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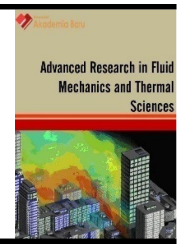

Combined Effect of Piezo-Viscous Dependency and Non-Newtonian Couple Stresses in Porous Squeeze-Film Circular Plate

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ABSTRACT

On the basis of Morgan and Cameron approach and the Barus experimental research, a mathematical model is considered for the combined effect of piezo-viscous dependency and non-Newtonian couple stresses on the squeeze-film characteristics of porous circular plates. An analytical solution for the mathematical model using the small perturbation technique is obtained. The results are presented graphically for selected parameter values. Results put forward to show that the effect of pressure dependent viscosity is to enhance the load carrying capacity significantly and lengthen the squeeze film time and the effect of permeability is to decrease the pressure, load carrying capacity and squeeze film time. It is seen that the present results are in good agreement with the earlier works in the limiting conditions of conventional circular bearing.

1. Introduction

An excellent review has been given on squeeze film lubrication of various porous bearings in the field of Tribology. For example, Morgan and Cameron [1], Rouleau [2], Wu [3], Cusano [4], Prakash and Vij [5], and Tian [6]. They have confined their investigations to Newtonian fluid as a lubricant and described that the bearing performance of various disks would be increased with use of porous bearings of different permeabilities. The generalised micro-continuum theory which permits the presence of couple stress, body couples and non-symmetric stress tensors proposed by Stokes [7]. Many researchers have made an attempt to study the effect of couple stress characteristics of distinct thin film bearing such as finite journal bearing, externally pressurized circular step Thrust bearing, rotor bearing system, and sphere and a flat plate by Lin [8-11]. Recently, Elsharkawy [12] studied the effect of misalignment on the performance of finite journal bearings lubricated with couple stress

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fluid using finite difference method. In recent years, a series of thin film bearing model have been produced by considering the combined effects of pressure dependent viscosity and couple stresses. For example, circular step plates by Hanumagowda [13], between a sphere and a rough flat plate by Naduvinamani, *et al.,* [14], a long cylinder and infinite plates by Lin and Lu [15]. All the investigations provide more insight into the influence of PVD on thin film bearing and also concerned with non-porous bearings.

Recently, Lin *et al.,* [16], studied the effect of pressure dependent viscosity on parallel circular plates with non-Newtonian fluid. Manjunatha Gudekote *et al.,* [18] investigated the combined effects of slip and inclination on peristaltic transport of Casson fluid in an elastic tube with porous walls. S. Hassan *et al.,* [19] have studied the effects of porous twisted plate as insert on heat transfer performance and flow characteristic in fire tube boiler. Rajashekhar Choudhari *et al.,* [20] investigated the role of hematocrit, slip and TPMA (Total Protein Minus Albumin) on the flow of blood in an axisymmetric inclined porous tube. Hence, the present work examines the combined effect of piezo-viscous dependency and non-Newtonian couple stress fluid on the squeeze film characteristics of porous circular plates.

A study of the squeeze film characteristics between porous circular plates lubricated with non-Newtonian fluids on the effect of pressure dependent viscosity is studied in this paper. It has been observed that as the permeability parameter approaches to zero, the squeeze film characteristics obtained in the present study reduces to the corresponding conventional bearing case studied by Lin [16] and same is presented in Table 1. It is observed that the results are well agreement with Lin [16] analysis.

2. Mathematical Formulation of the Problem

With the usual assumption of lubrication of fluid namely laminar and neglecting inertial body forces and body couples, Non-Newtonian couple stress fluid is considered to study the effect of pressure-dependent viscosity on the performance of porous circular plates. The configuration of porous circular plates under investigation is as shown in Figure 1.

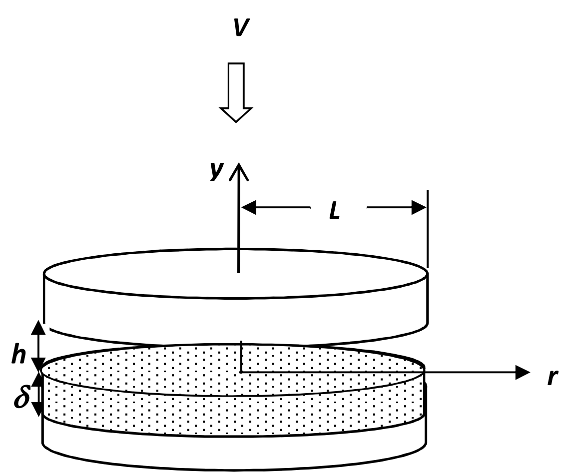

Fig. 1. Configuration of Porous parallel circular bearing lubricated with non-Newtonian fluid

The continuity and linear momentum equation are considered as

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\mu\frac{\partial^2 u}{dy^2}-\eta\frac{\partial^4 u}{\partial y^4}=\frac{\partial p}{\partial r} \tag{2}$$

$$\frac{\partial p}{\partial y}=0 \tag{3}$$

In the above equations, the symbols u and v are velocity components along the coordinate axes, the viscosity is μ , p is the pressure, η represent the constant associated with the couple stress fluid. The appropriate boundary conditions for velocities $u(r,y)$ and $v(r,y)$ at the lower and upper surfaces are expressed as

$$u=0,\ v=-v^* \text{ and } \frac{\partial^2 u}{\partial y^2}=0 \text{ at } y=0 \tag{4a}$$

$$u=0,\ v=-\frac{\partial h}{\partial t} \text{ and } \frac{\partial^2 u}{\partial y^2}=0 \text{ at } y=h \tag{4b}$$

The velocity components in the porous matrix are expressed as

$$u^*=\frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial r} \tag{5}$$

$$v^*=\frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial y} \tag{6}$$

Solving Equation (2) with conditions 4(a) and 4(b) we get

$$u=-\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[y^2-hy+2l^2\left\{1-\frac{Cosh\left((2y-h)/2l\right)}{Cosh(h/2l)}\right\}\right] \tag{7}$$

Substituting the expression for u in the continuity equation (1) and integrating using the conditions 4(a), 4(b), (5) and (6), yields

$$\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left\{h^3-12l^2h+24l^3\tanh\left(\frac{h}{2l}\right)+\frac{12\delta k}{(1-\beta)}\right\}\frac{\partial p}{\partial r}\right]=12r\frac{dh}{dt} \tag{8}$$

The pressure dependent viscosity relation was analysed by Barus [17] and the expression is obtained as

$$\mu = \mu_0 e^{\alpha p} \tag{9}$$

Substituting Equation (9) into Equation (8), the modified Reynold's equation governing pressure dependent viscosity for porous circular plates lubricated with non-Newtonian fluid is expressed as

$$\frac{\partial}{\partial r}\left\{ f(h,m,\alpha,p)\frac{\partial p}{\partial x}\right\} = 12 r\mu_0 \frac{dh}{dt} \tag{10}$$

Where

$$f(h,m,\alpha,p) = h^3 e^{-\alpha p} - 12m^2 h e^{-2\alpha p} + 24m^3 e^{-2.5\alpha p}\tanh\left(\frac{h e^{\alpha p/2}}{2m}\right) + \frac{12\delta k e^{-\alpha p}}{(1-\beta)}$$

$$l = \left(\frac{\eta}{\mu}\right)^{1/2} = m e^{-\alpha p/2}$$

For convenience, let us introduce the following non-dimensional scheme.

$$r^* = \frac{r}{L}, l^* = \frac{m}{h_0}, h^* = \frac{h}{h_0}, p^* = \frac{p h_0^3}{\mu_0 L^2(-dh/dt)}, G = \frac{\alpha\mu_0 L^2(-dh/dt)}{h_0^3} \text{ and } \psi = \frac{k\delta}{h_0^3}$$

The modified Reynolds equation in non-dimensional form is given by

$$\frac{\partial}{\partial r^*}\left\{ f^*(h^*, l^*, G, p^*, \psi).r^*.\frac{\partial p^*}{\partial r^*}\right\} = -12r^* \tag{11}$$

Where

$$f^*(h^*, l^*, G, p^*, \psi) = e^{-Gp^*} h^{*3} - 12(l^*)^2 e^{-2Gp^*} h^* + 24(l^*)^3 e^{-2.5Gp^*}\tanh(e^{0.5Gp^*} h^*/2l^*) + \frac{12\psi e^{Gp^*}}{(1-\beta)} \tag{12}$$

The non-dimensional Reynolds Equation (12) is observed to be highly non-linear and hence the small perturbation is adopted to find the analytical solution for the film pressure with the consideration of small values of the viscosity parameter $0 \le G \le 1$.The expression for p^* as follows.

$$p^* = p_0^* + G p_1^* \tag{13}$$

Substituting equation (13) into Reynolds Equation (11) and neglecting second and higher order terms of *G*. The following two equations responsible for pressure p_0^* and p_1^* are obtained as follows.

$$\frac{\partial}{\partial r^*}\left\{ r^* \frac{dp_0^*}{dr^*} \right\} = \frac{-12r^*}{f_0^*(h^*,l^*,\psi)} \tag{14}$$

$$\frac{d}{dr^*}\left\{ r^* \frac{dp_1^*}{dr^*} \right\} = -\frac{f_1^*(h^*,l^*,\psi)}{f_0^*(h^*,l^*,\psi)} \frac{d}{dr^*}\left\{ p_0^*.r^* \frac{dp_0^*}{dr^*} \right\} \tag{15}$$

Where,

$$f_0^*(h^*,l^*,\psi) = h^{*3} - 12(l^*)^2 h^* + 24(l^*)^3 \tanh(h^*/2l^*) + \frac{12\psi}{(1-\beta)} \tag{16}$$

$$f_1^*(h^*,l^*,\psi) = -h^{*3} + 6(l^*)^2 h^* \left(4 + \mathrm{sech}^2(h^*/2l^*)\right) - 60(l^*)^3 \tanh(h^*/2l^*) - \frac{12\psi}{(1-\beta)^2} \tag{17}$$

The solution for p_0^* and p_1^* are obtained by solving Equation (14) and (15) as

$$p_0^* = \frac{3(1-r^{*2})}{f_0^*(h^*,l^*,\psi)} \tag{18}$$

$$p_1^* = -\frac{9}{2}\frac{f_1^*(h^*,l^*,\psi)}{f_0^{*3}(h^*,l^*,\psi)}(1-r^{*2})^2 \tag{19}$$

Substituting these expressions into Equation (13) the dimensionless pressure is obtained as

$$p^* = \frac{3(1-r^{*2})}{f_0^*(h^*,l^*,\psi)} - G\frac{9}{2}\frac{f_1^*(h^*,l^*,\psi)}{f_0^{*3}(h^*,l^*,\psi)}(1-r^{*2})^2 \tag{20}$$

The load W^* on the bearing can be obtained by

$$W^* = \frac{Wh_0^3}{\mu_0 L^4 \left(-dh/dt\right)} = 2\pi\int_0^1 p^* r^* dr^* \tag{21}$$

After performing the integration, the load on the bearing in dimensionless form can be obtained as

$$W^* = \frac{3\pi}{2}\left\{ \frac{f_0^{*2}(h^*,l^*,\psi) - Gf_1^*(h^*,l^*,\psi)}{f_0^{*3}(h^*,l^*,\psi)} \right\} \tag{22}$$

The squeeze film time t can be obtained by integrating (22) with respect to *h** under the condition h* = 1 at t=0 as follows.

$$T^* = \frac{W\,t\,h_0^2}{\mu_0 L^4} = \frac{3\pi}{2}\int_{h_1^*}^{1}\left\{ \frac{f_0^{*2}(h^*,l^*,\psi) - Gf_1^*(h^*,l^*,\psi)}{f_0^{*3}(h^*,l^*,\psi)} \right\} dh^* \tag{23}$$

3. Results and Discussions

The combined effect of piezo – viscosity and couple stress on the squeeze film characteristics between porous circular plates is presented. According to Barus [17], the Stoke's continuum theory [7] for couple stress fluid and Morgan and Cameron approximation for permeability, the squeeze film characteristic of circular plates is discussed as below.

3.1 Pressure Distribution

The relationship between dimensionless pressure distribution P^* and r^* for various values of couple stress parameter l^* and G with $h^* = 0.4$, $\psi = 0.001$ and $\beta = 0.1$ is presented in Figure 2. The variation of non-dimensional pressure increases significantly with increasing the values of l^* and G. It is also noted that with increasing the values of permeability parameter ψ, the pressure P^* decreases which is represented in Figure 3.

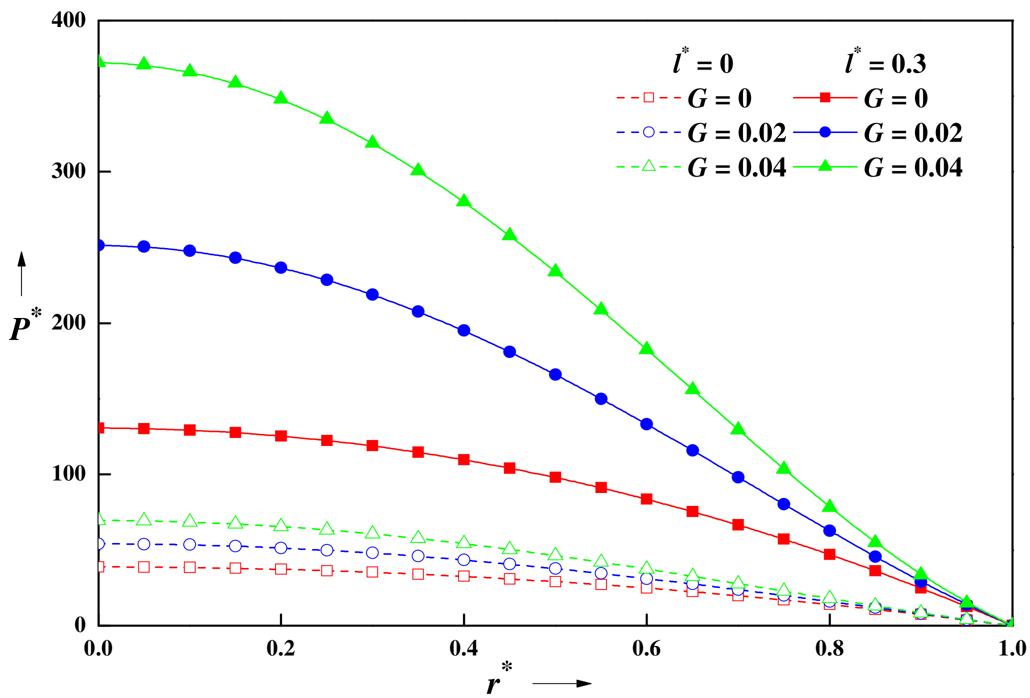

Fig. 2. Variation of non-dimensional pressure *P** with *r** for different values of *l** and *G* with *h**= 0.4, *ψ*= 0.001, *β*= 0.1

3.2 Load-Carrying Capacity

Figure 4 shows the variation of W^* with h^* for various values of l^* and G. It is noted from the figure that the effect of couple stress will increase the load-carrying capacity for different values of G. Figure 5 shows the dimensionless load capacity W^* as a function of permeability parameter ψ for different parameter. It is observed that the load-carrying capacity W^* is higher for the smaller permeability parameter ψ. The performance of the permeability of the bearing is analysed by relative load R_{W^*}.

The relative percentage decrease in dimensionless load-carrying capacity R_{W^*}. is characterized by $R_{W^*} = \left\{ \left(W^*_{Porous} - W^*_{Non-Porous} \right) / W^*_{Non-Porous} \right\} X\ 100$. Table 2 shows the values of R_{W^*}. with viscosity parameter $G$and ψ for $h = 1.2, \beta = 0.1$. It is interesting to note that there is a decrease of 9.17% in R_{W^*} for $l^* = 0.2,\ \psi = 0.01$ and $G = 0.04$.

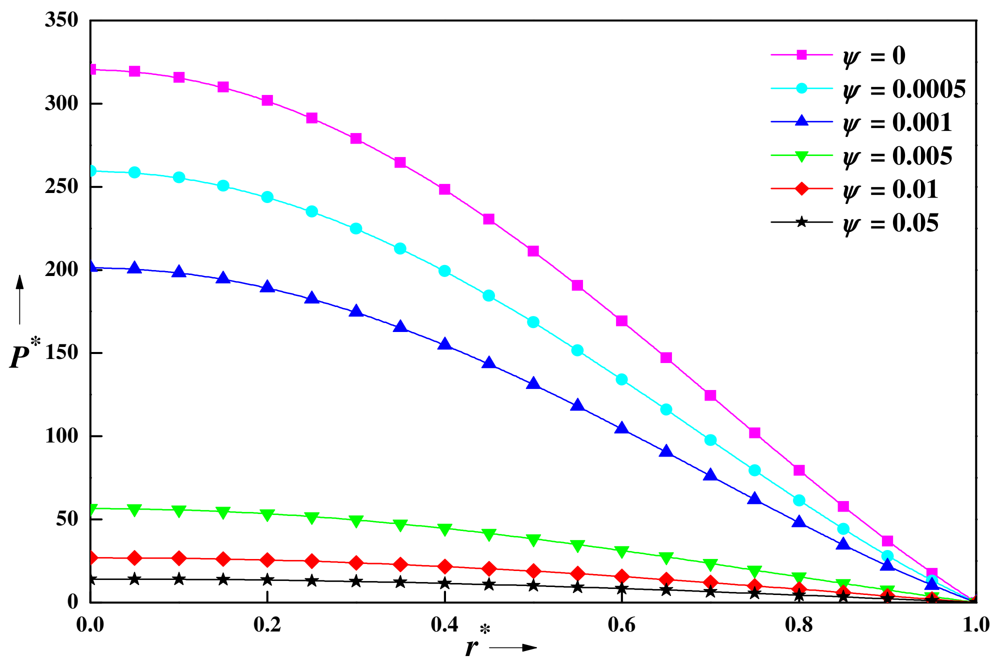

Fig. 3. Variation of non-dimensional pressure *P** with *r** for different values of *ψ* with *h**= 0.4, *C*= 0.2, *G*= 0.04, *β*= 0.1

3.3 Squeeze Film Time

For different values of couple stress parameter l^* and G, the variation of squeeze film time T^* for h_1^* with different parameters is depicted in Figure 6. It is noted that T^* is increasing with increasing the values of *l** and G. Figure 7 shows the variation of squeeze film time T^* with h_1^* for different values of ψ . It is observed that the squeeze film time T^* decreases when the permeability parameter ψ is increases.

Table 1 represents the comparison of viscosity and couple stress characteristics W^*, T^* with the case by Lin *et al.,* [16] by taking $l^* = 0.3, \beta = 0.1$. For $\psi = 0$ and $G = 0(0.2)0.6$, the load-carrying capacity and squeeze film time of the present study agrees well with the case of conventional bearing. Increasing the values of ψ and G, the load and squeeze film time decreases.

The performance of the permeability of the bearing is analysed by relative squeeze film time R_{T^*}. R_{T^*} Is characterized by $R_{T^*} = \left\{ \left(T^*_{Porous} - T^*_{Non-Porous} \right) / T^*_{Non-Porous} \right\} X\ 100$. Table 2 shows the values of R_{T^*} with viscosity parameter G and ψ for $h_1^* = 0.4,\ \beta = 0.1$. It is interesting to note that there is a decrease of 72.90% in R_{T^*} for $l^* = 0.2,\ \psi = 0.01$ and $G = 0.04$.

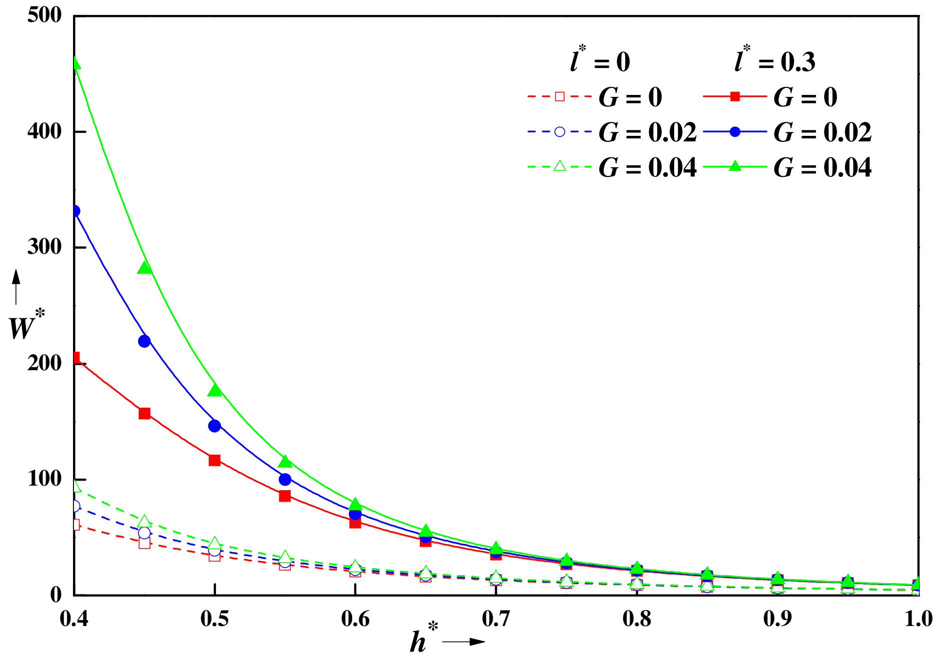

Fig. 4. Variation of non-dimensional Load carrying capacity W^* with h^* for different values of l^* and G with ψ= 0.001, β= 0.1

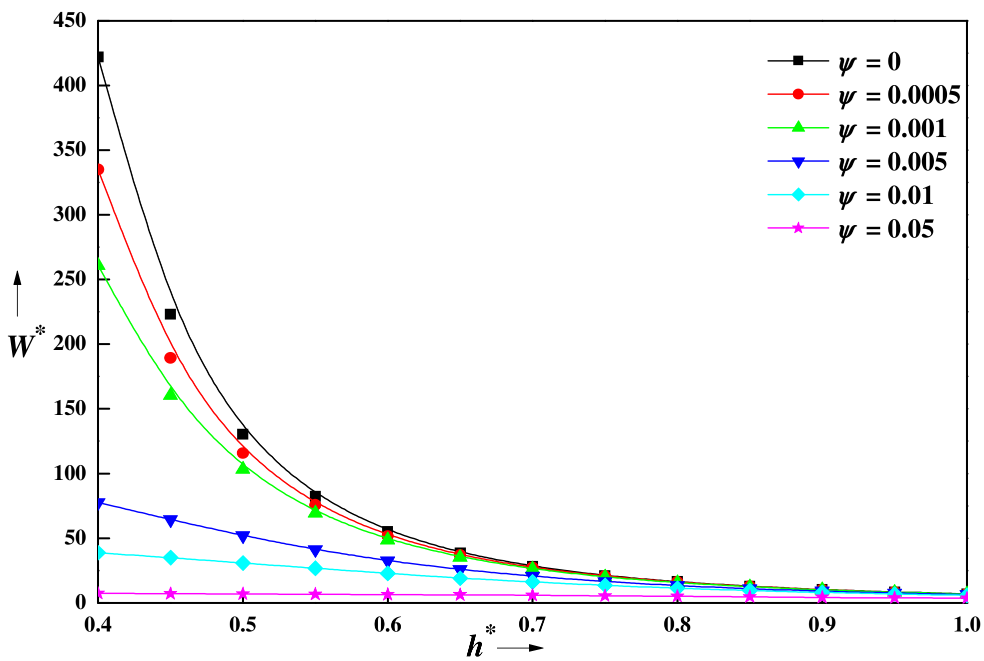

Fig. 5. Variation of non-dimensional Load carrying capacity W^* with h^* for different values of ψ, with G= 0.04, C= 0.2, β= 0.1

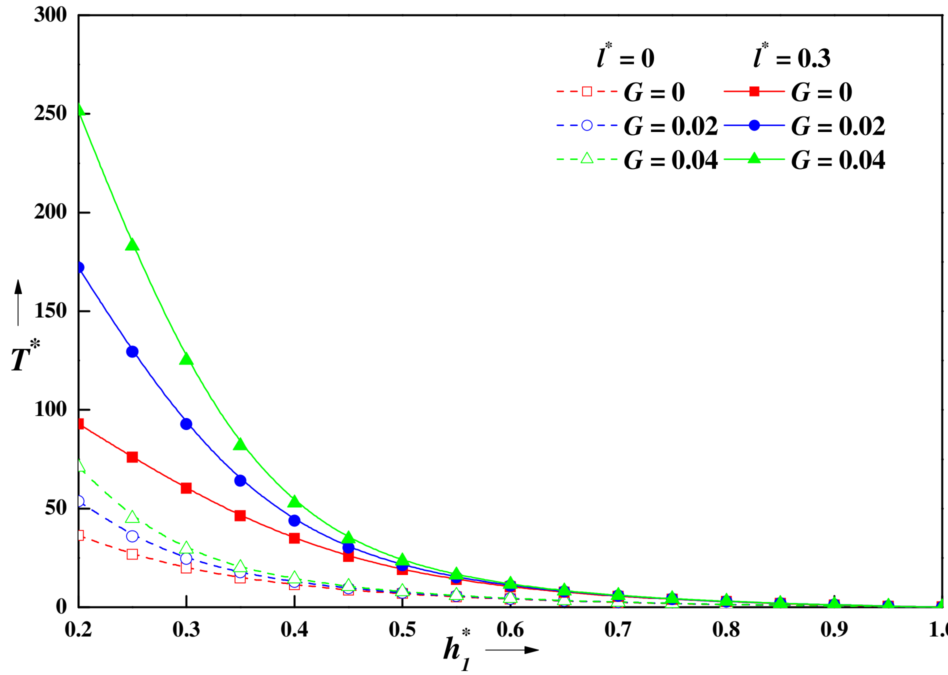

Fig. 6. Variation of squeeze film time T^* with h^*_1 for different values of l^* and G with ψ= 0.001, β= 0.1

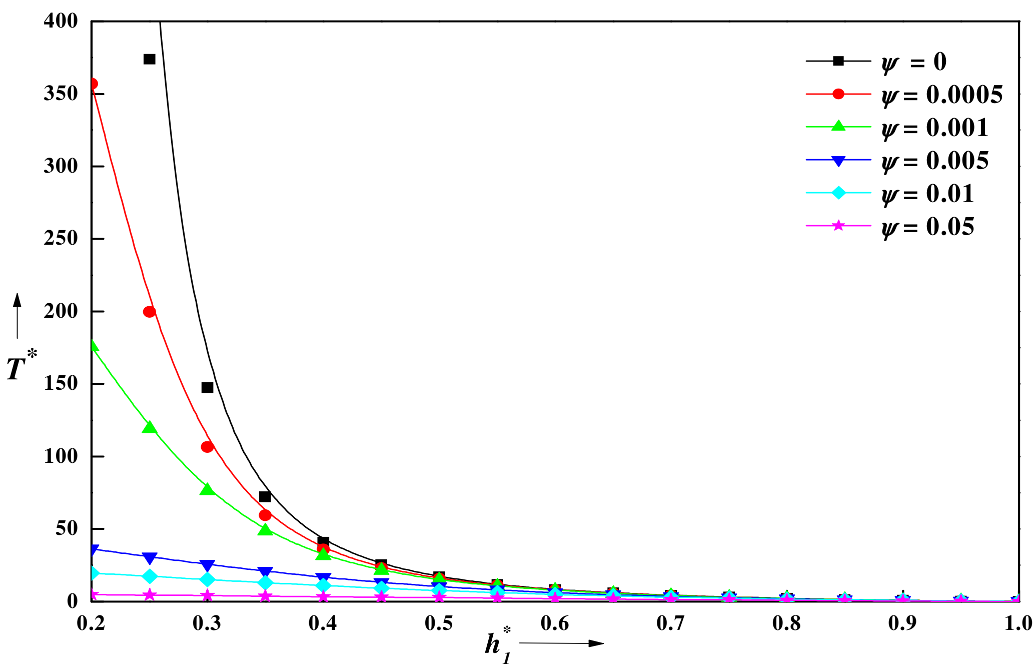

Fig. 7. Variation of squeeze film time T^* with h^*_1 for different values of ψ with l^*= 0.2, G= 0.04, β= 0.1

Table 1
Comparison of the present analysis with Lin et. al., [16] with l^*= 0.3, β = 0.1

		Lin *et. al.,* [16] analysis		Present analysis					
				ψ = 0, G = 0		ψ = 0.001		ψ= 0.01	
	G	h^*= 1.2	h^*= 1.5	h^*= 1.2	h^*= 1.5	h^*= 1.2	h^*= 1.5	h^*= 1.2	h^*= 1.5
W^*	0	4.45958	1.96815	4.45958	1.96815	4.40401	1.95725	3.95991	1.86433
	0.02	4.51165	1.9799	4.51165	1.9799	4.45519	1.9689	4.00383	1.8751
	0.04	4.56373	1.99166	4.56373	1.99166	4.50636	1.98055	4.04775	1.88587
	0.06	4.6158	2.00341	4.6158	2.00341	4.55754	1.9922	4.09166	1.89663
	G	h_1^*= 0.4	h_1^*= 0.8	h_1^*= 0.4	h_1^*= 0.8	h_1^*= 0.4	h_1^*= 0.8	h_1^*= 0.4	h_1^*= 0.8
T^*	0	52.8692	2.86819	52.8692	2.86819	34.7558	2.74927	11.5701	2.01208
	0.02	62.2007	2.95381	62.2007	2.95381	43.0436	2.83173	12.4754	2.06791
	0.04	71.5323	3.03943	71.5323	3.03943	51.3314	2.91418	13.3808	2.12374
	0.06	80.8639	3.12505	80.8639	3.12505	59.6193	2.99664	14.2861	2.17957

Table 2
Variation of R_W^* and R_T^* for different values G and ψ with h^*= 1.2, h_1^*= 0.4, β= 0.1

G	ψ	R_W^*	R_T^*	R_W^*	R_T^*
		l^* = 0.2		l^*= 0.4	
0	0.001	-0.98298	-22.6765	-1.61103	-44.7873
	0.01	-9.03093	-68.1232	-14.07	-84.3751
	0.1	-49.8185	-94.0334	-62.0836	-97.676
0.02	0.001	-0.9911	-22.6157	-1.60949	-38.1886
	0.01	-9.10187	-70.8627	-14.0776	-85.6146
	0.1	-50.0662	-94.8376	-62.2421	-98.0039
0.04	0.001	-0.99903	-22.5703	-1.60799	-33.3348
	0.01	-9.17092	-72.9066	-14.0849	-86.5263
	0.1	-50.3083	-95.4377	-62.397	-98.2452
0.06	0.001	-1.00679	-22.5352	-1.60669	-29.614
	0.01	-9.23841	-74.4902	-14.0923	-87.225
	0.1	-50.545	-95.9026	-62.5483	-98.4301

4. Conclusions

This paper has described a study of the squeeze film characteristics between porous circular plates lubricated with non-Newtonian fluids on the effect of pressure dependent viscosity. To obtain the first order solution of modified Reynold's equation, small perturbation technique is used. In the view of foregoing results, the following conclusions can be drawn.

- As the permeability parameter approaches to zero, the squeeze film characteristics obtained in the present study reduces to the corresponding conventional bearing case studied by Lin [16] and same is presented in Table 1. It is observed that the results are well agreement with Lin [16] analysis.
- The effects of permeability are to decrease the pressure, load-carrying capacity and squeeze film time as compared to the corresponding conventional bearing case.

- The pressure, load- carrying capacity and squeeze film time increases with increasing values of Pressure dependent viscosity *G* and couple stress parameter *l**.

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