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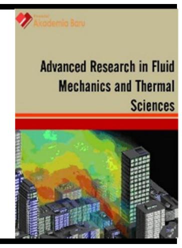


# Effect of MHD and Porosity on Exact Solutions and Flow of a Hybrid Casson-Nanofluid

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**ABSTRACT**

In this work, a novel hybrid nanofluid model with advanced thermophysical properties is considered for Casson fluid. The exact solutions are evaluated for non-integer derivative equations via Laplace technique. The unsteady flow, MHD, radiation and porous medium are considered. For the hybrid concept, alumina and copper nanoparticles are used in this research investigation. The problem is modeled using Caputo definition of non-integer derivatives. The influence of concerned parameters is depicted physically and graphically on the heat, concentration and flow of the hybrid nanofluid. The effect of volume fraction of both the nanoparticles on flow of hybrid nanofluids is observed. It is found that velocity increases with increasing values of $Gr$, $\beta$, and $k$, whereas $M$ decreases.



## 1. Introduction

The heat transfer of working fluids is improved using various techniques, one of which is to suspend nanoparticles to the working fluid. Maxwell's work [1] in this regard was the pioneer research. After that a practical implementation of this concept was carried out by Choi [2] to enhance thermal conductivity and rate of heat transfer. Azwadi *et al.*, [3] performed a process of preparation and factors affecting the performance of hybrid nanofluid. They found that the thermal characteristics of hybrid nanofluid were higher in comparison to the base fluid and fluid containing single nanoparticles respectively. Besides experimental studies, most of the researchers attracted towards the theoretical research work in this area in implementations of the proposed concepts and models in their work [4-9]. Another advanced concept of "hybrid nanofluids" was found to achieve better outcomes in this area. Those with the new thermophysical properties and modified models were expected to provide the desired efficiency both theoretically and experimentally. This work is fascinating to the researchers because of having a lot of space of new researches and investigations.

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Minea [10] studied hybrid nanofluids based on $Fe_3O_4$, MgO, Ag CuO, Cu and MWCNTs numerically along with the estimation of their viscosities. They found that maximum viscosity augmentation (i.e. 140.5% and 178.5% respectively) is obtained for CuO-Cu hybrid nanoparticles. Toghraie *et al*., [11] measured thermal conductivity of ZnO-$TiO_2$/EG hybrid nanoparticles experimentally at temperature 25 to 50$^oC$ and volume fraction 0-3.5%. They observed that at high temperature the variation of thermal conductivity enhancement with volume fraction is greater than at lower temperature. They discussed various correlations for thermal conductivities of nanofluids defined by previous researchers and proposed a new correlation for the thermal conductivity of ZnO-$TiO_2$/EG hybrid nanofluids with high accuracy, using experimental findings. Devi and Devi [12] numerically investigated two-dimensional flow of hydromagnetic hybrid nanofluid with suction using Nachtsheim-Swigert shooting iteration method along with R.K. Fehlberg integration method. They used Cu-$Al_2O_3$/water hybrid nanofluids. Since oxide nanoparticles have less thermal conductivities than metal nanoparticles. For this reason, a high-volume fraction of oxides nanoparticles is required to achieve the desired thermal efficiencies.

All the above discussed literature is based on experimental study of hybrid nanoparticles. Since there are some theoretical studies done in this area. Iqbal *et al*., [13] studied numerically the hybrid nanofluids in rotating transport in Oscillating vertical channel. They considered hall current thermal radiation with three different shapes of nanoparticles. They found that heat transfer augments with volume fraction and platelets shapes of hybrid nanoparticles are found to have the highest temperature. Cao *et al*., [14] examined on fractional Maxwell viscoelastic nanofluid over a moving plate. Zainal *et al*., [15] investigated the effect of using hybrid nanofluid, silver/graphene (Ag/HEG) in horizontal circular pipe of 0.01m diameter with constant heat flux, 1000W/m$^2$. They used AnsysFluent software to predict the heat transfer coefficient and Nusselt number for forced convection heat transfer of Ag/HEG+water nanofluid. Azhar *et al*., [16] studied fractional nanofluid over a moving vertical plate and concluded that fractional nanofluids have higher heat transfer rate compared to ordinary nanofluids. Few others recent studies on fractional nanofluids can be found in ref [17-19].

Studies are done in the area of hybrid nanofluids experimentally or frequently in numerical solutions while there is a lack for exact solutions research in this area. Moreover, problem on Casson hybrid nanofluids is not reported yet for non-integer derivative model. Thus, the aim of the present research work is to find analytical solution for flow of Casson hybrid nanofluids with Sodium Alginate base fluid using Caputo time fractional derivatives model. Here in this work, *Cu* and $Al_2O_3$ metal nanoparticles are considered because of their high thermal conductivities.

## 2. Formulation and Solution of the Problem

Consider unsteady flow of Sodium Alginate based hybrid nanofluid (Cu and $Al_2O_3$) modelled using Caputo time fractional derivative in a vertical channel. The flow is induced due to mixed convection. In vertical channel, $T_0$ and $T_d$ show lower and upper plate temperatures while $C_0$ and $C_d$ shows concentration at lower and upper plate. The governing equations of momentum, energy and mass are

$$\rho_{hbnf}\frac{\partial^{\alpha}u}{\partial t^{\alpha}}=-\frac{\partial p}{\partial x}+\mu_{hbnf}\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\left(\sigma_{hbnf}B_0^2+\frac{\mu_{hbnf}}{k_1}\right)u+(\rho\beta_T)_{hbnf}\,g\left(T-T_0\right)+(\rho\beta_c)_{hbnf}\,g\left(C-C_0\right), \tag{1}$$

$$(\rho c_p)_{hbnf}\frac{\partial^\alpha T}{\partial t^\alpha}=k_{hbnf}\frac{\partial^2 T}{\partial y^2}+4\alpha_0^2\left(T-T_0\right), \tag{2}$$

$$\frac{\partial^\alpha C}{\partial t^\alpha}=D_{hbnf}\frac{\partial^2 C}{\partial y^2}, \tag{3}$$

where $u=u\left(y,t\right)$, $T=T\left(y,t\right)$, $C=C\left(y,t\right)$, $\rho_{hbnf}$, $\mu_{hbnf}$, $\sigma_{hbnf}$, $\beta_T$, $\beta_C$, $g$, $\left(\rho c_p\right)_{hbnf}$, $k_{hbnf}$, $\alpha_0$, $D_{hbnf}$, are respectively fluid velocity in the *x*-direction, temperature, concentration, density, the dynamic viscosity, electrical conductivity of the base fluid, volumetric thermal expansion coefficient, gravitational acceleration, heat capacitance of nanofluids, thermal conductivity of nanofluid, radiation absorption and thermal diffusion coefficient. We consider $-\frac{\partial p}{\partial x}=H(t)[\lambda+\lambda_0\exp(iwt)]$, with boundary conditions

$$\begin{aligned}&u(0,t)=0,\;\; u(d,t)=0,\;\; T(0,t)=T_0,\;\; T(d,t)=T_d,\\&T(y,0)=T_0,\;\; C(0,t)=C_0,\;\; C(d,t)=C_d.\end{aligned} \tag{4}$$

The density $\rho_{hbnf}$, thermal expansion coefficient $\left(\rho\beta\right)_{hbnf}$, heat capacitance $\left(\rho c_p\right)_{hbnf}$ and thermal conductivity $\sigma_{hbnf}$, are derived by using the relations

$$\begin{aligned}&\left(\rho\beta\right)_{hnf}=(1-\phi_2)\left(\rho\beta\right)_f\left[(1-\phi_1)+\phi_1\left(\frac{\left(\rho\beta\right)_{s1}}{\left(\rho\beta\right)_f}\right)\right]+\phi_2\left(\rho\beta\right)_{s2},\;\; \mu_{hnf}=\frac{\mu_f}{(1-\phi_2)^{2.5}(1-\phi_1)^{2.5}},\\&\left(\rho c_p\right)_{hnf}=(1-\phi_2)\left(\rho c_p\right)_f\left[(1-\phi_1)+\phi_1\left(\frac{\left(\rho c_p\right)_{s1}}{\left(\rho c_p\right)_f}\right)\right]+\phi_2\left(\rho c_p\right)_{s2},\\&k_{hbnf}=\left(\frac{k_{s2}+(s-1)k_{bf}-(s-1)\phi_2(k_{bf}-k_{s2})}{k_{s2}+(s-1)k_{bf}+\phi_2(k_{bf}-k_{s2})}\right)k_{bf},\\&k_{bf}=\left[\frac{k_{s1}+(s-1)k_f-(s-1)\phi_1(k_f-k_{s1})}{k_{s1}+(s-1)k_f+\phi_1(k_f-k_{s1})}\right]k_f,\end{aligned} \tag{5}$$

where $\phi_1,\phi_2$, are the nanoparticles volume fraction, $\rho_f$, $\rho_{s1}$, and $\rho_{s2}$ is the density of the base fluid and hybrid nanoparticles, the volumetric coefficient of thermal expansions of nanoparticles and base fluids are denoted by $\beta_{s1}$, $\beta_{s2}$, and $\beta_f$ respectively, $(c_p)_{s1,}(c_p)_{s2,}$ and $(c_p)_f$ is the specific heat capacities of nanoparticles and base fluids at constant pressure. Here $k_f$ and $k_{s1},k_{s2}$ are thermal conductivities of base fluid and nanoparticles. Using the non-dimensional variables

$$u^*=\frac{u}{U_0},x^*=\frac{x}{d},\ t^*=\frac{tU_0}{d},y^*=\frac{y}{d},\ p^*=\frac{d}{\mu U_0}p,\ \lambda_0^*=\frac{\lambda_0 d^2}{\mu U_0},\ \lambda^*=\frac{\lambda d^2}{\mu U_0}$$
$$\theta=\frac{T-T_0}{T_d-T_0},\ C^*=\frac{C-C_0}{C_d-C_0},\ \omega^*=\frac{\omega d}{U_0},\ \ -\frac{\partial p^*}{\partial x^*}=\lambda_0^*+\lambda^* exp\left(i\omega^* t^*\right), \tag{6}$$

The following non-dimensional differential Eqs are obtained (asterisk* is omitted for convenience)

$$\psi_1 Re\frac{\partial^\alpha u(y,t)}{\partial t^\alpha}=H(t)\left[\lambda_0+\lambda exp\left(i\omega t\right)\right]+\psi_2\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u(y,t)}{\partial y^2}-\left(M+\frac{\psi_2}{K}\right)u(y,t)$$
$$+\psi_3 Gr\theta(y,t)+\psi_4 GmC(y,t), \tag{7}$$

$$\psi_5\frac{\mathrm{Pe}}{\lambda_{hbnf}}\frac{\partial^\alpha\theta(y,t)}{\partial t^\alpha}=\frac{\partial^2\theta}{\partial y^2}+\frac{N^2}{\lambda_{hbnf}}\theta, \tag{8}$$

$$Sc\frac{1}{(1-\phi)}\frac{\partial^\alpha C}{\partial t^\alpha}=\frac{\partial^2 C}{\partial y^2}, \tag{9}$$

with dimensionless boundary conditions

$$u(y,0)=0,\ u(0,t)=0,\ \ u(1,t)=0,$$
$$\theta(y,0)=0,\ \ \theta(0,t)=0,\ \theta(1,t)=1,$$
$$C(y,0)=0,\ \ C(0,t)=0,\ C(1,t)=1, \tag{10}$$

where

$$\psi_1=(1-\phi_2)\left[(1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right]+\phi_2\frac{\rho_{s2}}{\rho_f},\qquad \psi_2=\frac{1}{(1-\phi_2)^{2.5}(1-\phi_1)^{2.5}},$$
$$\psi_3=\left\{(1-\phi_2)\left[(1-\phi_1)+\phi_1\left(\frac{(\rho\beta_T)_{s1}}{(\rho\beta_T)_f}\right)\right]+\phi_2\frac{(\rho\beta_T)_{s2}}{(\rho\beta_T)_f}\right\},\quad \lambda_{hbnf}=\frac{k_{hbnf}}{k_{bf}},$$
$$\psi_4=\left\{(1-\phi_2)\left[(1-\phi_1)+\phi_1\left(\frac{(\rho\beta_c)_{s1}}{(\rho\beta_c)_f}\right)\right]+\phi_2\frac{(\rho\beta_c)_{s2}}{(\rho\beta_c)_f}\right\},\quad Gm=\frac{g\beta_c d^2\ (C_d-C_0)}{\nu U_0},$$
$$\psi_5=\left\{(1-\phi_2)\left[(1-\phi_1)+\phi_1\left(\frac{(\rho c_p)_{s1}}{(\rho c_p)_f}\right)\right]+\phi_2\frac{(\rho c_p)_{s2}}{(\rho c_p)_f}\right\},\quad N^2=\frac{4\alpha_0^{\ 2}d^2}{k},\ Pe=\mathrm{Pr}\,Re,$$
$$Re=\frac{U_0 d}{\nu},\ M=\frac{\sigma B_0^2 d^2}{\mu},\ K=\frac{k_1}{d^2},\ Gr=\frac{g\beta_T d^2\ (T_d-T_0)}{\nu U_0}, \tag{11}$$

and $D_t^\alpha u(y,t)$ represent the Caputo time-fractional derivative of $u(y,t)$, defined as

$$D_t^\alpha u(y,t)=\begin{cases}\frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{1}{(t-\tau)^\alpha}\frac{\partial u(y,\tau)}{\partial \tau}d\tau; & 0<\alpha<1,\\ \frac{\partial u(y,t)}{\partial t}; & \alpha=1.\end{cases} \tag{12}$$

Applying the Laplace transform to Eqs. (7)-(9) and using Eq. (10), we obtain

$$\left(\phi_1 \operatorname{Re} q^\alpha + M + \frac{\phi_2}{K}\right)\bar{u}(y,q)=\phi_2\frac{\partial^2 \bar{u}(y,q)}{\partial y^2}+\phi_3 Gr\frac{\sinh\left[y\sqrt{b_0}\sqrt{q^\alpha+b_2}\right]}{q\sinh\left[\sqrt{b_0}\sqrt{q^\alpha+b_2}\right]} + \phi_4 Gm\frac{\sinh\left[y\sqrt{b_3}\sqrt{q^\alpha}\right]}{\text{q}\sinh\left[\sqrt{b_3}\sqrt{q^\alpha}\right]}+\frac{\lambda_0}{q}+\frac{\lambda}{q-i\omega}, \tag{13}$$

$$\left(b_0 q^\alpha - b_1^2\right)\bar{\theta}(y,q)=\frac{\partial^2\bar{\theta}(y,q)}{\partial y^2}, \tag{14}$$

$$b_3 q^\alpha \bar{C}(y,t)=\frac{\partial^2 \bar{C}(y,t)}{\partial y^2}, \tag{15}$$

with boundary conditions

$$\bar{u}(0,q)=0,\ \bar{u}(1,q)=0. \tag{16}$$

$$\bar{\theta}(0,q)=0,\ \bar{\theta}(1,q)=\frac{1}{q}. \tag{17}$$

$$\bar{C}(0,q)=0,\quad \bar{C}(1,q)=\frac{1}{q}. \tag{18}$$

where $b_3=\frac{Sc}{(1-\phi)}$ and $a_0=\frac{1}{\operatorname{Re}}\left(M+\frac{\phi_2}{K}\right),\ b_2=-\frac{b_1^2}{b_0}.$

Solutions after Inverse Laplace transform:

$$u(y,t)=(\lambda_0+\lambda e^{iwt})\left[\frac{1}{2}\psi\left[(y+1),t\right]+\frac{1}{2}\psi\left[(y-1),t\right]\right]-a_4\int_0^\infty\left\{\begin{array}{l}\sum_{n=0}^{\infty}\left[erfc\left(\frac{1-y+2n}{2\sqrt{u}}\right)\right]-erfc\left(\frac{1+y+2n}{2\sqrt{u}}\right)\\ e^{a_0a_1u}t^{-1}\Phi\left(0,-\alpha,-a_1ut^{\frac{-\alpha}{2}}\right)\end{array}\right\}du$$
$$-\frac{a_1a_2}{(a_1-b_0)}\left(1+(a_0-c_0)t^\alpha E_{\alpha,\alpha+1}(-c_0t^\alpha)\right)\int_0^\infty\left\{\begin{array}{l}\sum_{n=0}^{\infty}\left[erfc\left(\frac{1-y+2n}{2\sqrt{u}}\right)\right]-erfc\left(\frac{1+y+2n}{2\sqrt{u}}\right)\\ e^{a_0a_1u}t^{-1}\Phi\left(0,-\alpha,-a_1ut^{\frac{-\alpha}{2}}\right)\end{array}\right\}du$$
$$-\frac{a_1a_3}{(a_1-b_3)}\left(1+(a_0-d_0)t^\alpha E_{\alpha,\alpha+1}(-d_0t^\alpha)\right)\int_0^\infty\left\{\begin{array}{l}\sum_{n=0}^{\infty}\left[erfc\left(\frac{1-y+2n}{2\sqrt{u}}\right)\right]-erfc\left(\frac{1+y+2n}{2\sqrt{u}}\right)\\ e^{a_0a_1u}t^{-1}\Phi\left(0,-\alpha,-a_1ut^{\frac{-\alpha}{2}}\right)\end{array}\right\}du \qquad (19)$$
$$-a_4(\lambda_0+\lambda e^{iwt})\frac{1}{2}\left[\int_0^\infty\left(2-erfc\frac{y}{2\sqrt{u}}\right)g(u,t)du+\int_0^\infty erfc\frac{y}{2\sqrt{u}}g(u,t)du\right]+\frac{a_2b_0}{(a_1-b_0)}\left(1+(b_2-c_0)t^\alpha E_{\alpha,\alpha+1}(-c_0t^\alpha)\right)$$
$$+\frac{a_3}{(a_1-b_3)}F_{\alpha,\alpha-1}(t,-d_0)\sum_{n=0}^{\infty}\left[erfc\left(\frac{1-y+2n}{2\sqrt{b_3t}}\right)\right]-erfc\left(\frac{1+y+2n}{2\sqrt{b_3t}}\right)+\frac{a_4}{a_1}F_\alpha(t,-a_0)(\lambda_0+\lambda e^{iwt}).$$

$$\theta(y,t)=\int_0^\infty f\left(y\sqrt{b_0},u,-\frac{b_1}{b_0},\sqrt{b_0}\right)D_t^\alpha\Phi(1,-\alpha,-ut^{-\alpha})du;\quad 0<\alpha<1. \qquad (20)$$

$$C(y,t)=\int_0^\infty f\left(y\sqrt{b_3},u,0,\sqrt{b_3}\right)D_t^\alpha\Phi(1,-\alpha,-ut^{-\alpha})du;\quad 0<\alpha<1. \qquad (21)$$

**Table 1**
Thermophysical properties of base fluid and graphene nanoparticles

| Physical properties | $\rho(kg/m^3)$ | $c_p(J/kgK)$ | $\sigma(S/m)$ | $k(W/mK)$ | $\beta\times10^5(K^{-1})$ |
|---|---|---|---|---|---|
| Sodium Alginate | 989 | 4175 | $5.5*10^{-6}$ | 0.6376 | 21 |
| Copper $\phi_1$ | 8933 | 385 | $59.6*10^{6}$ | 400 | 1.67 |
| Alumina $\phi_2$ | 3970 | 765 | $35*10^{6}$ | 40 | 0.85 |

## 3. Graphical Results and Discussion

Hybrid nanofluids flow with mixed convection is studied for analytical solutions in a channel under effect of MHD and porosity. Graphical illustrations are made for effect of different parameters on concentration, temperature and velocity of the hybrid nanofluid. For the graphical presentation, the thermophysical properties for base fluid and nanoparticles are taken from the Table 1.

The effect of embedded parameters on the flow, temperature and concentration of the fractional nanofluid $\alpha \in (0,1)$, is observed through figures 1-6. Figure 1 depicts the effect of radiation parameter on fluid temperature; the temperature of nanofluid is an increasing function of radiation parameter $N$. The increasing radiation parameter increases heat absorption which leads to increase the fluid temperature. Fig 2 illustrates the influence of nanoparticles volume fraction $\phi_2$ on concentration profile while keeping the copper's volume fraction constant as $\phi_1 = 0.04$. It depicts that concentration decreases when nanoparticles volume fraction increases. Fig 3 shows the behavior of nanofluid flow with the varying values of $M$. The resistive force emerges due to magnetic field causes a resistance in the flow, hence minimizes the flow of nanofluid. The viscous forces get dominant in this case.

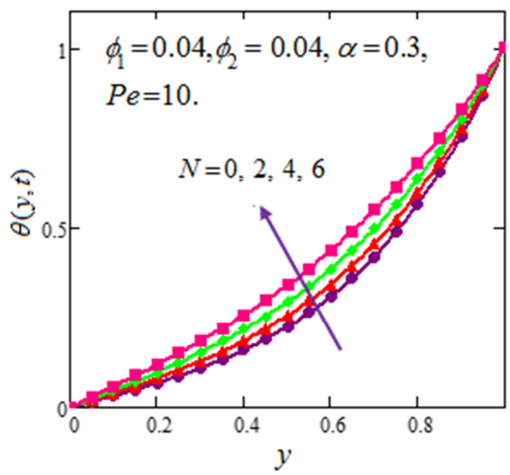


**Fig. 1.** Temperature profile for different values of radiation parameter N

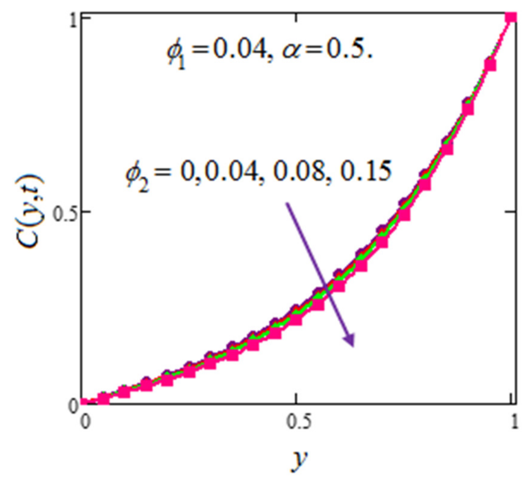


**Fig. 2.** Concentration profile for different values of volume fraction $\phi_2$.

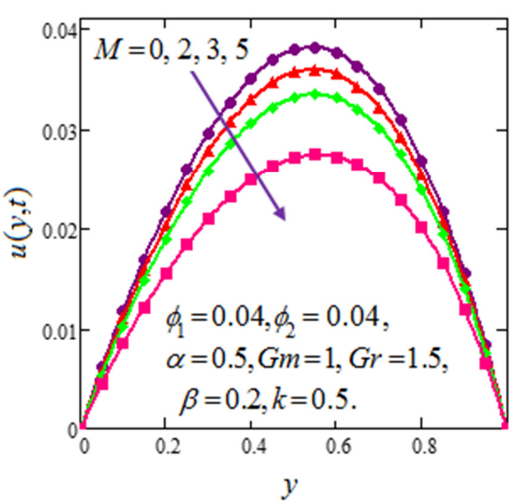


**Fig. 3.** Velocity profile for different values of magnetic parameter M

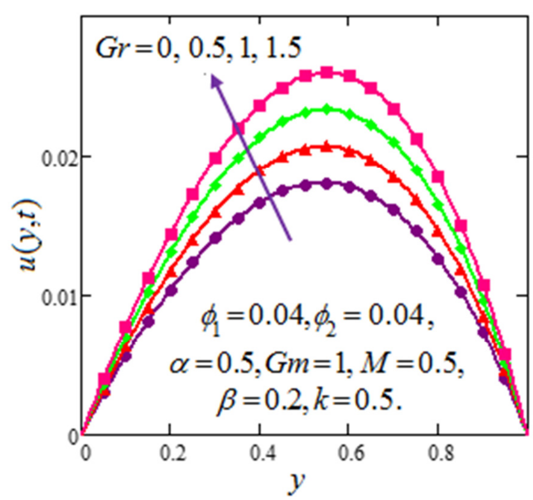


**Fig. 4.** Velocity profile for different values of volume fraction *Gr*

Figure 4 depicts that velocity of nanofluid is a maximizing function of volume fraction number $Gr$ for constant $\phi_1 = 0.04,\ \phi_2 = 0.03$. For fractional nanofluids the increasing buoyancy forces rises the nanofluid flow. The fluid flow is found to be increasing with increasing porosity parameter $k$ illustrated in Fig 5. Fig 6 is displayed to show the behavior of flow with varying values of Casson parameter. We can see that velocity is increasing function of Casson parameter, $\beta$.

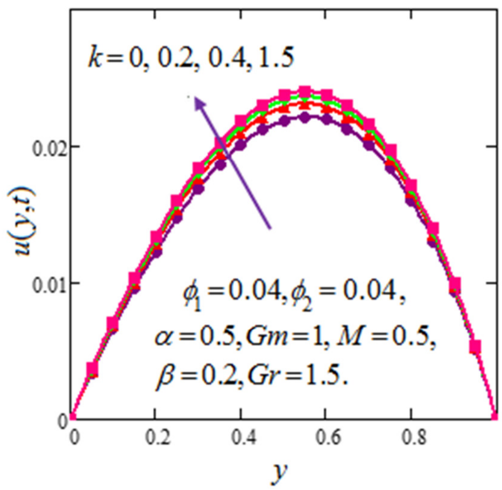


**Fig. 5.** Velocity profile for different values of porosity parameter *k*

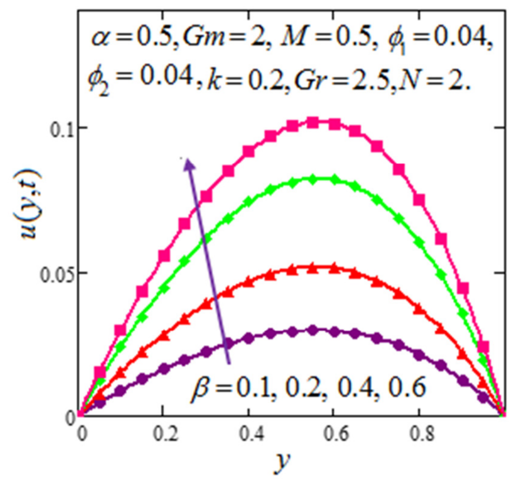


**Fig. 6.** Velocity profile for different values of Casson parameter $\beta$.

## 4. Conclusions

In this research work, the analytical solutions for hybrid nanofluid with copper and alumina oxide are obtained using Caputo-time fractional derivates. Exact expressions of velocity, concentration and temperature are evaluated using Laplace transform method and then depicted graphically for various parameters. Temperature increases with increase in $N$ at $t$. Velocity maximizes with increasing values of $Gr$, $\beta$, and $k$ while decreases with rising values of $M$. Furthermore, the analytical solutions obtained in this study not only because they are solutions of some fundamental flows, but also serve as accuracy standards for other methods, such as numerical, asymptotic or experimental.

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