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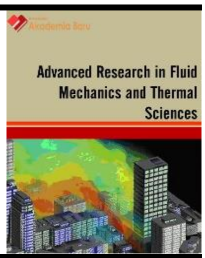

Characteristics of the Internal Fluid Flow Field Induced by an Oscillating Plate with the other Parallel Plate Stationary

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ABSTRACT

The analytical solution to the internal fluid flow induced by an oscillating plate with the other parallel plate stationary is derived and the results explored fully. The unsteady analytical solution is obtained by using Duhamel's theorem. The result is analysed non-dimensionally with respect to variations in the gap, the time and the parameter ω^*. Results reveal that the wide variation in fluid flow is strongly depending on the dimensionless parameter ω^* . The present governing equation is modelled numerically with same conditions and at selected $\omega^* = 1$ and $\omega^* = 10$, respectively, the numerical results are able to match with the analytical solution remarkably well.

1. Introduction

In the field of fluid mechanics, the analytical solutions to the Navier-Stokes equation constitute part of the educational syllabus in the subject, for the purpose of understanding the role of the terms in the equation, although possible only for the simplest cases. The Couette flow is such an example, with linear motion induced by one moving parallel plate while the other parallel plate is kept stationary [1–4].

In the extension of this classic Couette flow, the analytical solution for the case of an oscillating plate is often mentioned. However, they are usually referring to the case of a single plate oscillating upon an infinite initially stationary mass of viscous fluid, the so-called Stokes' second problem [5–7]. The case of an oscillating plate on a finite channel gap with a stationary parallel plate has been alluded but consequences of the full solution has seldom been discussed [8,9]. The insight of viscous effect in oscillating flow is important as it could vary the fluid flow significantly as compared to Couette flow or Couette-Poiseuille flow.

As the case of an oscillating plate on a finite channel gap with a stationary plate is not found in the literature, the novelty of the present study is to investigate this basic case. The full analytical solution is derived to study the characteristics of an internal Newtonian fluid flow induced by the oscillating lower plate and to analyse the fluid flow effect in general. For an application, the motion

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may be applicable to the synovial fluid within an artificial hip joint [10–14], where the lower part of the hip joint is somewhat oscillating sinusoidally and the upper part is kept at stationary. For general microchannel study, the fluid flow passages may be regarded to have hydraulic diameter within the range of 10 - 200 μm [15,16].

2. Mathematical Formulation and Solution

2.1 Problem Statement

The physical model considered in Figure 1, is the pair of infinite parallel plates distanced *W* apart, where lower plate is oscillating sinusoidally while the upper one is fixed. As shown in Figure 1, the *x*-*y* Cartesian coordinate system is taken with the origin at a fixed point in space located on the lower plate. Thus, the motion of the lower plate can be specified as $u = U\sin(\omega t)$, where *u* is the velocity component in the *x*-direction, *U* is a maximum velocity magnitude at the lower plate, ω is the angular frequency and *t* is the time.

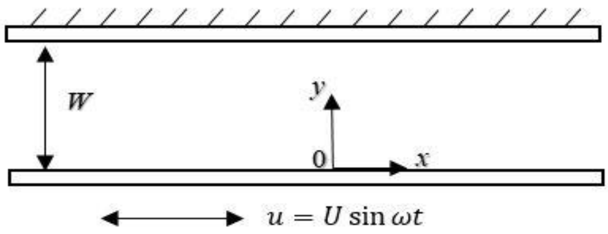


Fig. 1. Physical model

The general equation of motion for unsteady Newtonian fluids, for case of incompressible, laminar, constant property throughout, the Navier-Strokes equation with typical symbols, is applicable [17,18]

$$\rho\left(\frac{\partial\vec{u}}{\partial t} + \vec{u}\cdot\nabla\vec{u}\right) = -\nabla p + \mu\nabla^2\vec{u} + \rho\vec{F}. \tag{1}$$

For the assumptions of no pressure gradient and body force, with parallel flow in infinite plates, the velocity components along the y and z directions are neglected, and the Eq. 1 reduces to the following

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

where ν is the kinematic viscosity. The general solution of Eq. 2, that yields $u(y,t)$, is subjected to three conditions, with the appropriate non-slip conditions at the plates as the following

$$u(0,t) = U\sin\omega t\,, u(W,t) = 0 \text{ and } u(y,0) = 0. \tag{3}$$

2.2 Solution

A solution involving the time-dependent boundary condition can be obtained by applying the Duhamel's theorem [19,20] to the specific solution of the present problem statement, but replacing it with a time independent condition: at $y = 0, u = U$. This is just the standard Couette flow within a channel, and the set consists of Eq. 2 subjected to $u(W,t) = 0, u(0,t) = U$,and $u(y,0) = 0$. The solution to this simpler problem is well-known as an unsteady flows between two infinite plates

$$u_a(y,t) = U\left(1 - \frac{y}{W}\right) - \sum_{n=1}^{\infty} \frac{2U}{n\pi} sin\left(n\pi \frac{y}{W}\right) exp\left[-\left(\frac{n\pi}{W}\right)^2 \nu t\right], \tag{4}$$

where $u_a(y,t)$ in Eq. 4 denotes this specific solution.

Then, according to Duhamel's theorem, the solution to the original problem is

$$u(y,t) = \int_{\tau=0}^{t} u_a\,(y, t-\tau) \frac{df(\tau)}{d\tau} d\tau\,, \tag{5}$$

where τ is the running time variable, and $f(\tau) = U\sin(\omega\tau)$.

Inserting $f(\tau)$ into Eq. 5, we have

$$u(y,t) = \int_{\tau=0}^{t} \left\{U - \frac{Uy}{W} - \sum_{n=1}^{\infty} \frac{2U}{n\pi} sin\left(\frac{n\pi}{W} y\right) exp\left[-\nu\left(\frac{n\pi}{W}\right)^2 (t-\tau)\right]\right\} \omega\, cos(\omega\tau)\ d\tau, \tag{6}$$

Where,

$$\int_0^t \exp\left[-\nu\left(\frac{n\pi}{W}\right)^2 (t-\tau)\right] \cos(\omega\tau)\, d\tau = \frac{\nu\left(\frac{n\pi}{W}\right)^2 \cos(\omega t) + \omega\sin(\omega t) - \nu\left(\frac{n\pi}{W}\right)^2 \exp\left[-\nu\left(\frac{n\pi}{W}\right)^2 t\right]}{\left[\nu\left(\frac{n\pi}{W}\right)^2\right]^2 + \omega^2}. \tag{7}$$

Hence, it can be shown that

$$u(y,t) = U\left(1 - \frac{y}{W}\right) sin(\omega t) - \sum_{n=1}^{\infty} \frac{2\omega U\, sin\left(n\pi \frac{y}{W}\right)}{n\pi\left\{\omega^2 + \left[\nu\left(\frac{n\pi}{W}\right)^2\right]^2\right\}} \left\{\nu\left(\frac{n\pi}{W}\right)^2 cos(\omega t) + \omega\, sin(\omega t) - \nu\left(\frac{n\pi}{W}\right)^2 exp\left[-\nu\left(\frac{n\pi}{W}\right)^2 t\right]\right\}. \tag{8}$$

Eq. 8 is cast into a dimensionless form, using the following four dimensionless parameters

$$u^* = \frac{u}{U},\ y^* = \frac{y}{W}\ , t^* = \frac{\nu t}{W^2} \text{ and } \omega^* = \frac{\omega W^2}{\nu}, \tag{9}$$

giving finally

$$u^*(y^*, t^*) = (1 - y^*)\, sin(\omega^* t^*) - \sum_{n=1}^{\infty} \frac{2\omega^*\, sin(n\pi y^*)}{n\pi[\omega^{*2} + (n\pi)^4]} \{(n\pi)^2 cos(\omega^* t^*) + \omega^* sin(\omega^* t^*) - (n\pi)^2 exp[-(n\pi)^2 t^*]\}. \tag{10}$$

Eq. 10 shows the dimensionless form of fluid velocity variation for general Newtonian flow with an unsteady flow profile induced by a sinusoidal oscillation at the lower plate, with the upper plate fixed.

3. Results and Discussions

From Eq. 10, it is clear that the unsteady solution depends on the dimensionless variable $\omega^* = \frac{\omega W^2}{\nu}$, which contains information on geometry, fluid properties and the oscillation rate. It is then of interest to understand how the velocity profile varies with ω^*. Since all the dimensional parameters are always positive, ω^* must always be positive. From the defined ω^*, increasing ω or W and decreasing ν, will increase the ω^* ; the converse is also true. Hence in applying the results to the present problem, without referring to specific values of ω, W and ν, this section reveal and discuss the solution with respect to the single variable, ω^*.

3.1 Velocity Profiles with Varying ω^* from Analytical Solution

Figure 2 displays four different velocity profiles for fluid flow with selected $\omega^* = 1$, 10, 50 and 100. Since the dimensionless velocity $u^*$is oscillating sinusoidally with time and the reference origin is stationary, the range for u^* is clearly within -1 to 1. The eight whole numbers of dimensionless time, t^* are selected as to illustrate the change in velocity profiles over these periods of time.

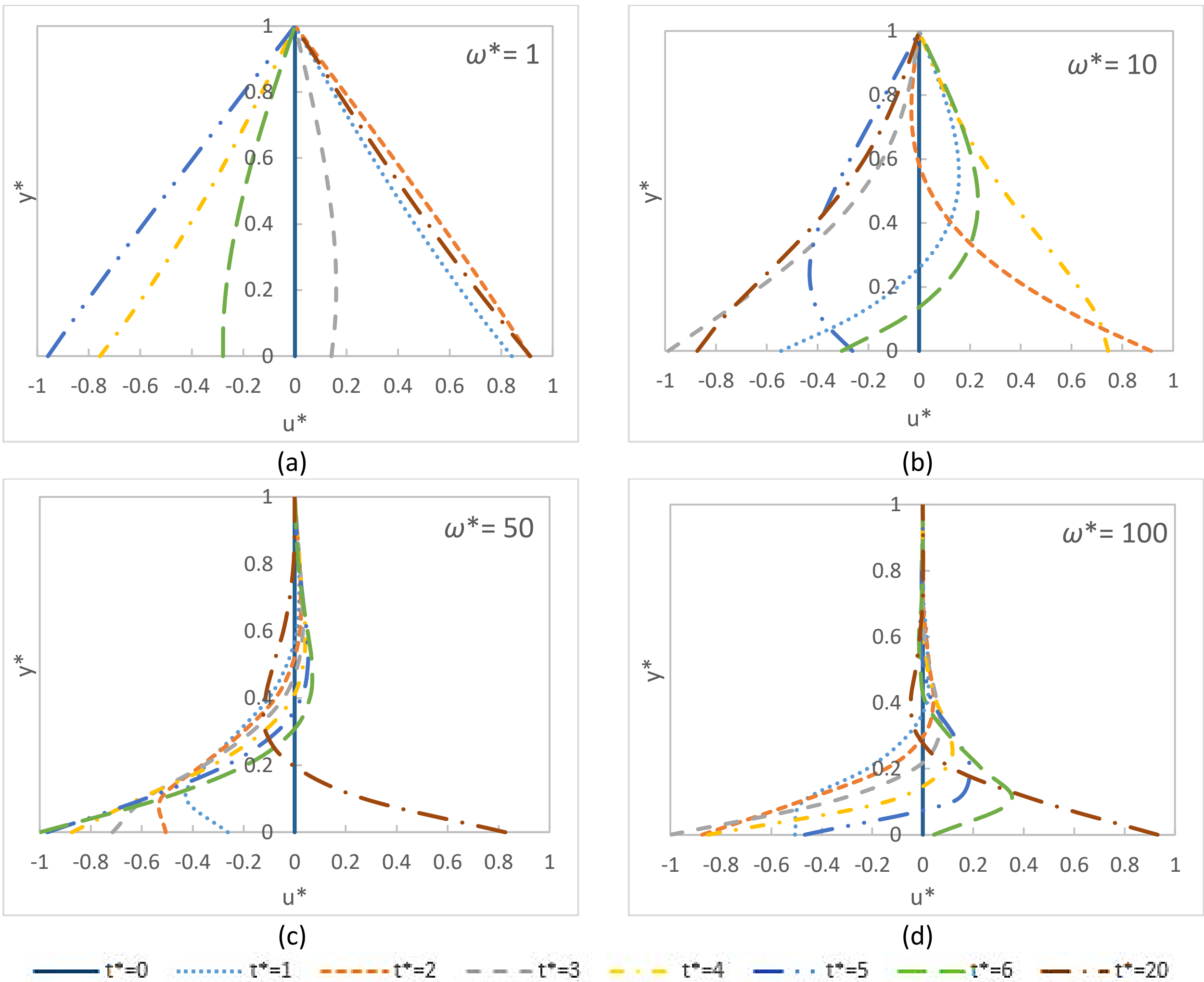


Fig. 2. Fluid velocity profiles at four dimensionless angular frequencies ω^*: (a) $\omega^* = 1$, (b) $\omega^* = 10$, (c) $\omega^* = 50$, (d) $\omega^* = 100$, from the analytical solution

It is worth mentioning that the terms under the summation sign from Eq. 10 seem to converge after *n* = 10, as the magnitude of values change after that will approximately less than 0.0002 for range $\omega^* \leq 10$ and 0.002 for range $50 \leq \omega^* \leq 100$.

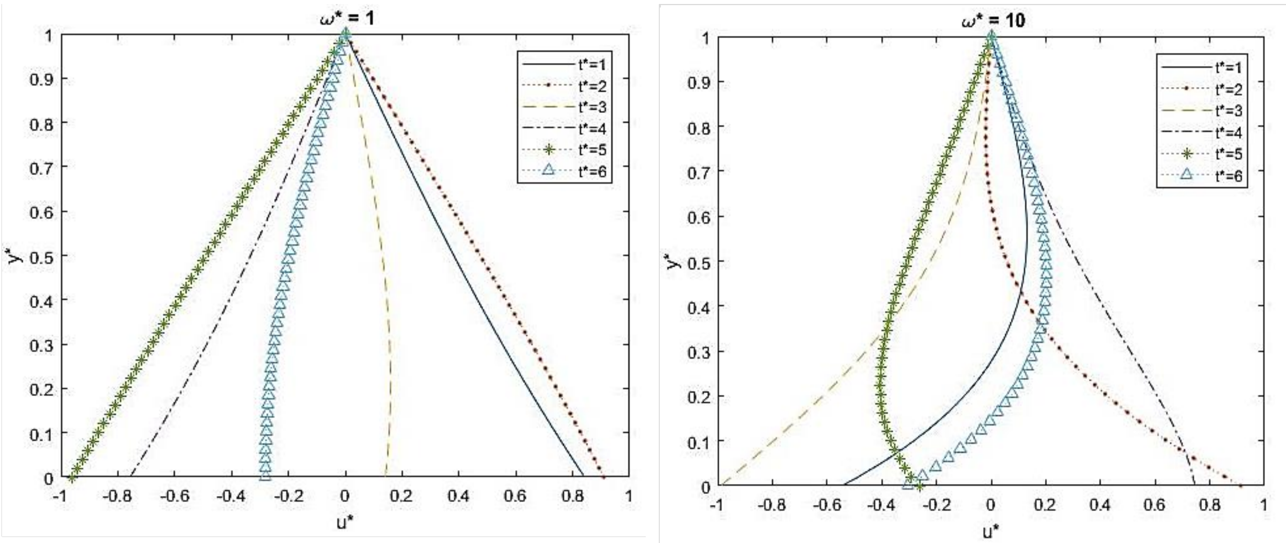
In Figure 2, the fluid velocity is always zero for all ω^* and t^* at the top plate, satisfying the no-slip boundary condition. And at the bottom plate, the fluid velocity always depending on the moving plate. The u^* is always zero at time $t^* = 0$, reflecting the initial condition assumed.

It is within the plates that the velocity profile depends on the value of ω^*. The fluid motion in Figure 2(a) shows that the motion is moving nearly with a linear velocity profile to the bottom moving plate. This is because the magnitude of values from the term under the summation sign is small and does not to influence the fluid velocity profile at all time. At $\omega^* \geq 10$, the fluid motion has a wide variation of curve-like pattern at all time across the gap of the channel. As time move on, the fluid velocities within the plates are varying due to the effect of the fluid inertia that influences the fluid acceleration and give rise to a significant velocity phase lag. This effect is buried inside the general solution. For example, at $\omega^* = 100$ and $t^* = 6$, the fluid near the bottom plate is having a $u^* \approx 0$ meanwhile at $y^* = 0.1$, the $u^* \approx 0.4$.

3.2 Velocity Profiles with Varying ω^* from Numerical Solution as Verification

The present problem from Eq. 2 is replicated numerically in Matlab software, with the same conditions stated in Eq. 3. Finite element method is used for all the computations that ran the model. From trial computations for $\omega^* \leq 10$, it is noteworthy that the optimum time step for these suggested ranges is found to be approximately $t^* = 0.02$. In numerical modelling, it is sensible to find an optimum time step as too small a time step requires additional time while a large time step gives an unpredictable error in solution.

Figure 3(a) and (b) illustrate the numerical solution of velocity profile at $\omega^* = 1$ and $\omega^* = 10$, respectively. For instance, Figure 2(a) and 3(a) are the results from both analytical and numerical solution respectively, when $\omega^* = 1$. The result shows that the numerical solution able to match well with the derived analytical solution. Figure 3(b) strengthens the verification at different selected ω^*, in this case, $\omega^* = 10$.



(a) (b)

Fig. 3. Fluid velocity profiles at two dimensionless angular frequencies ω^* : (a) $\omega^* = 1$ and (b) $\omega^* = 10$, from the numerical solution

4. Conclusions

This study reveals a derived mathematical formulation for equation of motion, focusing on viscous effect for infinite parallel plate channel with lower plate oscillate sinusoidally and upper plate is fixed. The equation derived intended to seek and analyse the behaviours of fluids velocity at different condition and properties of the fluids, such as angular velocity, fluid's viscosity and the distance between the parallel plates. The overall of fluid motion for selected range when $\omega^* = 1$ to $\omega^* = 100$ are shown in Figure 2(a) to 2(d). Moreover, the results from derived analytical solution is verified by numerical modelling remarkably. The obtained analytical equation can be further use for conservation of energy to study the thermo-fluids behaviour in future. One potential application using the derived solution is to analyse the motion of synovial fluids in the artificial hip joint.

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