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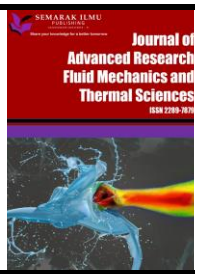

Flow Visualization on Submerged Flow Conditions in Open Channel Weir Downstream

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ABSTRACT

Submerged flow regimes are common phenomena observed in open channel flow, such as rivers and spillways, with or without weirs. These flow regimes involve a sudden rise in the liquid surface downstream when high-velocity liquid discharges into a region of lower velocity. The rapid deceleration of the flowing liquid results in an increase in height and a conversion of kinetic energy into potential energy, with some energy dissipated as turbulence and heat. In this study, flow visualization techniques were employed to investigate the two-dimensional hydrodynamic characteristics of submerged hydraulic jumps within the Froude number range of 1.0 to 5.4 over a fixed bed. The obtained results were compared with existing literature data, showing a good agreement. This research contributes to the understanding of submerged flow phenomena and provides valuable insights into the hydrodynamics of submerged hydraulic jumps.

1. Introduction
1.1 Flow Visualization on Downstream Channel

Understanding flow regime transitions is of paramount importance in the study of open channel hydraulics, particularly in the context of submerged hydraulic conditions. Flow visualization techniques play a pivotal role in the observation and analysis of various flow regimes, including tranquil, transition, and rapid flows. By employing flow visualization and characterizing these regimes, researchers can gain valuable insights into the hydraulic processes occurring in open channels, specifically those related to submerged hydraulic conditions.

Flow visualization enables the identification of critical flow conditions and the determination of parameters that influence regime transitions. One such crucial parameter is the Froude number (Fr), which holds significant relevance in open channel flow analysis. The Froude number is a

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dimensionless parameter that quantifies the relative significance of inertial forces and gravity in fluid flow. Higher values of the Froude number indicate a dominance of inertial forces, while lower values indicate a dominance of gravity. This parameter serves as a key tool in estimating the hydrodynamic performance of flow in submerged hydraulic conditions.

In this study, the Froude number is employed to assess the hydrodynamic performance of the flow in rectangular channels under submerged hydraulic conditions. The Froude number is a dimensionless parameter used to quantify the relative importance of inertial forces and gravity in fluid flow [1]. A higher Froude number indicates that inertial forces are dominant, while a lower Froude number indicates that gravity is dominant. The Froude number is calculated using Eq. (1)

$$Fr = \frac{V}{\sqrt{gd}} \tag{1}$$

In Eq. (1), the variable V (ms^{-1}) is the flow velocity; g (ms^{-2}) is the acceleration due to gravity; d (m) signifies the flow depth of the rectangular channel under investigation. The Froude number, derived from this equation, is a crucial parameter utilized to evaluate the stability and hydrodynamic performance of fluid flow in submerged hydraulic conditions.

By incorporating flow visualization techniques and the Froude number analysis, this study aims to enhance our understanding of submerged hydraulic conditions in open channels. The investigation will provide valuable insights into the flow regime transitions, stability, and hydrodynamic behavior in submerged hydraulic conditions, thus contributing to the broader field of open channel hydraulics.

1.2 Previous Research Works and Investigations.

The hydraulic characteristics and performance at the base of a straight drop spillway have been extensively studied by different researchers throughout decades. Wu and Rajaratnam [2] focused on the relationship between flow regime transitions and the Froude number in open channels. Their study stated that the flow in the study was categorized into four regimes: impinging jet, surface jump flow, surface wave, and surface jet. The classification was based on visual inspection. In the impinging jet regime, the flow plunges over the weir into the tailwater and diffuses as a submerged jet until it reaches the bed of the downstream channel. In the surface flow regime, the flow remains as a downstream surface jet, and the scour hole downstream of the weir is formed by the thickening of the jet and turbulence mixing with the tailwater. In contrast, the impinging jet regime creates a scour hole downstream of the weir through the direct impact of the plunging jet over the weir. This study contributed to understanding the role of the Froude number in characterizing flow regimes.

Azimi *et al.,* [3] conducted a study that examined the relationship between the Froude number and the behavior of flow in compound open channels. The researchers investigated the effects of different channel geometries, bed slopes, and flow discharges on the flow regime transitions and hydraulic characteristics. The study aimed to develop correlations between the Froude number and key flow parameters in compound channels, such as velocity distribution, flow resistance, and boundary shear stress. The findings of the study provided useful information for designing and managing flow in compound open channels.

The study by Hotchkiss and Comstock [4] examined the hydraulic characteristics and flow regimes in laboratory flumes with various channel slopes. They investigated the effects of bed slope on flow patterns, including tranquil, transition, and rapid flows. The researchers analysed the relationship between the Froude number and the critical slope at which flow regime transitions occurred. Their findings enhanced the understanding of flow behaviour and regime transitions in open channels with different bed slopes.

Fan [5] conducted a study focusing on the transition between tranquil and rapid flows in straight compound channels. The research aimed to identify the critical flow conditions for the transition and the influence of several factors, such as channel geometry, flow discharge, and sediment concentration. The study provided insights into the mechanisms and characteristics of flow regime transitions, particularly the transition from tranquil to rapid flow, in compound channels.

Leutheusser and Fan [6] investigated flow regime transitions in straight and meandering compound channels. Their study examined the effects of channel geometry, specifically the floodplain width, on flow behaviour and regime transitions. The researchers analysed the relationship between the Froude number and the critical width-to-depth ratio at which flow regime transitions occurred. The findings of the study contributed to understanding the impact of channel geometry on flow regime transitions in compound channels.

Collective information about previous and current research can be seen in Table 1. It contains a list of experimental studies on hydraulic parameters developed by various researchers. Numerous research studies have investigated the characteristics of fully developed, two-dimensional open channels over smooth and rough bottoms. These studies were aimed at improving understanding of turbulent flows and quantifying key flow parameters. Notable studies include those by Kline *et al.,* McQuivey and Richardson, Blinco and Partheniades, and Grass [7].

The findings derived from these studies have made a substantial and profound contribution to the scholarly understanding of the statistical properties of mean and turbulent flow, as well as the intricate interplay between flow structures existing on the channel bed and the flow conditions that manifest in open channel settings. It is noteworthy to mention that the investigations concerning the flow regime downstream of a weir with a rigid bed are characterized by an exceptional level of meticulousness, owing to the inherent complexity arising from the two-phase interaction involved.

In the realm of open channel hydraulics, an essential aspect that necessitates consideration pertains to the velocity distribution, which delineates the way flow velocities are distributed across the channel cross-section. This distribution is intricately influenced by a multitude of factors, including the Froude number, channel geometry, and boundary conditions. The Froude number, serving as a dimensionless parameter, offers crucial insights into the relative significance of inertial and gravitational forces within the flow dynamics.

The primary objective of these studies was to investigate the complex velocity distribution and its relationship with the flow regime downstream of a weir with a rigid bed. Through meticulous analysis and extensive experimental data, researchers gained deep insights into the behaviour of flow structures, such as the formation of hydraulic jumps, the onset of turbulence, and the intricate interaction between flow dynamics and the channel bed. The examination of the Froude number played a crucial role in unravelling these insights, providing a comprehensive understanding of the phenomena observed.

In summary, these studies enhanced our understanding of the mean and turbulent properties of flow, as well as the complex interactions between flow structures and flow conditions in open channels. They have shed light on the velocity distribution across the channel cross-section and its connection to the Froude number, contributing to our knowledge of flow dynamics in the specific context of the flow regime downstream of a weir with a rigid bed.

Table 1
Hydraulic parameters experimental investigations

Authors	*Fr* min	*Fr* max	Bed	Flow
McQuivey and Richardson (1969)	0.4	0.6	Smooth/rough	Uniform
Blinco and Partheniades (1971)	0.2	0.4	Smooth/rough	Uniform
Grass (1971)	0.2	0.2	Smooth/rough	Uniform
Nezu and Rodi (1986)	0.1	1.2	Smooth	Uniform
Cardoso *et al.,* (1989)	0.2	0.2	Smooth	Uniform
Tominaga and Nezu (1992)	0.3	3.1	Smooth/rough	Uniform
Prinos and Zeris (1995)	0.8	3.1	Smooth	Uniform
Li *et al.,* (1995)	1.6	2.7	Smooth	Uniform
Song and Graf (1996)	0.6	0.9	Rough	Gradually varied/unsteady
Nezu *et al.,* (1997)	0.1	0.7	Smooth	Gradually varied/unsteady
Song and Chiew (2001)	0.2	0.6	Rough	Gradually varied/unsteady
Nezu and Azuma (2004)	0.4	0.7	Smooth	Uniform
Rodriguez and Garcia (2008)	0.5	0.6	Rough	Uniform
Albayrak and Lemmin (2011)	0.3	0.3	Rough	Uniform
Auel *et al.,* (2014)	1.7	6.1	Transitional	Uniform/gradually varied/steady
Present study	1.0	5.2	Smooth	Gradually varied/unsteady

1.3 Velocity Distribution

Studies by Leutheusser and Birk [8], Hotchkiss and Comstock, Leutheusser and Fan, Olsen *et al.,* [9], and Fan explore the potential hazards associated with submerged flow patterns below a low head dam. They focus on identifying dangerous flow conditions and establishing criteria for determining the level of hazard to individuals within the flow.

These studies have provided a definition of dangerous flow in the context of submerged hydraulic jumps below weirs. The characteristic feature of this hazardous flow pattern is the presence of a forced vortex accompanied by a significant upstream-directed (countercurrent) free-surface velocity. It is worth noting that the depth and velocity of this submerged hydraulic jump can vary. To evaluate the potential risks associated with such flow patterns, researchers have conducted comparisons between the maximum counter-current velocity and human swimming capabilities. Fan developed sequent depth equations based on experiments with submerged hydraulic jumps below weirs.

The sequent depth in Eq. (2) allows for the estimation of the backward surface velocity under specific flow conditions.

$$\frac{Y_2}{Y_1} = \frac{1}{2}\left(\sqrt{1+8{F_1}^2}-1\right) \quad (2)$$

with y_1 is the depth of the upstream flow and y_2 is the depth of the downstream flow
F_1 was referring to the previous Eq. (1) which is.

$$Fr = \frac{V}{\sqrt{gd}}$$

The magnitude of upstream-directed surface velocities is directly related to the dimensionless degree of submergence, which is defined by Rao and Rajaratnam [10] in Eq. (3)

$$S = \frac{Y_t - Y_2}{Y_2} \quad (3)$$

The supercritical reference velocity was theoretically calculated using Eq. (4).

$$v_1 = \frac{q}{Y_1} \tag{4}$$

By comparing this estimated velocity to the typical swimming speeds of "ordinary persons," ranging from 0.25 ms^{-1} to 0.87 ms^{-1} for poor to good swimmers, respectively, Fan and other researchers have been able to assess the potential hazard level posed by the observed flow conditions. This approach enables a quantitative evaluation of the counter-current velocity in relation to human swimming abilities, thereby providing valuable insights into the safety implications of submerged hydraulic jumps below weirs. Leutheusser and Birk, on the other hand, considered the maximum human swimming speed to be approximately 2 ms-1, based on average velocities of Olympic swimmers. McArdle *et al.,* [11] reported a maximum velocity of about 0.35 ms-1 for an untrained swimmer, based on swimming speed studies.

These studies emphasize the importance of considering the Froude number, which characterizes the flow regime, when assessing the potential hazards of submerged flow below a low head dam. By comparing the counter-current velocity with human swimming capabilities, researchers aim to determine the level of danger associated with the flow pattern. While different studies provide varying estimates for human swimming speeds, they collectively underscore the need to consider the Froude number and its implications on the safety of individuals in these flow conditions.

The primary objective of this study is to comprehensively investigate the effects of flow rate and tail water depth on the counter-current flow phenomenon observed in a downstream channel. A crucial aspect of this research is to establish a deeper understanding of the relationship between flow rate, tail water depth, and the characteristics of the counter-current flow, while drawing parallels to the submerged hydraulic jump phenomenon.

The submerged hydraulic jump phenomenon represents a transition from supercritical to subcritical flow in an open channel, involving changes in flow depth and energy dissipation. In the context of this study, particular attention is given to the examination of counter-current flow, which entails the upstream flow that opposes the primary downstream flow. This counter-current flow is analogous to the recirculation zone typically observed in submerged hydraulic jumps.

By systematically analyzing the influence of flow rate and tail water depth on the counter-current flow, this study aims to bridge the existing knowledge gap pertaining to the intricate interplay between flow characteristics, such as flow rate and tail water depth, and the formation and behavior of the counter-current flow phenomenon. Moreover, the study seeks to establish meaningful information between the counter-current flow and the submerged hydraulic jump phenomenon, thereby advancing our understanding of both phenomena and their relationship within the broader context of open-channel hydraulics.

2. Materials and methods

2.1 Open Channel Set Up

In-house flow visualization facility was set up comprising an open channel setup with flow visualization technique for flow characterization on the downstream channel as shown in Figure 1. The open channel with a working section 53 mm wide, 120 mm deep, and 2500 mm long was built at the Universiti Tun Hussein Onn to perform flow visualization experiments. The flow was controlled by a flowmeter through an inlet at the upstream end with an adjustable tailgate at the downstream end, which was also used to control the tail water level. Tap water was used as the working fluid to characterize the flow feature on the downstream channel and water surface profiles. The water temperature inside the facility was held at indoor laboratory temperatures, typically ranging between 28 to 30 ℃. The equipment is designed primarily for use with volumetric Hydraulic Bench which provides the necessary pumped water supply, drain and a volumetric tank for flow measurement. The open channel flume used for this study has a height of 120 mm; therefore, a maximum water level of 83.2 mm, which is half the flume depth, was selected to achieve total submergence and to simulate the emerged condition of the downstream water level considering the height of the sharp crested weir used.

Figure 2 shows the open-channel setup for flow characterization with sharp crested weir. The sharp-crested weir was made of an aluminum alloy plate 13.2 mm thick. The thickness at the crest was reduced to 1 mm by chamfering at 45° on the downstream face as seen in Figure 2. When the hydraulic bench was turned on, the flow of water in the tank was pumped to the open channel flume which then flowed through the sharp crested weir. When the flow was stable, the dowel was released at the fixed point on the upstream flow with 3 P (10 cm) from the weir crest as seen in Figure 2. The water plunges into the downstream channel. The recovery point on the downstream was marked as an escape point from the flows.

2.2 Dowel Characteristics

Cylindrical dowel was manufactured from hardwood tree species was used as the flow marker for the experiment. The hardwood species were composed of Rengas and Arang Bunga. They covered a wide range of densities from 450 kg/m^3 to 1055 kg/m^3, spanning two density-range models as seen in Table 2. From Table 2, listed the Rengas dowel (640 – 960 kg/m^3) and Arang Bunga dowel (595 – 1055 kg/m^3). The densities were chosen such that they covered all the cases observed in real open channel.

Table 2
Two (2) dowels properties with their respective densities

Dowel species	Dowel scientific names	Density (kg/m^3)
Rengas	Gluta Rengas L.	705.2
Arang Bunga	Diospyros spp.	940.2

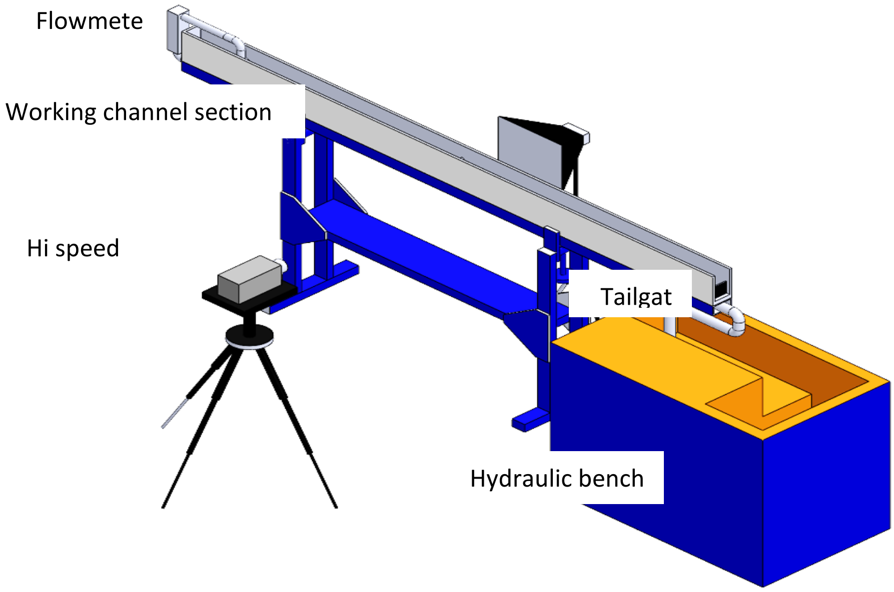

Fig. 1. Open channel flow visualization layout

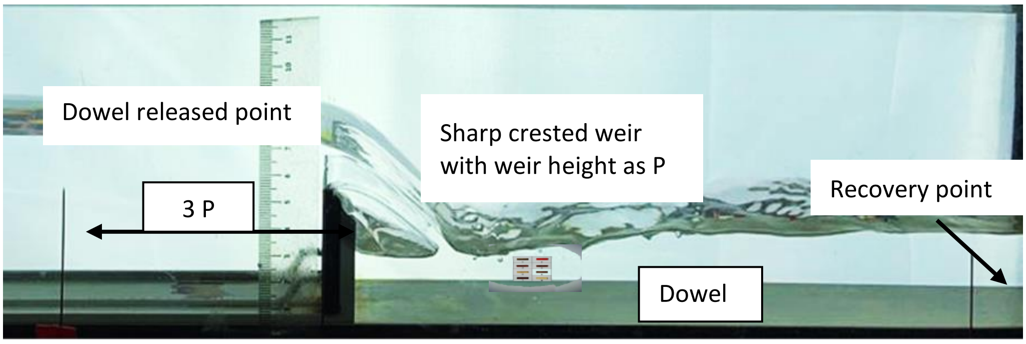

Fig. 2. Sharp crested weir flow visualization using dowel as a marker

2.3 Experiment Procedures

The flow visualization setup was based on several preliminary tests before running the overall experiment. The experiment condition was depending on the upstream level and tailwater level on downstream channel as shown in Table 3. For the upstream level the dimensionless parameter was the ratio of head over weir crest, (h_c) over weir height, (P) with weir height denotes as P was given as (h_c/P) = 0.5; 0.4 and 0.3 and the downstream level was given by the ratio for the tailwater height, (h_t) = 0.7 P, 0.8 P and 1.0 P. Each dowel was released individually from the upstream area, flowing over the crested weir, and plunging into the downstream channel as shown in Figure 2. Each experiment was recorded using a high-speed camera. The dowel coordinates were extracted through image analysis system using an open source digitize software which is known as Tracker.

Table 3
Experimental condition for flow path investigation

Upstream (h_c/P)	Downstream (h_t/P)	Flowrate (lpm)
0.3	1.0	12.0 ± 0.5
0.4	0.8	20.0 ± 0.5
0.5	0.7	30.0 ± 0.5

3. Results
3.1 Froude Number, Fr Classification

This work considers the flow regimes occurring just downstream of a sharp-crested weir for the entire range of tailwater levels. The experiments are performed in smooth horizontal channels, covering a wide range of water discharges. From the experiment, we classify the downstream flow regime based on the classical hydraulic jump's typology, taking into account the Froude number of the approaching flow. The hydraulic jump is classified into undular jump, weak jump, oscillating jump, steady jump, and strong jump, based on the Froude number. This classification serves as a rough guideline for low head jumps in rectangular horizontal channels. Several authors have defined various classifications, which listed in Table 3. Although the classifications differ slightly, they share some similarities in their descriptions of hydraulic jump typology [12], [13].

Figure 3 shows that the highest *Fr* is 5.4 for the upstream condition of 0.5, referring to the ratio of head over weir height (h_c/P), with a tailwater, h_t of 0.7 P. This flow regime is similar to a low head dam in nature and is supercritical. According to the classification provided in Table 4, this flow condition (*Fr* = 5.4) is identified as steady and stabilized. Moreover, this high *Fr* number suggests that the flow is fast-moving and energetic. The steady jump condition is often associated with rapid changes in flow velocity or flow properties, leading to complex phenomena such as hydraulic jumps.

For upstream condition (h_c/P) of 0.4, with its tailwater h_t of 0.8 P shows that the *Fr* is 3.7. At this condition, (*Fr*=3.7), the flow transitions into the oscillating condition. During this regime, the flow experiences periodic variations and oscillations, creating a dynamic and energetic environment.

Meanwhile, the *Fr* minimum is 1.0, also known as the critical condition. The flow regime is associated with a tailwater of 1.0 P. According to Table 4, for the critical condition (*Fr* = 1.0), it is classified as undular, indicating a calm regime. the flow exhibits smooth and uniform characteristics, without significant disturbances or oscillations. The undular condition is ideal for certain applications where stability and low turbulence are desirable.

Table 4
Hydraulic jump typology [14]

Fr range	Classification [[15], [16]	Classification [17,18]
1–1.7	Without roller	Undular
1.7–2.5	Pre-jump	Weak
2.5–4.5	Transition	Oscillating
4.5–9	Stabilized	Steady
>9	Choppy	Strong

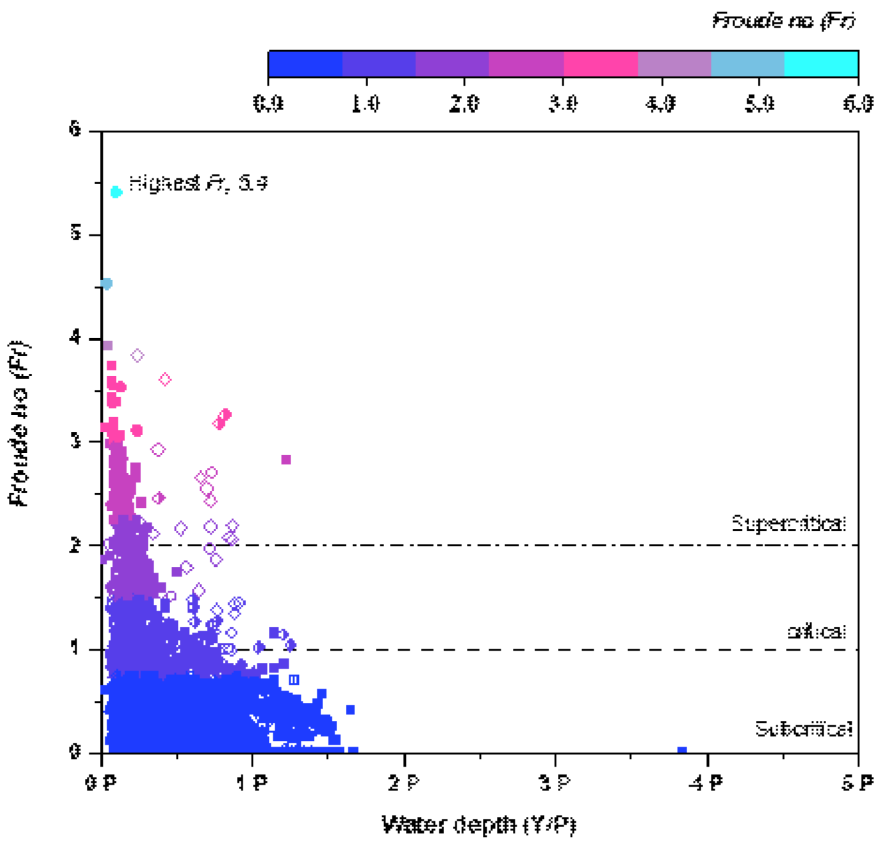

Fig. 3. Froude number, *Fr* distribution on downstream channel

3.2 Velocity Distribution on Downstream Channel

Figure 4 illustrates the velocity contour of steady jump at highest *Fr* which is 5.4, providing valuable insights into the velocity distribution within the flow. The three identified zones, also known as the impingement zone, the recirculation zone, and the recovery zone, represent the characteristic regions associated with a submerged hydraulic jump. As water flows over a weir crest and plunges into the downstream channel, the velocity increases, indicating the transition from subcritical flow (lower velocity) to supercritical flow (higher velocity) in the open channel. This transition typically occurs at a high Froude number, signifying fast-moving and energetic flow conditions.

The impingement zone (0 -1 P) suggests that the high-velocity flow impacts the downstream channel, resulting in turbulent and chaotic behaviour. The results showed at the impingement zone 0 – 1 P, the velocity were increased from 0.25 ms^{-1} to 0.5 ms^{-1}.

The recirculation zone (1 P to 3.7 P), the velocity increases, from 0.5 ms^{-1} to 0.75 ms^{-1}. The region represents the mixed zone area where water recirculates back upstream, causing the formation of eddies and vortices due to energy dissipation [19]. From visual observation, free surface roller presence from 2.5 P to 3.7 P extending towards 4.4 P. The mixed region velocity values range from 0.25 ms^{-1} to 0.89 ms^{-1}, which aligns well with the literature values. In the recirculation zone, the Froude number is lower than in the impingement zone. This is because the counter-current motion creates regions of slower flow, resulting in a reduced Froude number. Near the bed channel, the velocity is high, while it is relatively lower on the surface [20].

In the recovery zone (3.7 P to 5 P), the flow gradually regains its velocity and begins to stabilize. This zone signifies the region where the flow resumes its uniform characteristics. Notably, the highest Froude number is observed in this zone, primarily due to the flow accelerates to maintain continuity, which means that flows exit with high velocity. The presence of high turbulence suggests the occurrence of rapid changes in flow momentum. In the recovery zone at 4.4 P, the velocity started to decrease from 0.63 ms^{-1} to 0.5 ms^{-1} and keep reducing towards 0.1 ms^{-1} until being swept away towards the end.

Overall, the observed flow patterns and their corresponding Froude numbers align closely with literature data [21]. The Froude number provides a valuable perspective in open channel flow analysis by indicating the flow regime (subcritical or supercritical), the presence of eddies or counter-currents, and the intensity of the flow. These insights can be crucial for understanding factors such as sediment transport, flow stability, and the potential for erosion or deposition in open channel systems.

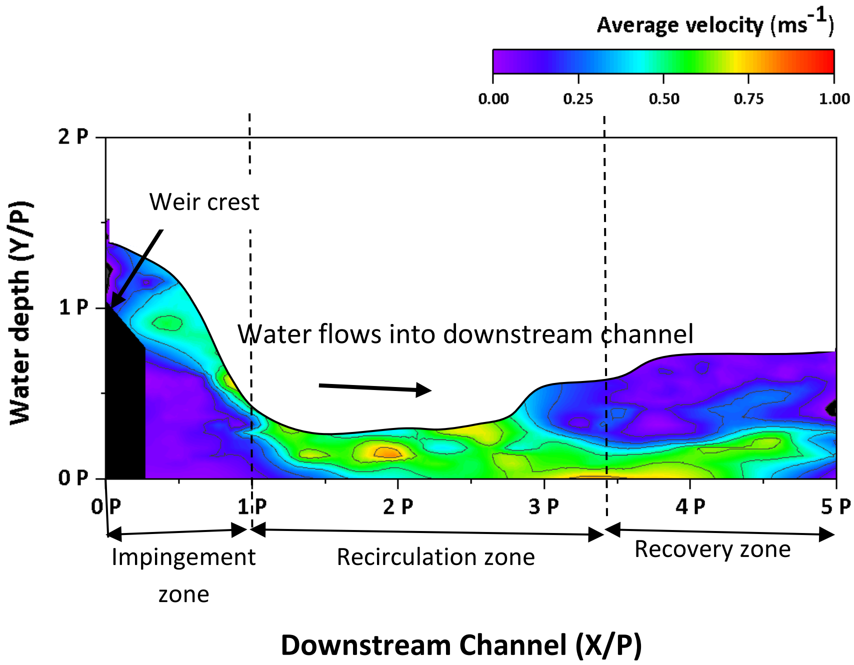

Fig. 4. Velocity contour at the highest Froude number, *Fr* = 5.4

4. Conclusions

In summary, this study has provided valuable insights into the behaviour of submerged hydraulic jumps in open channels, specifically focusing on the correlation between upstream and downstream conditions. By analysing flow visualization techniques, channel geometry, and hydraulic properties, we have enhanced our understanding of the complex fluid dynamics involved in these jumps. The findings of this research have important practical implications for the design, management, and predictive modelling of hydraulic systems with submerged hydraulic jumps. This knowledge can be utilized to optimize the design and operation of open channel systems, improving their efficiency and effectiveness. Further studies are needed to validate and refine our understanding of the intricate flow patterns and characteristics associated with submerged hydraulic jumps in open channels. Continued experimentation and analysis will contribute to expanding knowledge and refining the methods used to manage and design open channel systems with submerged hydraulic jumps.

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References
[1] Rocha Filho, Sergio Murilo Daruis, Roger Matsumoto Moreira, Marcio Zamboti Fortes, and Rafael Eitor dos Santos. "CFD Analysis of The Heave and Pitch Motion of Hull Model." *CFD Letters* 14, no. 4 (2022): 14-31. https://doi.org/10.37934/cfdl.14.4.1431

[2] Ohtsu, Iwao, Youichi Yasuda, and Yasushi Yamanaka. "Discussion of “Submerged flow regimes of rectangular sharp-crested weirs” by S. Wu and N. Rajaratnam." *Journal of Hydraulic Engineering* 123, no. 10 (1997): 927-929. https://doi.org/10.1061/(asce)0733-9429(1997)123:10(927.2)

[3] Azimi, Amir H., Nallamuthu Rajaratnam, and David Z. Zhu. "Water surface characteristics of submerged rectangular sharp-crested weirs." *Journal of Hydraulic Engineering* 142, no. 5 (2016): 06016001. https://doi.org/10.1061/(asce)hy.1943-7900.0001110

[4] Leutheusser, Hans J., and Warren M. Birk. "Drownproofing of low overflow structures." *Journal of Hydraulic Engineering* 117, no. 2 (1991): 205-213. https://doi.org/10.1061/(asce)0733-9429(1991)117:2(205)

[5] McGhin Ronald, F., H. Hotchkiss Rollin, and Kern Ed. "Submerged Hydraulic Jump Remediation at Low-Head Dams: Partial Width Deflector Design." American Society of Civil Engineers, 2018. https://doi.org/10.1061/(asce)hy.1943-7900.0001513

[6] Leutheusser, Hans J., and Jerry J. Fan. "Backward flow velocities of submerged hydraulic jumps." *Journal of Hydraulic Engineering* 127, no. 6 (2001): 514-517. https://doi.org/10.1061/(asce)0733-9429(2001)127:6(514)

[7] Auel, Christian, Ismail Albayrak, and Robert M. Boes. "Turbulence characteristics in supercritical open channel flows: Effects of Froude number and aspect ratio." *Journal of Hydraulic Engineering* 140, no. 4 (2014): 04014004. https://doi.org/10.1061/(asce)hy.1943-7900.0000841

[8] Leutheusser, Hans J., and Warren M. Birk. "Drownproofing of low overflow structures." *Journal of Hydraulic Engineering* 117, no. 2 (1991): 205-213. https://doi.org/10.1061/(asce)0733-9429(1991)117:2(205)

[9] Olsen, Riley, and Michael Johnson. n.d. “DigitalCommons@USU Hazards at Low-Head Dams.” Accessed June 14, 2023. https://damfailures.org/wp-content/uploads/2018/10/11-USU-Hazard-at-Low-Head-Dams.pdf.

[10] Rao, NS Govinda, and Nallamuthu Rajaratnam. "The submerged hydraulic jump." *Journal of the Hydraulics Division* 89, no. 1 (1963): 139-162. https://doi.org/10.1061/jyceaj.0000822

[11] McArdle, William D., Frank I. Katch, and Victor L. Katch. *Exercise physiology: nutrition, energy, and human performance*. Lippincott Williams & Wilkins, 2010.

[12] Ting, Wen Kiat, How Tion Puay, and Nor Azazi Zakaria. "Numerical Simulation of Hydraulic Jump with the Inclusion of Boussinesq Term." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 93, no. 1 (2022): 186-199. https://doi.org/10.37934/arfmts.93.1.186199

[13] Ohtsu, Iwao, Youichi Yasuda, and Hiroshi Gotoh. "Flow conditions of undular hydraulic jumps in horizontal rectangular channels." *Journal of hydraulic engineering* 129, no. 12 (2003): 948-955. https://doi.org/10.1061/(asce)0733-9429(2003)129:12(948)

[14] Valero, Daniel, Nicolò Viti, and Carlo Gualtieri. "Numerical simulation of hydraulic jumps. Part 1: Experimental data for modelling performance assessment." *Water* 11, no. 1 (2018): 36. https://doi.org/10.3390/w11010036

[15] Bradley, J. N., and A. J. Peterka. "Hydraulic design of stilling basins: Hydraulic jumps on a horizontal apron (Basin I)." *Journal of the Hydraulics Division* 83, no. 5 (1957): 1401-1. https://doi.org/10.1061/jyceaj.0000126

[16] Hager, Willi H. *Energy dissipators and hydraulic jump*. Vol. 8. Springer Science & Business Media, 2013.

[17] Chow, Ven Te. 1981. “Hydraulic Exponents.” *Journal of the Hydraulics Division* 107 (11): 1489–99. https://doi.org/10.1061/jyceaj.0005765

[18] Chanson, Hubert. "The hydraulics of open channel flow: an introduction. physical modelling of hydraulics." (1999): 261-283.

[19] Manogaran, Thiennieesh, Mohd Remy Rozainy Mohd Arif Zainol, Muhammad Khairi A. Wahab, Mohd Sharizal Abdul Aziz, Nurhanani Abd Aziz, Nazirul Mubin Zahari, Mohd Hafiz Zawawi, and Mohd Rashid Mohd Radzi. "Free-Surface Vortices Mitigation using Anti-Vortex Plates in Dam Intakes through CFD." *CFD Letters* 15, no. 6 (2023): 26-41. https://doi.org/10.37934/cfdl.15.6.2641

[20] Shinneeb, Monsif, Ghassan Nasif, and Ram Balachandar. "Secondary Currents at Very Low and High Aspect Ratios of Open Channels." *CFD Letters* 14, no. 5 (2022): 1-15. https://doi.org/10.37934/cfdl.14.5.115

[21] Ahmed, S., Y. Ye, H. Liu, and N. Rajaratnam. "Preliminary study of surface hydraulic jumps." *Journal of Irrigation and Drainage Engineering* 145, no. 3 (2019): 06019001. https://doi.org/10.1061/(asce)ir.1943-4774.0001379