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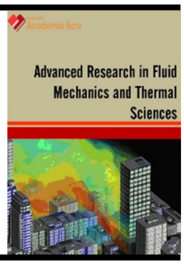

Peristaltic Flow of Herschel-Bulkley Fluid in an Elastic Tube with Slip at Porous Walls

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ABSTRACT

A mathematical model for the impact of slip velocity on peristaltic transport of blood flow has been investigated by utilizing the Herschel-Bulkley model in a flexible tube. The closed-form solutions are obtained for velocity, plug flow velocity, and volume flux. It is noticed that the impact of yield stress, amplitude ratio, Darcy number, velocity slip parameter, elastic parameters and fluid behavior index plays a vital role in controlling the flux in an elastic tube. The outcomes acquired from the flow quantities reveal that, the volume flux in a flexible tube decreases with an increase in the porous parameter and it increases with an increase in the slip parameter. Further, the results of Newtonian, Bingham plastic and Power-law models have been presented graphically and analysed.

1. Introduction

Peristalsis is a mechanism initiated by a dynamic influx of region compression and expansion along the walls of a distensible tube. Physiologically, peristaltic pumping is a trademark neuromuscular property of a biological system in which biofluids are transported along a tube by the propulsive developments of the tube wall. The peristaltic phenomenon can be seen in the movement of the bolus through the esophagus, chyme transport in the digestive tract, maturation of spermatozoa in the male reproductive tract, urine flow through the ureter and flow of blood through small veins. In recent years, the investigation of peristaltic transport of non-Newtonian fluids in various geometric conditions and assumptions has acquired the attention of researchers due to their application in developing biomedical devices such as blood pump and dialysis machine [1].

The mechanism of peristalsis has been of logical enthusiasm for some researchers. Since the initial examination by Latham [2], a few experimental and theoretical investigations have been done to investigate the peristaltic activity in various circumstances by assuming small wave

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number, amplitude ratio and Reynolds number [3,4]. By examining the two-layered power law model, Usha and Ramachandra [5] concluded that the positive or negative mean flow was due to the rheology of the peripheral layer. Also, comparative study was carried out by Misra and Pandey [6] for the axisymmetric and channel flow. Blood consists of plasma which is a suspension of cells and is responsible for the non-Newtonian nature at low shear rates. Thus, studies on blood flow, exhibiting the non-Newtonian behavior, have attracted several researchers. Moreover, at low shear rates, blood can be modeled either by Casson or Herschel-Bulkley model. Furthermore, the use of Herschel-Bulkley model over Casson model is more appropriate since it contains one extra parameter (varying fluid behavior index) and is valid for lower values of shear rates where the Casson model fails to explain the physiological behavior of blood. In addition to these, the Herschel-Bulkley model can be used to derive Bingham-plastic, Power-law and Newtonian models for particular values of yield stress and the fluid behavior index. The investigations on considering a Herschel-Bulkley fluid for different physiological situations has been reported by various authors [7-9]. Recently, a detailed survey regarding peristaltic transport of physiological fluids was done by Thanesh and Kavitha [10].

The Poiseuille's law indicates that for a fluid which is incompressible, the flux in the tube is a linear function of the pressure difference between the ends of the rigid tube through which it flows. Hence, the non-Newtonian fluids obey Poiseuille's law, in most of the theoretical as well as experimental studies. The nonlinearity in vascular beds of mammals is assigned to the elastic nature of blood vessels and their vast distensibility. This flexible property of blood vessels was first recognized by Young [11]. Later, Rubinow and Keller [12] demonstrated that the range of the tube can be controlled by the tension in the walls and the transmural pressure difference by assuming that the Poiseuille's law holds locally. Thus, there is a requirement for the subjective hypothesis of blood flow through tubes which are elastic in nature. Subsequently, Vajravelu *et al.*, [13-15] considered the flow of blood through arteries and studied the different physiological behaviours either by Casson or Herschel-Bulkley model. The flow patterns obtained by the models with rigid tube cannot explain the behavior of blood flowing through constricted arteries thoroughly. Henceforth, it becomes crucial to consider the elasticity in the present model. It is noticed that, in many application problems there exists a slip flow corresponding to the flow pattern due to which there is a loss of adhesion at the walls. Thus, the fluid slides along the walls of the tube. This slip flow of fluids is used in polishing of the internal cavities and artificial heart valves. Recently, several researchers have investigated the impact of slip and boundary conditions on biological and classical fluids in different geometries and configurations [16-38].

To the best of the author's knowledge, no attempt has been made to study the peristaltic transport of blood through an elastic tube under the effects of slip velocity. This specific examination is helpful in filling the gap in this direction. The present paper aims to investigate the effects of slip velocity on the peristaltic transport of blood, modelled as a Herschel-Bulkley fluid, through an elastic tube. The physical quantities associated with the mathematical model are examined in the non-dimensional form, and exact solutions are obtained for velocity, plug flow velocity, pressure and flux. The influence of yield stress, amplitude ratio, porous parameter, slip parameter, elastic parameters and fluid behavior index on flux and pressure are analysed and represented graphically. The outcomes of the investigation might be useful to additionally comprehend the peristaltic movement of non-Newtonian flow of blood in narrow arteries.

2. Formulation of the Problem

The flow of blood is modeled to be laminar, steady, incompressible, two-dimensional, fully-developed, axisymmetric and exhibiting peristaltic motion of Herschel-Bulkley fluid in an elastic tube of radius $a'(z)$ as shown in Figure 1. The region between $r=0$ and $r=r_p$ is called as plug flow region where $|\tau_{rz}| \leq \tau_0$. In the region between $r=r_p$ and $r=a(z,t)$, we have $|\tau_{rz}| \geq \tau_0$. The change in radius of the tube due to the elastic nature is given by $a''(z)$ and the change due to the peristaltic nature is given as

$$a'(z,\, t) = a_0 + b\, \sin\left[\frac{2\pi}{\lambda}(z-ct)\right] \tag{1}$$

where, a_0 is the radius of the tube in the absence of elasticity, b is the amplitude, λ is the wavelength, z is the axial direction, c is the wave speed and t is the time.
The flow becomes steady in the wave frame (r,z) moving with velocity c away from the fixed frame (R,Z) given by

$$r = R,\quad z = Z - ct,\quad \psi = \Psi - \frac{R^2}{2},\quad p(\mathrm{z},t) = P(\mathrm{Z}), \tag{2}$$

where p and P are pressures, ψ and Ψ are stream functions, in the wave and fixed frames of references, respectively.

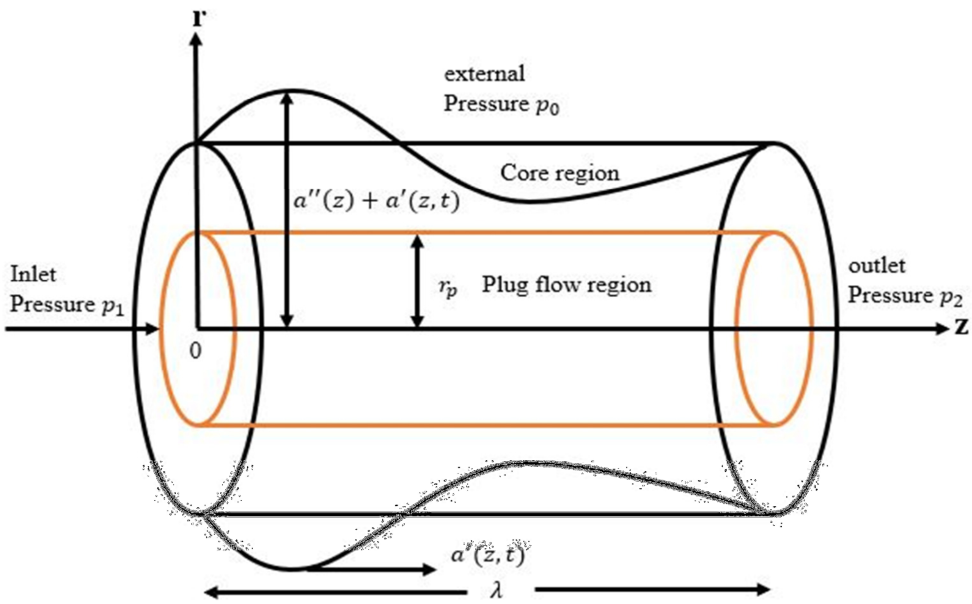

Fig. 1. Geometrical representation of Peristaltic waves in an elastic tube

3. Mathematical Model and Closed Form Solutions

Considering the long wavelength approximation by neglecting the inertial terms and wall slope, the equations of motion in the wave frame of reference which is moving with wave speed c is given by

$$\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\tau_{r'z'}\right) = -\frac{\partial p'}{\partial z'}, \tag{3}$$

$$\frac{\partial p'}{\partial r'} = 0. \tag{4}$$

Where $\tau_{r'z'}$ for Herschel-Bulkley fluid given as [39]

$$-\frac{\partial u'}{\partial r'} = f(\tau) = \left[\frac{1}{\mu}\left(\tau'_{r'z'} - \tau'_0\right)\right]^{\frac{1}{n}}, \quad \tau'_{r'z'} \geq \tau'_0, \tag{5}$$

$$-\frac{\partial u'}{\partial r'} = f(\tau) = 0, \qquad \tau'_{r'z'} \leq \tau_0. \tag{6}$$

The variables are rendered dimensionless by the following transformations

$$p = \frac{a_0^{n+1} p'}{\lambda\,\mu\,c^n},\ r = \frac{r'}{a_0},\ z = \frac{z'}{\lambda},\ u = \frac{u'}{c},\ \varepsilon = \frac{b}{a_0},\ r_p = \frac{r_p'}{a_0},\ \tau_0 = \frac{a_0^n\,\tau_0'}{c^n\,\mu},\ \tau_{rz} = \frac{\tau'_{r'z'}\,a_0^n}{\mu\,c^n}. \tag{7}$$

Making use of the non-dimensional quantities in Equation (7), the governing equations (3) and (4) (after dropping the primes) takes the form as

$$\frac{1}{r}\frac{\partial}{\partial r} r\left[\left(-\frac{\partial u}{\partial r}\right)^n + \tau_0\right] = -\frac{\partial p}{\partial z}, \quad \tau_{rz} \geq \tau_0 \tag{8}$$

$$\frac{\partial u}{\partial r} = 0, \quad \tau_{rz} \leq \tau_0 \tag{9}$$

where τ_{rz} and τ_0 are dimensionless shearing and yield stresses, respectively. The corresponding non-dimensional boundary conditions are [16]

$$a'\frac{\partial u}{\partial r} = \frac{-\alpha u}{\sqrt{Da}} \text{ at } r = a'(z,\,t) \tag{10}$$

$$\tau_{rz} \text{ is finite at } r = 0 \tag{11}$$

Solving Equation (8) and (9) under the boundary conditions (10) and (11), the expression for velocity so obtained is given by

$$u=\frac{2}{P(K+1)}\left[\left(\frac{Pa'}{2}-\tau_0\right)^{K+1}-\left(\frac{P\mathrm{r}}{2}-\tau_0\right)^{K+1}\right]-\frac{a'\sqrt{Da}}{\alpha}\left(\frac{Pa'}{2}-\tau_0\right)^{K} \tag{12}$$

where $P=-\frac{\partial p}{\partial z},\ K=\frac{1}{n}$.

Using the condition $\tau_0=\frac{Pr_p}{2}$ at $r=r_p$, the upper limit of plug flow region is obtained as $r_p=\frac{2\tau_0}{P}$. Also, by using the condition $\tau_{rz}=\tau_{a'}$ at $r=a'$ (Bird *et al.,* [40]), we obtain

$$P=\frac{2\tau_{a'}}{a'} \text{ and } \frac{r_p}{a'}=\frac{\tau_0}{\tau_{a'}}=\tau. \tag{13}$$

Using relation (13) and by taking $r=r_p$ in Equation (12), the plug flow velocity is obtained as

$$u_p=a'^{K+1}\left(\frac{P}{2}\right)^{K}(1-\tau)^{K}\left[\frac{(1-\tau)}{K+1}-\frac{\sqrt{Da}}{\alpha}\right] \tag{14}$$

The instantaneous volume flux Q across any cross section of the artery is defined as

$$Q=2\left[\int_0^{r_p}u_p\,r\,dr+\int_{r_p}^{a'}u\,r\,dr\right] \tag{15}$$

$$Q=\frac{(1-\tau)^{K+1}}{2^K(K+1)}\left[1-\frac{2(1-\tau)(\tau+K+2)}{(K+2)(K+3)}-\frac{\sqrt{Da}\,(K+1)}{\alpha(1-\tau)}\right]a'^{K+3}P^K \tag{16}$$

4. Theoretical Determination of Flux with an Application to Flow through an Artery

A theoretical calculation of the flux Q is carried out for an incompressible Herschel-Bulkley fluid through an elastic tube of radius $a(t,\,z)=a'(t,\,z)+a''(z)$ where $a'(t,\,z)$ is the change in radius of the tube due to peristalsis and $a''(z)$ is the change in radius due to the elastic nature. The fluid is assumed to enter the tube with a pressure p_1 and leaves the tube with pressure p_2, while the pressure outside the tube is p_0. If z denotes the distance along the tube from the inlet end, then the pressure $p(z)$ in the fluid at z diminishes from $p(0)=p_1$ to $p(\lambda)=p_2$. The tube may contract or expand due to the difference in pressure of the fluid $p(z)-p_0$. Subsequently, the cross section of the tube may have a deformation due to the elastic property of the walls. Thus, the difference in pressure influences the conductivity σ_1 of the tube at z. We consider the conductivity $\sigma_1=\sigma_1[p(z)-p_0]$ to be a known function of the pressure difference $(p(z)-p_0)$.

This conductivity is assumed to be the same as that of a uniform tube having an identical cross section at z. It may be the conductivity for either laminar or turbulent flow, depending upon the type of the flow occurring at z in the non-uniform tube. The relation between Q and the pressure gradient is

$$Q = \sigma_1 (p - p_0) P^K \tag{17}$$

This connection, which includes Poiseuille's law, is precisely right for a uniform tube. It is approximately right for a non-uniform one in which the cross-section changes bit by bit along the tube. It can be reasoned, together with correction terms, by an asymptotic analysis of stream in such tubes, in any case, we shall not present that analysis.

Under the considerations of peristaltic motion and the elastic property of the tube wall, we can rewrite Equation (17) as

$$\sigma_1 (p - p_0) = F(a' + a'')^{K+3} \tag{18}$$

where, a' is the change in radius due to the peristalsis and a'' is the change in radius due to the elastic nature of the tube. The pressure $(p - p_0)$ at each cross section due to the Poiseuille flow i.e. $[a''(p - p_0)]$. By taking the inlet condition $p(0) = p_1$ and integrating Equation (17) gives

$$Q^n z = \int_{p(z)-p_0}^{p_1-p_0} (\sigma_1(p'))^n \, dp' \tag{19}$$

where, $p' = p(z) - p_0$. This equation gives $p(z)$ in terms of z and Q. Setting $z = 1$ and $p(1) = p_2$ in Equation (19), we get Q as,

$$Q = \left[\int_{p_2-p_0}^{p_1-p_0} (\sigma_1(p'))^n \, dp' \right]^{\frac{1}{n}} \tag{20}$$

Now, using Equation (18) in Equation (20), we have

$$Q = F \left[\int_{p_2-p_0}^{p_1-p_0} (a' + a'')^{3n+1} \, dp' \right]^{\frac{1}{n}} \tag{21}$$

Equation (21) can be solved if we explicitly know the function $a''(p - p_0)$. If a'' is known as a function of the tension $T(a'')$ or stress, then $a''(p')$ can be determined from the equilibrium condition given by [12]

$$\frac{T(a'')}{a''} = p - p_0. \tag{22}$$

Rubinow and Keller [12] carried out experimental investigations by controlling static pressure volume connection of a 4-cm long piece of a human iliac artery and gave an expression for tension in an elastic tube as

$$T(a'') = t_1(a'' - 1) + t_2(a'' - 1)^5 \tag{23}$$

Using Equation (23) with $t_1 = 13$ and $t_2 = 300$, Equation (22) takes the following form

$$dp' = \left[\frac{t_1}{a''^2} + t_2 \left(4a''^3 - 15a''^2 + 20a'' - 10 + \frac{1}{a''^2} \right) \right] da'' \tag{24}$$

Equation (21) can be written as

$$Q = F\left[\int_{p_2-p_0}^{p_1-p_0} (a'+a'')^{3n+1}\left[\frac{t_1}{a''^2}+t_2\left(4a''^3-15a''^2+20a''-10+\frac{1}{a''^2}\right)\right]da''\right]^{\frac{1}{n}} \tag{25}$$

Letting $p = p_1$ and $p = p_2$ in Equation (22) the solutions are obtained for a_1'' and a_2'' respectively. Equation (25) can be rewritten as

$$Q = F[g(a_1'')-g(a_2'')]^{\frac{1}{n}} \tag{26}$$

Where

$$F = \frac{(1-\tau)^{K+1}}{2^K(K+1)}\left[1-\frac{2(1-\tau)(\tau+K+2)}{(K+2)(K+3)}-\frac{\sqrt{Da}\,(K+1)}{\alpha(1-\tau)}\right] \text{ and}$$

$$g(a) = t_1\left(a''+2a'\log a''-\frac{a'^2}{a''}\right)+t_2\left(\frac{2a''^6}{3}+\frac{a''^5}{5}(8a'-15)+\frac{a''^4}{4}(4a'^2-30a'+20)\right.$$
$$\left.+\frac{a''^3}{3}(-15a'^2+40a'-10)+\frac{a''^2}{2}(20a'^2-20a')+a''(-10a'^2+1)-\frac{a'^2}{a''}+2a'\log a''\right).$$

It is worth noticing that from Equation (26) one can obtain the results of Rubinow and Keller [12] as a special case of the present model by substituting $a'=0,\ \tau=0,\ Da=0$ and $n=1$. Also, in the absence of peristalsis and porous parameter $(a'=0$ and $Da=0)$ the present results are in well agreement with the results of Vajravelu *et al.*, [13, 14].

5. Results and Discussion

The present paper focuses on the peristaltic transport of blood in a flexible tube, modeled as a Herschel-Bulkley fluid. From the present investigation, one can obtain the results of Rubinow and Keller [12], and Vajravelu *et al.*, [14] as a special case (in the absence of velocity slip and porous walls). The consequences of the model are investigated graphically by using the fixed values for physiological parameters such as $t_2 = 300,\ \varepsilon = 0.6,\ n = 3,\ a_1'' = 0.6,\ \tau = 0.5,\ \alpha = 0.2,\ a_2'' = 0.5,\ Da = 0.0002$ and $t_1 = 13$.

The comparison between present and Vajravelu *et al.,* [14] model is shown in Figure 2. It is clear from Figure 2 that the results of the present model exactly match with their results, which also validate our results. Further, the slight variation is observed in the current model because of the considered sinusoidal wave form. Figure 3 shows the variation of yield stress (τ) on volume flux (Q). It is observed from the figure that an increase in the value of τ decreases the flux in an elastic tube. This behavior is expected due to the presence of yield stress in the model which requires more amount of energy to begin the fluid flow and hence it decreases the flux. Figure 4 depicts the variation of flux along the axis of the tube for varying amplitude ratio (ε). It is seen that an increase in ε increases the flux in an elastic tube. Since ε is the amplitude ratio, an increase in the value of ε results in an increase in the wave height which in turn increases the flux. Figure 5 illustrates the effect of shear thickening $(n>1)$ behavior of blood on the flux. It can be observed that the flux increases with the axial distance and decreases with increase in the fluid behavior index (n). This decrease in flux is due to the shear thickening behavior of blood (viscosity increases with increasing shear stress).

Figure 6 and 7 shows the behavior of flux with axial locations for different values of elastic parameters. For a shear thickening fluid $(n > 1)$, the flux increases with an increase in the elastic parameter t_1 (Figure 6). The same trend holds good for the other parameter, namely t_2 (Figure 7). Figure 8 and 9 respectively explore the effects of Darcy number (Da) and slip parameter (α) on the flux. From Figure 8, it is noticed that an increase in the value of Darcy number decreases the flux. This is mainly because of an increase in the Darcy number, the porosity of the wall increases and therefore, the flux decreases. Further, an increase in the slip parameter increases the flux in an elastic tube (Figure 9). The flux profiles with inlet and outlet elastic radius variations are shown graphically in Figure 10 and 11. For a fixed value of outlet radius, the effect of increasing values of inlet elastic radius makes the flux to increase and hence flux increases as the elastic radius increases (Figure 10). However, the opposite behavior is observed when we fix the inlet elastic radius and varying outlet elastic radius (Figure 11). The impact of flux with axial locations for different fluids is plotted in Figure 12. From the geometrical portrayal, it is noticed that the flux in the case of a Newtonian fluid is more than that of the Bingham, Power-law, and Herschel-Bulkley fluid. Additionally, the flux in the Herschel-Bulkley model is less when contrasted with alternate models (Newtonian, Power-law, and Bingham). This is due to the presence of yield stress, and fluid behavior index (shear thickening) present in the Herschel-Bulkley model reduces the flux.

The effects of τ, ε, Da, n and α on Pressure gradient along the axis are plotted in Figures 13-17. From Figure 13 and 14, the magnitude of pressure gradient increases with an increase in the values of τ and ε. Moreover, an increase in the value of Da (porosity of the walls) is accompanied with an increase in the magnitude of the pressure gradient (Figure 15). Figure 16 gives the variation of pressure gradient along the axis for different values of n. It is noticed that an increase in the value of fluid behavior index increases the magnitude of pressure gradient. Further, the effect of α on pressure gradient shows the opposite behavior as that of n (Figure 17).

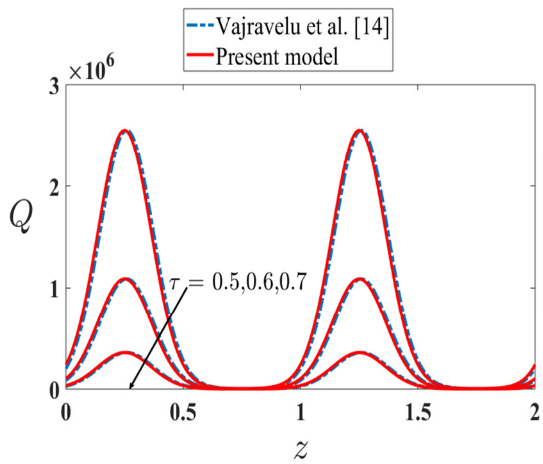

Fig. 2. Q versus z for varying τ

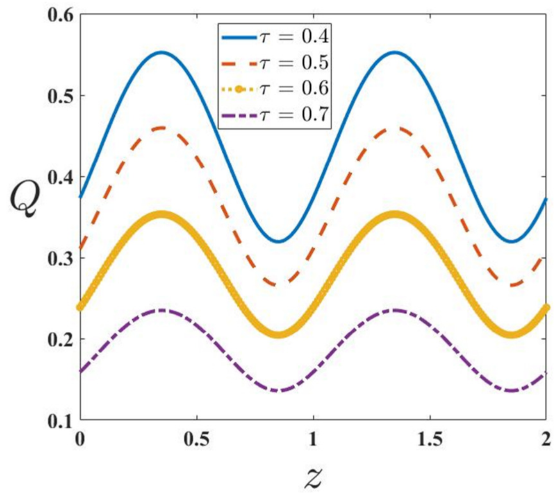

Fig. 3. Q versus z for varying τ

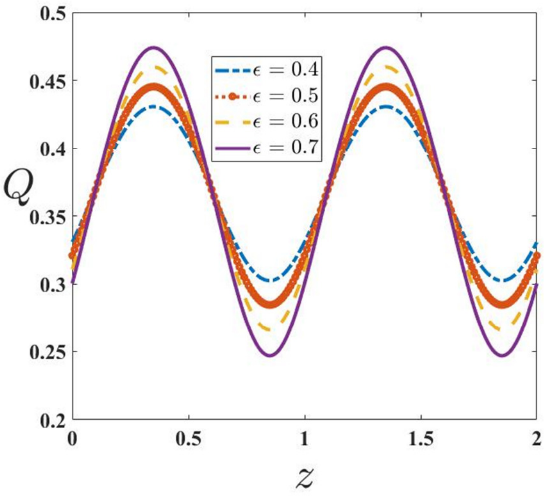

Fig. 4. Q **versus** z **for varying** ε

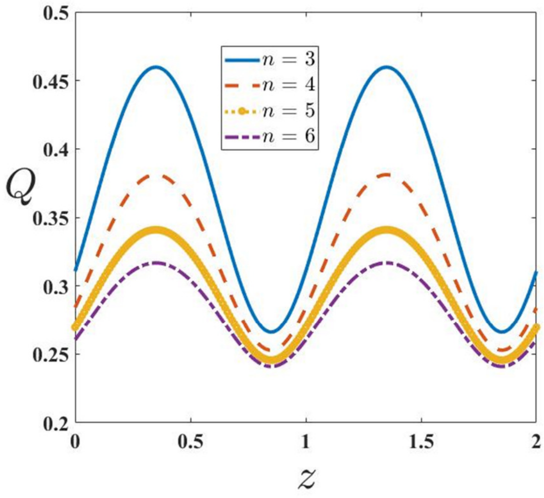

Fig. 5. Q **versus** z **for varying** n

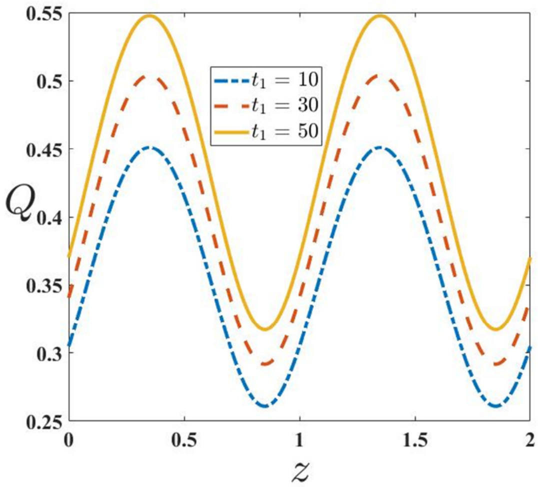

Fig. 6. Q **versus** z **for varying** t_1

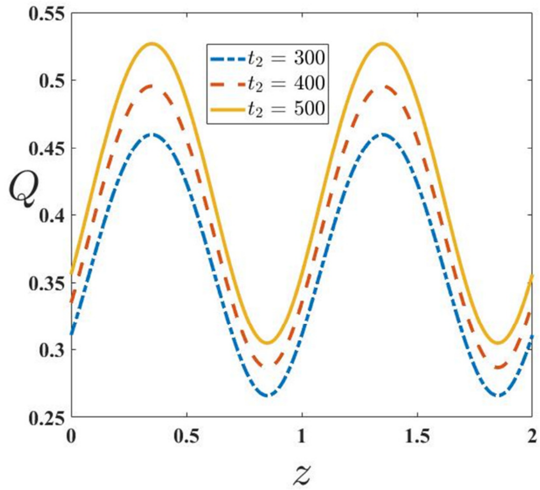

Fig. 7. Q **versus** z **for varying** t_2

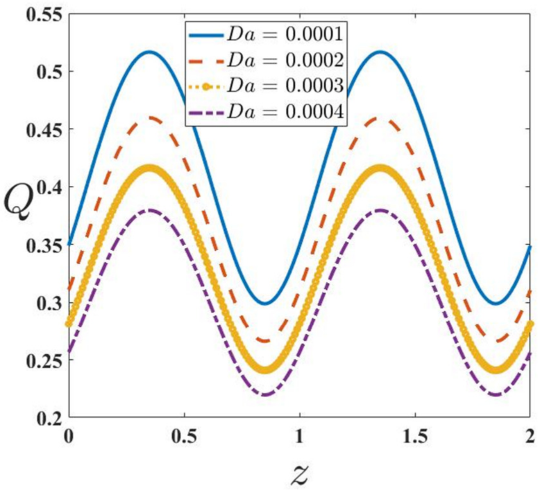

Fig. 8. Q **versus** z **for varying** Da

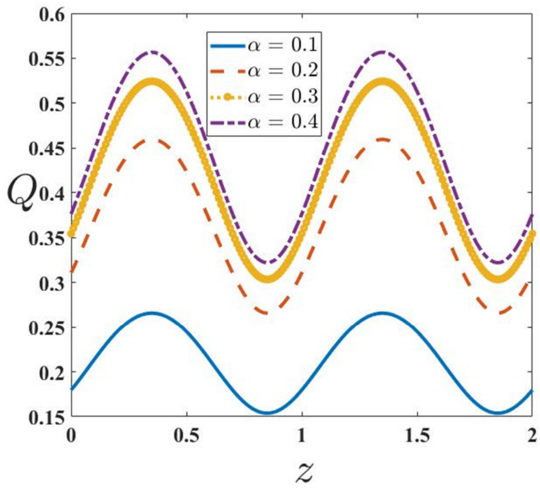

Fig. 9. Q **Versus** z **for varying** α

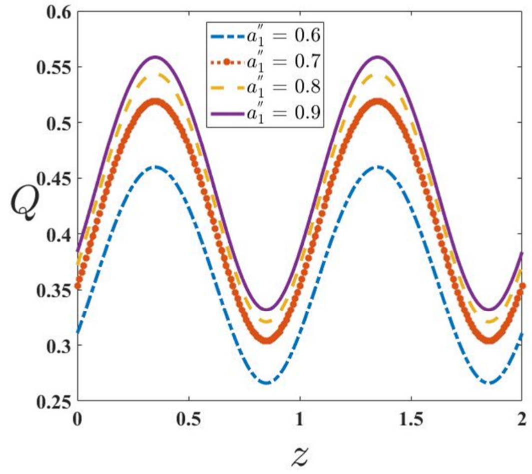

Fig. 10. Q **versus** z **for varying** a_1''

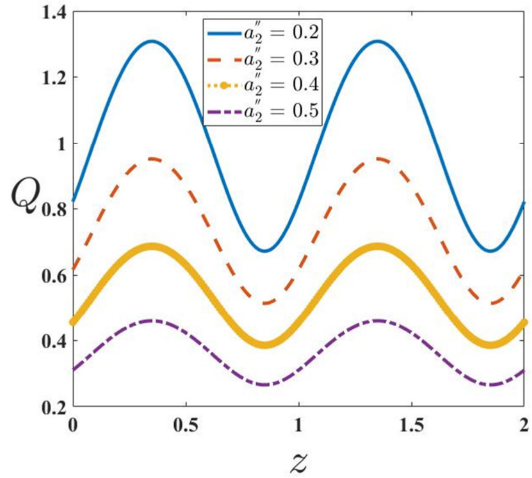

Fig. 11. Q **versus** z **for varying** a_2''

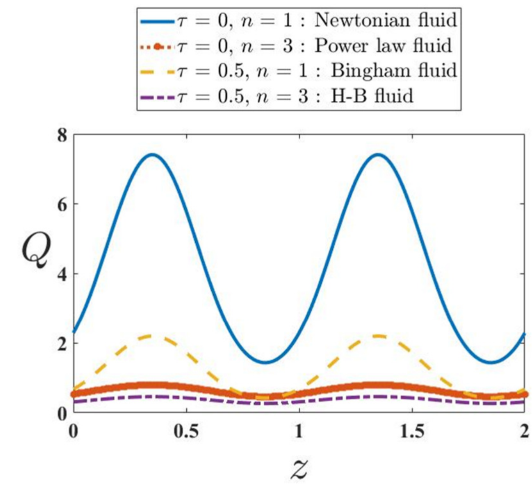

Fig. 12. Q **versus** z **for different types of fluids**

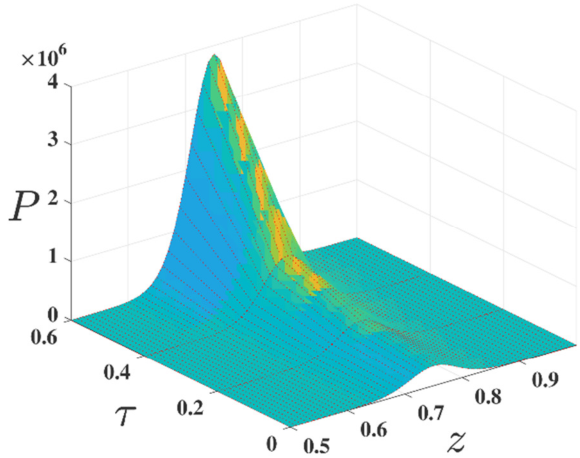

Fig. 13. P **versus** z **for varying** τ

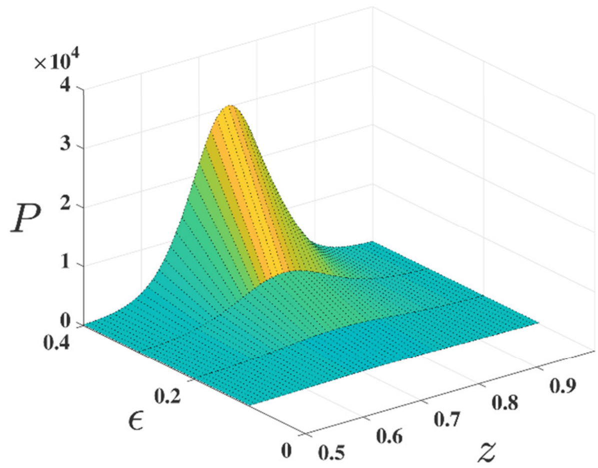

Fig. 14. P **versus** z **for varying** ε

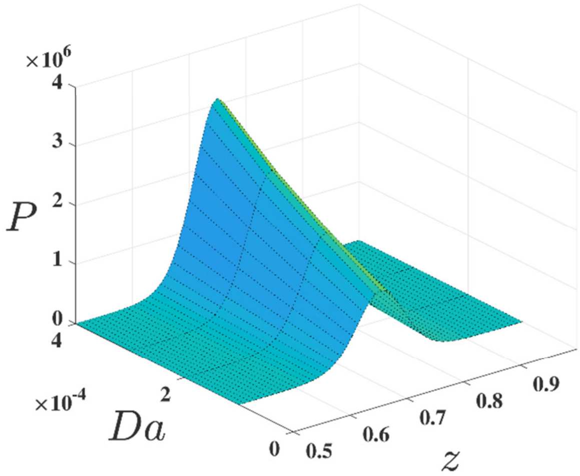

Fig. 15. P **versus** z **for varying** Da

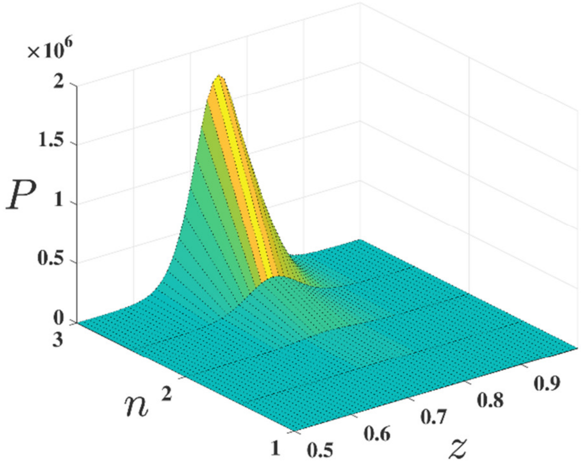

Fig. 16. P **versus** z **for varying** n

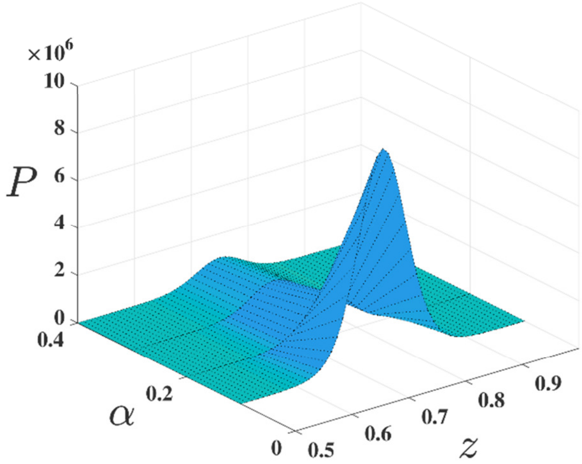

Fig. 17. P **versus** z **for varying** α

6. Conclusions

The present paper emphasizes on the peristaltic flow of blood in the human circulatory system by using Herschel-Bulkley Model in an elastic tube with porous walls. The study provides a satisfactory outcome that represents some of the natural phenomena, especially, the flow of blood in narrow arteries which can be handled and processed in case of dysfunction. The conclusions can be summarized as follows.

- The volumetric flow rate increases with an increase in slip parameter and decreases with an increase in the porous parameter.
- The presence of elastic parameters has a vital role in enhancing the flux.
- The flux in an elastic tube decreases with an increase in the values of yield stress, fluid behavior index and outlet elastic radius, and it increases with an increase in the values of inlet elastic radius and amplitude ratio.
- The magnitude of pressure gradient increases with an increase in the values of yield stress, amplitude ratio, fluid behavior index and Darcy number, and it decreases with an increase in the slip parameter.

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