

# Comparative Study of Sound Wave Propagation in Single-Walled Carbon Nanotubes Using Nonlocal Elasticity for Two Materials (Al) and (Ni)

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**Abstract** -In this work we have begun a comparative study of the propagation of sound waves in carbon nanotubes has single wall using nonlocal elasticity for two different materials such as aluminum designated by (AL) and Nickel (Ni ), one based on the theories of beams of Euler-bernoulli and Timoshenko, the constructions are based on these two materials grace to its lightness and hardness it Frequency equations and modal shape functions of Timoshenko beams structures with some typical boundary conditions are also derived from nonlocal elasticity. The research work reveals the significance of the small-scale effect on wave propagation in single-walled CNTs. **Copyright © 2016 Penerbit Akademia Baru - All rights reserved.**

**Keywords:** Single-Walled Carbon Nanotubes, Sound Wave, Timoshenko Beam Theory, Euler-Bernoulli, Non-Local Elasticity

## 1.0 INTRODUCTION

Since the discovery of carbon nanotubes (CNTs) in the early 1990's by Iijima [1], the CNTs has attracted worldwide attention in many areas of science and industry and stimulated extensively experimental and theoretical studies [2]. Numerous studies showed that CNTs possess extraordinary physical properties such as the ratios of high stiffness-to-weight and strength-to-weight and enormous electrical and thermal conductivities over other known materials. CNTs are cylindrical macromolecules composed of a highly ordered sheet of carbon atoms in a periodic hexagonal arrangement rolled into a tube.

Many believe that carbon nanotubes may provide good reinforcing materials for the development of a new class of nanocomposites [3,4]. In particular, CNTs expect to have lots of uses in nano-electron such as nano-switch, nano-devices, sensors and (high frequency) micromechanical oscillators [5–9]. In recent years, the micro-wave absorbing effect and electromagnetic characteristics of CNTs has also attracted considerable interest for theoretical and practical importance in fundamental science and application [10–12]. So, understanding the effect of magnetic field on the characteristics of wave propagation in CNTs is essential and may give a useful help in applications for nano-engineering. Carbon nanotubes (CNTs) possess remarkable electronic, thermal and mechanical properties [1–3], leading to many potential applications for nanoelectronics, nanodevices and nanocomposites [4–7].

Hence, understanding the mechanical and physical properties of CNTs is essential to their applications in engineering. The study of vibration and wave propagation in CNTs is a major topic of current interest [8]. Many theoretical and experimental [9–16] methods are used to estimate and measure the mechanical and physical properties of CNTs. Since controlled experiments at nanoscales are difficult, it is nearly impossible direct measurement of their properties [17,18]. It is well known that a molecular dynamics method (MD) [19,20] has been highly developed to simulate the properties of the material with microstructures. However, MD simulations remain expensive and formidable especially for large-scale systems. The study of wave propagation in carbon nanotubes is only recently things.

Sudak [15] studied infinitesimal column buckling of CNTs, incorporating van der Waals forces and small-scale effects, and showed that the critical axial strain decreases, compared to the results with classical continuum beam model, where the small length scale increases in magnitude. Zhang et al. [16] proposed a nonlocal multi-shell model for the axial buckling of CNTs under axial compression. Their results showed that the effect of the small-scale on axial buckling strain is related to the buckling mode and the length of tubes. Wang [17] studied the dispersion relations for CNTs considering small-scale effects.

Wang and Hu [18] studied the flexural wave propagation in a single-walled CNT (SWCNT) through the use of the continuum mechanics and molecular dynamic simulation based on the Tersoff–Brenner potential. Lu et al. [19,20] analyzed dynamic properties of flexural beams using a nonlocal elasticity model. A qualitative validation study showed that results based on the nonlocal continuum mechanics are in agreement with the published experimental reports in this field. In this paper, based on the theory of nonlocal elasticity, a single-elastic beam model is developed for transverse propagation wave in SWCNTs, which considers the scale effect in the formulation of stress tensors. The wave characteristic solution is studied with respect to the vibrational mode, the scale coefficient, and diameters of SWCNTs.

## 2.0 MATHEMATICAL FORMULATION

### 2.1 Theory of Nonlocal Elasticity

The length scales associated with nano structures like CNTs are such that to apply any classical continuum techniques, we need to consider the small length scales such as lattice spacing between individual atoms, grain size, etc. This makes the consistent classical continuum model formulation very challenging.

This theory assumes that the stress state at a reference point, in the body is regarded to be dependent not only on the strain state at  $x$  but also on the strain states at all other points  $x=0$  of the body. This is in accordance with atomic theory of lattice dynamics and experimental observations on phonon dispersion.

The most general form of the constitutive relation in the nonlocal elasticity type representation involves an integral over the entire region of interest. The integral contains a nonlocal kernel function, which describes the relative influences of the strains at various locations on the stress at a given location. The constitutive equations of linear, homogeneous, isotropic, non-local elastic solid with zero body forces are given by

$$\sigma_{ij,j} = 0$$

$$\sigma_{ij} = \int_V \alpha(|x-x'|, \tau) C_{ijkl} \varepsilon_{kl}(x') dV(x'), \quad \forall x \in V \quad (1)$$

$$\varepsilon_{ij,j} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

where  $C_{ijkl}$  is the elastic modulus tensor of classical isotropic elasticity,  $\sigma_{ij}$  and  $\varepsilon_{ij}$  are stress and strain tensor, respectively, and  $u_i$  is the displacement vector.  $\alpha(|x-x'|, \tau)$  is the nonlocal modulus or attenuation function which incorporates into the constitutive equations the nonlocal effects at the reference point  $x$  produced by local strain at the source  $x'$ .  $|x-x'|$  is Euclidean distance. In  $\tau = e_0 a/l$  [12],  $e_0$  is a constant appropriate to each material,  $a$  is an internal characteristic length (e.g. length of C–C bond, lattice parameter, granular distance), and  $l$  is an external characteristic length (e.g. crack length, wavelength). It is noted that the value of  $e_0$  needs to be determined from experiments or by matching dispersion curves of plane wave with those of atomic lattice dynamics. In the limit when the effect of strains at points other than  $x$  is neglected, one obtains classical (local) model of elasticity by setting  $e_0 = 0$ .

For the nonlocal Timoshenko beam theory, the Hook's law of carbon nanotube can be expressed as the following partial differential forms:

$$\left[ \sigma_x - (e_0 a)^2 \frac{\partial^2 \sigma_x}{\partial x^2} \right] = E \varepsilon_x$$

$$\left[ \tau_{xy} - (e_0 a)^2 \frac{\partial^2 \tau_{xy}}{\partial x^2} \right] = G \gamma_{xy} \quad (2)$$

Thus, the scale coefficient  $e_0 a$  in the modeling will lead to small-scale effect on the response of structures in nano-size. To investigate the small-scale effect on the wave solutions of SWCNTs, nonlocal Euler–Bernoulli and Timoshenko beam models are proposed hereinafter.

### 3.0 NONLOCAL TIMOSHENKO BEAM MODEL

According to the Timoshenko beam theory, the displacement field of any point in the beam writes:

$$u(x, z) = u_0(x) - z \frac{\partial w_0(x)}{\partial x} - z \gamma_0^{xz}(x)$$

$$w(x, z) = w_0(x) \quad (3)$$

Where  $x$  is the longitudinal coordinate measured from the left end of the beam,  $z$  the coordinate measured from the mid-plane of the beam,  $w$  the transverse displacement,  $u$  the longitudinal displacement,  $u_0(x)$  and  $w_0(x)$  are the displacement components of a point located on the neutral axis and  $\gamma_0^{xz}$  is the transverse shear strain measured on the mean-line of the beam (Eq. 7):

$$\gamma_0^{xz}(x) = z \frac{dw_0(x)}{dx} - \phi_0(x) \quad (4)$$

where  $\phi_0(x)$  is the total section rotation measured on the mean-line of the beam. The linear Green–Lagrange strain tensor writes:

$$\begin{aligned}\varepsilon_{xx}(z) &= \frac{du_0(x)}{dx} - z \frac{d\phi_0(x)}{dx} \\ \gamma^{xz}(x) &= \gamma_0^{xz}(x)\end{aligned}\quad (5)$$

The bending moment can be defined by

$$M = \int_{A_c} z \sigma_x dA_c \quad (6)$$

$A$  is the cross-section area of the beam, It should be pointed out that for the Timoshenko beam theory, both bending moment  $M$  and the shear force  $V$  are independent. Therefore, for the nonlocal model, these resultant forces are linked to the nonlocal stress components  $\tau_{xy}$  through the relations (9) and (11) and the constitutive relations (4) and (5). The similar remarks have been reported by Lu et al. [20]. Substituting Eq. 8 and 9 into the nonlocal constitutive relation Eq. 4 leads to

$$M - (e_0 a)^2 \frac{\partial^2 M}{\partial x^2} = -EI \frac{d\phi_0}{dx} \quad (7)$$

$I$  is the moment of inertia, the shear force can be defined by

$$V = \int_{A_c} \tau_{xy} dA_c \quad (8)$$

From the relations Eqs. 5, 7, 8, and 11, the shear force for the nonlocal model can be expressed as

$$V - (e_0 a)^2 \frac{\partial^2 V}{\partial x^2} = k A_c G \left( \frac{dw_0}{dx} - \phi_0 \right) \quad (9)$$

where  $\beta$  is the form factor of shear depending on the shape of the cross section. The recommended value of  $\beta$ , the adjustment coefficient, is  $10/9$  for a circular shape of the cross area [26]. Now, it is straightforward to write out the dynamic equation for the beam element of length  $dx$  subjected to bending  $M$  and shear force  $V$  as follows:

$$\frac{\partial V}{\partial x} dx - \rho A \frac{\partial^2 w}{\partial t^2} dx = 0 \quad (10)$$

and

$$V dx - \frac{\partial M}{\partial x} M - \rho I \frac{\partial^2 \phi_0}{\partial t^2} dx = 0 \quad (11)$$

The explicit expressions of the nonlocal bending moment  $M$  and the nonlocal shear force  $V$  can be obtained by substituting Eqs. 13, and 14 into Eqs. 10 and 12 as

$$M = -EI \frac{d\phi_0}{dx} + (e_0a)^2 [\rho A \frac{\partial^2 w}{\partial t^2} - \rho I \frac{\partial^3 \phi_0}{\partial x \partial t^2}] \quad (12)$$

$$V = \beta AG \left( \frac{dw}{dx} - \phi_0 \right) + (e_0a)^2 [\rho A \frac{\partial^3 \phi_0}{\partial x \partial t^2}] \quad (13)$$

Substitution of Eqs. 15, and 16 into 13, and 14 leads to the following nonlocal Timoshenko beam model

$$\beta AG \left( \frac{\partial \phi_0}{\partial x} - \frac{\partial^2 w}{\partial x^2} \right) + \rho A \frac{\partial^2}{\partial t^2} [w - (e_0a)^2 \frac{\partial^2 w}{\partial x^2}] = 0 \quad (14)$$

$$\beta AG \left( \phi_0 - \frac{\partial w}{\partial x} \right) - EI \frac{\partial^2 \phi_0}{\partial x^2} + \rho I \frac{\partial^2}{\partial t^2} [\phi_0 - (e_0a)^2 \frac{\partial^2 \phi_0}{\partial x^2}] = 0 \quad (15)$$

Eqs. 17, and 18 are the consistent basic equations of the nonlocal Timoshenko beam model based on the constitutive relations (4) and (5). The similar equations are found recently by Lu et al. [20]. Consequently, in the nonlocal Timoshenko beam model, both the nonlocal bending moment and the nonlocal shear force need to be determined based on relations (4) and (5). This conclusion is confirmed in literature [18,20].

#### 4.0 FLEXURAL WAVE DISPERSION IN DIFFERENT BEAM MODELS

To study the flexural wave propagation in an infinitely long beam, let the dynamic deflection and slope be given by [27,28]

$$w(x,t) = \bar{W} e^{i\bar{\omega}t} \sin\left(\frac{n\pi x}{L}\right) \quad (16)$$

$$\phi = \bar{\phi} e^{i\bar{\omega}t} \cos\left(\frac{n\pi x}{L}\right) \quad (17)$$

where  $\bar{W}$  is the amplitude of deflection of the beam, and  $\bar{\phi}$  is the amplitude of the slope of the beam due to bending deformation alone. In addition,  $\bar{\omega}$  is the frequency of the wave motion. Substitution of Eqs. 19 into Eqs. 17 and 18 leads to the following two equations:

$$[\beta AG \left(\frac{n\pi}{L}\right)^2 - \rho A \bar{\omega}^2 \frac{\partial^2}{\partial t^2} (1 + (e_0a)^2 \left(\frac{n\pi}{L}\right)^2)] \bar{W} - \beta AG \frac{n\pi}{L} \bar{\phi} = 0 \quad (18)$$

$$[\beta AG \left(\frac{n\pi}{L}\right) \bar{W} - [EI \left(\frac{n\pi}{L}\right)^2 + \beta AG + \rho I \bar{\omega}^2 (1 + (e_0a)^2 \left(\frac{n\pi}{L}\right)^2)] \bar{\phi} = 0 \quad (19)$$

From Eqs. 20, and 21, the solution for the wave propagation in CNTs via nonlocal Timoshenko beam model can be derived from an eigenvalue problem searching for nontrivial solution of the variables of  $\bar{W}$ , and  $\bar{\phi}$ . The wave solution based on the nonlocal Timoshenko beam model, ignoring rotary effect, is thus obtained as

$$\frac{v_{nT}}{v_{IT}} = \frac{1}{\sqrt{1 + (e_0 a)^2 \left(\frac{n\pi}{L}\right)^2}} \quad (20)$$

The above equation is identical to that given by Lu et al. [20]. And  $v_{IT}$  is CNTs phase velocity based on the local Timoshenko beam model which is given by

$$v_{IT} = \frac{\sqrt{EI / \rho A}}{\sqrt{(EI / \beta AG) + (L / n\pi)^2 + (\rho I / \rho A)}} \quad (21)$$

The asymptotic phase velocity  $v_{aIT}$  at  $n \rightarrow \infty$  based on the local Timoshenko beam model can be determined as well from Eq. 23 as follows:

$$v_{aIT} = \frac{\sqrt{EI / \rho A}}{\sqrt{(EI / \beta AG) + (\rho I / \rho A)}} \quad (22)$$

The corresponding frequency-vibrational mode numbers  $n$  from the relation on  $\omega_{nT} = v_{nT} (n\pi / L)$ , and are given by

$$\frac{\omega_{nT}}{v_{IT}} = \frac{1}{\sqrt{1 + (e_0 a)^2 \left(\frac{n\pi}{L}\right)^2}} \quad (23)$$

As is remarked in Eq. 22, the Eq. 25 is identical to that given by Lu et al. [20]. The asymptotic phase frequency  $\omega_{aIT}$  at  $n \rightarrow \infty$  based on the nonlocal Timoshenko beam model can be determined as well from Eq. 25 as follows:

$$\omega_{aIT} = \frac{1}{e_0 a} \frac{\sqrt{EI / \rho A}}{\sqrt{(EI / \beta AG) + (\rho I / \rho A)}} \quad (24)$$

If neither the rotary inertial nor the shear deformation is taken into account, Eqs. 17, and 18 lead to the dynamic equation of a nonlocal elastic Euler beam as follows:

$$EI \frac{\partial^4 w}{\partial x^4} + \rho A \frac{\partial^2 w}{\partial t^2} \left( w - (e_0 a)^2 \frac{\partial^2 w}{\partial x^2} \right) = 0 \quad (25)$$

Eq. 27 is identical to the equation of motion for the nonlocal Euler beam model given by Lu et al. [19,20] in the case where the distributed transverse force is zero. Solving Eq. 27 for the nonlocal phase velocity  $v_{nE}$  give

$$\frac{v_{nT}}{v_{IT}} = \frac{1}{\sqrt{1 + (e_0 a)^2 \left(\frac{n\pi}{L}\right)^2}} \quad (26)$$

where  $v_{IE}$  is the wave phase velocity based on the local Euler–Bernoulli beam model given by

$$\frac{v_{nT}}{v_{IT}} = \frac{n\pi}{L} \sqrt{\frac{EI}{\rho A}} \quad (27)$$

It can be observed from Eqs. 22, and 28 that the wave velocity ratio by Timoshenko model is identical to Euler–Bernoulli model:

$$\frac{v_{nT}}{v_{IT}} = \frac{v_{nE}}{v_{IE}} \quad (28)$$

Eq. (30) shows that the velocity ratio between nonlocal model and local model using Euler–Bernoulli or Timoshenko theory can be investigated in the same way. The asymptotic phase velocity  $v_{anE}$  at  $n \rightarrow \infty$  based on the nonlocal Euler–Bernoulli beam model can be determined as well from Eq. (28) as follows:

$$v_{anE} = \frac{1}{e_0 a} \sqrt{\frac{EI}{\rho A}} \quad (29)$$

## 5.0 GENERAL SOLUTIONS FOR DIFFERENT BOUNDARY CONDITIONS

For harmonic flexural wave propagation in an infinite beam governed by the equation of motion (17) and (18), the solution can be assumed in the form:

$$w(x,t) = W e^{i\alpha x} \quad \phi_0(x,t) = \Phi e^{i\alpha x} \quad (30)$$

Using Eq. 32, Eqs. 17, and 18 can be expressed as

$$(\alpha^2 b^2 c^2 - 1) \frac{d^2 W}{d\xi^2} + a^2 c^2 W + L \frac{d\Phi}{d\xi} = 0 \quad (31)$$

$$c^2 (\alpha^2 b^2 c^2 - 1) \frac{d^2 \Phi}{d\xi^2} + (1 - a^2 b^2 c^2) \Phi - \frac{dW}{L d\xi} = 0 \quad (32)$$

where

$$a^2 = \frac{\rho A L^4 \omega^2}{EI}, \quad b^2 = \frac{\rho l}{\rho A L^2}, \quad \xi = \frac{x}{L}, \quad c^2 = \frac{EI}{\beta A G L^2} \quad \text{and} \quad \alpha = \frac{e_0 a}{L} \quad (33)$$

Eqs. 33, and 34 can be transformed into two uncoupled differential equations by eliminating  $\bar{W}$  or  $\bar{\phi}$  as follows:

$$(\alpha^2 a^2 c^2 - 1) \frac{d^4 W}{d\xi^4} - (a^2 c^2 - \frac{1 + (1 - a^2 b^2 c^2)(\alpha^2 a^2 c^2 - 1)}{c^2 (\alpha^2 a^2 c^2 - 1)}) \frac{d^2 W}{d\xi^2} - \frac{a^2 (1 - a^2 b^2 c^2)}{(\alpha^2 a^2 c^2 - 1)} W = 0 \quad (34)$$

$$(\alpha^2 a^2 c^2 - 1) \frac{d^4 \Phi}{d\xi^4} - (a^2 c^2 - \frac{1 + (1 - a^2 b^2 c^2)(\alpha^2 a^2 c^2 - 1)}{c^2 (\alpha^2 a^2 c^2 - 1)}) \frac{d^2 \Phi}{d\xi^2} - \frac{a^2 (1 - a^2 b^2 c^2)}{(\alpha^2 a^2 c^2 - 1)} \Phi = 0 \quad (35)$$

The general solutions for Eqs. 36, and 37 are, respectively, given by [30]

$$W = C_1 \cosh \chi \xi + C_2 \sinh \chi \xi + C_3 \cos \gamma \xi + C_4 \sin \gamma \xi \quad (36)$$

$$\Phi = C_1 \sinh \chi \xi + C_2 \cosh \chi \xi + C_3 \sin \gamma \xi + C_4 \cos \gamma \xi \quad (37)$$

Where

$$\frac{\chi}{\gamma} = \frac{1}{\sqrt{2}} \left\{ \begin{array}{l} - \\ + \end{array} (b^2 + c^2) + \left[ (b^2 - c^2)^2 + \frac{4}{a^2} \right]^{\frac{1}{2}} \right\}^{\frac{1}{2}} \quad (38)$$

The constants  $C_i$  and  $D_i$  are, however, not independent of one another. Through Eq. 33, they are related as follows:

$$D_1 = C_1 \Psi_\chi, \quad D_2 = C_2 \Psi_\chi, \quad D_3 = C_3 \Psi_\gamma, \quad D_4 = C_4 \Psi_\gamma \quad (39)$$

where

$$\Psi_\chi = -\frac{a^2 c^2 (\alpha^2 \chi^2 - 1) - \chi^2}{L \chi} \quad (40)$$

$$\Psi_\gamma = \frac{a^2 c^2 (\alpha^2 \gamma^2 - 1) - \gamma^2}{L \gamma} \quad (41)$$

Now, the constants must be determined using the proper boundary conditions in the same way as is discussed in Ref. [19].

## 5.1 SIMPLY SUPPORTED BEAM

The simply supported boundary condition is specified by  $w = 0$  and  $M = 0$ , where  $M$  is the bending moment given in Eq. (15). The condition  $M = 0$  cannot be simply replaced by  $\frac{\partial^2 w}{\partial x^2} = 0$ . It is correct for the classical beam model but is wrong under the nonlocal beam model. In some of published work (e.g. Ref. [31]), the boundary conditions have been incorrectly defined. Therefore, the formulas and results obtained therein should be rechecked. For harmonic free vibration, the boundary condition of the simply supported nonlocal Timoshenko beam in the non-dimensional form is

$$\text{At } \xi = 0,1 \quad \begin{cases} W = 0 \\ M = \frac{ML}{EI} (1 - \alpha^2 a^2 b^2) \frac{d\Phi}{d\xi} + \alpha^2 a^2 W = 0 \end{cases} \quad (42)$$

In view of  $W = 0$ , the non-dimensional moment condition can be simplified to

$$\frac{d\Phi}{d\xi} = 0 \quad (43)$$

By substituting Eqs. 38, and 39 into Eqs. 43, and 44, and then by using the relationships between the constants  $C_i$  and  $D_i$  in Eq. (41), we arrive at an eigen value problem defined by the following matrix equation:



$$\begin{bmatrix} \sinh \chi & \sinh \gamma \\ \chi \Psi_\chi \sinh \chi & \gamma \Psi_\gamma \sinh \gamma \end{bmatrix} \begin{Bmatrix} C_2 \\ C_4 \end{Bmatrix} = 0 \quad (44)$$

Since  $C_1 = C_3 = 0$ . The eigen values (frequencies) are obtained by setting the determinant of the matrix to zero and then solving the characteristic equation. In this stage, we will find the same frequencies as illustrated in Section 4. The corresponding mode shapes for simply supported beams are given by:

$$W = \sin \gamma \xi \quad (45)$$

with

$$\gamma = n\pi \quad (46)$$

It can be noticed from Eq. 46 that the vibration modes do not include any scale effect parameter.

## 5.2 Clamped–Clamped Beam

The boundary conditions of clamped–clamped beam are specified by  $W = 0$  and  $\Phi = 0$  at  $\xi = 0, 1$ . 47 By substituting the general solutions Eqs. 38, and 39 into the boundary conditions and noting the relationships between the constants  $C_i$  and  $D_i$  in Eq. 41, we arrive at an eigen value problem defined by the following matrix equation:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \Psi_\chi & 0 & -\Psi_\gamma \\ \cosh \gamma & \sinh \chi & \cos \gamma & \sin \gamma \\ \Psi_\chi \sinh \chi & \Psi_\chi \sinh \chi & \Psi_\gamma \sin \gamma & -\Psi_\gamma \cos \gamma \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = 0 \quad (47)$$

The characteristic equation is obtained by setting the determinant of the matrix to zero. The eigen values (frequencies) are computed from solving the characteristic equation. The corresponding modal shape function are obtained as

$$W = C_1 \cosh \chi \xi + C_2 \sinh \chi \xi + C_3 \cos \gamma \xi + C_4 \sin \gamma \xi \quad (48)$$

## 5.3 Cantilever Beam

For harmonic free vibration, the boundary conditions of a cantilever beam, according to Eqs. 15, and 16 in the non-dimensional form, are specified by  $W = 0$ , and  $\Phi = 0$  at  $\xi = 0, 1$ , and

$$\begin{cases} \bar{V} = \frac{VL}{\beta AG} = (1 - \alpha^2 a^2 b^2) \frac{dW}{d\xi} - \Phi = 0 \\ \bar{M} = \frac{ML}{EI} (1 - \alpha^2 a^2 b^2) \frac{d\Phi}{d\xi} + \alpha^2 a^2 W = 0 \end{cases} \quad (49)$$

By substituting the general solutions Eqs. 38, and 39 into the boundary conditions and noting the relationships between the constants  $C_i$  and  $D_i$  in Eq. (41), we arrive at an eigen value problem defined by the following matrix equation:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \Psi_\chi & 0 & -\Psi_\gamma \\ H_1 \cosh \gamma & H_1 \sinh \chi & H_2 \cos \gamma & H_2 \sin \gamma \\ H_3 \sinh \chi & H_3 \sinh \chi & H_4 \Psi_\gamma \sin \gamma & -H_4 \Psi_\gamma \cos \gamma \end{bmatrix} \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{Bmatrix} = 0 \quad (50)$$

where

$$H_1 = (\alpha^2 a^2 b^2 - 1) \chi \Psi_\chi - \alpha^2 a^2 \quad (51)$$

$$H_2 = (\alpha^2 a^2 b^2 - 1) \gamma \Psi_\gamma - \alpha^2 a^2 \quad (52)$$

$$H_3 = (1 - \alpha^2 a^2 c^2) \chi - \Psi_\chi \quad (53)$$

$$H_4 = -(1 - \alpha^2 a^2 c^2) \gamma - \Psi_\gamma \quad (54)$$

The characteristic equation is obtained by setting the determinant of the matrix to zero. The eigenvalues (frequencies) are computed from solving the characteristic equation.

## 6.0 DISCUSSION

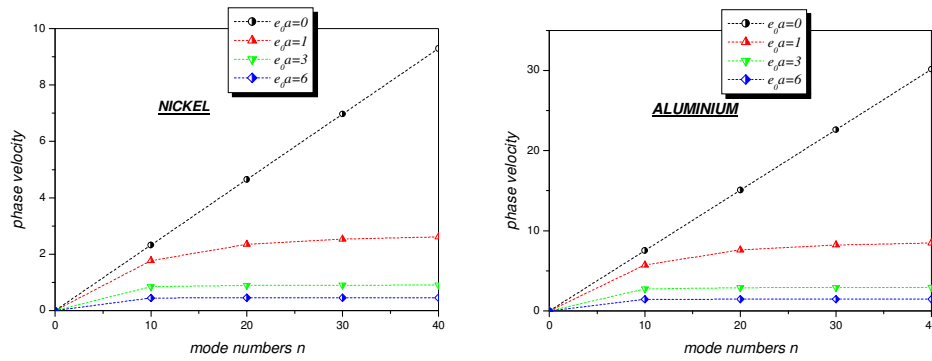
Based on the formulations obtained above with the nonlocal beam models, the wave properties of single-walled nanotubes are discussed here. The material and geometry constants of CNTs are given in Table 1 [25,29], and the shear module can be determined from the relation  $G=0.5E/(1+\nu)$ . The thickness of CNT  $t=0.34$  nm is chosen [25] with diameter  $d = 5$  nm and the length of nanotube is 36.8 nm. Parameter  $a$  describes internal characteristic length. The length of a C– C bond, which is 0.142 nm, is chosen for the analysis of CNTs [14,15]. On the other hand, parameter  $e_0$  was given as 0.39 by Eringen [12].

**Table 1:** Material constants of *AL* and *Ni*

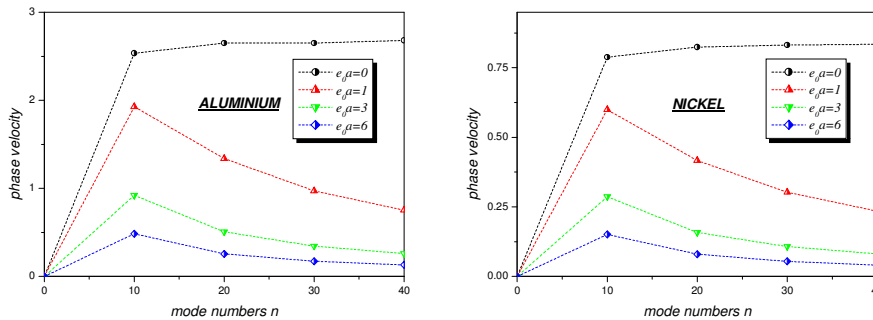
<i>Aluminum</i>	$v=0.364, \rho=2700\text{Kg/m}^3$ $,E=67000\text{Mpa}$
<i>Nickel</i>	$v=0.312, \rho=8900\text{Kg/m}^3$ $,E=21000\text{Mpa}$

To examine the influence of the small scale on vibration of single-walled nanotubes, let us compare the local and nonlocal results. The dispersion curves of the transverse wave propagation of the SWNT with  $d = 5$  nm, based on local and nonlocal Euler–Bernoulli beam models, respectively, are shown in Fig. 1 at different values of the nonlocal parameter, namely,  $e_0 a = 1, 3, \text{ and } 6$  nm. It is seen that the difference of the phase velocities becomes more obvious at higher vibrational mode numbers  $n$ , although this difference is almost invisible at small vibrational mode numbers  $n = 10$  for the given domain of scale parameter  $e_0 a$ . The local Euler–Bernoulli beam model shows a virtual linear variation, whereas the velocity from the nonlocal model have their asymptotic values as given in Eq. 31. On the other hand, the derived phase velocities diminish with increasing the scale parameter  $e_0 a$ .

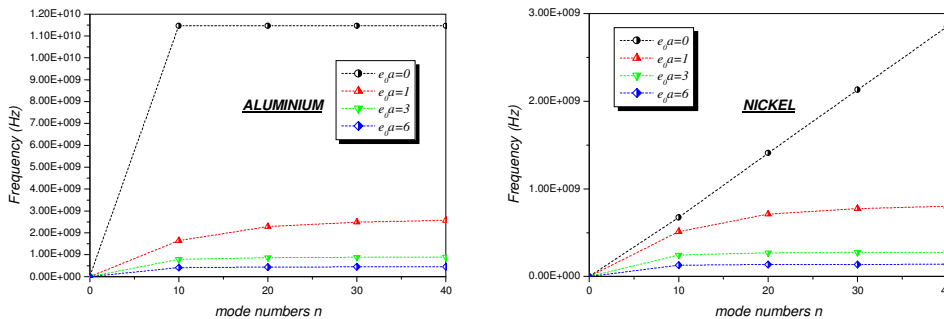
Figures 2, and 3 give a comparison of the CNTs phase velocity and frequency based on local and nonlocal Timoshenko beam model. It is seen that both frequency and phase velocity decrease with increasing the scale parameter  $e_0a$ . This means that the dynamical properties (frequency, phase velocity, etc.) of the nanotubes based on the classical beam theories are over estimated. Figure 2 shows that the phase velocity from the local Timoshenko beam model has its asymptotic value as given in Eq. (24).



**Figure 1:** Phase velocity of flexural wave versus vibrational mode numbers by Euler-Bernoulli model.



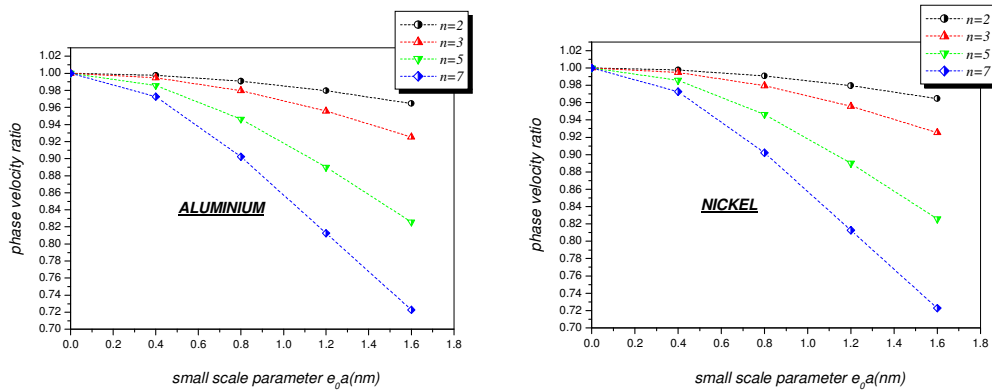
**Figure 2:** Phase velocity of flexural wave versus vibrational mode numbers by Timoshenko model.



**Figure 3:** Wave frequency versus vibrational mode numbers by Timoshenko model.

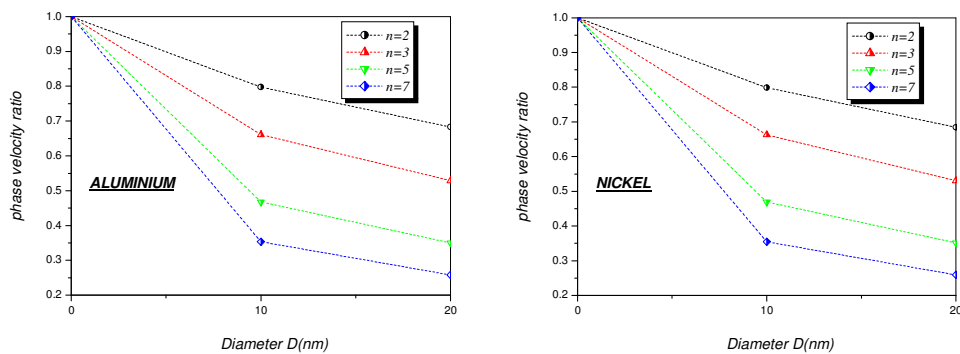
Figure 3 shows a linear variation of frequency value versus the vibrational mode numbers for the local Timoshenko model. Further, a constant value of the frequency at higher vibrational mode numbers is observed for the nonlocal Timoshenko model. This constant value decreases with high scale parameter  $e_0a$ . The asymptotic value of frequency is given in Eq. 26.

Figure 4 shows the variation of phase velocity ratio versus the scale parameter  $e_0a$  for different values of the vibrational mode number  $n$ . It can be observed that the velocity ratio reaches unit at  $e_0a= 0$ . At higher vibrational mode number, the ratio is seen to virtually approach unit. This investigation further demonstrates the conclusion that the phase velocity decreases as scale parameter increases and vibrational mode number decreases.



**Figure 4:** Small effect on phase velocity ratio of flexural wave versus vibrational mode.

In Fig. 5, the ratio of the phase velocity versus the diameter of the CNT at  $e_0a = 3\text{nm}$  is plotted. The four curves represent the variation of the velocity ratio at vibrational mode numbers  $n = 10, 20, 30,$  and  $40$ . It is clearly seen that the velocity ratio is lower at smaller diameters. Therefore, it can be concluded that CNTs wave solutions are diameter dependent based on nonlocal elasticity. The small-scale effect on diameter becomes almost unnoticeable at larger diameters. The diameter dependence of [32].



**Figure 5:** Phase velocity ratio of flexural wave versus diameter at  $e_0a= 3 \text{ nm}$ .

Wave solution for CNTs is first observed in the manuscript as all previous studies [33] indicated the diameter independent buckling solution of CNTs via nonlocal elasticity [34].

## 7.0 CONCLUSION

The main contribution in this paper is to describe the sound wave propagation in single walled for two materials (Al) and (Ni), the free vibration of short SWCNTs is studied in the present research via the nonlocal continuum beam models. Nonlocal Euler–Bernoulli and Timoshenko beam models enable the investigation of small-scale effects on a CNT's dispersion solutions. The characteristics of sound wave propagation in single-walled carbon nanotubes is very significant, It is shown that the dynamical properties of the nanotubes based on the classical beam theories are over estimated. Hence, the work in the manuscript not only reveals the significance of the small-scale effect on CNTs mechanical response, but also points out the limitation of the applicability and feasibility of local continuum models in analysis of CNTs mechanical behaviors.

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