. An electrical generator is then utilised to convert the oscillatory airflow established into electrical energy. The pneumatic gearing provided by the air coupling allows the conversion of low frequency wave power into high frequency electrical power.

Oscillating water column type wave energy conversion devices can be located near-shore as a fixed structure or offshore in a floating moored-structure configuration. Much analytical, numerical and experimental work has been undertaken on fixed (e.g. [1]) and floating (e.g. [2]) oscillating water column wave energy conversion. A number of concepts have been demonstrated at scale prototype including the Limpet [3], Oceanlinx's near-shore OWC [4] and the Pico plant [5].

The analysis of floating oscillating water column wave energy conversion devices involves the coupled dynamics of the water column and the floating structure. Mechanical oscillator models have seen considerable use in the study of wave energy converters including

oscillating water column wave energy devices (e.g. [6-8]). This simplified approach, which does not analyse the full hydrodynamic complexity of the situation, provides clear indication of device performance trends and is particularly useful in the preliminary design and model testing development phases. It can provide a more general description of the system behaviour compared to complex numerical approaches (e.g. [9, 10]), allowing for greater ease in determining optimal performance or the efficacy of control strategies [11].

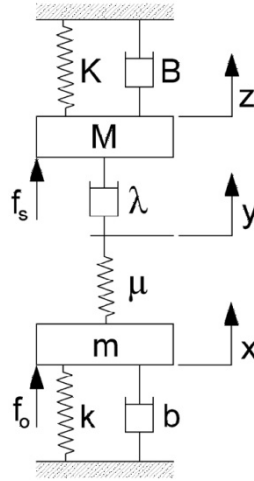


**Figure 1:** Floating Oscillating Water Column Wave energy device (air flow arrows indicate the exhalation phase).

The aim of the present study was to perform system identification on simple OWC geometries to ascertain frequency dependent parameters. The results were used to introduce realistic hydrodynamic characteristics to a discrete parameter mechanical oscillator model. Specifically, this paper presents a novel method for obtaining the hydrodynamic mass and damping coefficients characterizing specific floating OWCs through several forced vibration experiments. Amalgamation of these in the mechanical oscillator model then allows more meaningful investigation of the system behaviour.

### 1.1 Mechanical oscillator model

The basis of the floating OWC heave motion mechanical oscillator model utilised in the present study (figure 2) was the fixed OWC model proposed by Szumko [12] and more recently adopted by Folley and Whittaker [6] with the inclusion of air compressibility. The lower-case variables  $k$ ,  $b$  and  $m$  are the OWC water plane stiffness, radiation damping and mass respectively. The corresponding upper-case parameters for the floating structure are  $K$ ,  $B$  and  $M$ . It must be noted that for the floating structure,  $K$  also includes the mooring line stiffness. The turbine damping is modeled by the linear damping parameter  $\lambda$  and the air compressibility by the linear stiffness  $\mu$ . The  $x$  coordinate is the OWC mean free surface elevation and  $z$  is the floating structure displacement relative to the still-water level (see also figure 1).



**Figure 2:** OWC mechanical oscillator model

The equation of motion for the system illustrated in figure 2 is

$$\mathbf{M}\ddot{\tilde{x}} + \mathbf{B}\dot{\tilde{x}} + \mathbf{K}\tilde{x} = \mathbf{F} \quad (1)$$

where the displacement vector is

$$\tilde{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

and the wave force on the structure and water column are represented by the force vector

$$\mathbf{F} = \begin{bmatrix} f_0 \\ 0 \\ f_s \end{bmatrix} \quad (2)$$

The mass matrix is

$$\mathbf{M} = \begin{bmatrix} m+a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M+A \end{bmatrix}, \quad (3)$$

the linear damping matrix is

$$\mathbf{B} = \begin{bmatrix} b & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & -\lambda & B + \lambda \end{bmatrix} \quad (4)$$

and the stiffness matrix is

$$\mathbf{K} = \begin{bmatrix} k + \mu & -\mu & 0 \\ -\mu & \mu & 0 \\ 0 & 0 & K \end{bmatrix} \quad (5)$$

The parameters  $a$  and  $A$  are the added mass for the water column and floating structure respectively. The wave forcing on the OWC and the structure are  $f_0$  and  $f_s$ . The wave forces on the OWC,  $f_0$ , and the floating structure,  $f_s$ , are assumed to be related via the parameter  $r$  (see equation (6)). In general  $r$  is complex, allowing for both a magnitude and phase difference between the forces. In the present study, the floating OWC is axisymmetric. For linear waves, using the Froude-Krylov assumption as a first estimate of the wave induced heave force, it may be shown that the parameter  $r$  is therefore real. In the limit of large wavelength, or small wave number,  $r$  can also be shown to be equivalent to the area ratio of the OWC opening to the total base area of the floating wave energy converter.

$$\begin{aligned} f_0 &= rf \\ f_s &= (1-r)f \end{aligned} \quad (6)$$

where

$$0 \leq r \leq 1$$

The system power capture,  $P$ , may be determined through the analytic solution of equation 1 using the relationship

$$P = \frac{\lambda \omega^2}{2} |Z - Y|^2 \quad (7)$$

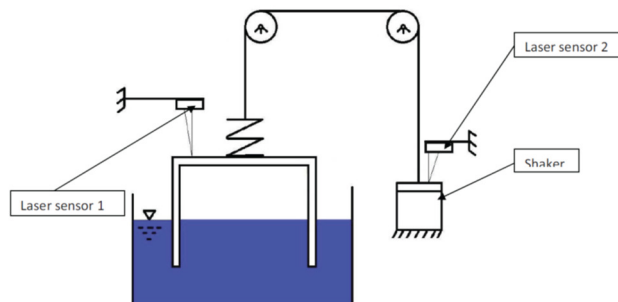
In the expression of equation 7, the terms  $Z$  and  $Y$  are the harmonic solution amplitudes (i.e.  $y = Y e^{i\omega t}$  and  $z = Z e^{i\omega t}$ ) determined from the solution of the system equation of motion.

The characteristics of this system with constant parameter values have been thoroughly investigated analytically in the studies by Stappenbelt and Cooper [13, 14]. In order to introduce realistic hydrodynamic behaviour in this model, the system identification process needs to determine experimental values for the relevant frequency dependent parameters for both the structure and the water column.

## 2.0 METHODOLOGY

The intention of the experimental work conducted was to determine the added mass and the linear damping coefficients of two OWC device structures and their water columns at various frequencies. Forced vibration testing was performed using the arrangement depicted in figure 3.

The model used for the experimental work consisted of a circular cylinder with a removable top. The cylinders were moored with eight horizontal elastic mooring lines configured such that the system had high stiffness in all but the heave degree of freedom. The linearity of the heave restoring force was tested under static loading, displaying a linear curve fit of ( $R^2=0.997$ ). The air compressibility stiffness at the scale of the present experiment is of the order of 105 N/m. Other key experimental parameter values for both OWC devices are presented in tables 1 and 2.



**Figure 3:** Forced vibration experimental arrangement

**Table 1:** Experimental parameter values; OWC 1

<b>Parameter</b>	<b>Value</b>
Cylinder mass	0.296 kg
Cap mass	0.150 kg
Water column mass	0.314 kg
Stiffness of forcing spring	37.1 N/m
Total stiffness (chamber open)	53.3 N/m
Total stiffness (chamber closed)	122.0 N/m
Cylinder external diameter	0.11 m
Cylinder internal diameter	0.10 m
Structural damping ratio	0.7 %

**Table 2:** Experimental parameter values; OWC 2

<b>Parameter</b>	<b>Value</b>
Cylinder mass	0.700 kg
Cap mass	0.558 kg
Water column height	0.118 m
Stiffness of forcing spring	111 N/m
Total stiffness (chamber open)	144 N/m
Total stiffness (chamber closed)	755 N/m
Cylinder external diameter	0.250 m
Cylinder internal diameter	0.242 m
Structural damping ratio	0.1 %

Forced vibration tests were conducted by sweeping linearly through the range of 0.8 to 4 Hz. The length of each test was 780s, of which only 500s was considered in order to exclude the transient phase. The resulting frequency domain data therefore has a frequency resolution of 0.002Hz. With the aid of multiple wave absorption layers, a maximum reflection coefficient of the order of 1% was achieved to reduce the presence of reflected wave energy near the test section.

To ascertain the value of the various parameters required for the model, two sweep tests were required for each OWC; the first with the chamber completely open (i.e. zero turbine damping with the two bodies completely detached), to obtain the parameters of the structure and a second test with the chamber completely closed (i.e. two bodies rigidly attached). Using the difference between the values of the first test and the second, it is possible to estimate the frequency dependent parameters for the water columns.

The frequency dependent physical parameters of interest in the present OWC floating structure system are the added mass and the linear and non-linear damping. The application of the reverse multiple inputs, single output (MISO) technique described for example by Falzarano, Cheng and Rodrigues [15] is therefore most appropriate and is also most commonly employed.

The system identification procedure comprises of the examination of two limiting cases to deduce the hydrodynamic mass and damping coefficients for both the structure and the water column. The two cases considered are an OWC with no orifice (i.e. closed chamber) and an OWC with zero turbine damping (i.e. a completely open chamber). In the closed chamber case the relatively incompressible air at the small scale of the experiment allows the system, comprising of the chamber and water column, to move as a single rigid body. In the chamber open configuration the chamber alone moves as a single rigid body. The forced vibration experimental analysis conducted therefore needs only to consider a single degree of freedom system.

For a single degree of freedom system the differential equation of motion of a body with viscous damping can be expressed as

$$(M + A(\omega))\ddot{z} + B_1(\omega)\dot{z} + B_2(\omega)|\dot{z}|\dot{z} + Kz = f(t) \quad (8)$$

where  $B_1$  is the linear damping coefficient and  $B_2$  is the nonlinear damping coefficient.

The nonlinear heave integro-differential equation of motion for linear and nonlinear damping terms with memory (i.e. they are frequency dependent) [16] is then given by

$$\begin{aligned} (M + A(\infty))\ddot{z}(t) + \int_0^t B_1(\tau)\dot{z}(t-\tau)d\tau + \dots \\ \int_0^t B_2(\tau)|\dot{z}(t-\tau) - w(t-\tau)|(\dot{z}(t-\tau) - w(t-\tau))d\tau + \dots \\ Kz(t) = f(t) \end{aligned} \quad (9)$$

Note that  $B_1(\omega)$  and  $B_2(\omega)$  are the Fourier transform of  $B_1(\tau)$  and  $B_2(\tau)$ ,  $f(t)$  is the heave excitation and  $z(t)$  is the heave response. To reduce the correlation between the terms in equation 9, generally a relative vertical velocity term including the vertical water particle velocity  $w(t)$  is introduced.

The Fourier transform of equation 9 gives the reverse dynamic system

$$H_1(\omega)Z_1(\omega) + H_2(\omega)Z_2(\omega) = F(\omega) \quad (10)$$

where  $Z_1(\omega)$  is the Fourier transform of  $z(t)$ ,  $Z_2(\omega)$  is the Fourier transform of  $z'(t-\tau)-w(t-\tau)$  and  $F(\omega)$  is the Fourier transform of  $f(t)$ . This system may be readily solved to determine the frequency dependent parameters of interest.

However, since the vertical water particle velocity is zero when the OWC undergoes forced vibration, the displacement and velocity terms are unfortunately perfectly correlated. With a correlation of one, the reverse MISO method cannot therefore be used to determine the system parameters (see [17], p227) and the non-linear damping parameter must be ignored. The system then reverts to a linear system and the reverse single input, single output (SISO) analysis may be employed. The low Reynolds numbers present at the small laboratory scale go some way to justifying the consideration of linear damping only. Later experiments with wave excitation of the structure (where the reverse MISO method was successfully employed) indicated that the non-linear damping coefficients were indeed relatively small.

The system identification process used must consider the linear system equivalent of equation 10, expressed as

$$H(\omega)Z_1(\omega) = F(\omega) \quad (11)$$

The frequency dependent added mass,  $A(\omega)$ , and linear damping,  $B(\omega)$ , parameters may then be determined from the relationships

$$A(\omega) = \frac{B - \text{Re}[H(\omega)]}{\omega^2} - M \quad (12)$$

$$B(\omega) = \frac{\text{Im}[H(\omega)]}{\omega} \quad (13)$$

These coefficients need to be calculated for both the floating structure (i.e. the chamber) and the water column.

### 3.0 RESULTS AND DISCUSSION

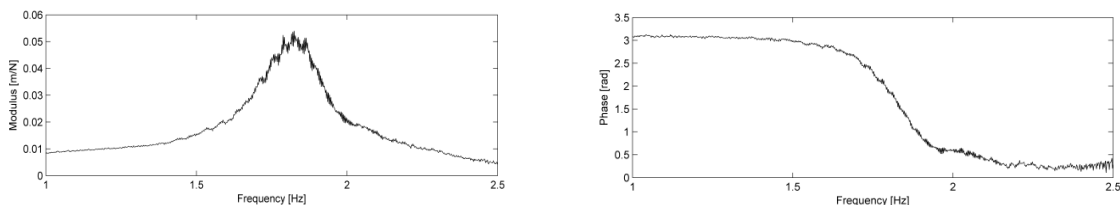
In the presentation of results, all parameters have been normalised by the mass of displaced water,  $m_d$ , as shown in equations 14 and 15. The heave displacements plotted throughout this paper are the displacements per unit force applied, either with the shaker or through wave excitation.

$$A^*(\omega) = \frac{A(\omega)}{m_d} \quad (14)$$

$$B^*(\omega) = \frac{B(\omega)}{\omega m_d} \quad (15)$$

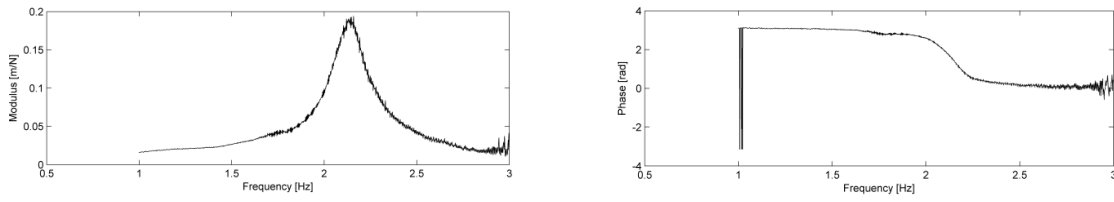
#### 3.1 Oscillating Water Column 1 (OWC 1)

The Fourier transform of the frequency sweeps for OWC 1, with an open and closed chamber are presented in figures 4 and 5 respectively. Clearly visible is the amplitude resonance peak and the associated near 180 degree phase shift. The natural frequency of the closed chamber is higher than the corresponding open chamber frequency. This indicates that for the present system, the relative increase in the water plane stiffness (due to an increase in the water-plane area when the chamber is closed) is more significant than the increase in the mass of the body due to the water column.

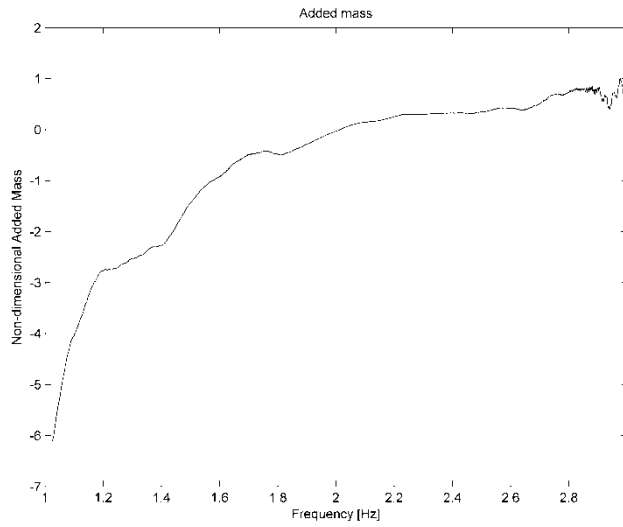


**Figure 4:**OWC 1 Forced response frequency sweep; open chamber.

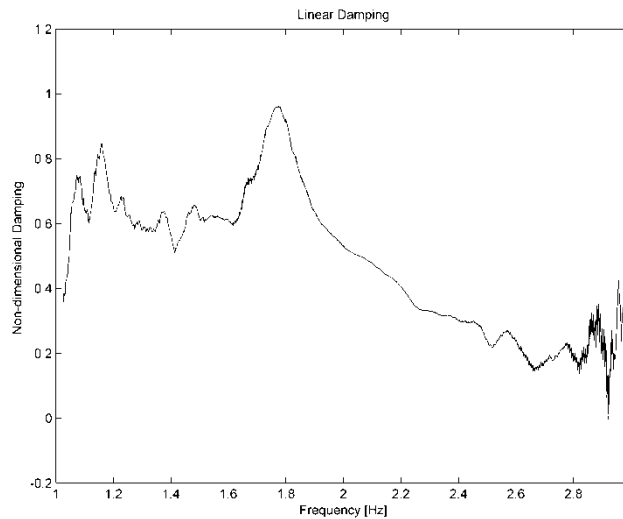




**Figure 5:**OWC 1 Forced response frequency sweep; closed chamber.



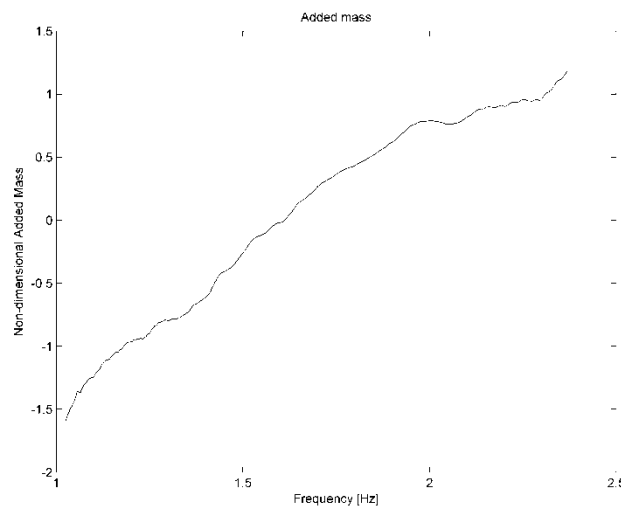
**Figure 6:**OWC 1 Dimensionless added mass; open chamber.



**Figure 7:**OWC 1 Dimensionless linear damping; open chamber.

Dimensionless parameters for OWC 1 with an open chamber are plotted in figures 6 and 7. Note that a negative added mass is observed at low frequencies. A negative added mass, indicating a hydrodynamic force in phase with acceleration that is assisting the motion of the OWC, is similarly found in a number of other vibrating marine systems (e.g. [18, 19]). The linear damping coefficient shows a general decline with increasing frequency. As expected, it remains positive, indicating an energy loss rather than source for the system.

Similarly, the results for the two bodies rigidly linked (i.e. with a closed chamber) are shown in figures 8 and 9. A similar trend is seen with regard to the added mass although negative added mass is not observed until much lower frequencies than in the open chamber case.



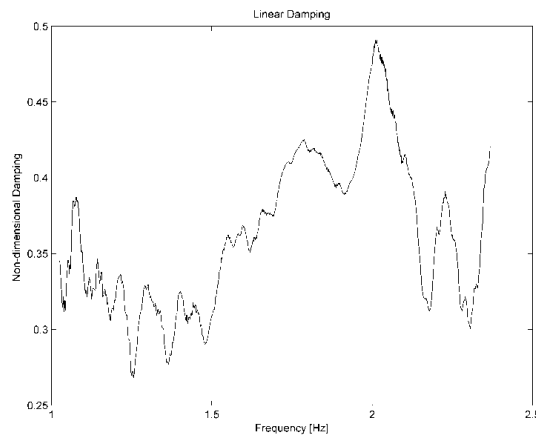
**Figure 8:**OWC 1 Dimensionless added mass; closed chamber.

From the results of the analysis of the open and closed chamber tests on OWC 1, the hydrodynamic mass and damping parameters for the floating structure and the water column may be determined. These are of course necessary inputs for the model presented in figure 2. The hydrodynamic mass and damping of the OWC 1 structure were taken directly from figure 6 and 7 respectively. The corresponding parameters for the water column were estimated by the difference of the closed chamber and open chamber results obtained.

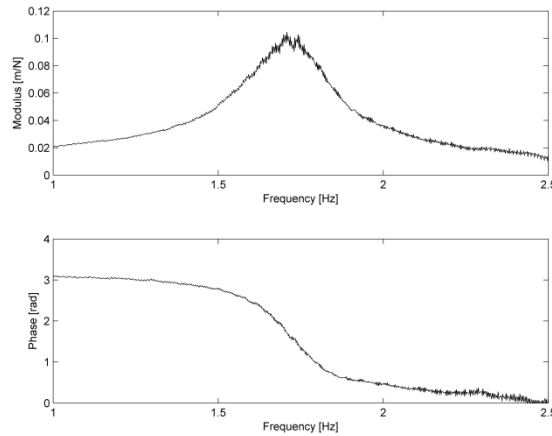
To determine the accuracy of the mechanical oscillator model using the frequency dependent parameters determined, forced vibration experiments were performed on the floating OWC 1 system with a finite, non-zero power take-off damping. A 20mm orifice in the top of the OWC was used to simulate a non-linear turbine damping of  $\lambda=14.38|z'-x'|$  Ns/m (with an orifice flow coefficient of 0.64). The Fourier transform of the frequency sweep is presented in figure 10.

The response of the structure may be predicted by solving the equations of motion of the three degree of freedom mechanical oscillator model with the frequency dependent parameters obtained. A mean power take-off damping value over the operating range of the OWC was used as the linear turbine damping input to the model. The predicted and experimental heave response curves of the OWC 1 structure are plotted together for

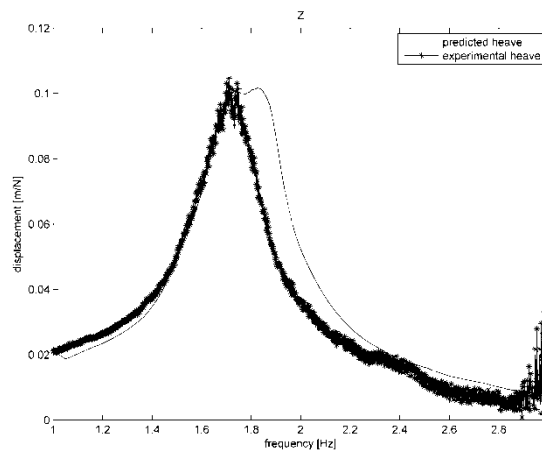
comparison in figure 11 and show reasonable agreement. It is believed that the deviation between the model and the experimental results are due to the non-linear power take-off characteristics inherent in an orifice turbine simulation.



**Figure 9:**OWC 1 Dimensionless linear damping; closed chamber.



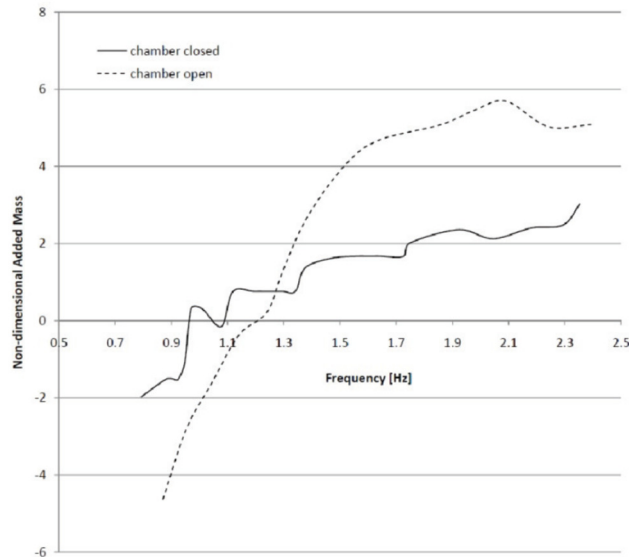
**Figure 10:**OWC 1 Forced response frequency sweep; 20mm orifice



**Figure 11:**OWC 1 theoretical and experimental forced vibration heave response; 20mm orifice

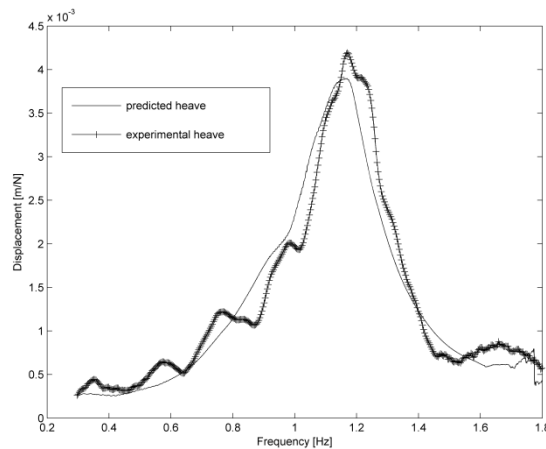
### 3.2 Oscillating Water Column 2 (OWC 2)

Dimensionless parameters for OWC 2 with an open and closed chamber were also determined using the forced vibration method. The form of these plots is very similar to those for OWC 1. See for example the added mass plots in figure 12. The linear damping observed was approximately uniform across the frequency range of interest.



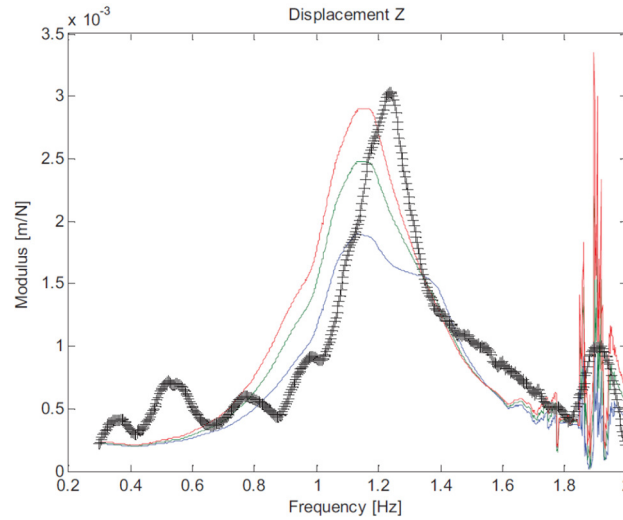
**Figure 12:**OWC 2 Dimensionless added mass; open and closed chamber.

Further experimental validation of the mechanical oscillator modelling capability was undertaken by examining the predicted OWC 2 heave behaviour under wave excitation. The OWC 2 system with a 32mm orifice (equating to a damping value of  $\lambda=34.22|z'-x'|$  Ns/m) was tested.

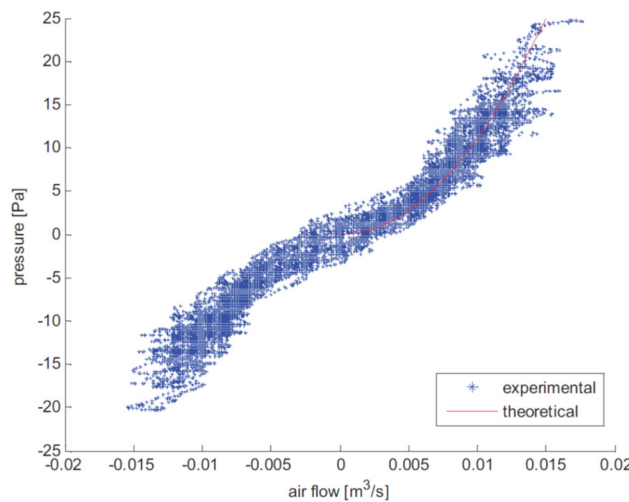


**Figure 13:**OWC 2 theoretical and experimental wave forcing heave response; 32mm orifice

The predicted and experimental heave response curves of the structure with a 32 mm orifice are plotted together for comparison in figure 13 and show reasonable agreement. The difference in the predicted and measured responses is again believed to be primarily due to the non-linearity of the experimental power take-off damping. The orifice, with its quadratic pressure-flow rate behaviour, is commonly employed to simulate turbine loads in OWC experimentation.



**Figure 14:**OWC 2 theoretical and experimental wave forcing heave response; 64mm orifice



**Figure 15:**OWC 2 pressure-flow rate relationship; 64mm orifice

The predicted and experimental heave response curves of the structure with a 64 mm orifice are plotted together for comparison in figure 14. This figure illustrates the potential influence

of the power take-off nonlinearity by plotting the model predictions using the two linear curve fit extremes (i.e. the red and blue lines) to the orifice behaviour (see figure 15). The response in figure 14 may be seen to align better with one or other curve (produced by the power take-off damping at either extreme) at various frequencies throughout the range of interest. It is worth noting that typical OWC turbine behaviour more closely follows a linear pressure-flow rate relationship [20]. Future experimental modelling should therefore aim to reduce the non-linearity in the power take-off damping.

#### **4.0 CONCLUSION**

A novel approach using forced vibration experimental data for open and closed OWC chambers (i.e. a single degree of freedom floating OWC system) has been introduced. The system identification process utilises the reverse SISO method. The frequency dependent parameters of interest in the associated mechanical oscillator model were estimated using a combination of these results.

The mechanical oscillator modelling accuracy using these parameters was assessed against the multiple degree of freedom motion of the coupled floating OWC system by introducing power take-off damping. Comparison of the model predictions of the heave motion of the structure with experimental results under forced vibration and wave excitation conditions were favourable.

The primary problem identified in the modelling is the linearisation of the non-linear power take-off damping introduced by the orifice employed to experimentally simulate the turbine load. This damping is underestimated in the model around the resonant peak, where the actual flow velocity is higher than the mean and overestimated in the low frequency region, where the flow velocity from the chamber is low. Since the mechanical oscillator model linear damping and not the orifice turbine simulation most closely mimics the pressure-flow rate behaviour of a real turbine [20], future experimental modelling should aim to produce a more linear power take-off damping.

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