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Flows Generated by Critical Opposing Thermosolutal Convection in Fluid Annular Cavities

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ABSTRACT

A two-dimensional numerical study has been performed to investigate thermosolutal convection in an annular space, bounded by two very long cylinders subject to opposing heat and solute gradients, the inner and outer walls being submitted to constant temperature and concentration. Three values of the radii ratio of the cavity have been investigated: R = 1.4, R = 1.6 and R = 1.8. The binary fluid (Pr = 0.7 and Le = 3) is set in motion under the effect of the competition of the opposing buoyancy forces with a ratio N = −1. The coupled governing equations written in vorticity-stream function form are solved by ADI scheme with the finite difference method. We have obtained multicellular regimes due to the severe competition between buoyancy forces of thermal and mass origin. In addition, the increase in the aspect ratio influences the gradual disappearance of the small cells. Moreover, it appears that the multicellular regime and the increase in the aspect ratio both contribute to the improvement of thermal and mass transfer.

Keywords:
Annular cavity, thermosolutal convection, multicellular flows, opposing buoyancies, finite differences, moderate aspect ratio

1. Introduction

Thermosolutal convection in binary fluids has been extensively investigated, given its involvement in technological applications and natural phenomena, such as geophysics, metallurgy, electrochemistry and drying processes, such as geophysical flows presented by Clark *et al.*, [10], crystal growth and solidification studied by Beckermann and Viskanta [3]. Much research has been done on modeling the thermosolutal convection in various types of cavities: Han and Kuehen [12] performed a finite difference algorithm on double diffusive natural convection fluid flow in a vertical rectangular cavity of aspect ratio $A=4$. Different flow regimes are obtained as a function of the Grashof number for aiding and opposing buoyancy conditions. Béghein *et al.*, [4] studied the double diffusion convection and the influence of the buoyancy ratio on the rate of heat and mass transfer,

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in a square cavity filled with air. Mamou *et al*., [13] presented an analytical modeling and numerical study of double diffusion natural convection in a fluid contained in a rectangular cavity. Beji *et al*., [5] used the finite volume method to study double diffusion natural convection in a vertical annulus. Bardan *et al*., [2] have investigated the nonlinear states in doubly diffusive convection in a rectangular cavity driven by lateral temperature and concentration differences. This study is organized around a special case that allows a static equilibrium, and it is shown that stationary states bifurcate from this equilibrium and are either symmetric or antisymmetric. Sezai and Mohamad [18] attempted to model the three-dimensional aspects of thermosolutal natural convection in a cubic enclosure subject to horizontal and opposing gradients of heat and solute. They identified the different flow patterns and bifurcations as functions of the governing parameters. Papanicolaou and Belessiotis [16] studied the natural convection, heat and mass transfer in a trapezoidal enclosure to highlight the effect of control parameters on the flow, and on the rate of heat and mass transfer. Sankar and Venkatachalappa [17] have investigated a numerical study to understand the effect of axial or radial magnetic field on heat and mass transfer in a vertical cylindrical annular cavity. Chen *et al*., [9] used Boltzmann model to study the double diffusion natural convection in a vertical annular system with opposite gradients of temperature and concentration. Nakamura *et al*., [15] found the approximate solution (theoretically) for the vertical ice melting with the transient double effects of temperature and concentration, and proposed the simple formula to evaluate the melting mass. Masuda *et al.,* [14] studied analytically the double-diffusive convection in a porous medium due to the opposing heat and mass fluxes on the vertical walls. In the horizontal annular geometry, Cheddadi *et al.,* [7] described the heat and mass transfer by thermosolutal convection for different modes obtained in the cases of co-operating and opposed forces for a single radii ratio $R=2$. Numerical aspects such as the influence of the mesh on the results have been discussed by Cheddadi *et al.,* [8] for the same aspect ratio. Annular spaces with low and moderate radii ratios have been considered by El moustaine and Cheddadi [11] using the finite difference method with ADI scheme in the case of cooperating thermosolutal convection. The present study is a continuation of these investigations dealing with horizontal cavities. We are interested in showing the development of multicellular regimes generated by the thermosolutal convection in the case of the critical value of buoyancy ratio $N=-1$ with a value of Le different from unity. The cylindrical annular geometry is widely involved in industrial devices, and study of the thermosolutal transfer in this geometry has recently gained interest (see for example Sorour *et al.,* [19] and Afifah *et al.,* [1]).

2. Physical Model and Mathematical Formulation

2.1 System Description

We study the problem of thermosolutal convection in an annular space bounded by two very long cylinders, coaxial, horizontal and isotherm, filled with a binary fluid considered incompressible and viscous. It is assumed that both gradients of temperature and concentration are negligible along the axis direction of the annular medium, that is to say the problem is considered bidimensional as shown in Figure 1. The surface of the inner cylinder is maintained at constant uniform temperature and solute concentration T_i, C_i respectively. The surface of the outside cylinder is maintained at constant uniform temperature and solute concentration T_o, C_o, with $T_i > T_o$ and $C_i > C_o$.

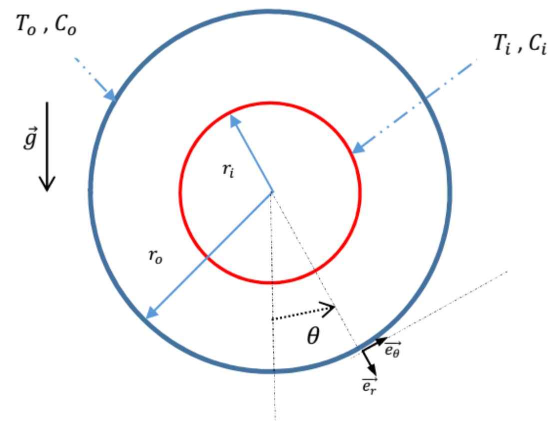


Fig. 1. Problem scheme

2.2 Dimensionless Equations

We consider the problem to be symmetrical, hence the study is restricted to the half-annulus: $0 \le \theta \le \pi$ and $r_i \le r \le r_o$. The fluid is Newtonian and incompressible, satisfying the Boussinesq hypothesis, with

$$\rho = \rho_o(1 - \beta_T(T - T_o) - \beta_C(C - C_o)) \tag{1}$$

Where T_o, C_o and ρ_o are the reference values of temperature, concentration and density.

The non-dimensional equations governing the two-dimensional problem are written in polar coordinates (r,θ) using the stream function-vorticity formulation (ψ,ω). We consider the problem to be symmetrical, hence the study is restricted to the half-annulus: $0 \le \theta \le \pi$ and $r_i \le r \le r_o$. The equations write as follows.

- Momentum conservation

$$\frac{\partial \omega}{\partial t} + U\frac{\partial \omega}{\partial r} + \frac{V}{r}\frac{\partial \omega}{\partial \theta} = Pr\left(\frac{\partial^2 \omega}{\partial t^2} + \frac{1}{r}\frac{\partial \omega}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \omega}{\partial \theta^2}\right) + Ra_T Pr\left[\left(\frac{1}{r}\frac{\partial T}{\partial \theta}\cos\theta + \frac{\partial T}{\partial r}\sin\theta\right) + N\left(\frac{1}{r}\frac{\partial C}{\partial \theta}\cos\theta + \frac{\partial C}{\partial r}\sin\theta\right)\right] \tag{2}$$

- Energy conservation

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial r} + \frac{V}{r}\frac{\partial T}{\partial \theta} = \frac{\partial^2 T}{\partial t^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} \tag{3}$$

- Solute conservation

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial r} + \frac{V}{r}\frac{\partial C}{\partial \theta} = \frac{1}{Le}\left(\frac{\partial^2 C}{\partial t^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2}\right) \tag{4}$$

The vorticity ω is related to the stream function ψ by the relation

$$\omega = -\frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} - \frac{\partial^2\psi}{\partial r^2} \tag{5}$$

In these equations, four charac teristic dimensionless parameters appear. These are $Ra_T = \beta_T g \Delta T r_i^3 / \nu\alpha$ the thermal Rayleigh number, $N = \beta_C \Delta C / \beta_T \Delta T$ the ratio of buoyancy forces, $Pr = \nu / \alpha$ the Prandtl number and the Lewis number defined by $Le = \alpha / D$.

- Boundary conditions. The above equations are subject to the following conditions.

$$\begin{cases} r = 1, \quad \psi = 0, \quad T = C = 1 \text{ and } U = V = 0 \\ r = R, \quad \psi = 0, \quad T = C = 0 \text{ and } U = V = 0 \end{cases} \tag{6}$$

where $R = r_o / r_i$ is a fifth dimensionless parameter, representing the radial aspect ratio. On the vertical plane containing the axis of the cylinders, due to symmetry

$$\theta = 0 \text{ or } \theta = \pi \text{ and } r_i \le r \le r_o : \quad \frac{\partial T}{\partial\theta} = \frac{\partial C}{\partial\theta} = 0 \text{ and } \psi = 0 \tag{7}$$

For the vorticity ω we consider the following boundary conditions.

$$\begin{cases} r = 1 \text{ or } r = R \text{ and } 0 \le \theta \le \pi : \quad \dfrac{\partial^2\psi}{\partial r^2} + \omega = 0 \\ \theta = 0 \text{ or } \theta = \pi \text{ and } r_i \le r \le r_o : \ \omega = 0 \end{cases} \tag{8}$$

2.3 Numerical Method and Study Objectives

The governing equations with boundary conditions are solved numerically using centered finite difference method with the Alternating Direction Implicit (ADI) scheme. The results will be presented using a thermal Rayleigh number based on the gap width Ra_T^L, defined by

$$Ra_T^L = (R-1)^3 Ra_T \tag{9}$$

The heat and mass transfer rates are evaluated by averaging local Nusselt and Sherwood numbers on hot and cold walls, as follows.

$$\begin{cases} Nu = \dfrac{-LogR}{2\pi}\left(\displaystyle\int_0^\pi \left.\frac{\partial T}{\partial r}\right|_{r=1} d\theta + R\int_0^\pi \left.\frac{\partial T}{\partial r}\right|_{r=R} d\theta\right) \\ Sh = \dfrac{-LogR}{2\pi}\left(\displaystyle\int_0^\pi \left.\frac{\partial C}{\partial r}\right|_{r=1} d\theta + R\int_0^\pi \left.\frac{\partial C}{\partial r}\right|_{r=R} d\theta\right) \end{cases} \tag{10}$$

The validation of the numerical code developed by our research team has been examined in a previous work recently by El moustaine and Cheddadi [11]. In this study, the numerical results are obtained using mesh (60x60), which gives accurate results with optimal calculation time.

The present study is motivated by the need of examining the opposing thermosolutal convection in annular cavities with aspect ratio $R = 1.4$, $R = 1.6$ and $R = 1.8$. The value of the modified thermal Rayleigh number was set at $Ra_T^L = 5 \times 10^3$ that guarantees the development of both heat and mass convections. The Lewis number is set to the value $Le = 3$, in order to investigate the strong imbalance between the thermal and mass diffusivities.

The main aim of our study is to show in stationary mode the existence of multicellular flows in the case of buoyancy forces with the critical ratio $N = -1$, as well as to examine the corresponding thermal and mass transfer rates in the cavities with moderate aspect ratios cited above.

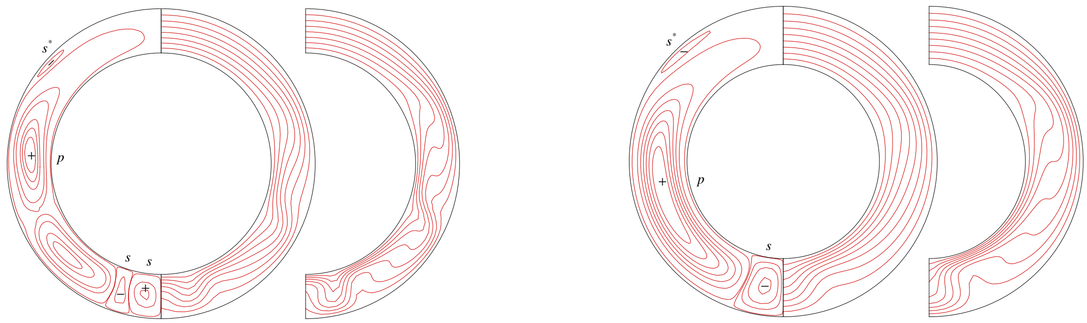
3. Results and Discussion

The following notations are used: the index "p" is assigned to the main cell (large size), while the index "s" is for the secondary cell (small size). Regarding the rotation of fluid particles in the left half-annulus, a cell turning in the trigonometric direction will be designated by the sign "+" and the clockwise direction will be designated by the sign "−". We have plotted in Figure 2 the Streamlines, the isotherms and the isomass contours. In both annular cavities with aspect ratio $R = 1.4$ and $R = 1.6$, thermosolutal convection in the case $N = -1$ generates multicellular flow structures of different type, size and location of cell centers. Considering the left half-annulus, it appears clearly that the large cell is of thermal type rotating in the trigonometric direction and its shape widens with the increase in the aspect ratio. This is a main characteristic of predominance of buoyancy forces of thermal origin. In the annular cavity $R = 1.4$, the main thermal cell contains two vortices rotating in the same direction. With the increase in the ratio R, the bi-vortex effect disappears in the same way as the number of cells of the multicellular regime decreases as the fluid tends to be driven towards the lower part of the annular cavity of larger aspect ratio. On the other hand, the small clockwise rotating cell trapped in the upper part of the cavity between the cold wall and the main thermal cell remains present with the same intensity during the variation of the aspect ratio. It is a cell of very low intensity and does not influence the deformation of the isotherms and the isomass contours.

As the radius ratio is increased to $R = 1.8$ (Figure 3), the regime becomes bicellular, with a trigonometric main cell. It could be considered as almost unicellular, as the secondary cell is of low intensity. In the lower part of the annular cavities $R = 1.4$ and $R = 1.6$ where there are counter-rotating cells, there is a strong deformation of the isotherms and isomasses forming plumes centred between two counter-rotating cells of different types.

However, in the top part of these two cavities, we observe a quasi-stratification of the isotherms and the isoconcentrations. It is a quasi-stagnant zone with quasi-absence of natural convection and the fluid motion is induced only by the process of pure diffusion, It is noted that the counter-rotating cells are located in the lower part of the annular cavity, signifying a main character of predominance of the buoyancy forces of solutal origin.

The conclusion that could be drawn is that the thermosolutal convection for the critical value $N = -1$ is generated simultaneously by the two types of forces, of thermal and mass origin, which are in strong competition. This is the reason why one cannot estimate a priori which type of forces is predominant, and the value of Le seems to be here a decisive parameter. Indeed, in the annular cavities, a strong deformation of the isoconcentrations with respect to the isotherms is noticed (Figure 2 and Figure 3), due to the fact that the value of Lewis number is large enough to strengthen the mass gradients compared to the thermal ones. Similarly, the mass transfer remains higher than the thermal transfer $Sh > Nu$ (see Tab.1 where values of Nu and Sh are presented and compared to the case of pure thermal convection, i.e. $N = 0$). On the other hand, the increase in the aspect ratio improves the thermal and mass transfer except for a very low attenuation found for the cavity with aspect ratio $R=1.8$. This remark could be linked to the disappearance of the multicellular regime (the counter-rotating cells in the lower part of the cavity) (Figure 3) that carries generally an improvement of the thermal and mass transfer.



(a): $R=1.4$
$\psi_p^+=4.519$, $\psi_s^+=1.477$
$\psi_s^-=-0.549$, $\psi_{s^*}^-=-0.011$

(b): $R=1.6$
$\psi_p^+=4.488$
$\psi_s^-=-1.159$, $\psi_{s^*}^-=-0.01$

Fig. 2. Streamlines (left), isotherms (middle) and isomass lines (right)

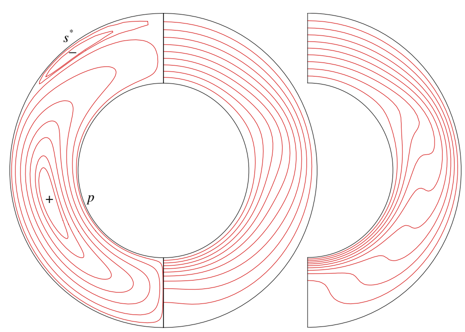


Fig. 3. Streamlines (left), isotherms (middle) and isomass lines (right) for $R=1.8$ with: $\psi_p^+=4.547$, $\psi_{s^*}^-=-0.01$

On the other hand, we plotted (Figure 4) the evolution of Nusselt and Sherwood numbers according to the iteration number n_I. We notice that for $n_I \le n^R$, (where n^R is a number of iterations depending on R), $Nu=Sh=1$. The transfer is first exclusively due to pure conduction and diffusion transport modes used as initial conditions in our calculations. Subsequently, during this phase, $N=-1$ signifies that the term generator of motion in the Navier-Stokes Equations is null.

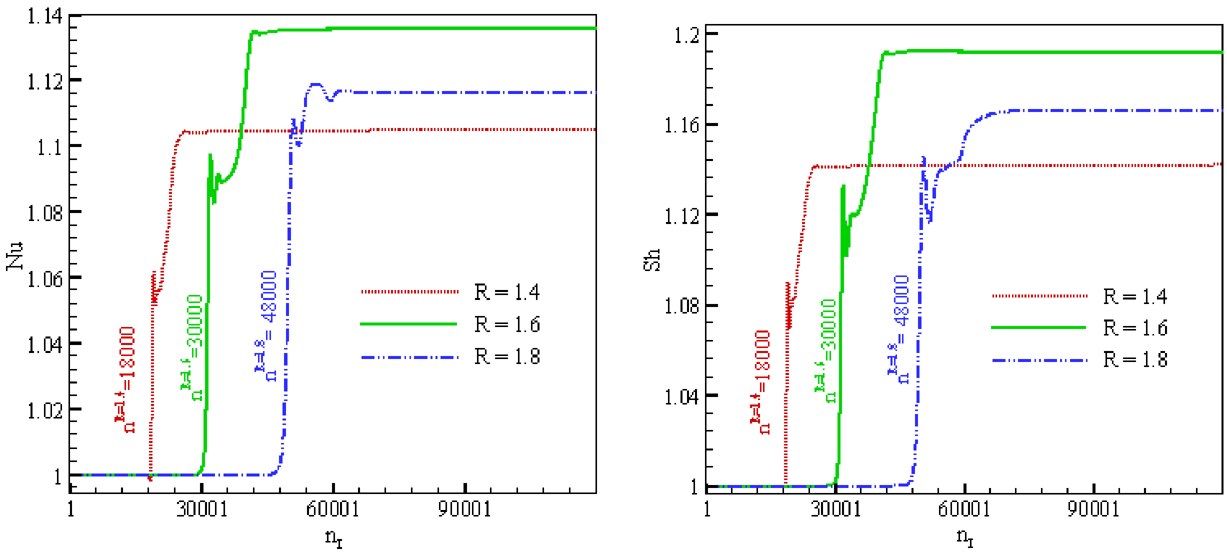


Fig. 4. Convection start delay depending on the aspect ratio of annular cavity. n_I : Number of iterations

In a second phase, the appearance of the thermosolutal convection is favored by the fact that the thermal and solute gradients are no longer identical, since the value of Le is different from 1 (compare Equation 3 and 4). Hence, although $N = -1$, the term generator of natural convection in Equation 2 is not null. Thermal and mass transfer is achieved concomitantly with the circulation of fluid within the annular cavity, leading to significant transfer rates with Nu and $Sh > 1$ in the steady state. It is worth noting that the increase of the aspect ratio results in a delay of the development of thermosolutal convection. This result could be interesting as it reflects a temporal constraint in order to optimize the thermal and mass transfer rates in the annular space.

This delay could be related to the important attenuation of the real value of the thermal Rayleigh number Ra_T (based on the inner radius) when R increases, as is obvious from Equation 9 knowing that in the present study the modified Rayleigh number (based on the gap width) has a constant value $(Ra_T^L = 5\times10^3)$.

Finally, according to Table 1, it is found that the thermal and mass transfer in the case of thermosolutal convection with buoyancy ratio $N = -1$ is smaller than that obtained in the case of pure thermal convection $N = 0$ (see for this case Cheddadi *et al.,* [6]), and the difference varies depending on the annular radii ratio.

Table 1
Comparison of thermal and mass transfer with pure thermal convection

	$R = 1.4$		$R = 1.6$		$R = 1.8$	
	Nu	Sh	Nu	Sh	Nu	Sh
$N = -1$	1.116	1.470	1.124	1.548	1.119	1.536
$N = 0$	1.230	1.954	1.339	2.158	1.424	2.289

4. Conclusion

Two-dimensional natural thermosolutal convection in concentric horizontal annulus filled with a binary fluid with Prandtl number $Pr = 0.7$ and Lewis number $Le = 3$ has been investigated numerically. Three moderate values of the radii ratio have been investigated: $R = 1.4$, $R = 1.6$ and $R = 1.8$. Multicellular regimes have been obtained, induced by the severe competition between buoyancy forces of thermal and solutal origin for the critical value $N = -1$. It has been shown that the increase in the aspect ratio influences the gradual disappearance of the small cells and causes a delay in the development of the thermosolutal convection. The multicellular regime and aspect ratio both contribute to the improvement of thermal and mass transfer, but the transfer rates are still inferior to that obtained in the case of pure thermal convection.

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