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Slip and Thermo Diffusion Effects on the Flow Over an Inclined Plate

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ABSTRACT

An exact analysis of unsteady free convection flow of fractionalized viscous fluid over an oscillating vertically inclined plate is obtained. The phenomenon of exponential heating is added into account for thermal aspects of an inclined plate. Moreover, in the model of problem, additional effects like thermo diffusion and slip are also used. Caputo fractional derivative is used in the model. The novelty of present study is to analyze the effect of angle of inclination on the flow phenomena and the model is generalized by using Fourier's and Fick's laws. The governing dimensionless equations for velocity, concentration, and temperature profiles are solved using Laplace transform method and compared graphically. The effects of different parameters like fractional parameter, thermo diffusion parameter, and slip parameter are discussed through numerous graphs. From figures, it is observed that Prandtl and Smith numbers have decreasing effect on velocity profile, whereas thermo diffusion and mass Grashof numbers have increasing effect on velocity of fluid.

1. Introduction

Convection flow with porous media has numerous applications such as flows in soils, solar power collectors, heat transfer correlated with geothermal systems, heat source in the field of agricultural storage system, heat transfer in nuclear reactors, heat transfer in aerobic and anaerobic reactions, heat evacuation from nuclear fuel detritus, and heat exchangers for porous material. Mass and heat transfer occur mostly in nature due to temperature and concentration differences respectively. Today research work in Magnetohydrodynamics (MHD) has substantial significance as these flows are absolutely prevailing in nature.

The important significance of non-Newtonian fluids can be seen in applied mathematics, engineering, and physics. It has various significance in many areas, such as uses of lubricants, biological fluids food processing, or plastic manufacturing. Some commonly examples of non-Newtonian fluids are custard, colloids, melted butter, paint, ketchup, starch suspensions, blood, toothpaste, gels, shampoo, and corn starch.

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MHD flow has many implementation in meteorology, distillation of gasoline, energy generators, geophysics, accelerators, petroleum industry, astrophysics, polymer technology, aerodynamics, boundary layer control, and for material process such as glass fiber drawing, extrusion, and casting wire. Kai-Long Hsiao [1] studied the MHD heat transfer thermal extrusion system with Maxwell fluid. The solution for unsteady flow of magnetic field in cylinder is obtained by Shah *et al.,* [2]. The influence of slip and heat transfer effect on MHD flow with porosity has been observed by Das. [3]. Author in [4] discussed the solution of Jeffrey fluid flow with thermal radiation.

Furthermore, convection flow in the existence of porosity has wide applications such as ground water hydrology, oil extraction, geothermal systems, cooling system, store of nuclear waste materials, energy efficient drying processes, solid matrix heat exchangers, and wall cooled catalytic reactors. Exact solution for magnetohydrodynamics flow through a plate in the existence of porosity is obtained by Ahmad *et al.,* [5]. Fractional differential equation are analyzed by [6,7].

Kumar *et al.,* [8] studied the influence of thermo-diffusion and radiation on MHD free convection flow. Sandeep *et al.,* [9] worked on the flow of fluid with heat source. Authors in [10] studied the heat and mass transfer through an inclined plate. Ali *et al.,* [11] analyzed the conjugate effects of heat and mass transfer over an inclined vertical plate. He obtained the solution for time dependent concentration and temperature. Convection flow immersed in a porous media through a plate is discussed in [12-14].

The impact of conjugate flow of MHD fluid is discussed by Khan *et al.,* [15]. Aladig *et al.,* [16] focused on the stream analysis model. MHD flow through an accelerated surface in the existence of porous media is discussed by Chaudhary *et al.,* [17]. Authors in [18] also analyzed the solution of velocity field graphically. Ramzan *et al.,* [19] examined the solution of Brinkman fluid with heat generation and chemical reaction, whereas, the solution for convection flow with non-uniform temperature through a moving plate is obtained by Seth *et al.,* [20]. The solution of nanofluid with ramped temperature is studied by Khalid *et al.,* [21].

The discussion of mass diffusion has empirical uses in numerous areas of engineering and applied sciences. This phenomena plays a vital role in cooling of nuclear reactor, tabular reactor, chemical industry, mixture of terracotta material, petroleum industry, and decomposition of rigid materials. Seddeek *et al.,* [22] examined the MHD fluid flow with thermal radiation. An intensive study of chemical reaction with heat source/sink is studied by Shah *et al.,* [23]. Seth *et al.,* [24] obtained the solution of unsteady magnetohydrodynamic flow of fluid over a plate with ramped condition. The solution of convection flow of MHD Casson fluid through a channen with heat generation/obsorption is obtained by [25]. The exact solution of MHD fluid with mass transfer immersed in a porous media is studied by Ali *et al.,* [26]. The exact solution of magnetohydrodynamic flow of a Brinkman fluid perpendicular to the plate is focused by Khan *et al.,* [27]. The analytical investigation of Brinkman fluid flow with variable concentration, temperature, and velocity is obtained by Ali *et al.,* [28]. Authors in [29] studied the Brinkman type nanofluid with slip effect. The unsteady flow of fractional fluid is analyzed by Shah *et al.,* [30]. The study of nanofluids over a plate is discussed in [31,32]. The effects of Hall and ion slip on unsteady MHD free convective rotating flow is studied by [33,34].

In this problem, the model of unsteady free convection flow of viscous fluid over an inclined plate is considered. The impact of slippage and thermo diffusion is added into account. Firstly, the governing equations have been made non-dimensional and then solved analytically. The model is fractionalized by using Fourier's and Fick's Laws. The results for velocity profile, temperature profile, and concentration profile are obtained and then analyzed graphically. Various graphs are plotted and discussed for different parameters, which are used in the flow model.

2. Mathematical Description of the Model

Let us consider the unsteady free convection flow of an incompressible viscous fluid over an inclined plate. The fluid is flowing vertically upward along $y^{\cdot}$-axis and the $x^{\cdot}$-axis is normal to the plate. The plate is inclined to vertical direction with an angle A. The plate and fluid have concentration $C_{\infty}^{\cdot}$ and temperature $T_{\infty}^{\cdot}$ at time $t_1^{\cdot} = 0$ with zero velocity. But for $t_1^{\cdot} > 0$, the plate starts to oscillate in the plane with uniform velocity $U_1 f(t^{\cdot})$. The concentration and temperature of the plate is increased to $C_w^{\cdot}$ and $T^{\cdot} = T_w^{\cdot}(1 - ae^{-b^{\cdot}t^{\cdot}}) + T_{\infty}^{\cdot}$ respectively. We made the following assumption:

i. The Fluid's conducting property is supposed to be slight and hence the magnetic Reynolds number is negligible.
ii. Viscous dissipation and Joule heating in energy Eq. are neglected.
iii. Electric field is neglected.
iv. It is further supposed that there is no applied voltage, as the electric field is absent.

In view of above assumption and using Boussinesq's approximation, the convection flow of viscous fluid with Soret effect through a plate, linear momentum equation [12,19,35] is

$$\rho \frac{\partial u_1(x^{\cdot}, t_1^{\cdot})}{\partial t_1^{\cdot}} = \frac{\partial \tau(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}} + \rho g \beta_{T^{\cdot}} (T^{\cdot} - T_{\infty}^{\cdot}) cos(A) + \rho g \beta_{C^{\cdot}} (C^{\cdot} - C_{\infty}^{\cdot}) cos(A), \quad (1)$$

shear stress τ is

$$\tau = \mu \frac{\partial u_1(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}}. \quad (2)$$

thermal equation is

$$\rho C_p \frac{\partial T^{\cdot}(x^{\cdot}, t_1^{\cdot})}{\partial t_1^{\cdot}} = -\frac{\partial q(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}}, \quad (3)$$

where $q_1(x^{\cdot}, t_1^{\cdot})$ is the thermal flux, its constitutive equation is obtained by Fourier's law given by

$$q_1(x^{\cdot}, t_1^{\cdot}) = -\alpha_0 \frac{\partial T^{\cdot}(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}}. \quad (4)$$

Diffusion equation is

$$\frac{\partial C^{\cdot}(x^{\cdot}, t_1^{\cdot})}{\partial t_1^{\cdot}} = -\frac{\partial J(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}} - \frac{D_{KT}}{Tm} \frac{\partial q(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}}. \quad (5)$$

where $J_1(x^{\cdot}, t_1^{\cdot})$ is mass flux rate. The constitutive Eq. of molecular diffusion is obtained by Fick's Law

$$J_1(x^{\cdot}, t_1^{\cdot}) = -D_m \frac{\partial C^{\cdot}(x^{\cdot}, t_1^{\cdot})}{\partial x^{\cdot}}. \quad (6)$$

where D_m is the diffusivity constant.

The initial and boundary conditions for the flow model are [14,29]

$$u_1(x^{\cdot}, t_1^{\cdot}) = 0, \;\; T^{\cdot}(x^{\cdot}, t_1^{\cdot}) = T_{\infty}, \;\; C^{\cdot}(y^{\cdot}, t_1^{\cdot}) = C_{\infty}^{\cdot}, \;\; y^{\cdot} > 0, \quad t_1^{\cdot} = 0, \quad (7)$$

$$u_1(0,t_1^{\cdot}) - R_1 \frac{\partial u_1}{\partial x^{\cdot}} = U_1 f(t_1^{\cdot}),\ \ T^{\cdot}(0,t_1^{\cdot}) = T_\infty^{\cdot} + T_w^{\cdot}(1 - ae^{-b^{\cdot}t^{\cdot}}),\ \ C^{\cdot}(0,t_1^{\cdot}) = C_w^{\cdot},\ \ t_1^{\cdot} > 0, \tag{8}$$

$$u_1(x^{\cdot},t_1^{\cdot}) \to 0,\ \ T^{\cdot}(x^{\cdot},t_1^{\cdot}) \to 0,\ \ C^{\cdot}(x^{\cdot},t_1^{\cdot}) \to 0, \quad x^{\cdot} \to \infty, \quad t_1^{\cdot} > 0. \tag{9}$$

3. Generalized Model

To write the flow model in dimensionless form, we used the following dimensionless variables

$$x^* = \frac{Ux^{\cdot}}{\nu}, \qquad t^* = \frac{U^2 t_1^{\cdot}}{\nu}, \qquad T^* = \frac{T^{\cdot} - T_\infty^{\cdot}}{T_w^{\cdot} - T_\infty^{\cdot}}, \qquad u^* = \frac{u_1}{U_{\cdot}},$$
$$Gr^* = \frac{\nu\beta_{T^{\cdot}}(T_w^{\cdot} - T_\infty^{\cdot})}{U^3}, \qquad C^* = \frac{C^{\cdot} - C_\infty^{\cdot}}{C_w^{\cdot} - C_\infty^{\cdot}}, \qquad Gm^* = \frac{\nu\beta_{C^{\cdot}}(C_w^{\cdot} - C_\infty^{\cdot})}{U^3}. \tag{10}$$

Eq. (1) is fractionally generalized by Blair and Caffyn [36]

$$\tau = L_\beta D_t^{1-\beta} \frac{\partial u(x,t)}{\partial x}, \qquad 1 \geq \beta > 0, \tag{11}$$

where $L_\beta = n_1 K_{1-\beta} = 1$ when $\beta \to 1$. Put Eq. (11) into Eq. (1) and using non-dimensional parameters from Eq. (7), we have

$$\frac{\partial u(x,t)}{\partial t} = L_\beta D_t^{1-\beta} \frac{\partial^2 \underline{u}(x,t)}{\partial x^2} + GrT(x,t)cos(A) + GmC(x,t)cos(A), \tag{12}$$

Eq. (3) is generalized by using Fourier's Law defined by Povstenko and Hristov [37,38]

$$q = -K_\gamma D_t^{1-\gamma} \frac{\partial T(x,t)}{\partial x}, \qquad 1 \geq \gamma > 0, \tag{13}$$

where thermal conductivity has generalized coefficient K_γ. Put Eq. (13) into Eq. (3) and making non-dimensional results, we have

$$\frac{\partial T(x,t)}{\partial t} = \frac{1}{Pr} D_t^{1-\gamma} \frac{\partial^2 T}{\partial x^2}, \tag{14}$$

where $Pr = \frac{\rho\nu C_p}{K_\gamma}$ is the generalized Prandtl number.

Eq. (5) is generalized by using Fick's Law defined by

$$J = -D_\alpha D_t^{1-\alpha} \frac{\partial C(x,t)}{\partial x}, \qquad 1 \geq \gamma > 0. \tag{15}$$

where molecular diffusion has generalized coefficient D_α. Put Eq. (15) into Eq. (5) and making non-dimensional results, we have

$$\frac{\partial C(x,t)}{\partial t} = \frac{1}{Sc} D_t^{1-\alpha} \frac{\partial^2 C(x,t)}{\partial x^2} + Sr D_t^{1-\gamma} \frac{\partial^2 T(x,t)}{\partial x^2}, \tag{16}$$

where $Sc = \frac{\nu}{D_\alpha}$ is the generalized Schimdt number.

Taking inversion left operator on Eq. (12), (14), and (16), we obtain

$$I_t^{1-\beta}\frac{\partial}{\partial t}u(x,t) = D_t^{\beta}u(x,t) = L_\beta\frac{\partial^2 u(x,t)}{\partial x^2} + I_t^{1-\beta}GrT(x,t)cos(A) + I_t^{1-\beta}GmC(x,t)cos(A), \tag{17}$$

$$I_t^{1-\gamma}\frac{\partial T(x,t)}{\partial t} = D_t^{\gamma}T(x,t) = \frac{1}{Pr}\frac{\partial^2 T(x,t)}{\partial x^2}, \tag{18}$$

for $\alpha = \gamma$

$$I_t^{1-\alpha}\frac{\partial C(x,t)}{\partial t} = D_t^{\alpha}C(x,t) = \frac{1}{Sc}\frac{\partial^2 C(x,t)}{\partial x^2} + Sr\frac{\partial^2 T(x,t)}{\partial x^2}. \tag{19}$$

with dimensionless initial and boundary conditions are

$$u(x,t) = T(x,t) = C(x,t) = 0,\ \ x > 0,\quad t = 0, \tag{20}$$

$$u(0,t) - R\frac{\partial u}{\partial x} = f(t),\ T(0,t) = 1 - ae^{-bt},\ \ C(0,t) = 1,\ \ t > 0, \tag{21}$$

$$u(x,t) \to 0,\quad T(x,t) \to 0,\quad C(x,t) \to 0,\quad x \to \infty,\ \ t > 0, \tag{22}$$

where $D_t^{\alpha}u(x,t)$ represents the Caputo fractional derivative of $u(x,t)$ as

$$D_t^{\alpha}u(x,t) = \{\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{1}{(t-s)^{\alpha}}\frac{\partial u(x,s)}{\partial s}ds,\ 0 \le \alpha < 1;\ \frac{\partial u(x,t)}{\partial t},\ \alpha = 1. \tag{23}$$

4. Solution of Problem

Eq. (17)-(19) with initial and boundary conditions has been solved analytically.

4.1 Calculation of Temperature

By applying Laplace transform on Eq. (18), we have

$$s^{\gamma}\underline{T}(x,s) = \frac{1}{Pr}\frac{\partial^2 \underline{T}(x,s)}{\partial x^2}. \tag{24}$$

Boundary conditions satisfying Eq. (24) are

$$\underline{T}(0,s) = \frac{1}{s} - \frac{a}{s+b},\qquad \underline{T}(x,s) \to 0,\ \ x \to \infty. \tag{25}$$

Eq. (24) is solved using conditions given in Eq. (25), which results in

$$\underline{T}(x,s) = (\frac{1}{s} - \frac{a}{s+b})e^{-x\sqrt{Prs^{\gamma}}}, \tag{26}$$

Suitable form of Eq. (26) is

$$\underline{T}(x,s) = (\frac{s^{\gamma}}{s} - \frac{as^{\gamma}}{s+b})\frac{e^{-x\sqrt{Prs^{\gamma}}}}{s^{\gamma}}. \tag{27}$$

Analytical solution of Eq. (27) is

$$T(x,t) = \int_0^t \quad [H_1(t-p) - aH_2(t-p)]f_1(x,p)dp, \tag{28}$$

where

$$f_1(x,p) = \int_0^\infty \quad erfc(\frac{x\sqrt{Pr}}{2\sqrt{u}})\frac{1}{p}\phi(0,-\gamma,-up^{-\gamma})du, \tag{29}$$

x is the space coordinate, Pr is the generalized Prandtl number, γ is fractional parameter, and by using formula $L^{-1}(\frac{e^{-a\sqrt{s^\alpha}}}{s^\alpha}) = \int_0^\infty \quad erfc(\frac{a}{2\sqrt{u}})\frac{1}{t}\phi(0,-\alpha,-ut^{-\alpha})du$,

$$H_1(t) = \frac{t^{-\gamma}}{\Gamma(1-\gamma)}, \tag{30}$$

$$H_2(t) = -t^\gamma E_{1,1-\gamma}(-bt). \tag{31}$$

4.2 Nusselt Number

From Eq. (27), the Nu can be calculated in the following way

$$Nu = -\frac{\partial T}{\partial x}|_{x=0} = -L^{-1}\{\frac{\partial \underline{T}}{\partial x}|_{x=0}\} = \int_0^t \quad [H_1(t-p) - aH_2(t-p)]\frac{p^{\frac{\gamma}{2}-1}\sqrt{Pr}}{\Gamma(\frac{\gamma}{2})}dp. \tag{32}$$

4.3 Calculation of Concentration

By applying Laplace transform on Eq. (19), we have

$$s^\alpha \underline{C}(x,s) = \frac{1}{Sc}\frac{\partial^2 \underline{C}(x,s)}{\partial x^2} + Sr\frac{\partial^2 \underline{T}(x,s)}{\partial x^2}. \tag{33}$$

Boundary conditions satisfying Eq. (33) are

$$\underline{C}(0,s) = s^{-1}, \qquad \underline{C}(x,s) \to 0, \;\; x \to \infty. \tag{34}$$

Eq. (33) is solved by using conditions given in Eq. (34), we have

$$\underline{C}(x,s) = s^{-1}e^{-x\sqrt{Scs^\alpha}} + \frac{SrScPrs^\alpha(\frac{1}{s}-\frac{a}{s+b})}{s^\alpha(Pr-Sc)}[e^{-x\sqrt{Scs^\alpha}} - e^{-x\sqrt{Prs^\gamma}}]. \tag{35}$$

Suitable form of Eq. (35) is

$$\underline{C}(x,s) = \frac{1}{s^{1-\alpha}}\frac{e^{-x\sqrt{Scs^\alpha}}}{s^\alpha} + [\frac{SrScPr}{Pr-Sc}][\frac{1}{s^{1-\alpha}} - \frac{as^\alpha}{s+b}][\frac{e^{-x\sqrt{Scs^\alpha}}}{s^\alpha} - \frac{e^{-x\sqrt{Prs^\gamma}}}{s^\alpha}]. \tag{36}$$

By taking $\alpha = \gamma$, Eq. (36) is solved analytically as

$$C(x,t)=\int_0^t H_1(t-p)f_2(x,p)dp+[\frac{ScSrPr}{Pr-Sc}]\int_0^t [H_1(t-p)-aH_2(t-p)]\times [f_2(x,p)-f_1(x,p)]dp, \tag{37}$$

where

$$f_2(x,p)=\int_0^\infty erfc(\frac{x\sqrt{Sc}}{2\sqrt{u}})p^{-1}\phi(0,-\gamma,-up^{-\gamma})du. \tag{38}$$

4.4 Sherwood Number

In order to calculate the Sherwood number, we use Eq. (36) in the following relation

$$Sh=-\frac{\partial C}{\partial x}|_{x=0}=-L^{-1}\{\frac{\partial \underline{C}}{\partial x}|_{x=0}\}=\int_0^t H_1(t-p)\frac{p^{\frac{\gamma}{2}-1}\sqrt{Sc}}{\Gamma(\frac{\gamma}{2})}dp+ [\frac{ScSrPr}{Pr-Sc}]\int_0^t [H_1(t-p)-aH_2(t-p)][\frac{p^{\frac{\gamma}{2}-1}\sqrt{Sc}}{\Gamma(\frac{\gamma}{2})}-\frac{p^{\frac{\gamma}{2}-1}\sqrt{Pr}}{\Gamma(\frac{\gamma}{2})}]dp. \tag{39}$$

4.5 Calculation of Velocity

By applying Laplace transform on Eq. (17), we have

$$s^{\beta}\underline{u}(x,s)=L_{\beta}\frac{\partial^2\underline{u}(x,s)}{\partial x^2}+\frac{1}{s^{1-\beta}}Gr\underline{T}(x,s)cos(A)+\frac{1}{s^{1-\beta}}Gm\underline{C}(x,s)cos(A), \tag{40}$$

Boundary conditions satisfying Eq. (40) are

$$\underline{u}(0,s)-R\frac{\partial\underline{u}(0,s)}{\partial x}=\frac{s}{w^2+s^2},\qquad \underline{u}(x,s)\to 0,\ \ x\to\infty. \tag{41}$$

Eq. (40) is solved by using conditions given in Eq. (41), we obtain

$$\underline{u}(x,s)=\frac{\frac{s}{w^2+s^2}}{1+R\sqrt{L_{\beta}^{-1}s^{\beta}}}e^{-x\sqrt{L_{\beta}^{-1}s^{\beta}}}+[\frac{(\frac{1}{s}-\frac{a}{s+b})cos(A)}{s^{1-\beta}(Prs^{\gamma}-L_{\beta}^{-1}s^{\beta})}][Gr-\frac{GmSrPrSc}{Pr-Sc}][\frac{1+R\sqrt{Prs^{\gamma}}}{1+R\sqrt{L_{\beta}^{-1}s^{\beta}}}e^{-x\sqrt{L_{\beta}^{-1}s^{\beta}}}- e^{-x\sqrt{Prs^{\gamma}}}]+[\frac{cos(A)}{s^{1-\beta}(Scs^{\alpha}-L_{\beta}^{-1}s^{\beta})}][\frac{Gm}{s}+(\frac{1}{s}-\frac{a}{(s+b)})(\frac{GmSrPrSc}{Pr-Sc})][\frac{1+R\sqrt{Scs^{\alpha}}}{1+R\sqrt{L_{\beta}^{-1}s^{\beta}}}\times e^{-x\sqrt{L_{\beta}^{-1}s^{\beta}}}-e^{-x\sqrt{Scs^{\alpha}}}]. \tag{42}$$

By taking $\alpha=\beta=\gamma$, suitable form of Eq. (42) is

$$\underline{u}(x,s)=[\frac{s}{w^2+s^2}[\frac{R\sqrt{L_{\beta}^{-1}s^{\frac{3\alpha}{2}}}-s^{\alpha}}{R^2L_{\beta}^{-1}(s^{\alpha}-R^{-2}L_{\beta})}]+[\frac{cos(A)}{L_{\beta}^{-1}R^2(Pr-L_{\beta}^{-1})}][Gr-\frac{GmScSrPr}{Pr-Sc}][\frac{s^{\alpha}}{s^2}-\frac{a}{bs^{1-\alpha}}+ \frac{as^{\alpha}}{b(s+b)}][\frac{R^2\sqrt{L_{\beta}^{-1}Prs^{\alpha}}}{s^{\alpha}-L_{\beta}R^{-2}}+\frac{(R\sqrt{L_{\beta}^{-1}}-R\sqrt{Pr})s^{\frac{\alpha}{2}}}{s^{\alpha}-L_{\beta}R^{-2}}-\frac{1}{s^{\alpha}-L_{\beta}R^{-2}}]+[\frac{cos(A)}{L_{\beta}^{-1}R^2(Sc-L_{\beta}^{-1})}][\frac{Gm}{s^{2-\alpha}}+\frac{GmScSrPr}{Pr-Sc}(\frac{1}{s^{2-\alpha}}-\frac{a}{bs^{1-\alpha}}+ \frac{as^{\alpha}}{b(s+b)})][\frac{R^2s^{\alpha}\sqrt{L_{\beta}^{-1}Sc}}{s^{\alpha}-L_{\beta}R^{-2}}+\frac{s^{\frac{\alpha}{2}}(R\sqrt{L_{\beta}^{-1}}-R\sqrt{Sc})}{s^{\alpha}-L_{\beta}R^{-2}}-\frac{1}{s^{\alpha}-L_{\beta}R^{-2}}]][\frac{e^{-x\sqrt{s^{\alpha}L_{\beta}^{-1}}}}{s^{\alpha}}]-[\frac{cos(A)}{(Pr-L_{\beta}^{-1})}][Gr-\frac{GmScSrPr}{Sc-Pr}](\frac{1}{s^{2-\alpha}}-$$

$$\frac{a}{bs^{1-\alpha}}+\frac{as^{\alpha}}{b(s+b)})[\frac{e^{-x\sqrt{Prs^{\alpha}}}}{s^{\alpha}}]-[\frac{cos(A)}{(Sc-L_{\beta}^{-1})}][\frac{Gm}{s^{2-\alpha}}+\frac{GmScSrPr}{Sc-Pr}(\frac{1}{s^{2-\alpha}}-\frac{a}{bs^{1-\alpha}}+\frac{as^{\alpha}}{b(s+b)})][\frac{e^{-x\sqrt{s^{\alpha}Sc}}}{s^{\alpha}}]. \tag{43}$$

Analytical solution of Eq. (43) is

$$u(x,t)=\int_0^t [G_1(t-p)-G_2(t-p)+(\frac{cos(A)}{R^2L_{\beta}^{-1}(Pr-L_{\beta}^{-1})})(Gr-\frac{GmScSrPr}{Pr-Sc})G_3(t-p)+ (\frac{Gmcos(A)}{R^2L_{\beta}^{-1}(Sc-L_{\beta}^{-1})})G_4(t-p)]f_3(x,p)dp-(\frac{cos(A)}{Pr-L_{\beta}^{-1}})(Gr-\frac{GmScSrPr}{Pr-Sc})\int_0^t G_5(t-p)f_1(x,p)dp- (\frac{Gmcos(A)}{Sc-L_{\beta}^{-1}})\int_0^t G_6(t-p)f_2(x,p)dp. \tag{44}$$

where

$$f_3(x,p)=\int_0^{\infty} erfc(\frac{x\sqrt{L_{\beta}^{-1}}}{2\sqrt{u}})p^{-1}\phi(0,-\gamma,-up^{-\alpha})du, \tag{45}$$

$$G_1(t)=\frac{\sqrt{L_{\beta}}}{R}\int_0^t (t-p)^{-\frac{\alpha}{2}-1}E_{\alpha,-\frac{\alpha}{2}}^{1}(L_{\beta}R^{-2}(t-p)^{\alpha})cos(wp)dp, \tag{46}$$

$$G_2(t)=\frac{1}{L_{\beta}^{-1}R^2}\int_0^t (t-p)^{-1}E_{\alpha}^{1}(R^{-2}L_{\beta}(t-p)^{\alpha})cos(wp)dp, \tag{47}$$

$$G_3(t)=\int_0^t [\frac{(t-z)^{1-\alpha}}{\Gamma(2-\alpha)}-\frac{a}{b}\frac{(t-z)^{-\alpha}}{\Gamma(1-\alpha)}+\frac{a}{b}(-(t-z)^{\alpha}E_{1,1-\alpha}(-b(t-z)))] [R^2\sqrt{PrL_{\beta}^{-1}}z^{-1}E_{\alpha}^{1}(R^{-2}L_{\beta}z^{\alpha})+(R\sqrt{L_{\beta}^{-1}}-R\sqrt{Pr})z^{\frac{\alpha}{2}-1}E_{\alpha,\frac{\alpha}{2}}^{1}(R^{-2}L_{\beta}z^{\alpha})- (z^{\alpha-1}E_{\alpha,\alpha}(R^{-2}L_{\beta}z^{\alpha}))]dz, \tag{48}$$

$$G_4(t)=\int_0^t [\frac{(t-z)^{1-\alpha}}{\Gamma(2-\alpha)}+\frac{SrScPr}{Pr-Sc}(\frac{(t-z)^{1-\alpha}}{\Gamma(2-\alpha)}-\frac{a}{b}\frac{(t-z)^{-\alpha}}{\Gamma(1-\alpha)}+ \frac{a}{b}(-(t-z)^{\alpha}E_{1,1-\alpha}(-b(t-z))))][R^2\sqrt{ScL_{\beta}^{-1}}z^{-1}E_{\alpha}^{1}(R^{-2}L_{\beta}z^{\alpha})+ (R\sqrt{L_{\beta}^{-1}}-R\sqrt{Sc})z^{\frac{\alpha}{2}-1}E_{\alpha,\frac{\alpha}{2}}^{1}(R^{-2}L_{\beta}z^{\alpha})-(z^{\alpha-1}E_{\alpha,\alpha}(R^{-2}L_{\beta}z^{\alpha}))]dz, \tag{49}$$

$$G_5(t)=\frac{t^{1-\alpha}}{\Gamma(2-\alpha)}-\frac{a}{b}\frac{t^{-\alpha}}{\Gamma(1-\alpha)}+\frac{a}{b}(-t^{\alpha}E_{1,1-\alpha}(-bt)) \tag{50}$$

$$(t)=\frac{t^{1-\alpha}}{\Gamma(2-\alpha)}+\frac{SrScPr}{Pr-Sc}(\frac{t^{1-\alpha}}{\Gamma(2-\alpha)}-\frac{a}{b}\frac{t^{-\alpha}}{\Gamma(1-\alpha)}+\frac{a}{b}(-t^{\alpha}E_{1,1-\alpha}(-bt))). \tag{51}$$

4.6 Skin friction

In order to find the Skin friction, we use Eq. (43) in the following relation

$$\tau=-\frac{\partial u}{\partial x}|_{x=0}=-L^{-1}\{\frac{\partial u}{\partial x}|_{x=0}\}=\int_0^t [G_1(t-p)-G_2(t-p)+(\frac{cos(A)}{R^2L_{\beta}^{-1}(Pr-L_{\beta}^{-1})})$$

$$(Gr - \frac{GmScSrPr}{Pr-Sc})G_3(t-p) + (\frac{Gmcos(A)}{R^2L_{\beta}^{-1}(Sc-L_{\beta}^{-1})})G_4(t-p)]\frac{p^{\frac{\alpha}{2}-1}\sqrt{L_{\beta}^{-1}}}{\Gamma(\frac{\alpha}{2})}dp - (\frac{\sqrt{Pr}cos(A)}{Pr-L_{\beta}^{-1}})(Gr - \frac{GmScSrPr}{Pr-Sc})\int_0^t G_5(t-p)\frac{p^{\frac{\alpha}{2}-1}}{\Gamma(\frac{\alpha}{2})}dp - (\frac{Gmcos(\sqrt{Sc}A)}{Sc-L_{\beta}^{-1}})\int_0^t G_6(t-p)\frac{p^{\frac{\alpha}{2}-1}}{\Gamma(\frac{\alpha}{2})}dp. \quad (52)$$

5. Results and Discussion

Analytical solution for free convection flow of viscous fluid with combined concentration and temperature gradient over an oscillating vertically inclined plat is obtained. The generalized model is solved with Caputo fractional derivative. The graph of concentration profile, temperature profile, and velocity profile are plotted for different parameters.

Figure 1 represents the effect of Sr on $u(x,t)$ without slippage. It is noted that the $u(x,t)$ increases with increasing values of Sr. Physically, mass buoyancy force increases with increasing values of Sr which raises the fluid motion. Figure 2 represents the effect of Sr on fluid velocity with slippage. The behavior of Gm on $u(x,t)$ with non slippage is reported in Figure 3. From this graph, it is concluded that the magnitude of fluid velocity rises by raising the values of Gm. Gm is the relative strength of viscous force and concentration buoyancy force. As Gm increases, the motion of fluid is accelerated due to an increment of buoyancy force. The behavior of Gm on $u(x,t)$ with slip effect is reported in Figure 4. Figure 5 represents the impact of different values of Gr on $u(x,t)$ with slip and non-slip effects. From this graph, it is noted that velocity distribution is directly proportional with Gr. Physically, Gr is a relation between viscous force and buoyancy force. Therefore, with an increment in the values of Gr, buoyancy force is increased which raises the magnitude of $u(x,t)$.

Figure 6 represents the behavior of Pr on the $u(x,t)$ with slippage and non-slippage. Pr represents the ratio of momentum (product of mass and velocity) diffusion to thermal diffusion. In the problems of heat transfer, Pr manages the thickness of boundary layer and momentum (velocity). For larger values of Pr, diffusion of heat becomes slow as compared to the fluid momentum (velocity) which decreases the thermal conductivity (thickness) and raises the boundary layer momentum. Figure 7 shows the influence of Sc on $u(x,t)$. Graph shows that for increasing values of Sc, the diffusion of molecule increases which reduces the fluid level. Figure 8 shows the influence of angle of inclination A on $u(x,t)$. Graph shows that for increasing values of A, velocity distribution is decreased. Figure 9 shows the influence of α= β= γ on $u(x,t)$ with non-slip and slip effect. Graph shows that for increasing values of fractional parameters, fluid velocity is increased. The behavior of fractional parameter and Pr on $T(x,t)$ is displayed in Figure 10. This figure shows that temperature increases with increment in the values of fractional parameter. Figure 10 indicates the influence of Pr on temperature $T(y,t)$. Temperature distribution decreases with increasing values of Pr as shown in graph.

The behavior of Pr and Sc on $C(x,t)$ are shown in Figure 11. The concentration level is accelerated with decreasing Pr as depicted in graph. Figure 11 shows the influence of Sc on $C(x,t)$. The concentration level increases with reducing values of Sc as highlighted in figure. The behavior of Sr and fractional parameter on $C(x,t)$ are shown in Figure 12. The concentration level is accelerated with increasing values of Sr as depicted in graph. Figure 12 shows the influence of fractional parameter on $C(x,t)$. The concentration level increases with increasing values of fractional parameter as highlighted in figure. Figure 13 shows the comparison of present work with Khalid *et al.,* [35]. If we take fractional parameters $\beta = \gamma = \alpha \to 1$, $Gm = Sr = 0$, inclined angle $=$ 0 and in the absence of Casson parameter of Khalid *et al.,* [35] work, the fluid profiles are identical which shows the authenticity of present work.

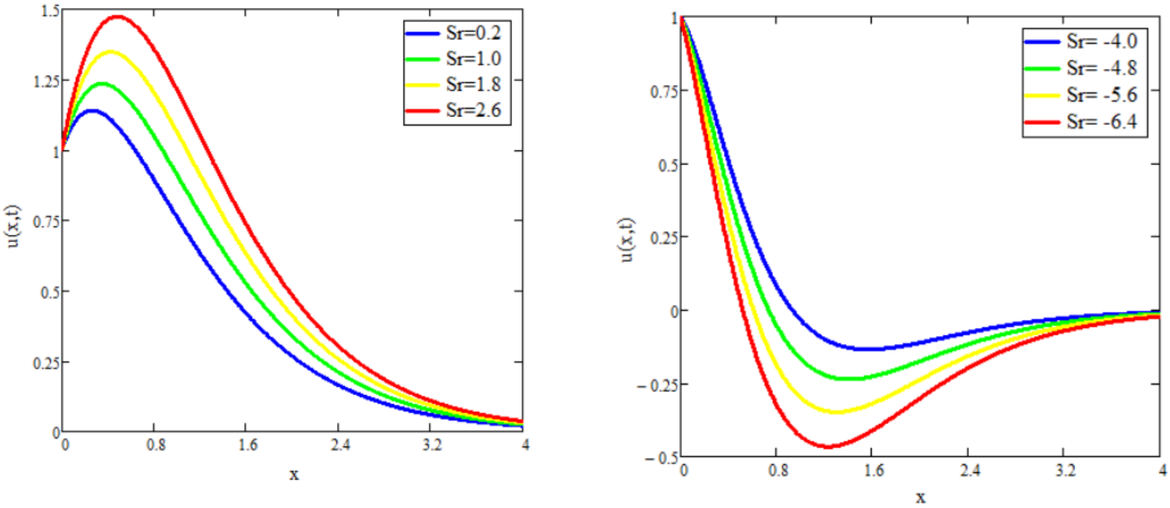

Fig. 1. Velocity diagram $u(x,t)$ for various values of parameter Sr at $R = 0, Gr = 4, A = 0, Gm = 4, Sc = 2.5, Pr = 16$

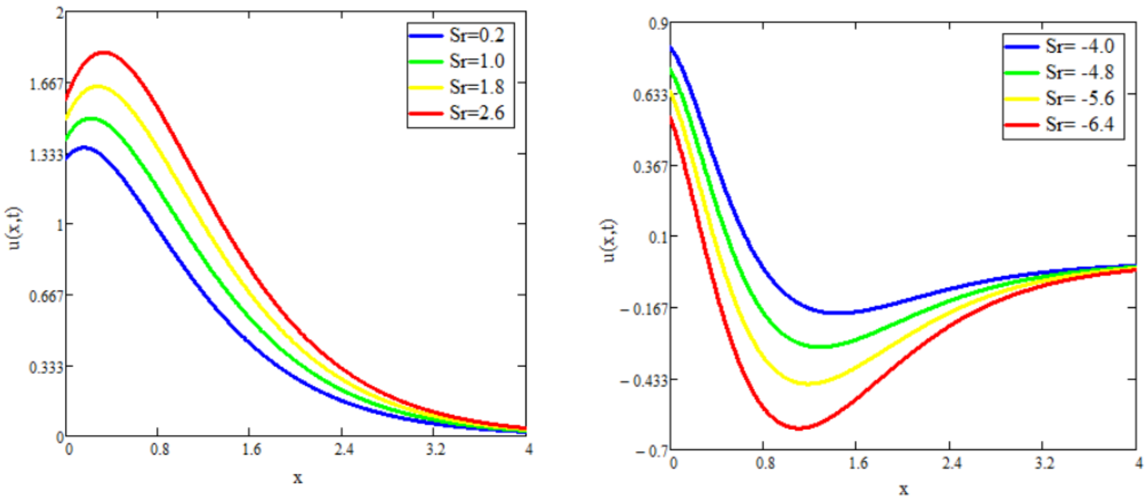

Fig. 2. Velocity distribution $u(x,t)$ for various values of parameter Sr at $R = 0.4, Gr = 4, A = 0, Gm = 4, Sc = 2.5, Pr = 16$

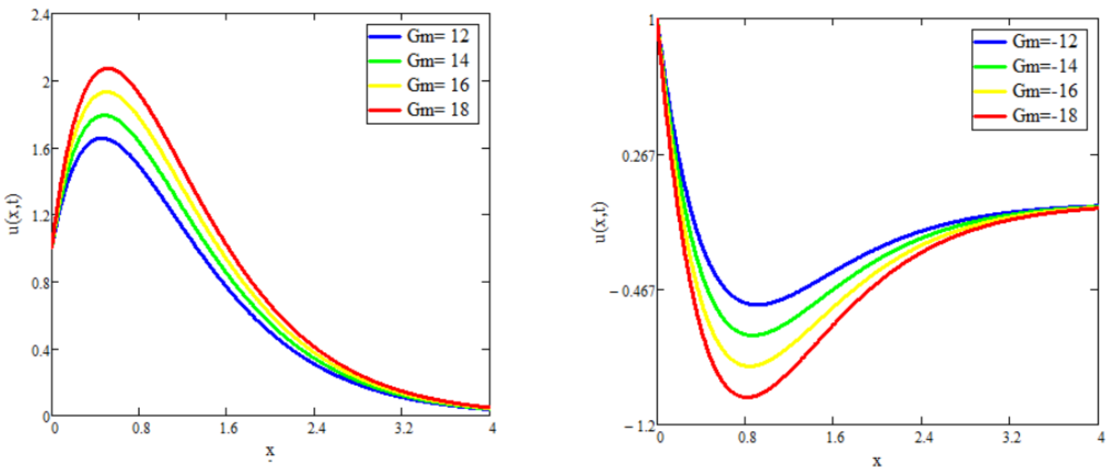

Fig. 3. Velocity diagram $u(x,t)$ for various values of parameter Gm at $R = 0, w = 0, Gr = 4, A = 0, Sr = 0.3, Sc = 2.5, Pr = 16.0$

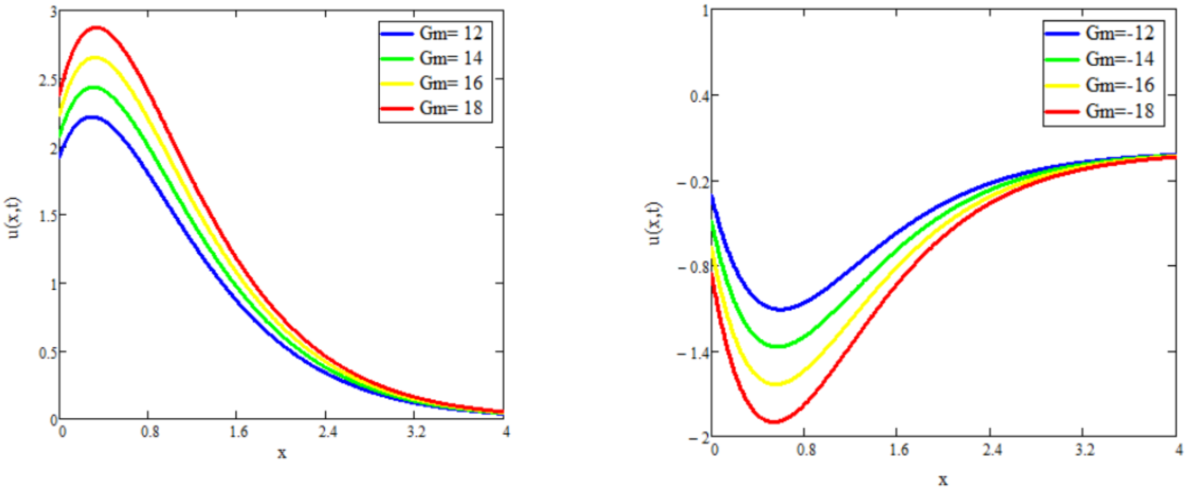

Fig. 4. Velocity distribution $u(x,t)$ for various values of parameter Gm at $R = 0.4, w = 0, Gr = 4, A = 0, w = 0, Sr = 0.3, Sc = 2.5, Pr = 16.0$

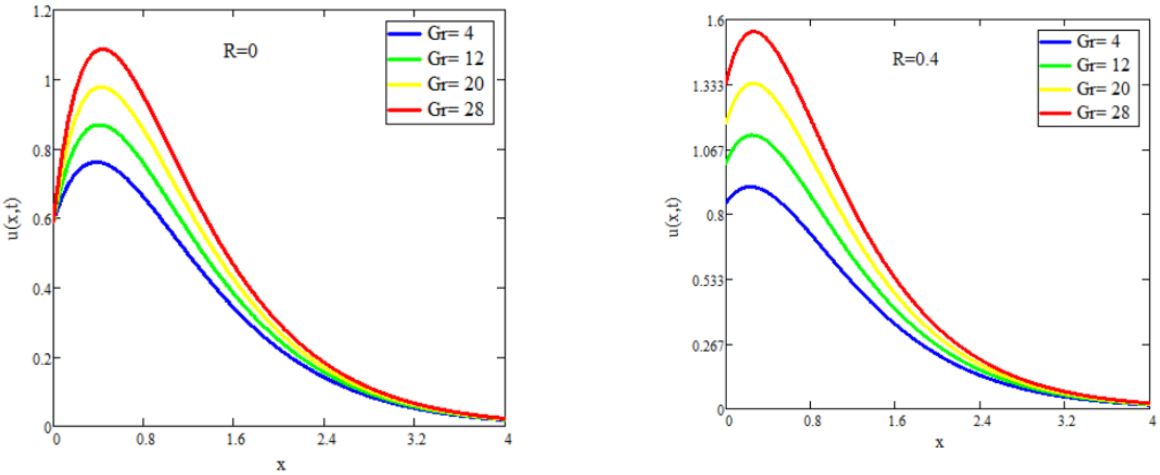

Fig. 5. Velocity diagram $u(x,t)$ for various values of parameter Gr at $A = 0, w = \frac{\pi}{3}, Gm = 4, Sr = 0.3, Sc = 2.5, Pr = 16.0$

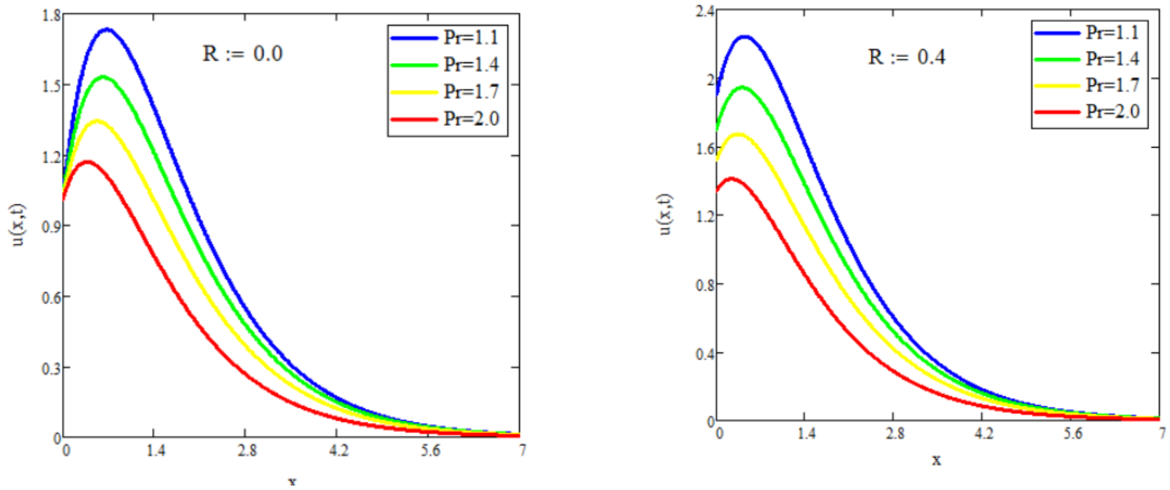

Fig. 6. Velocity distribution $u(x,t)$ for various values of parameter Pr at $A = 0.0, w = 0.0, Gm = 7, Sr = 0.3, Gr = 6, Sc = 2.5$

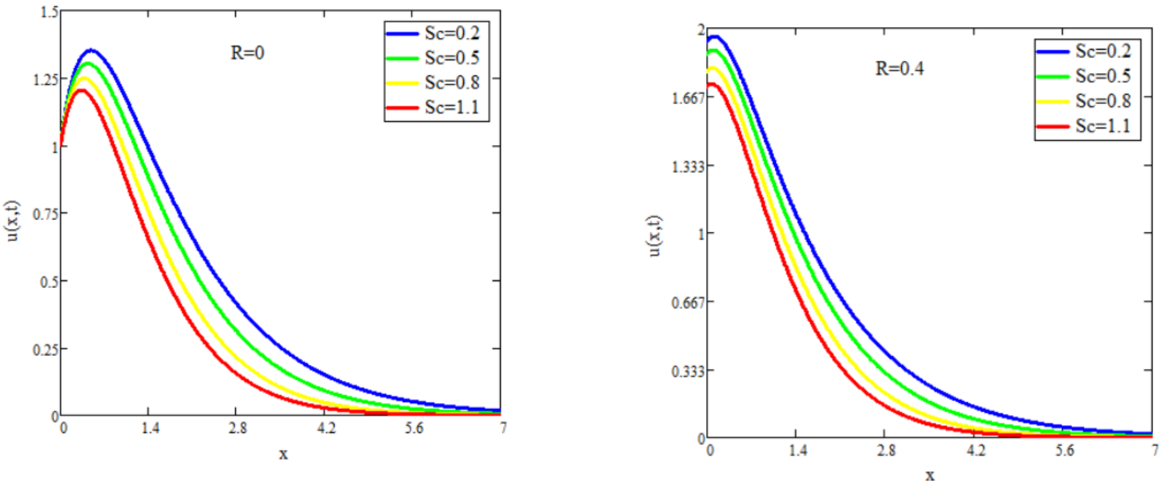

Fig. 7. Velocity diagram $u(x, t)$ for various values of parameter Sc at $A = 0, w = 0.0, Gm = 4, Sr = 0.3, Gr = 4, Pr = 16.0$

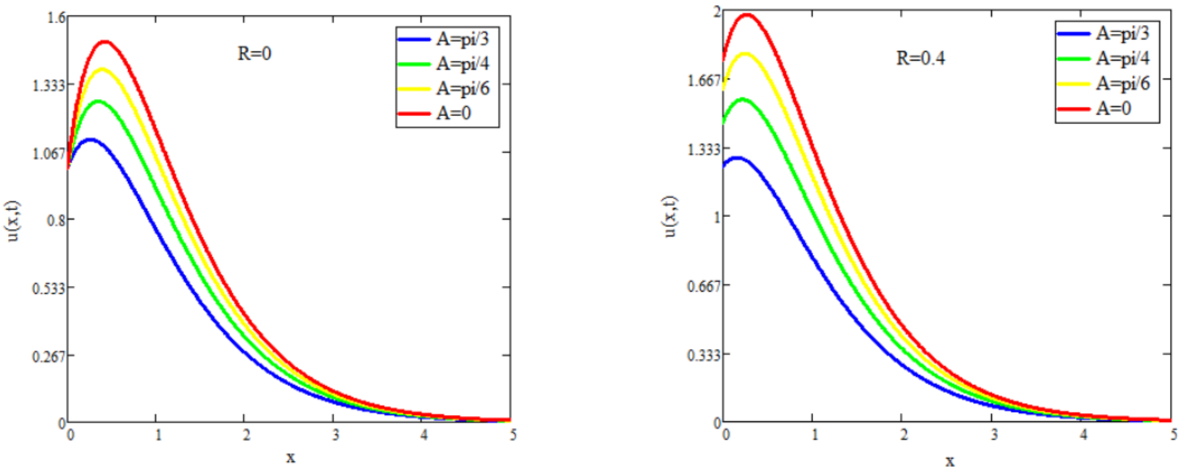

Fig. 8. Velocity distribution $u(x, t)$ for various values of parameter A at $Gm = 4, Sr = 0.3, Gr = 4, Sc = 2.5, Pr = 16.0$

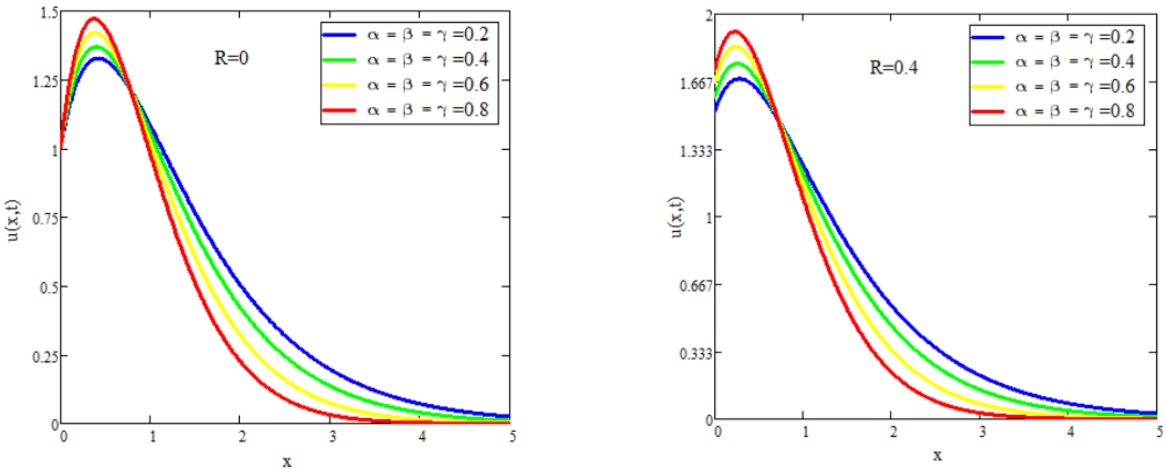

Fig. 9. Velocity diagram $u(x, t)$ for various values of fractional parameter at $A = 0, Gm = 4, Sr = 0.3, Gr = 4, Sc = 2.5, Pr = 16.0$

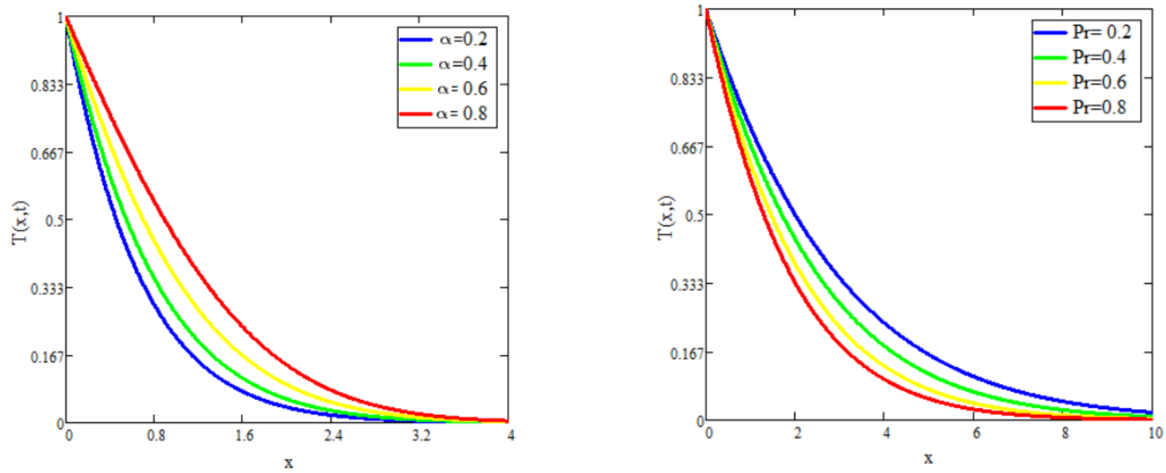

Fig. 10. Temperature distribution $T(x,t)$ for various values of fractional parameter and Pr

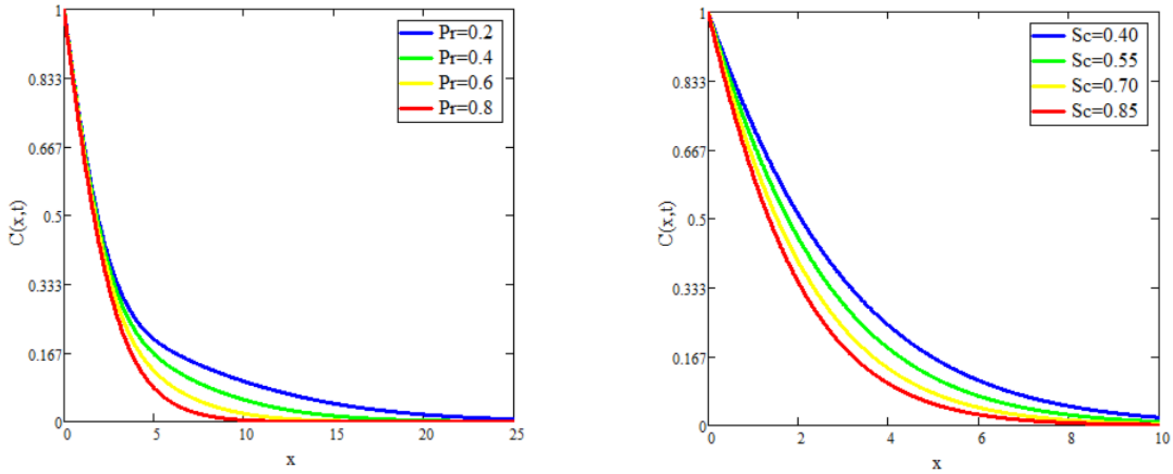

Fig. 11. Concentration distribution $C(x,t)$ for various values of parameter Pr and Sc

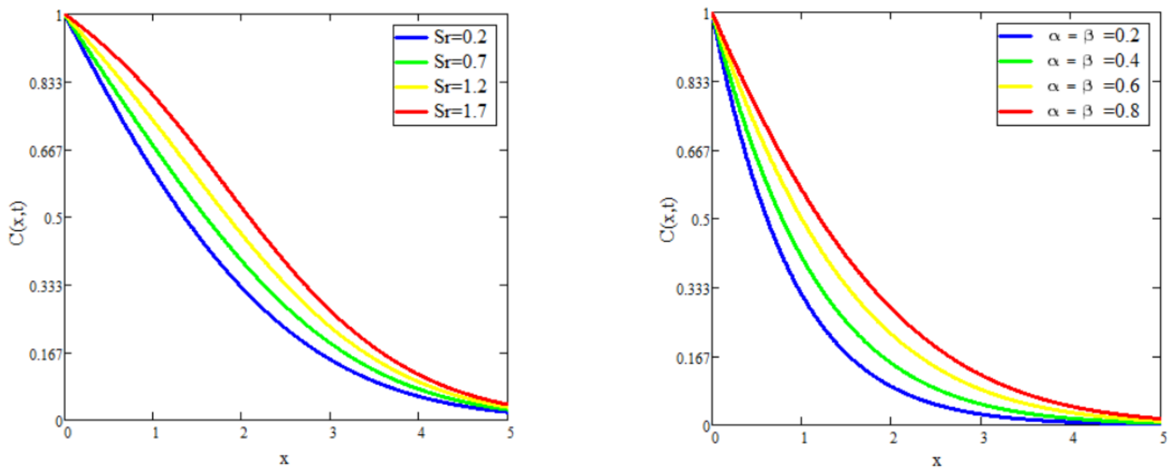

Fig. 12. Concentration distribution $C(x,t)$ for various values of Sr and fractional parameter

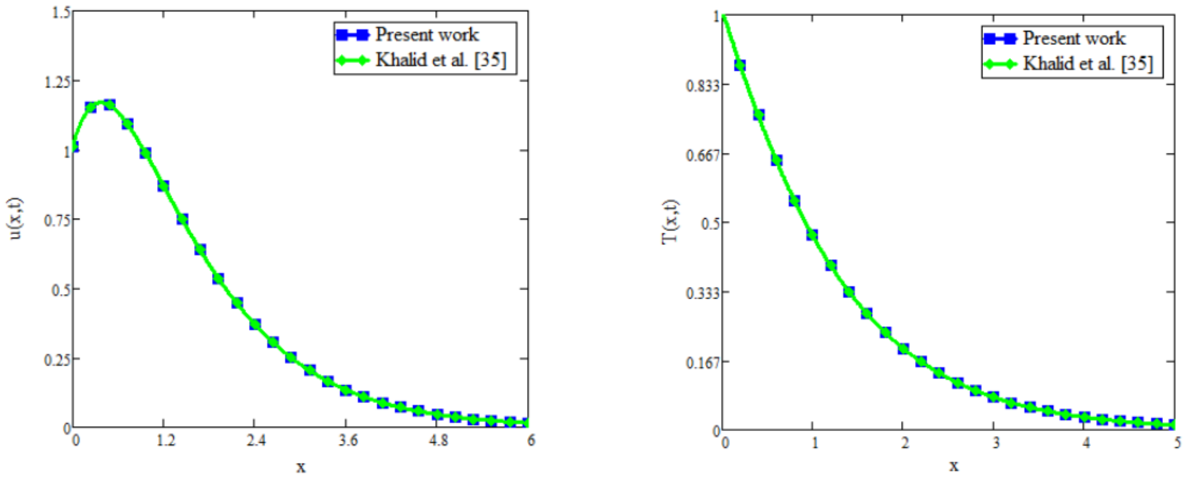

Fig. 13. Velocity and Temperature distribution for comparison of our work with Asma *et al.,* [35]

6. Conclusion

Analytical solution of free convection flow of viscous fluid has been obtained via Laplace transform. Different parameters used in the model are plotted and discussed. The model is solved with fractional derivative known as Caputo fractional derivative.

Here are the following main points which have been summarized for this model:

i. Thermal buoyancy forces lead to accelerate the $u(x,t)$.
ii. Velocity distribution retards with decreasing values of fractional parameter.
iii. The $u(x,t)$ decreases as Prandtl number increases.
iv. The $u(x,t)$ increases as values ofSc decreases.
v. The $u(x,t)$ increases with increasing values of Sr.
vi. The $u(x,t)$ increases as inclined angle A decreases.
vii. The temperature of fluid increases with decreasing vales of Pr.
viii. Temperature of fluid is an increasing function of fractional parameter.
ix. The concentration level of the fluid decreases with increasing values of Pr.
x. The concentration level is a decreasing function of Sc.
xi. The concentration level is an increasing function of fractional parameter.
xii. The concentration level is an increasing function of Sr.

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