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Natural Convection Flow of Visco-elastic Fluid Along on an Infinite Vertical Porous Plate Effects on Grashof Number, Magnetic Field and Visco–elastic Parameter

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Abstract – *Natural convection is widely studied heat transfer phenomenon. The reason for this particular interest is in the everyday importance of natural convection. An analysis is presented to study the effects on Grashof number, magnetic field and visco–elastic parameter on MHD natural convection flow with visco-elastic fluid. Results are presented for the zero order perturbation velocity profile, the first order perturbation velocity and temperature profiles with the help of Local non-similarity method. Numerical results show that effect of the dimensionless parameters have significant influences on the zero order perturbation velocity profile, the first order perturbation velocity and temperature profiles.*

Keywords: Natural convention flow, Visco-elastic Fluid, Porous plate, Grashof number, Magnetic field

1.0 INTRODUCTION

Magnetofluid dynamics is in the study of flow of electrically conducting fluids in electric and magnetic fields. Reineke [1] and Steinberner and Reineke [2] experimentally and numerically investigated the natural convection of Jule-heated fluids in rectangular and semicircular cavities. They found that the natural convection flows internally heated cavities are thermally stratifield in the lower regions, whereas the upper part is unsteady due to a multi-vortex flow field. It unifies in a framework the electromagnetic and fluid dynamic theories to yield a description of the concurrent effects of magnetic field on the flow and the flow on the magnetic field. There are many natural phenomena and engineering problems susceptible to Magnetofluid dynamics analysis. It is useful in astrophysics because much of the universe is filled with widely spaced, charged particles and permeated by magnetic fields, and so the continuum assumption becomes applicable. The presence of roughness elements on the flat surface disturbs the flow and alters the heat transfer rate. Using a simple transformation on the governing equation for the flow considered here, non-similar boundary layer equations for a wavy surface are derived which are proposed to be integrated by Keller box method. Of interest are the effects of the Grashof number *Gr*, magnetic field parameter *M* and, Visco-elasticity parameter *S*.

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A study of considerable importance in areas concerned with the energy generation and its utilization is that of heat transfer. Atmospheric, geophysical and environmental problems in connection with heat rejection, space research and manufacturing system require such type of studies. In the diversity of studies related to heat transfer, considerable effort directed at connective mode, in which relative motion of the fluid provides an additional mechanism for the transfer of energy.

The connective mode of heat transfer is divided into two basic processes. If the motion of the fluid arises due to an external agent, such as the externally-imposed flow of a fluid stream over a heated object, the process is termed as forced convection. This type of fluid flow is caused in general by a fan, blower, the wind or the motion of the heated object itself. Such problems are very frequently encountered in technology, where the heat transfers to, or from a body is often due to imposed flow of a fluid at a temperature different from that of the body. If, on the other hand, no such externally induced flow is provided and the flow arises "naturally" due to the effect a density difference, resulting from a temperature in a body, the process is termed natural or free convection. The density difference gives rise to buoyancy effects due to which the flow is generated. A heated body cooling in ambient air generates such a flow in the region surrounding it. Similarly buoyant flow arising from heat rejection to the atmosphere and to other ambient media, circulation's arising in heated rooms in the atmosphere, and in bodies of water, causes thermal stratification of the medium. Many other such heat transfer processes, in our natural environment as well as in many technological applications, are included in the area of natural convection. The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate, in its own plane was studied first by stokes. Because of its particular importance, it has been extended to bodies of different shapes by a number of researchers. Amongst them are Illingworth [3], Stewartson [4] Hell [5] and Elliot [6]. Illingworth [3] considered the flow of a compressible gas with variable viscosity near an impulsively started vertical plate and the problem was solved by the method of successive approximation. Analytical studies on unsteady laminar free-convection problems have received much attention by many research workers such as Surgawara and Michiyoshi [7]. Siegel [8], Gebhart [9], Chung and Anderson[10], Sparrow and Gregg [11] and Yang [12]. Exact solutions were available only for the infinite vertical plate with Prandtl number of unity and under transient conditions of step change in either the surface temperature or the surface heat flux. These represent asymptotic solutions expected to be valid not only at large distances away from the leading edge, but also during a short time interval after the communication of the free convection flow along a finite plate. Exact asymptotic solutions for the same problem with arbitrary surface temperature or heat flux variations were obtained by Menold and Yang [13] and Soundalgeker [14]. In these studies, solutions for the coupled equations governing the flow field were obtained in exact form using the Laplace transform technique. Study of the flow of electrically conducting fluid in presence of magnetic field across a surface is also important from the technological point of view. Fluid flowing across a transverse magnetic field produces an electromagnetic force. The current and the magnetic field combine to produce force that resists the fluid's motion. The current also generates itself magnetic field which distorts the original magnetic field. An opposing or pumping force on the fluid can be produced by applying an electric field perpendicular to the magnetic field. The disturbance created in the magnetic field or the fluid can produce magneto-fluid dynamics waves in upstream and downstream wake phenomena Cramer and Pai [15]. In natural convection boundary layer flow of a electrically conducting fluid up a vertical wall in presence of a strong cross-magnetic field has been studied by Singh and Cowling [16]. Sparrow and Cess [17], Riley [18], Kuike [19], Wilks [20], Hossain and Ahmed [21] and many others. All the above studies were confined to forced, free and combined forced and free convection flow of an electrically conducting fluid along a vertical surface in presence of a transverse strong magnetic field. Compared with the steady state situation, there are relatively few solutions available for the transient flow of the electrically conducting fluid in presence of a magnetic field. Rossow [22] investigated the problem of an infinite flat plate given impulsive motion in presence of transverse magnetic field. Later Singh [23] studied the transient free convection flow in presence of magnetic field. Recently, Seethamahalakshmi, Prasad and Ramana Reddy [24], Sushil and Gopal [25], Choudhury and Kumar Das [26] studied MHD free convection flow effects on the unsteady heat convective mass transfer flow past an infinite vertical porous plate with variable suction parameter. Islam [28] has investigated for the free convection flow of visco-elastic fluid past an infinite vertical with time dependent suction for an arbitrary Prandtl number. In the effect of elasticity on MHD flow of an elastico-viscous fluid past an accelerated plate has been investigated, and in , analysis of the stokes problem for the MHD natural convection flow of a visco-elastic fluid past an impulsively started vertical plate has been performed employing the Laplace transform technique for Prandtl number Pr = 1. In this work, it is proposed to study the effects of free convection flow of an electrically conducting visco- elastic fluid past an infinite vertical porous plate in presence of transverse magnetic fluid. The results for the velocity fields are shown graphically. The for the zero order perturbation velocity profile, the first order perturbation velocity and temperature profiles is represented graphically and in tabular form for different values of the Grashof number Gr, Magnetic field parameter M and visco-elastic parameter S.

2. 0 THE GOVERNING EQUATIONS AND TRANSFORMATIONS

Consider unsteady laminar, incompressible, viscous, electrically conducting fluid flowing past an infinite plate. The flow is assumed to be in the x-direction which is along the vertical plate in the upward direction and y-axis is taken to be normal to the plate. Initially the temperature of the plate and the fluid are same. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. We assume that all the fluid properties are constant and the influence of density variation with temperature is considered only in the body force term. Under the usual boundary layer approximation, the flow, heat and mass transfer in the presence of parameter are governed by the following equations:

Mass equation

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation

$$\frac{\partial u}{\partial t} - \lambda \frac{\partial u}{\partial y} = \beta g (T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_0 \beta_0^2 u}{\rho} - \kappa_0^* \frac{\partial^3 u}{\partial y^2 \partial t} \tag{2}$$

Energy equation

$$\frac{\partial T}{\partial t} - v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Here u and v are the velocity components in x and y-directions, respectively, ν is the kinematic viscosity T is the temperature of the fluid in the boundary layer, g is the acceleration due to gravity, β is the volumetric coefficient of expansion, κ is the thermal conductivity, ρ is the density of the fluid, σ_0 is the electric conductivity, C_P is the specific heat at constant pressure, T_∞ is the temperature of the ambient fluid, κ_0^* is the rotational viscosity coefficient and λ is the suction parameter.

The appropriate boundary conditions are

$$\begin{aligned} &t \le 0; \quad u(y,t) = 0; \; T(y,t) = T_\infty, \\ &t > 0; \quad u(0,t) = u_0 f(t); \; T(0,t) = T_W, \\ &t > 0; \quad u(\infty,t) = 0; \quad T(\infty,t) = T_\infty. \end{aligned} \tag{4}$$

The temperature at the plate is instantaneously raised or lowered to T_w, which is thereafter maintained constant in order to produce bouyancy effect. The heat due to viscous and joule dissipation are neglected in the energy equation because of small velocity usually encounters in free convection flow.

In order to reduce the number of independent variables to make the governing equations dimensionless, we apply the following transformations:

$$W = \frac{u}{u_0}, \quad \eta = \frac{u_0}{v} y, \quad \tau = \frac{u_0^2}{v} t, \quad \theta = \frac{T - T_\infty}{T_W - T\infty} \tag{5}$$

With the help of these non-dimensional variables the momentum equation (2) takes the form:

$$\frac{\partial w}{\partial \tau} - \frac{\lambda}{u_0}\frac{\partial w}{\partial \eta} = \frac{\partial^2 w}{\partial \eta^2} - S\frac{\partial^3 w}{\partial \eta^2 \partial \tau} - Mw + G_r\theta \tag{6}$$

Energy equation (3) takes the form

$$\frac{\partial \theta}{\partial \tau} - \frac{\lambda}{u_0}\frac{\partial \theta}{\partial \eta} = \frac{1}{p_r}\frac{\partial^2 \theta}{\partial \eta^2} \tag{7}$$

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where $Gr = \dfrac{g\beta\Delta T x^3}{\nu^2}$ is the Grashof Number, $\Pr = \dfrac{\mu C_P}{\kappa}$ is the Prandtl Number,

$S = k_0^*\left(\dfrac{u_0}{\nu}\right)^2$ is the visco- elasticity parameter, $M = \dfrac{\nu\sigma_0\beta_0^2}{u_0^2\rho}$ is the magnetic field Parameter.

The boundary conditions become

$$\begin{aligned} &\tau \le 0, \quad w(\eta,\tau) = \theta(\eta,\tau) = 0 \\ &\tau > 0, \quad w(0,\tau) = \tau^n, \quad \theta(0,\tau) = 1 \\ &\qquad\quad w(\infty,\tau) = 0, \quad \theta(\infty,\tau) = 0 \end{aligned} \tag{8}$$

where n is an exponent.

4.0 SOLVE OF THE TEMPERATURE EQUATION

We have applied the Laplace transform technique equation (7) subject to boundary condition,

$$\bar{\theta}(0,\tau) = \frac{1}{q}, \bar{\theta}(\infty,\tau) = 0 \tag{9}$$

we get

$$\bar{\theta} = \frac{e^{-\sqrt{P_r}\left(\alpha + \sqrt{\alpha^2 + q}\right)\eta}}{q} \tag{10}$$

where q is the Laplace transformation parameter and $a\sqrt{P_r} = \alpha$. The temperature distribution θ is now obtained by taking the inverse Laplace transformation of equation (10) to obtain.

$$\theta(\eta,t) = \frac{1}{2}e^{-\alpha\sqrt{P_r}\eta}\left[e^{\alpha\sqrt{P_r}\eta}erfc\left(\frac{\eta\sqrt{P_r}}{2\sqrt{t}} + \sqrt{\alpha t}\right) + e^{-\alpha\sqrt{P_r}\eta}erfc\left(\frac{\eta\sqrt{P_r}}{2\sqrt{t}} - \sqrt{\alpha t}\right)\right] \tag{11}$$

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For P_r=1, equation (11) turns into

$$\theta(\eta,t)=\frac{1}{2}e^{-\alpha\eta}\left[e^{\alpha\eta}erfc\left(\frac{\eta}{2\sqrt{t}}+\sqrt{\alpha t}\right)+e^{-\alpha\eta}erfc\left(\frac{\eta}{2\sqrt{t}}-\sqrt{\alpha t}\right)\right] \tag{12}$$

From equation (12) we get the rate of heat transfer defined by

$$q_w=-\frac{ku_0}{\nu}\Delta T\frac{\sqrt{P_r}}{\sqrt{(\pi T)}} \tag{13}$$

where $\Delta T=T_w-T_\infty$. It follows from equation (13) that the rate of heat transfer is directly proportional to the square root of the Prandtl number and inversely proportional to the square root of the time variable t.

4.0 SOLVE OF THE MOMENTUM EQUATION

Momentum equation (6) can be written as

$$\frac{\partial w}{\partial\tau}-a\frac{\partial w}{\partial\eta}=\frac{\partial^2 w}{\partial\eta^2}-S\frac{\partial^3 w}{\partial\eta^2\partial\tau}-Mw+G_r\theta \tag{14}$$

subject to the boundary conditions,

$$\begin{aligned}&\tau\le 0,\quad w(\eta,\tau)=0,\\ &\tau>0,\quad w(0,\tau)=\tau^n,\\ &\tau>0,\quad w(\infty,\tau)=0.\end{aligned} \tag{15}$$

Equation (14) is a third order partial differential equation. For S=0, it reduces to equation governing the Newtonian fluid. Hence, the presence of elastic parameter increases the order of the governing equation from two to three. Thus to overcome the difficulty we adopt the perturbation technique in which the elastic parameter S can be regarded as a small quantity. We assume the solution of (15) in the form $w=w_0+Sw_1$

$$\frac{\partial\left(w_0+Sw_1\right)}{\partial\tau}-a\frac{\partial\left(w_0+Sw_1\right)}{\partial\eta}=\frac{\partial^2\left(w_0+Sw_1\right)}{\partial\eta^2}-M\left(w_0+Sw_1\right)-S\frac{\partial^3\left(w_0+Sw_1\right)}{\partial\eta^2\partial\tau}+G_r\theta$$

Equating the coefficient of free S and those of S, we obtain the following equations:

$$\frac{\partial w_0}{\partial\tau}-a\frac{\partial w_0}{\partial\eta}=\frac{\partial^2 w_0}{\partial\eta^2}-Mw_0+G_r\theta, \tag{16}$$

$$\frac{\partial w_1}{\partial\tau}-a\frac{\partial w_1}{\partial\eta}=\frac{\partial^2 w_1}{\partial\eta^2}-Mw_1-\frac{\partial^3 w_0}{\partial\eta^2\partial\tau}. \tag{17}$$

with the boundary conditions (15) becoming

$$\tau\le 0,\quad w_0\left(\eta,\tau\right)=0,\quad w_1\left(\eta,\tau\right)=0,$$

$$\tau>0,\quad w_0\left(0,\tau\right)=\tau^n,\quad w_1\left(0,\tau\right)=0, \tag{18}$$

$$w_0\left(\infty,\tau\right)=w_1\left(\infty,\tau\right)=0.$$

Applying Laplace transform on (16), we get

$$\frac{\partial^2\overline{w}_0}{\partial\eta^2}+a\frac{\partial\overline{w}_0}{\partial\eta}-\left(q+M\right)\overline{w}_0=-G_r\overline{\theta}. \tag{19}$$

and the boundary conditions (18) take the form

$$\overline{w}_0\left(0,q\right)=\frac{1}{q},\quad \overline{w}_0\left(\infty,q\right)=0. \tag{20}$$

Solution of the differential equation (19) subject to (20) is

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$$\overline{w}_0 = Ae^{\frac{-a+\sqrt{a^2+4(q+M)}}{2}\eta} + Be^{\frac{-a+\sqrt{a^2+4(q+M)}}{2}\eta} - \frac{G_r}{q}\frac{e^{-\sqrt{P_r}\left(\alpha+\sqrt{\alpha^2+q}\right)\eta}}{P_r\left(\alpha+\sqrt{\alpha^2+q}\right)^2 - a\sqrt{P_r}\left(\alpha+\sqrt{\alpha^2+q}\right)-\left(M+q\right)} \tag{21}$$

for $P_r \neq 1$, where $Q_r^2 = \frac{M}{P_r - 1}$ .Using inverse Laplace transform on (21) we obtain

$$w_0 = I_0 + \frac{G_r}{P_r - 1}I_1 - \frac{G_r}{P_r - 1}I_2 \tag{22}$$

for $P_r \neq 1$, where

similarly application of Laplace transform on equation (17), gives

$$\frac{d^2\overline{w}_1}{d\eta^2} + a\frac{d\overline{w}_1}{d\eta} - (M+q)\overline{w}_1 = q\frac{d^2\overline{w}_0}{d\eta^2} \tag{23}$$

subject to the boundary conditions

$$\overline{w}_1(o,q) = \frac{1}{q} \tag{24}$$

Substituting w_0 from (22) in (23) we get

$$\frac{d^2\overline{w}_1}{d\eta^2} + a\frac{d\overline{w}_1}{d\eta} - \left(M+q\right)\overline{w}_1 = q\frac{d^2}{d\eta^2}\left[\frac{1}{q} + \frac{G_r e^{-\frac{1}{2}\left(a+\sqrt{a^2+4(q+M)}\right)\eta}}{q\left(P_r-1\right)\left[\left(\alpha+\sqrt{\alpha^2+q}\right)^2 - Q_r^2\right]} - \frac{G_r}{q\left(P_r-1\right)}\frac{e^{-\sqrt{P_r}\left(\alpha+\sqrt{\alpha^2+q}\right)\eta}}{\left[\left(\alpha+\sqrt{\alpha^2+q}\right)^2 - Q_r^2\right]}\right]$$

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$$= \left[1+\frac{G_r}{(P_r-1)\left[\left(\alpha+\sqrt{\alpha^2+q}\right)^2-Q_r^2\right]}\right]\left[-\frac{1}{2}\left\{a+\sqrt{a^2+4(q+M)}\right\}\right]^2 e^{-\frac{1}{2}\left\{a+\sqrt{a^2+4(q+M)}\right\}\eta}$$

$$-\frac{G_r}{(P_r-1)}\frac{P_r\left(\alpha+\sqrt{\alpha^2+q}\right)^2}{\left[\left(\alpha+\sqrt{\alpha^2+q}\right)^2-Q_r^2\right]}e^{-\sqrt{P_r}\left(\alpha+\sqrt{\alpha^2+q}\right)\eta} \quad (25)$$

Using inverse Laplace transform on (25) we obtain

$$w_1=\frac{G_rP_r}{(P_r-1)^2}L^{-1}\frac{\left(\alpha+\sqrt{\alpha^2+q}\right)^2}{\left\{\left(\alpha+\sqrt{\alpha^2+q}\right)^2-Q_r^2\right\}^2}e^{-\frac{a+\sqrt{a^2+4(q+M)}}{2}\eta}$$

$$-\frac{\eta}{4}L^{-1}\frac{\left[a+\sqrt{a^2+4(q+M)}\right]^2}{\sqrt{a^2+4(q+M)}}\left[1+\frac{G_r}{(P_r-1)\left\{\left(\alpha+\sqrt{\alpha^2+q}\right)^2-Q_r^2\right\}}\right]e^{-\frac{a+\sqrt{a^2+4(q+M)}}{2}\eta}$$

$$-\frac{G_rP_r}{(P_r-1)^2}L^{-1}\frac{\left(\alpha+\sqrt{\alpha^2+q}\right)^2}{\left\{\left(\alpha+\sqrt{\alpha^2+q}\right)^2-Q_r^2\right\}^2}e^{-\sqrt{P_r}\left(\alpha+\sqrt{\alpha^2+q}\right)\eta}$$

(26)

This solution can be written as

$$w_1=\frac{G_rP_r}{(P_r-1)^2}J_1-\frac{\eta}{4}J_2-\frac{G_rP_r}{(P_r-1)^2}J_3 \quad (27)$$

for $P_r \neq 1$, We obtain from equation(22)

$$w_0=\frac{e^{-\frac{a\eta}{2}}}{2}\left[e^{\frac{\sqrt{a^2+4M}}{2}\eta}erfc\left(\frac{\eta}{2\sqrt{\tau}}+\frac{\sqrt{\left(a^2+4M\right)\tau}}{2}\right)+e^{-\frac{\sqrt{a^2+4M}}{2}\eta}\right.$$

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$$erfc\left(\frac{\eta}{2\sqrt{\tau}}-\frac{\sqrt{\left(a^2+4M\right)\tau}}{2}\right)\Bigg]+\frac{Ge^{-\frac{a\eta}{2}}}{4\left(P_r-1\right)Q_r}\int_0^{\tau}\Bigg[e^{\frac{\sqrt{a^2+4M}}{2}\eta}\,erfc\left(\frac{\eta}{2\sqrt{v}}+\frac{\sqrt{\left(a^2+4M\right)v}}{2}\right)$$

$$+e^{-\frac{\sqrt{a^2+4M}}{2}\eta}\,erfc\left(\frac{\eta}{2\sqrt{\tau}}-\frac{\sqrt{\left(a^2+4M\right)}}{2}\eta\right)\Bigg]\Big[\left(\alpha+Q_r\right)e^{2\alpha Q_r(\tau-v)}\,erfc\left\{\left(\alpha+Q_r\right)\sqrt{\tau-v}\right\}$$

$$-\left[\left(\alpha-Q_r\right)e^{-2\alpha Q_r(\tau-v)}\,erfc\left\{\left(\alpha-Q_r\right)\sqrt{\tau-v}\right\}\right]e^{Q_r^2(\tau-v)}\,dv-\frac{Ge^{-\sqrt{P_r}\alpha\eta}}{2Q_r\left(P_r-1\right)\left(4\alpha^2-Q_r^2\right)}$$

$$\left[Q_r^2 e^{-\sqrt{P_r}\alpha\eta}\,erf\,c\left(\frac{\sqrt{P_r}\,\eta}{2\sqrt{\tau}}-\alpha\sqrt{\tau}\right)-\left(4\alpha^2-Q_r^2\right)e^{\sqrt{P_r}\alpha\eta}\,erfc\left(\frac{\sqrt{P_r}\,\eta}{2\sqrt{\tau}}+\alpha\sqrt{\tau}\right)\right] \tag{28}$$

for $P_r \neq 1$. Finally we obtained from equation (27)

$$w_1=\frac{G_r P_r \eta e^{-\frac{a\eta}{2}}}{2\left(P_r-1\right)^2\sqrt{\pi}}\int_0^{\tau}\Bigg[\left\{\frac{2Q_r+\alpha}{4Q_r}+\frac{\left(\alpha+Q_r\right)^2}{2}v\right\}e^{\left(2\alpha Q_r+Q_r^2\right)v}\,erfc\left(\alpha\sqrt{v}+Q_r\sqrt{v}\right)$$

$$-\left\{\frac{\left(\alpha-2Q_r\right)}{4Q_r}-\frac{\left(\alpha-Q_r\right)^2}{2}v\right\}e^{\left(-2\alpha Q_r+Q_r^2\right)v}\,erfc\left(\alpha\sqrt{v}-Q_r\sqrt{v}\right)-\alpha e^{-\alpha^2 v}\sqrt{\frac{v}{\pi}}\Bigg]$$

$$\frac{e^{-\frac{\left(a^2+4M\right)}{4}(\tau-v)-\frac{\eta^2}{4(\tau-v)}}}{\sqrt{(\tau-v)^3}}\,dv-\frac{\eta}{8\sqrt{\pi\tau}}e^{-\frac{a\eta}{2}-\frac{a^2+4M}{4}\tau-\frac{\eta^2}{4\tau}}\left\{a^2+\frac{2a\eta}{\tau}+\frac{2}{\tau}\left(\frac{\eta^2}{2\tau}-1\right)\right\}$$

$$-\frac{\eta G_r e^{-\frac{a\eta}{2}}}{2.8\left(P_r-1\right)\sqrt{\pi}Q_r}\int_0^{\tau}\Bigg[\frac{e^{-\frac{a^2+4M}{4}v-\frac{\eta^2}{4v}}}{\sqrt{v}}\left\{a^2+\frac{2a\eta}{v}+\frac{2}{v}\left(\frac{\eta^2}{2v}-1\right)\right\}\Big\{\left(\alpha+Q_r\right)e^{2\alpha Q_r(\tau-v)}$$

$$erfc\left(\alpha\sqrt{\tau-v}+Q_r\sqrt{\tau-v}\right)-\left(\alpha-Q_r\right)e^{-2\alpha Q_r(\tau-v)}erfc\left(\alpha\sqrt{\tau-v}-Q_r\sqrt{\tau-v}\right)\Big\}e^{Q_r^2(\tau-v)}\Bigg]dv$$

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$$-\frac{G_r P_r}{\left(P_r-1\right)^2}\left[\left\{\frac{\alpha+2Q_r}{4Q_r}+\frac{\left(\alpha+Q_r\right)\left(2\alpha\tau+2Q_r\tau+\sqrt{P_r}\eta\right)}{4}\right\}e^{2\alpha\tau Q_r+Q_r^2\tau+Q_r\eta\sqrt{P_r}}\right.$$

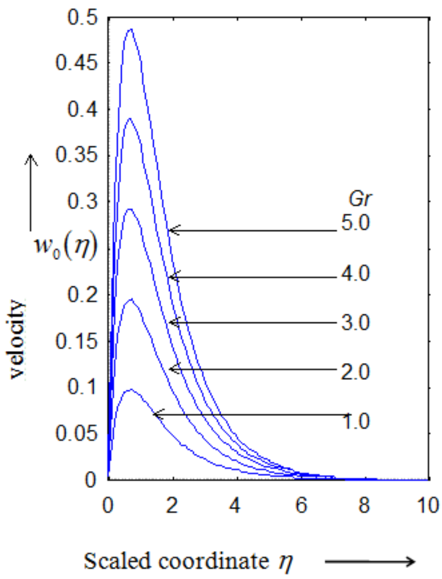
$$erfc\left(\frac{\eta}{2\sqrt{\tau}}+\alpha\sqrt{\tau}+Q_r\sqrt{\tau}\right)-\left\{\frac{\alpha-2Q_r}{4Q_r}-\frac{\left(\alpha-Q_r\right)\left(2\alpha\tau-2Q_r\tau-\sqrt{P_r}\eta\right)}{4}\right\}e^{-2\alpha\tau Q_r+Q_r^2\tau-Q_r\eta\sqrt{P_r}}$$

$$\left.erfc\left(\frac{\eta}{2\sqrt{\tau}}+\alpha\sqrt{\tau}-Q_r\sqrt{\tau}\right)-\alpha\sqrt{\frac{\tau}{\pi}}e^{-\alpha^2\tau-\alpha\sqrt{P_r}\eta-\frac{\eta^2}{4\tau}}\right] \qquad (29)$$

for $\Pr \neq 1$.

5.0 RESULTS AND DISCUSSIONS

The governing boundary layer equations (28-29 and 11) for the velocity and temperature. The present work generalized the problem of natural convection flow of an electrically conducting, viscous, incompressible fluid near an infinite vertical plate. Fig1-2 shows that the zero-order perturbation velocity profile, first order perturbation velocity profile and temperature profile against η for varies values of Grashof number while Prandtl number Pr =1.0, the magnetic field parameter M=4.0, transpiration parameter a=0.8 and visco-elasticity parameters S=0.4. From Fig.1 it is clear that velocity profile increases with the increase of the Grashof number Gr and near the surface of the plate the velocity profile increases and again decreases. From Fig. 1 also show that the first order perturbation velocity profile increases as Grashof number increase. From Figure 3 we may observe that the effect of Grashof number on temperature distribution is constant.



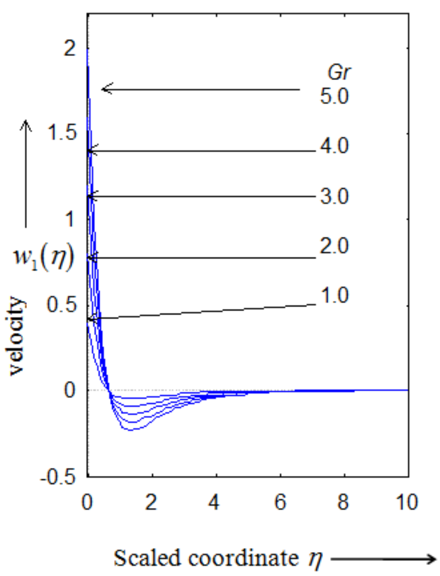


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Figure 1: Zero (left) and first (right) order perturbation velocity perturbation profiles for varies values of *Gr* while *Pr*=1.0, M=4.0, *a*=0.8 and *S*=0.4

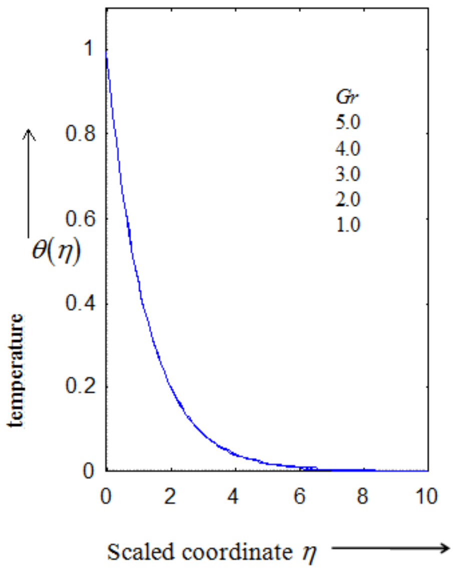

Figure 2: Temperature profiles for varies values of *Gr* while *Pr*=1.0, M=4.0, *a*=0.8 and *S*=0.4

Figs. 3-4 describes the behavior of the zero-order perturbation velocity profile, first order perturbation velocity profile and temperature profile against η for varies values of magnetic field parameter with Grashof number *Gr*=2.0, Prandtl number *Pr* =0.72, transpiration parameter *a*=0.5 and visco-elasticity parameters *S*=0.4. From Fig.3 we observe that the zero-order perturbation velocity profile decreases with increasing the value of magnetic field parameter *M*. Near the surface of the plate velocity profile increases and becomes maximum and then decreases and finally takes asymptotic values. From Fig.3 also shows that when the magnetic field parameter *M* increases the first order perturbation velocity profile decreases. For vertical porous plate the first order perturbation velocity profile is markedly affected by magnetic field parameter. From Fig. 4 we can see that the temperature profile remains unchanged for different values of the magnetic field parameter.

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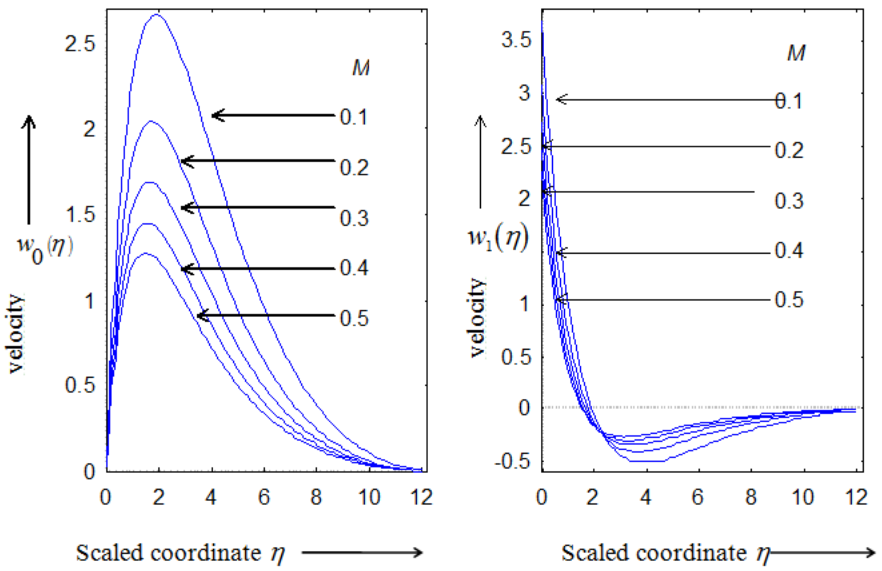

Figure 3: Zero (left) and first (right) order perturbation velocity perturbation profiles for varies values of *M* while *Pr*=0.72, *Gr*=2.0, *a*=0.5 and *S*=0.4

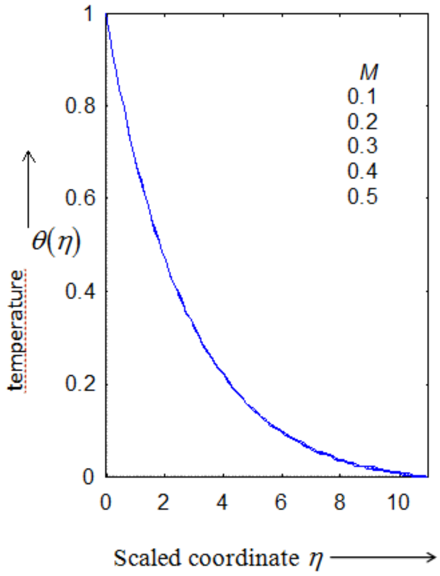

Figure 4: Temperature profile for varies values of *M* while *Pr*=0.72, *Gr*=2.0, *a*=0.5 and *S*=0.4

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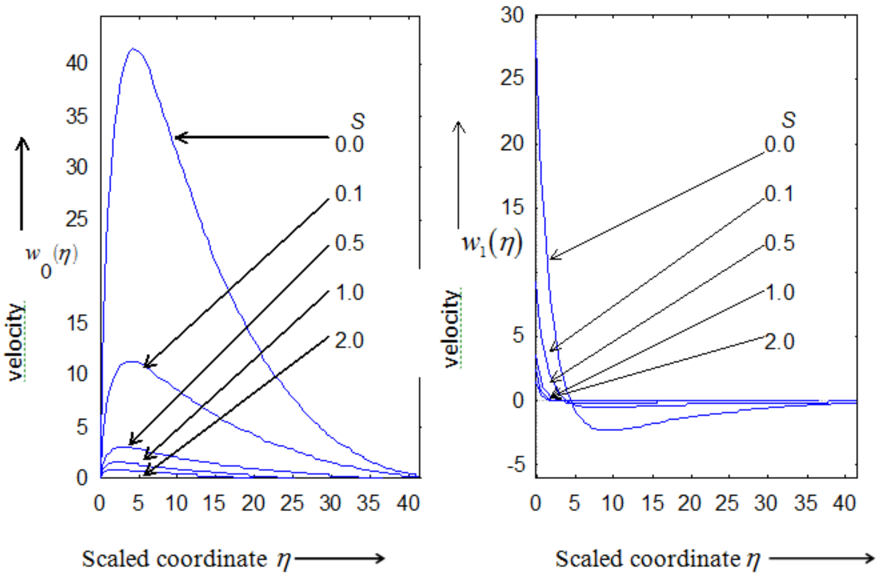

Figure 5: Zero (left) and first (right) order perturbation velocity perturbation profiles for varies values of *S* while *Pr*=0.72, *Gr*=2.0, *a*=0.5 and *S*=0.4

Figure 5 shows that the zero-order perturbation velocity profile and first order perturbation velocity profile against η for varies values of visco-elasticity parameter while Prandtl number *Pr* =0.72, Grashof number *Gr*=2.0, magnetic field parameter *M*=1.0 and transpiration parameter *a* =0.5. The figure describes the behavior of velocity profile with changes in the values of the visco-elastic parameter *S*. It is clearly observe that the zero order perturbation decreases as the visco-elasticity parameters increases. From Fig. 5 it is also observed that first order perturbation velocity profile decreases with increasing the visco-elasticity parameter.

7.0 CONCLUSSION

In this study, the effect of grashof number, magnetic field parameter and visco-elastic parameter is change then the velocity profile is change but temperature profile is unchanged. Both the zero and first order perturbation velocity profiles increase with the increase of the Grashof number *Gr*. The zero and first order perturbation velocity profiles decrease with the increase of the magnetic field parameter *M*. The values of the zero-order perturbation velocity profile and first order perturbation velocity profile increase the decrease of the visco-elastic parameter.

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