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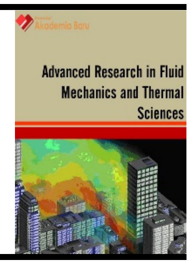

Analysis of Porous Eliptical Slider through Semi-Analytical Technique

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ABSTRACT

A non-Newtonian fluid of constant density is forced through the porous bottom of elliptic slider. The semi-major axis of the slider is assumed to be much longer than the gap width between the slider and the plane. The similarity transformation reduces the equations of motion to a set of nonlinear ordinary differential equations which are solved using a semi numerical technique for smaller and moderately large Reynolds numbers. In this method we develop the series expansion with polynomial coefficients for the solution, we calculate few terms manually and for obtaining a large number of terms we use computer. The region of validity of the series representing drag and lift is increased by Euler's transformation. The results so obtained are compared with earlier findings.

1. Introduction

Theoretical and practical study about porous slider are important in fluid cushioned moving pads and it is fact that fluid cushioned porous sliders are useful in reducing the frictional resistance between two solid surfaces moving relative to each other. Hydrostatic thrust bearing and air cushioned vehicles are the practical applications of such models. The application of porous bearing in mounting horsepower motors includes vacuum cleaners, coffee grinders, hair dryers, shaving machines, sewing machines, water pumps, record players etc. are the industrial applications. Developing these types of models for non-Newtonian [1,2] is very essential since the non-Newtonian fluids has many applications in various fields like, viscous coupling, military suits, sports shoes etc.

For Newtonian fluids Berman [3], Proudman [4], Terrill [5,6], Elkouh [7], Murti [8] and Rassmussen [9] studied the two dimensional or axis symmetry flow between porous plates. In each case the flow is either two dimensional or axisymmetric. Series of paper published by Wang [10-12] and his associates in

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the 19th centuries on porous slider problems. Wang [12] studied the porous circular slider. A fluid is forced through the porous bottom of the slider and thus separates the slider from the ground and analysed the problem through perturbation method for small Reynolds numbers and matched asymptotic expansions for higher Reynolds number. Wang and Skalak [13], studied fluid injection through one side of a long vertical channel, the resulting nonlinear ordinary differential equations are analyzed by numerical integration technique and power series method.

The perturbation method [14] was used earlier to solve the algebraic equations, next it used to solve the differential equations. In 1938 Goldstein [15] and his associate used this technique to solve the differential equation obtained in different fields. The numerical method are comparatively tedious and difficult to implement on computers due to nonlinearity of the equations. For this kind of geometry the method proposed here provide accurate results and it has advantages over numerical methods single computer run yields the results. Van Dyke [16-17] and his co-workers also used perturbation method to solve the differential equations along with gives the efficient techniques to improve the convergence region of the series, this work makes the perturbation techniques to stronger in solving differential equations. Bujurke [18-20], Yasir Khan [21] and his associates successfully applied these techniques to solve different classes of problems occur in different engineering fields.

In this paper we consider and discuss the problem of elliptic porous slider with special reference to non-Newtonian fluid model. The present analysis is primarily the extension of perturbation series by using computer. The forms of few manually calculated terms allow to propose generation of universal functions in compact form. Which are solutions of infinite sequence of linear problems. Using these universal coefficient functions, we generate series associated with various physical parameters. The region of validity of the series for drag and lift is further increased by Euler transformation

2. Problem formulation

A fixed porous elliptic plate is situated at $z = d$, $x^2+\beta y^2=D^2$ and an infinite moving plate is at $z = 0$ We assume that $D \geq d$ so that the edge effects can be neglected. A constant injection velocity W is developed through the porous plate. The equations of motion and continuity for the problem are as below [22].

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=\frac{\partial P}{\partial x}+\upsilon_1\frac{\partial^2 u}{\partial z^2}+\upsilon_2\begin{pmatrix} u\dfrac{\partial^3 u}{\partial x\partial z^2}+3\dfrac{\partial u}{\partial x}\dfrac{\partial^2 u}{\partial z^2}+\dfrac{\partial u}{\partial z}\dfrac{\partial^2 w}{\partial z^2} \\ +w\dfrac{\partial^3 u}{\partial z^3}+2\dfrac{\partial w}{\partial z}\dfrac{\partial^2 u}{\partial z^2} \end{pmatrix}+\upsilon_3\begin{Bmatrix} \dfrac{\partial u}{\partial z}\left(\dfrac{\partial^2 u}{\partial x\partial z}+\dfrac{\partial^2 w}{\partial z^2}\right) \\ -2\dfrac{\partial v}{\partial y}\dfrac{\partial^2 u}{\partial z^2} \end{Bmatrix} \quad (1)$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{\partial P}{\partial y}+\upsilon_1\frac{\partial^2 v}{\partial z^2}+\upsilon_2\begin{pmatrix} v\dfrac{\partial^3 v}{\partial y\partial z^2}+3\dfrac{\partial v}{\partial y}\dfrac{\partial^2 v}{\partial z^2}+\dfrac{\partial v}{\partial z}\dfrac{\partial^2 w}{\partial z^2} \\ +w\dfrac{\partial^3 v}{\partial z^3}+2\dfrac{\partial w}{\partial z}\dfrac{\partial^2 v}{\partial z^2} \end{pmatrix}+\upsilon_3\begin{Bmatrix} \dfrac{\partial v}{\partial z}\left(\dfrac{\partial^2 v}{\partial y\partial z}+\dfrac{\partial^2 w}{\partial z^2}\right)- \\ 2\dfrac{\partial u}{\partial x}\dfrac{\partial^2 v}{\partial z^2} \end{Bmatrix} \quad (2)$$

when

$$P=-\frac{P}{\rho}+(2\upsilon_2+\upsilon_3)\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right\} \quad (3)$$

Then the equation of motion governing the flow in z direction gives

$$P=\frac{1}{2}W^2-\upsilon_1\frac{\partial w}{\partial z}-\upsilon_2\left\{4\left(\frac{\partial w}{\partial z}\right)^2+w\frac{\partial^2 w}{\partial z^2}\right\}-3\upsilon_3\left(\frac{\partial w}{\partial z}\right)^2+\frac{W^2K(x^2+\beta y^2)}{2d^2R}-\frac{P_0}{\rho} \quad (4)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \quad (5)$$

Here K is a constant υ_1, υ_2, and υ_3 are respectively the coefficients of kinematic viscosity, viscoelasticity and cross-viscosity.

The boundary conditions are taken as

$$u=0=v,\ w=-W \text{ at } z=d \quad (6)$$

$$u=U,\ v=V,\ w=0 \text{ at } z=0 \quad (7)$$

By using the following transformations

$$u=Uf(\eta)+\frac{Wx}{d}h'(\eta)$$

$$v=Vg(\eta)+\frac{Wy}{d}k'(\eta)$$

$$w=-W(h+k),\ \eta=\frac{z}{d},$$

equations (1) and (2) can be simplified by using (4) as

$$R\left[fh'-(h+k)f'-M\left\{\begin{matrix}fh'''-(h+k)f'''+3h'f''\\-2f''(h'+k')-f'(h''+k'')\end{matrix}\right\}+N(f'k''+2k'f'')\right]=f'' \quad (8)$$

$$R\left[h'^2-(h+k)h''-M\left\{\begin{matrix}4h'h'''-h''(h''+k'')-(h+k)h^{iv}\\-2(h'+k')h'''\end{matrix}\right\}+N(h''k''+2k'h''')\right]=K+h''' \quad (9)$$

$$R\left[gk'-(h+k)g'-M\left\{\begin{matrix}gk'''-(h+k)g'''+3k'g''\\-2g''(h'+k')-g'(h''+k'')\end{matrix}\right\}+N(g'h''+2h'g'')\right]=g'' \quad (10)$$

$$R\left[k'^2-(h+k)k''-M\left\{\begin{matrix}4k'k'''-k''(h''+k'')-\\(h+k)k^{iv}-2(h'+k')k'''\end{matrix}\right\}+N(h''k''+2h'k''')\right]=\beta K+k''' \quad (11)$$

where

$M = \frac{\upsilon_2}{d^2}, N = \frac{\upsilon_3}{d^2}$.

The boundary conditions becomes

$$h(0) = k(0) = h'(0) = k'(0) = h'(1) = k'(1) = 0 \tag{12}$$

$$h(1) + k(1) = 1 \tag{13}$$

$$f(0) = g(0) = 1,\ f(1) = g(1) = 0\ . \tag{14}$$

3. Method of Solution

Assuming the solution of (8) to (11) as infinite series

$$f(\eta) = \sum_{n=0}^{\infty} R^n f_n(\eta),\ g(\eta) = \sum_{n=0}^{\infty} R^n g_n(\eta),\ h(\eta) = \sum_{n=0}^{\infty} R^n h_n(\eta)$$

$k(\eta) = \sum_{n=0}^{\infty} R^n k_n(\eta)$ and $K(\beta) = \sum_{n=0}^{\infty} R^n K_n(\beta)$ respectively. Substituting the assumed series in (8) to (11) and equating the various powers of R to zero, we get zeroth and first order equations as follows.

$$\begin{aligned} &f_0(\eta) = 0 \\ &K + h_0''' = 0 \\ &g_0'' = 0 \\ &\beta K + k_0''' = 0 \end{aligned} \tag{15}$$

$$\begin{aligned}
&\left[f_0 h_0' - (h_0 + k_0) f_0' - M \begin{Bmatrix} f_0 h_0''' - (h_0 + k_0) f_0''' + 3h_0' f_0'' \\ -2f_0''(h_0' + k_0') - f'(h_0'' + k_0'') \end{Bmatrix} + N(f_0' k_0'' + 2k_0' f_0'') \right] = f_1'' \\
&\left[h_0'^2 - (h_0 + k_0) h_0'' - M \begin{Bmatrix} 4h_0' h_0''' - h_0''(h_0'' + k_0'') - (h_0 + k_0) h_0^{iv} \\ -2(h' + k') h''' \end{Bmatrix} + N(h_0'' k_0'' + 2k_0' h_0''') \right] = K_1 + h_1''' \\
&\left[g_0 k_0' - (h_0 + k_0) g_0' - M \begin{Bmatrix} g k_0''' - (h_0 + k_0) g_0''' + 3k_0' g_0'' \\ -2g_0''(h_0' + k_0') - g_0'(h_0'' + k_0'') \end{Bmatrix} + N(g_0' h_0'' + 2h_0' g_0'') \right] = g_1'' \\
&\left[k_0'^2 - (h_0 + k_0) k_0'' - M \begin{Bmatrix} 4k_0' k_0''' - k_0''(h_0'' + k_0'') - \\ (h_0 + k_0) k_0^{iv} - 2(h_0' + k_0') k_0''' \end{Bmatrix} + N(h_0'' k_0'' + 2h_0' k_0''') \right] = \beta K_1 + k_1'''
\end{aligned} \tag{16}$$

Solving the above system of equations (15) and (16) subjected to the conditions

$$h_0(0) = k_0(0) = h_0'(0) = k_0'(0) = h_0'(1) = k_0'(1) = 0$$
$$h_0(1) + k_0(1) = 1$$
$$f_0(0) = g_0(0) = 1,\ f_0(1) = g_0(1) = 0$$

and

$$h_n(0) = k_n(0) = h_n'(0) = k_n'(0) = h_n'(1) = k_n'(1) = 0$$
$$h_n(1) + k_n(1) = 1$$
$$f_n(0) = g_n(0) = 1,\ f_n(1) = g_n(1) = 0$$

for $n \geq 1$, we get

$$f_0 = 1 - \eta$$
$$g_0 = 1 - \eta$$
$$h_0 = \frac{1}{12}\left(\frac{36\eta^2}{1+\beta} - \frac{24\eta^3}{1+\beta}\right)$$
$$k_0 = \frac{1}{12}\left(\frac{36\beta\eta^2}{1+\beta} - \frac{24\beta\eta^3}{1+\beta}\right)$$

and

$$f_1 = \frac{1}{20(1+\beta)}\begin{bmatrix} -9\eta - 60M\eta - 3\beta\eta + 20N\beta\eta + 20M\beta\eta + 60M\eta^2 - 60N\beta\eta^2 \\ -60M\beta\eta^2 + 20\eta^3 + 40N\beta\eta^3 + 40M\beta\eta^3 - 15\eta^4 + 5\beta\eta^4 + 4\eta^5 - 2\beta\eta^5 \end{bmatrix}$$

$$g_1 = \frac{1}{20(1+\beta)}\begin{bmatrix} -3\eta + 20N\eta + 20M\eta - 9\beta\eta - 60M\beta\eta - 60N\eta^2 - 60M\eta^2 + 60M\beta\eta^2 + 40N\eta^3 \\ +40M\eta^3 + 20\beta\eta^3 + 5\eta^4 - 15\beta\eta^4 - 2\eta^5 + 4\beta\eta^5 \end{bmatrix}$$

$$h_1 = \frac{1}{70(1+\beta)^3}\eta^2\begin{bmatrix} 16 + \beta(-1 + 37\beta + 42N(-13+5\beta) + 168M(-7+5\beta)) - 27\eta + 3\beta(-6 + 448M - 9\beta \\ +28N(11+5\beta))\eta - 840(N+M)\beta(1+\beta)\eta^2 + 21(1+\beta)(1 + (-1+16N+16M)\beta)\eta^3 \\ +14(-1+\beta+2\beta^2)\eta^4 - 4(-1+\beta+2\beta^2)\eta^5 \end{bmatrix}$$

$$k_1 = \frac{1}{70(1+\beta)^3}\eta^3\begin{bmatrix} 37 + 210N + 840M - \beta - 546N\beta - 1176M\beta + 16\beta^2 + 3(-9 + (-6 + 448M - 9\beta)\beta \\ +28N(5+11\beta))\eta - 840(N+M)(1+\beta)\eta^2 + 21(1+\beta)(-1+16N+16M+\beta)\eta^3 \\ +14(2+\beta-\beta^2)\eta^4 + 4(-2+\beta)(1+\beta)\eta^5 \end{bmatrix}$$

It is very much essential to get higher order approximations in the series if it has to reveal the true nature of the function represented by it. As we move to higher approximations the algebra becomes tedious and difficult to calculate the terms manually. So, we use a systematic scheme to generate the terms of the order $n = 15$.

4. Analysis and Improvement of the Series

The expressions for drag on the slider may be obtained as

$$\overline{Dx} = \frac{Dx}{A\rho WU} = -\frac{f'(1)}{R} - Mf(1)h''(1) \quad (17)$$

$$\overline{Dy} = \frac{Dy}{A\rho WV} = -\frac{g'(1)}{R} - Mg(1)k''(1) \quad (18)$$

As the series $f(1),\ g(1),\ f'(1),\ h''(1),\ g'(1)\ and\ k''(1)$ are slow converging it is essential to get higher approximations to analyze the problem. By using the Mathematica programing, we generated the 15 approximations.

The coefficients of the series $f'(1),\ h''(1)\ and\ g'(1)$ (Table 1) are decreasing in magnitude and have alternate sign pattern. The nearest singularity, lying on the negative axis has no direct physical significance. In this case, the simplest device to use is an Euler transformation based on estimate of ε_0, the radius of convergence of the series $f'(1),\ h''(1)\ and\ g'(1)$ .With this transformation, the singularity is mapped to infinity. The transformation envisages using the new variable ε^* such that

$$\varepsilon^* = \frac{R}{R+\varepsilon_0} \quad or\ R = \frac{\varepsilon_0 \varepsilon^*}{1-\varepsilon^*}$$

The series $f'(1),\ h''(1)\ and\ g'(1)$ takes the new form

$\sum_{n=1}^{15} b_n \varepsilon^*$, where $b_0 = -1, -6, -1$

For

$f'(1),\ h''(1),\ g'(1)$ respectively and

$$b_n = \sum \frac{(n-1)!}{(n-j)!(j-1)!} C_j \varepsilon_0{}^j$$

This new series can be used to approximate the solution for moderately higher values of R. The similar analysis is carried for $\beta = 0.5$ and $\beta = 1$.

Table 1
Coefficient of series

Coefficient of the series $f'(1)$ when $\beta = 0$		Coefficient of the series $h''(1)$ when $\beta = 0$		Coefficient of the series $g'(1)$ when $\beta = 0$	
n	C_n	n	C_n	n	C_n
0	-1	0	-6	0	-1
1	0.25	1	5.4286E-01	1	0.4000
2	2.3174E-02	2	-3.7647E-02	2	-8.0905E-02
3	8.5677E-03	3	5.9955E-02	3	4.9949E-02
4	-5.7328E-03	4	-3.1705E-02	4	-2.8078E-02
5	4.4600E-03	5	2.4335E-02	5	1.8277E-02
6	-3.8261E-03	6	-1.8309E-02	6	-1.2887E-02
7	3.0570E-03	7	1.4417E-02	7	9.6922E-03
8	-2.3594E-03	8	1.1371E-02	8	-7.7676E-03
9	1.7934E-03	9	9.0114E-03	9	6.5853E-03
10	-1.4521E-03	10	-7.2388E-03	10	-5.7601E-03
11	1.3146E-03	11	6.02787E-03	11	4.9927E-03
12	-1.2242E-03	12	-5.2405E-03	12	-4.1808E-03
13	9.3504E-04	13	4.5444E-03	13	3.5794E-03
14	-3.8985E-04	14	-3.5466E-03	14	-3.6764E-03
15	1.4871E-04	15	2.3288E-03	15	4.3073E-03

5. Results and Discussion

The elliptic porous slider at low cross-flow Reynolds number using a non-Newtonian second-order fluid is governed by a system of nonlinear ordinary differential equations (8)–(11) together with boundary conditions (12)-(14). The proposed perturbation series scheme enables us to obtain the large number of coefficients. A carefully written Mathematica code makes it possible to perform the complex algebra. The coefficients of the series decreases in magnitudes and alternate in sign. This indicates the presence of a singularity.

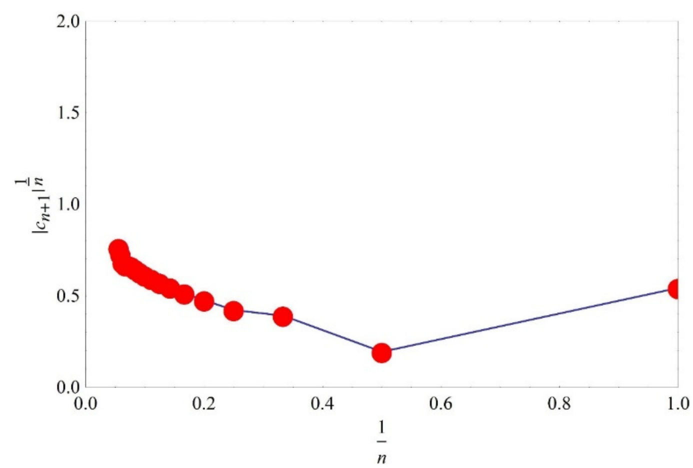

Fig. 1. Domb-Sykes plot for $f'(1)$ when $\beta=0$

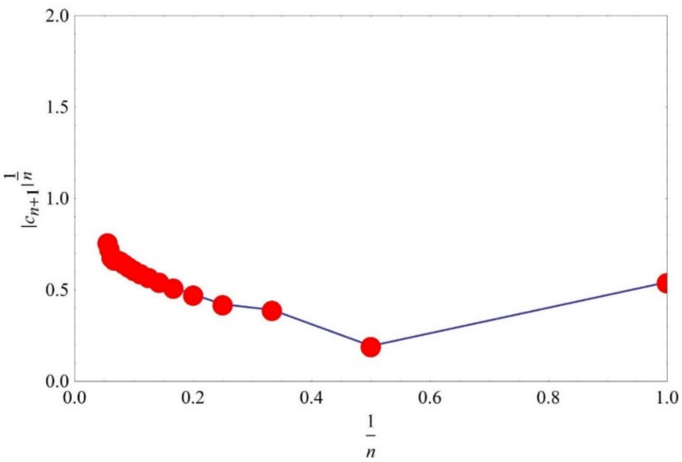

Fig. 2. Domb-Sykes plot for $h''(1)$ when $\beta=0$

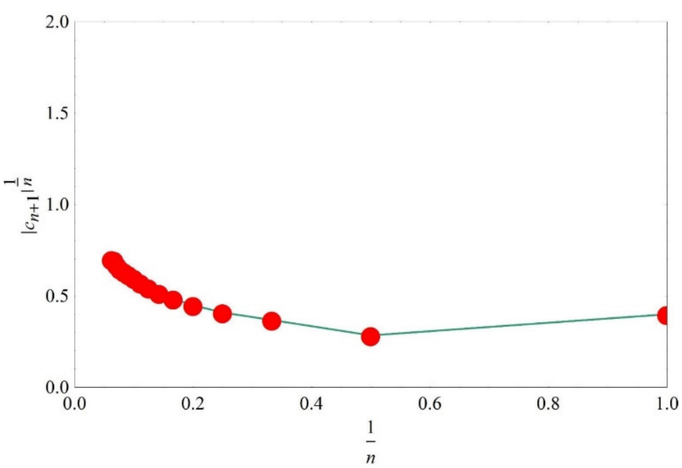

Fig. 3. Domb-Sykes plot for $g'(1)$ when $\beta=0$

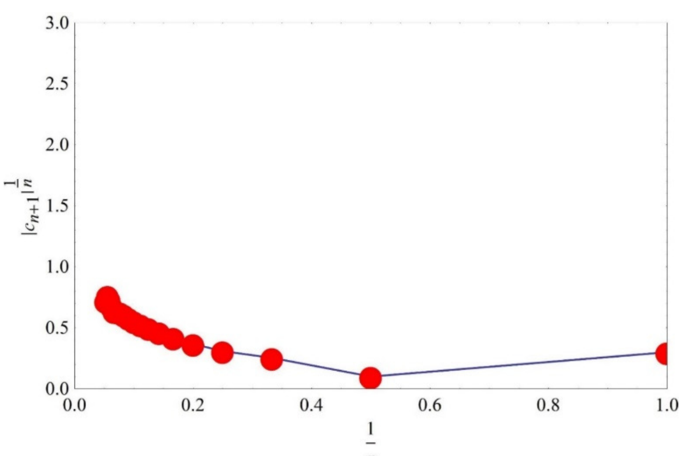

Fig. 4. Domb-Sykes plot for $f'(1)$ when $\beta=0.5$

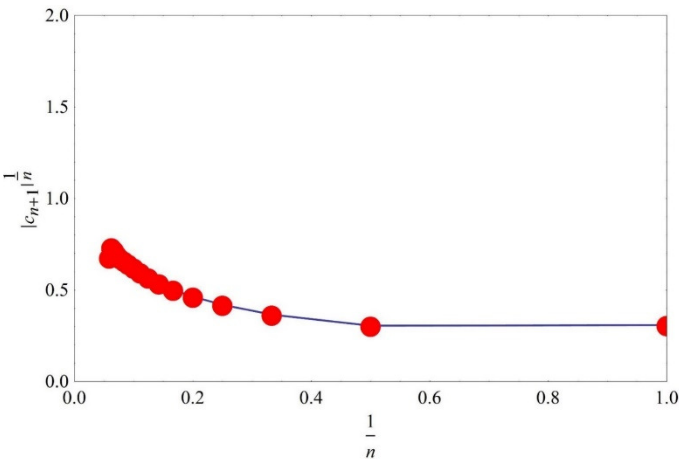

Fig. 5. Domb-Sykes plot for $k''(1)$ when $\beta = 0.5$

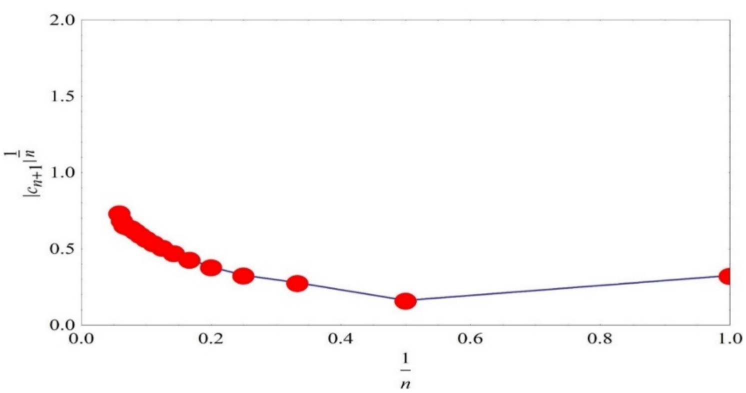

Fig. 6. Domb-Sykes plot for $f'(1)$ when $\beta = 1$

Table 4
Values of $\overline{D_x}$ and $\overline{D_y}$ non-Newtonian fluids when $\beta = 0, M = -0.1$ *and* $N = 0.15$

R	$\overline{D_x}$	$\overline{D_y}$
1	0.721645	0.6482
2	0.19043	0.16296
3	-0.00264	-0.00189
4	-0.110416	-0.08706
5	-0.16436	-0.13836
6	-0.20154	-0.17111
7	-0.22459	-0.19234
8	-0.23851	-0.20596
9	-0.24635	-0.21435
10	-0.25009	-0.21910

Table 5
Values of $\overline{D_x}$ and $\overline{D_y}$ non-Newtonian fluids when $\beta = 0.5, M = -0.1\ and\ N = 0.15$

R	$\overline{D_x}$	$\overline{D_y}$
1	0.69920	0.67383
2	0.18611	0.17631
3	0.00192	0.001765
4	-0.09631	-0.09334
5	-0.15498	-0.15476
6	-0.19052	-0.19712
7	-0.21135	-0.22696
8	-0.22253	-0.24799
9	-0.22736	-0.26260
10	-0.22799	-0.27245

Table 6
Values of $\overline{D_x}$ and $\overline{D_y}$ non-Newtonian fluids when $\beta = 1, M = -0.1\ and\ N = 0.15$

R	$\overline{D_x}$	$\overline{D_y}$
1	0.686733	0.686733
2	0.181813	0.181813
3	0.001477	0.001477
4	-0.098722	-0.098722
5	-0.164615	-0.164615
6	-0.210772	-0.210772
7	-0.243750	-0.243750
8	-0.267302	-0.267302
9	-0.283901	-0.283901
10	-0.295292	-0.295292

A Domb-Sykes plot (Figure 1 to 6) provides nearest singularity. In each case singularity is identified, restricting the convergence of the series, which happen to be in the complex plane in all cases. The series is recast into a new form using Euler transformation. This singularity has no physical significance. Our results, drag on the slider (Table 2 to 4) are good in agreements with results obtained by Bhatt [22]. Once the universal coefficients of the series are generated the rest of the analysis can be done at a single stretch, taking hardly any computer time and storage, while other numerical methods require huge storage and long computer time.

6. Conclusion

In this work we analysed the problem of porous elliptic slider using semi analytical technique. The models associated non-Newtonian fluids are near to reality and the drag evaluated is constant after one stage. The method used here is flexible and very efficient compared to pure numerical methods. Once the coefficients generated, the rest of the analysis can be done efficiently. The method gives the analytic structure of the solution. The derived quantity can be obtained very easily, unlike numerical schemes in which a separate scheme is to be developed. The method required less time and storage.

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