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Unsteady Magnetohydrodynamic Flow of Second Grade Nanofluid (Ag-Cu) With CPC Fractional Derivative

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ABSTRACT

Unsteady magnetohydrodynamics (MHD) flow of fractionalized second grade nanofluid (Ag-Cu) over a vertical plate is obtained. The model is solved by using CPC fractional derivative. The novelty of present study is to generalized the model by using Fourier's and Fick's laws. The nanofluids are formed by dispersing two different nanoparticles, silver (Ag) and copper (Cu), into a based fluid. The governing dimensionless equations for velocity, concentration, and temperature profiles are solved using Laplace transform method and compared graphically. The effects of different parameters like fractional parameter, second grade parameter and magnetic parameter M are discussed through numerous graphs. From figures, it is observed that second grade and magnetic field have decreasing effect on velocity profile, whereas mass Grashof number have increasing effect on velocity of fluid.

1. Introduction

Convection flow in the presence of porosity has numerous important applications such as flows in soils, solar power collectors, heat transfer correlated with geothermal systems, heat source in the field of agricultural storage system, heat transfer in nuclear reactors, heat transfer in aerobic and anaerobic reactions, heat evacuation from nuclear fuel detritus, and heat exchangers for porous material. Heat and mass transfer occurs mostly in nature due to temperature and concentration differences respectively. Magnetohydrodynamics (MHD) is branch of Mechanics which examines the movement of electro-conductive fluid in the existence of magnetic field. Today research work in Magnetohydrodynamics (MHD) has substantial significance as these flows are absolutely prevailing in nature. Hsiao [1] worked on MHD heat transfer thermal extrusion system using non-Newtonian Maxwell fluid with radiative and viscous dissipation effects. Chamkha *et al.,* [2] discussed the hall and ion slip effects on MHD rotating boundary layer flow of nanofluid. Chamkha *et al.,* [3] analyzed the hall and ion slip effects on MHD free convective rotating flow through a porous medium. Ramzan *et*

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al., [4] analyzed the MHD flow of fractionalized Jeffrey fluid with newtonian heating and thermal radiation over a vertical plate. Ahmad *et al.,* [5] compared the generalized form of Jeffrey fluid flow acquired by contemplating fractional derivative of singular kernel (Caputo) and non-singular kernel (Caputo-Fabrizio). During the last decade, different generalized fractional derivatives have appeared in the literature that are derivatives of Caputo, Caputo-Fabrizio, constant proportional Caputo [6-7]. Some studies of free convection on an inclined plane in various thermal and mechanical situations have recently been presented by mathematicians [8-14]. Some mathematical models of second grade fluids are industrial oils, slurry streams, and dilute polymer solutions with different geometry and boundary conditions. The solution of unsteady second grade fluid at plate with the assistance of the Fourier sine transformation was described by Fetecau *et al.,* [15]. Ahmed *et al.,* [16] has analyzed MHD heat transfer into convective boundary layer with a minimal pressure gradient. Convective mixed MHD flow studied by Narayana [17], while Chamkha *et al.,* [18] worked Hall and ion slip impacts on unsteady MHD convective rotating flow of heat generation/absortion second grade fluid. Because of its rising significance, engineering needs to incorporate non-Newtonian fluid. Khan *et al.,* [19] presented a fractional flow of the second grade fluid on a vertical surface driven by temperature as well as concentration gradients. Chamkha *et al.,* [20] discussed heat transfer analysis of unsteady hybrid nanofluid flow with thermal radiation. Seth *et al.,* [21] discussed the MHD convection flow over a vertical plate with ramped temperature. Tran *et al.,* [22] studied on mandatory stability of fractional derivatives for fractional calculus equations, and the mathematical model used for transference of COVID-19 with Caputo fractional derivatives also discussed by Tuan *et al.,* [23].

Shateyi *et al.,* [24] analyzed the free convection flow of fluid past an infinite vertical plate using a heat generation. Ramzan *et al.,* [25] analyzed the problem of Casson fluid through a channel. Khalid *et al.,* in [26] investigated the Casson fluid effect. Kataria *et al.,* [27] worked on the MHD fluid with heat and mass transport immersed in a porous medium. Authors in [28,29] discussed the Brinkman fluid over a plate. Siyal *et al.,* and Shah *et al.,* [30,31] discussed Casson fluid over a vertical plate.

In this problem, the model of MHD flow of second grade nanofluid (Ag-Cu) over a plate is considered. Firstly, the governing equations have been made non-dimensional and then solved semi-analytically. The results for velocity profile, temperature profile, and concentration profile are obtained and then analyzed graphically. Various graphs are plotted and discussed for different parameters, which are used in the flow model.

2. Mathematical Description of The Model

The magnetohydrodynamic flow of second grade nanofluid over a plate is considered. The fluid is flowing along y_1^*-axis. The motion of fluid depends on x_1^*-axis and time t_1^*. The plate and fluid have concentration C_∞^* and temperature T_∞^* at constant $t_1^* = 0$ with zero velocity. But for $t_1^* > 0$, the plate starts to move in the plane with uniform velocity $U_1 e^{a^* t_1^*}$. The level of concentration is lowered or raised to $t_1^*(C_\infty^* + C_w^*)/t_0 + C_w^*$ for $t_0 \geq t_1^*$ and C_w^* for $t_0 \leq t_1^*$. The temperature of the plate is lowered or raised to $t_1^*(T_\infty^* + T_w^*)/t_0 + T_w^*$ for $t_0 \geq t_1^*$ and T_w^* for $t_0 \leq t_1^*$. A uniform magnetic field β_0 is acting in the transverse direction to the flow. The transversely applied magnetic field and magnetic Reynolds number are assumed to be very small so that the induced magnetic field the Hall effect are negligible. normally. In view of above assumption and using Boussinesq's approximation, the convection flow of second grade fluid with chemical reaction, and magnetic field through a plate, linear momentum equation by using [28,32] is

$$\rho_{nf}\frac{\partial u_1(x_1^*,t_1^*)}{\partial t_1^*}=\mu_{nf}(1+\frac{\alpha_1}{\rho}\frac{\partial}{\partial t_1^*})\frac{\partial^2 u(x_1^*,t_1^*)}{\partial x_1^*}+(\rho\beta_{T^*})_{nf}g(T^*-T_\infty^*)-\frac{\sigma_{nf}\beta_0^2 u_1(x_1^*,t_1^*)}{\rho}$$
$$+(\rho\beta_{C^*})_{nf}g(C^*-C_\infty^*) \quad (1)$$

Thermal equation is

$$(\rho c_p)_{nf}\frac{\partial T^*(x_1^*,t^*)}{\partial t_1^*}=-\frac{\partial q(x_1^*,t^*)}{\partial x_1^*} \quad (2)$$

According to Fourier's Law, $q_1(x_1^*,t_1^*)$ is given by

$$q_1(x_1^*,t_1^*)=-K_{nf}\frac{\partial T^*(x_1^*,t_1^*)}{\partial x_1^*} \quad (3)$$

Diffusion Eq. is

$$\frac{\partial C^*(x_1^*,t_1^*)}{\partial t_1^*}=-\frac{\partial J(x_1^*,t_1^*)}{\partial x_1^{\cdot}} \quad (4)$$

According to Fick's Law, $J_1(x_1^*,t_1^*)$ is given by

$$J_1(x_1^*,t_1^*)=-D_{nf}\frac{\partial C^*(x_1^*,t_1^*)}{\partial x_1^*} \quad (5)$$

The conditions for the model [27] are

$$u_1(x_1^*,t_1^*)=0,\ \ T^*(x_1^*,t_1^*)=T_\infty,\ \ C^*(x_1^*,t_1^*)=C_\infty^*,\ \ x_1^*>0,\quad t_1^*=0, \quad (6)$$

$$u_1(0,t^*)=U_1 f(t_1^*),\ \ T(0,t_1^*)=\begin{cases}T_W^*, & t_1^*>t_0;\\ T_W^*+\frac{t_1^*(T_W^*-T_\infty^*)}{t_0}, & 0<t_1^*\le\ t_0,\end{cases} \quad (7)$$

$$C(0,t_1^*)=\begin{cases}C_W^*, & t_1^*>t_0;\\ C_\infty^*+\frac{t_1^*(C_W-C_\infty)}{t_0}, & 0<t_1^*\le\ t_0,\end{cases} \quad (8)$$

$$u_1(x_1^*,t_1^*)\to 0,\ \ T^*(x_1^*,t_1^*)\to 0,\ \ C^*(x_1^*,t_1^*)\to 0,\quad x_1^*\to\infty,\quad t_1^*>0 \quad (9)$$

Dimensionless form of the variables are

$$x^{\cdot}=\frac{Ux_1^*}{\nu},\qquad t^{\cdot}=\frac{U^2t_1^*}{\nu},\qquad T^{\cdot}=\frac{T^*-T_\infty^*}{T_W^*-T_\infty^*},\qquad u^{\cdot}=\frac{u_1}{U_*},$$
$$\mathrm{Gr}^{\cdot}=\frac{\nu\beta_{T^*}(T_W^*-T_\infty^*)}{U^3},\qquad C^{\cdot}=\frac{C^*-C_\infty^*}{C_W^{\cdot}-C_\infty^*},\qquad \mathrm{Gm}^{\cdot}=\frac{\nu\beta_{C^*}(C_W^*-C_\infty^*)}{U^3} \quad (10)$$

Using parameter of Eq. (10) into Eq. (1)-(9), we have

$$\frac{\partial u(x,t)}{\partial t}=a_0(1+S\frac{\partial}{\partial t})\frac{\partial^2 u(x,t)}{\partial x^2}-M_0u(x,t)+\mathrm{Gr}_0T(x,t)+\mathrm{Gm}_0C(x,t), \quad (11)$$

where $a_0 = ((1-\phi+\frac{\phi(\rho_s)}{\rho_f})(1-\phi)^{2.5})^{-1}$, $\mathrm{Gr}_0 = \frac{\mathrm{Gr}}{\phi_0}$, $\mathrm{Gm}_0 = \frac{\mathrm{Gm}}{\phi_0}$, $M_0 = \frac{M}{\phi_0}$,

$$\frac{\partial T(x,t)}{\partial t} = \frac{1}{b_0}\frac{\partial A}{\partial x}, \tag{12}$$

with

$$A(x,t) = -\frac{\partial T}{\partial x}, \tag{13}$$

where $b_0 = \frac{\phi_1 \mathrm{Pr}}{\phi_2}$, $\mathrm{Pr} = \frac{\nu\rho c_p}{K_f}$, $A = \frac{q\nu}{UK_{nf}(T_w - T_\infty)}$

$$\frac{\partial C(x,t)}{\partial t} = \frac{1}{c_0}\frac{\partial B}{\partial x}, \tag{14}$$

with

$$B(x,t) = -\frac{\partial C}{\partial x}, \tag{15}$$

where $c_0 = \frac{\mathrm{Sc}}{1-\phi}$, $\mathrm{Sc} = \frac{\nu}{D_{nf}}$, $B = \frac{q\nu}{UD_{nf}(C_w - C_\infty)}$, $\phi_0 = 1-\phi+\frac{\phi(\rho_s)}{\rho_f}$,
$\phi_1 = 1-\phi+\frac{\phi(\rho c_p)_s}{(\rho c_p)_f}$, $\phi_2 = \frac{-2k_f+k_s-2(k_f-k_s)\phi}{2k_f+k_s+(k_f-k_s)\phi}$

3. Generalized Model

Eq. (11) is

$$\frac{\partial u(x,t)}{\partial t} = a_0\left(1+S\frac{\partial}{\partial t}\right)\frac{\partial^2 u(x,t)}{\partial x^2} - M_0 u(x,t) + \mathrm{Gr}_0 T(x,t) + \mathrm{Gm}_0 C(x,t), \tag{16}$$

Eq. (13) and Eq. (15) is generalized fractionally by [33,34]

$$A(x,t) = -D_t^{\gamma}\frac{\partial T(x,t)}{\partial x}, \quad 1 \geq \gamma > 0, \tag{17}$$

$$B = -D_t^{\alpha}\frac{\partial C(x,t)}{\partial x}, \quad 1 \geq \gamma > 0 \tag{18}$$

Put Eq. (17) into Eq. (12) and Eq. (18) into Eq. (14) and making non-dimensional results, we have

$$\frac{\partial T(x,t)}{\partial t} = \frac{1}{b_0}D_t^{\alpha}\frac{\partial^2 T(x,t)}{\partial x^2}, \tag{19}$$

$$\frac{\partial C(x,t)}{\partial t} = \frac{1}{c_0}D_t^{\alpha}\frac{\partial^2 C(x,t)}{\partial x^2}, \tag{20}$$

Initial and boundary conditions are

$$u(x,t) = T(x,t) = C(x,t) = 0, \quad t = 0, \tag{21}$$

$$u(0,t) = f(t),\ T(0,t) = C(0,t) = \begin{cases} 1, & t > 1; \\ t, & 0 < t \leq\ 1, \end{cases} \tag{22}$$

$$u(x,t) \to 0, \quad T(x,t) \to 0, \quad C(x,t) \to 0, \quad x \to \infty,\ t > 0, \tag{23}$$

where $\mathrm{Gm}, S, M, \mathrm{Gr}$, and u represents the mass Grashof number, second grade parameter, magnetic field, mass Grashof number, and motion of fluid respectively and $D_t^{\beta} u(x,t)$ is the CPC derivative of $u(x,t)$ given by

$$D_t^{\beta} u(x,t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \left[K_1(\beta) u(x,\tau) + K_0(\beta) u'(x,\tau)\right](t-\tau)^{-\beta} d\tau \tag{24}$$

4. Solution of Problem

Eq. (16), (19), (20) with conditions have been solved semi-analytically via technique of Laplace transform can be solved numerically by using Stehfest's and Tzou's algorithms [35,36] in case of complex expression..

4.1 Calculation of Temperature

By applying Laplace transform on Eq. (19), we have

$$q\bar{T}(x,q) = \frac{1}{b_0}\left[\frac{K_1(\alpha)}{q} + K_0(\alpha)\right] q^{\alpha} \frac{\partial^2 \bar{T}(x,q)}{\partial x^2} \tag{25}$$

By applying Laplace transform on Eq. (19), we have with boundary conditions

$$\bar{T}(0,q) = \frac{1-e^{-q}}{q^2}, \qquad \bar{T}(x,q) \to 0,\ \ x \to \infty \tag{26}$$

Put Eq. (26) in Eq. (25)

$$\bar{T}(x,q) = \left(\frac{1-e^{-q}}{q^2}\right) e^{-x\sqrt{\frac{(qb_0)}{\left(\frac{K_1(\alpha)}{q} + K_0(\alpha)\right) q^{\alpha}}}}, \tag{27}$$

4.2 Calculation of Concentration

By applying Laplace transform on Eq. (20), we have

$$q\bar{C}(x,q) = \frac{1}{c_0}\left[\frac{K_1(\alpha)}{q} + K_0(\alpha)\right] q^{\alpha} \frac{\partial^2 \bar{C}(x,q)}{\partial x^2} \tag{28}$$

with boundary conditions

$$\bar{C}(0,q) = \frac{-e^{-q}+1}{q^2}, \qquad \bar{C}(x,q) \to 0,\ \ x \to \infty \tag{29}$$

Put Eq. (29) in Eq. (28)

$$\bar{C}(x,q) = (\frac{-e^{-q}+1}{q^2})e^{-x\sqrt{\frac{(qc_0)}{(\frac{K_1(\alpha)}{q}+K_0(\alpha))q^{\alpha}}}}, \tag{30}$$

4.3 Calculation of Velocity

By applying Laplace transform on Eq. (16), we have

$$q\bar{u}(x,q) = a_0(1+Sq)\frac{\partial^2\bar{u}(x,q)}{\partial x^2} - M_0\bar{u}(x,q) + \mathrm{Gr}_0\overline{\mathrm{T}}(x,q) + \mathrm{Gm}_0\bar{C}(x,q), \tag{31}$$

with boundary conditions

$$\bar{u}(0,q) = f(q), \quad \bar{u}(x,q) \to 0, \ \ x \to \infty \tag{32}$$

Putting Eq. (32) in Eq. (31) we have

$$\bar{u}(x,q) = \frac{1}{q-a}e^{-x\sqrt{\frac{q+M_0}{a_0[1+Sq]}}} + \frac{\mathrm{Gr}_0\frac{1-e^{-q}}{q^2}}{a_0[1+Sq][\frac{b_0q}{\left(\frac{K_1(\alpha)}{q}+K_0(\alpha)\right)q^{\alpha}}]-[q+m_0]}[e^{-x\sqrt{\frac{q+M_0}{a_0[1+Sq]}}} - e^{-x\sqrt{\frac{(qb_0)}{\left(\frac{K_1(\alpha)}{q}+K_0(\alpha)\right)q^{\alpha}}}}] +$$
$$\frac{\mathrm{Gm}_0\frac{1-e^{-q}}{q_2}}{a_0[1+Sq][\frac{c_0q}{(\frac{K_1(\alpha)}{q}+K_0(\alpha))q^{\alpha}}]-[q+m_0]}[e^{-x\sqrt{\frac{q+M_0}{a_0[1+Sq]}}} - e^{-x\sqrt{\frac{(qc_0)}{(\frac{K_1(\alpha)}{q}+K_0(\alpha))q^{\alpha}}}}] \tag{33}$$

5. Results and Discussion

The solution for the impact of second grade nanofluid past over a vertical plate are developed by using Laplace transform technique. The effect of numerous parameters used in the governing equations of velocity fields have been analyzed in figures.

The impact of Gm and Gr on fluid velocity $u(x,t)$ is illustrate in Figure 1. It is highlighted that fluid motion raises as values of Gm increasing. Physically higher the values of Gm increase the concentration gradients which make the buoyancy force significant and hence it is examined that velocity field is raising. And fluid motion rises up with maximizing the values of Gr, and it represents the impact of thermal buoyancy force to viscous force. Therefore maximizing the values of Gr exceed the temperature gradient due to which velocity field rises.

The impact of M and Pr on $u(x,t)$ is reported in Figure 2. Graph shows that fluid velocity $u(x,t)$ is reduced with accelerating values of parameter M. Resistivity becomes dominant with raising M which reduced the speed of fluid. The $u(x,t)$ increases with decreasing the values of Pr. As Pr is the ratio of momentum diffusivity to the thermal diffusivity. Physics behind this effect is that larger the momentum diffusion, slows down the fluid motion. From the graph. Figure 3 represents the effect of S and t on $u(x,t)$. The $u(x,t)$ increases with decreasing the values of S and increases with increasing values of time t. Figure 4 indicates the effect of Sc and ϕ on $u(x,t)$. The fluid velocity rises by falling the magnitude of Sc and ϕ. Physics behind this fact is that increase in Schmidt number, Kinematic viscosity rises which declines the molecular diffusion, therefore, velocity of the fluid falls down and it is found that enhancing volume fraction of nanoparticles boosts the viscous effect for the nanofluid consequently it decreases the fluid velocity. Figure 5 indicates the effect of α on $u(x,t)$ and $T(x,t)$. It is clear that as α increases, the fluid velocity and temperature are increase. Figure 6 indicates the impact of Pr and ϕ on $T(x,t)$. As we increased the value of Pr, heat diffusion is reduced which slow down the fluid motion and temperature increases with increasing values of ϕ as reported in figure. Because thermal conductivity of nanofluid increases with the increasing volume fraction ϕ of nanoparticles. The behavior of α and Sc on $C(x,t)$ are shown in Figure 7. The $C(x,t)$ is increases with increasing value of α and decreases with increasing values of Sc as depicted in graph. Figure 8 and 9 show the comparison of present work with Khalid *et al.,* [26]. If we put $\beta = \gamma = \alpha \to 1, S = \mathrm{Gm} = 0$, and in the absence of Casson parameter of Khalid *et al.,* [26] work, the both fluid are identical. The velocity distributions overlap which shows the authenticity of inversion algorithms as presented in Figure 10. The authenticity of inverse algorithms for temperature and concentration distributions as presented in Figure 11.

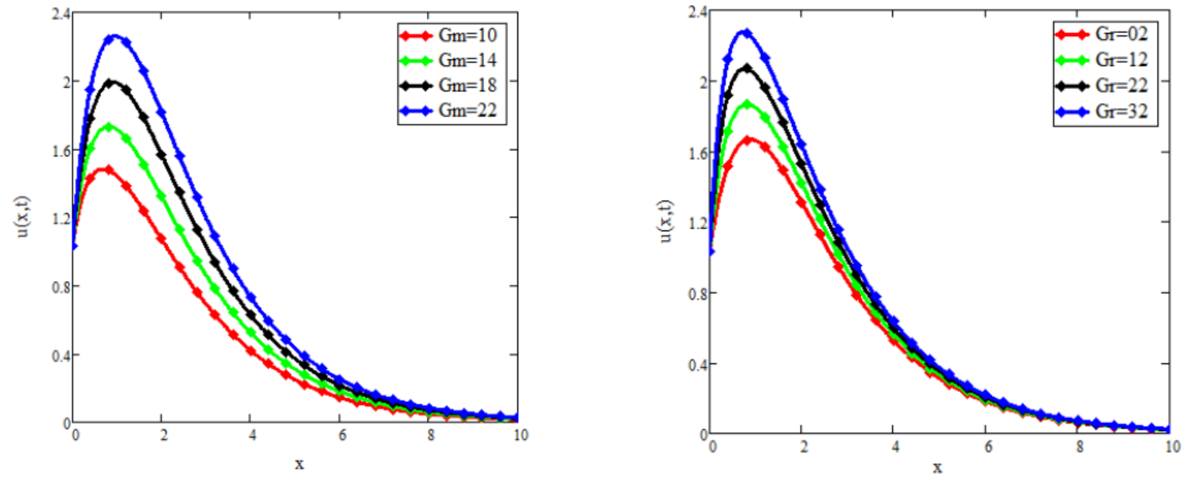

Fig. 1. Velocity diagram $u(x,t)$ for various values of parameters Gm and Gr at $S = 2, M = 0.3$, $\mathrm{Sc} = 1.2,\ \mathrm{Pr} = 2.5,\ \mathrm{t} = 1.5$

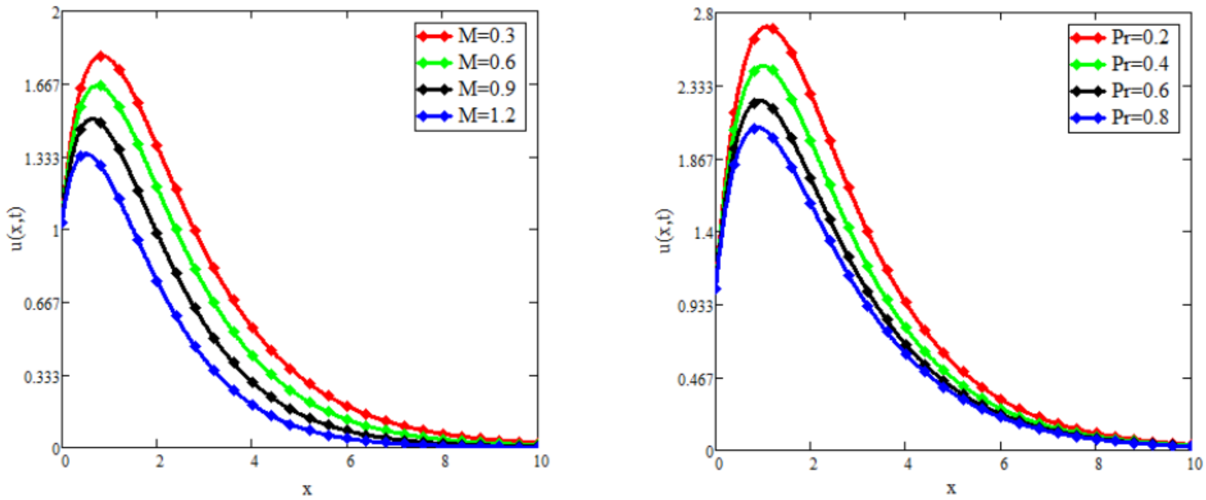

Fig. 2. Velocity distribution $u(x,t)$ for various values of parameter M and Pr at $S = 2, \mathrm{Gm} = 15, \mathrm{Sc} = 1.2,\ \mathrm{Gr} = 15,\ \mathrm{t} = 1.5$

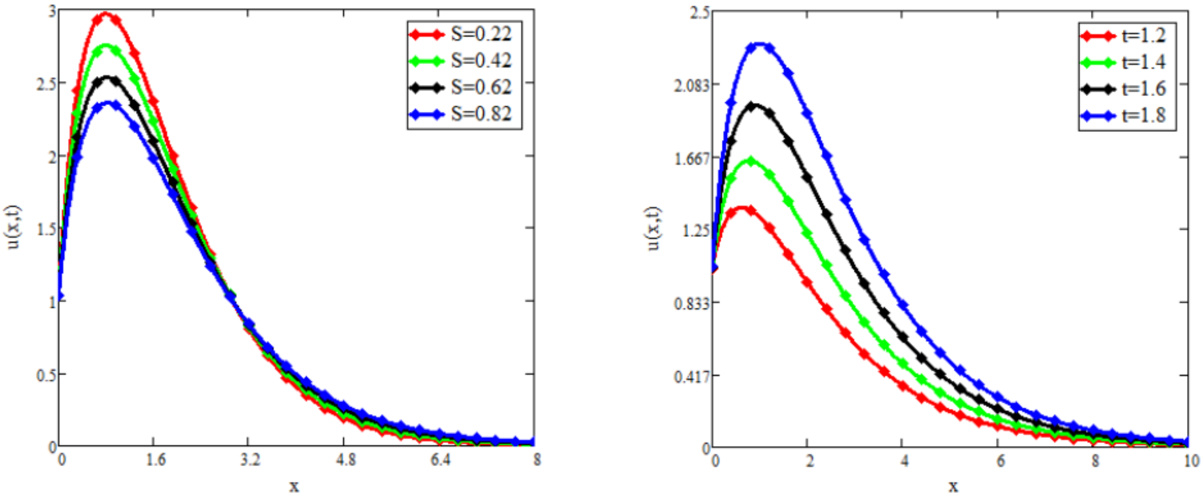

Fig. 3. Velocity distribution $u(x,t)$ for various values of parameter S and t at $M = 0.3, \mathrm{Gm} = 15, \mathrm{Sc} = 1.2,\ \mathrm{Gr} = 15,\ \mathrm{Pr} = 2.5$

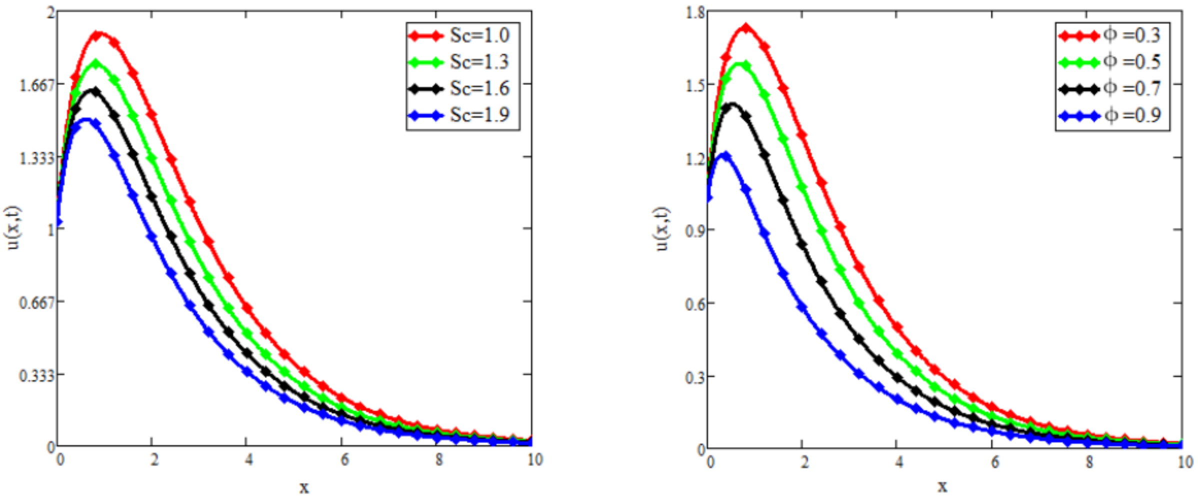

Fig. 4. Velocity diagram $u(x,t)$ for various values of parameters Sc and ϕ at $S = 2,\ M = 0.3,\ \mathrm{Gr} = 15,\ \mathrm{Pr} = 2.5,\ \mathrm{t} = 1.5,\ \ \mathrm{Gm} = 15$

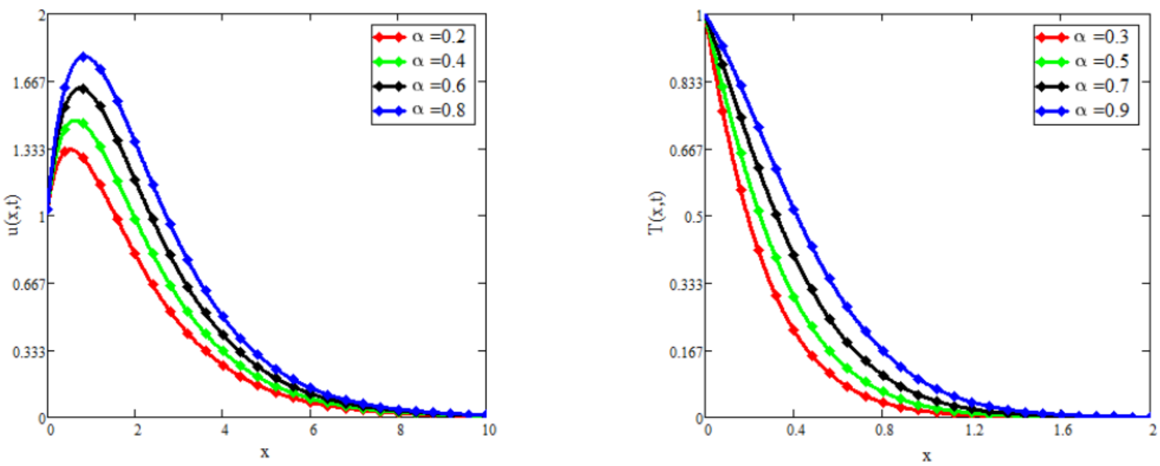

Fig. 5. Graph of fractional parameter α for Velocity and temperature distribution at $S = 2, \mathrm{Pr} = 2.5, \mathrm{Sc} = 1.2,\ \mathrm{Gr} = 15,\ \mathrm{t} = 1.5$

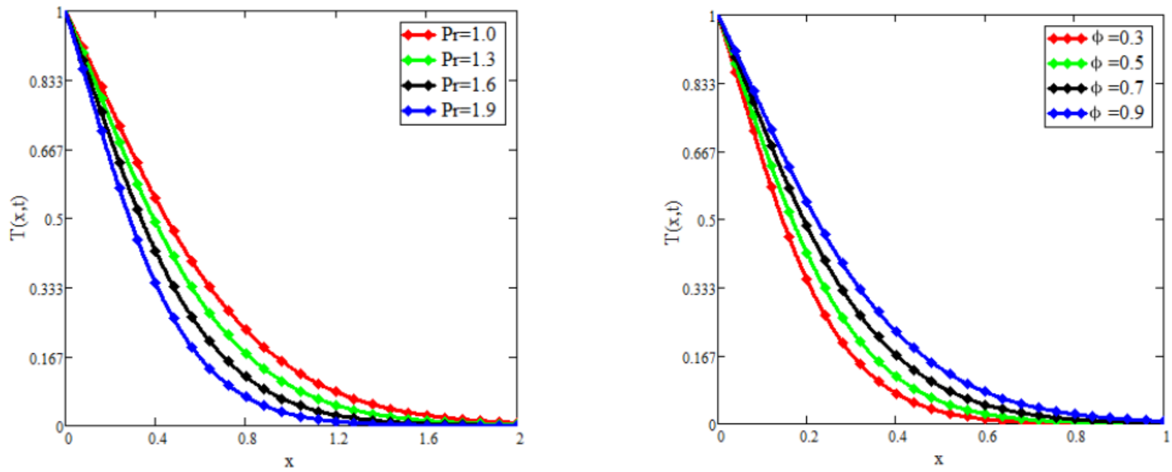

Fig. 6. Temperature distribution $T(x,t)$ for various values of parameter Pr and ϕ

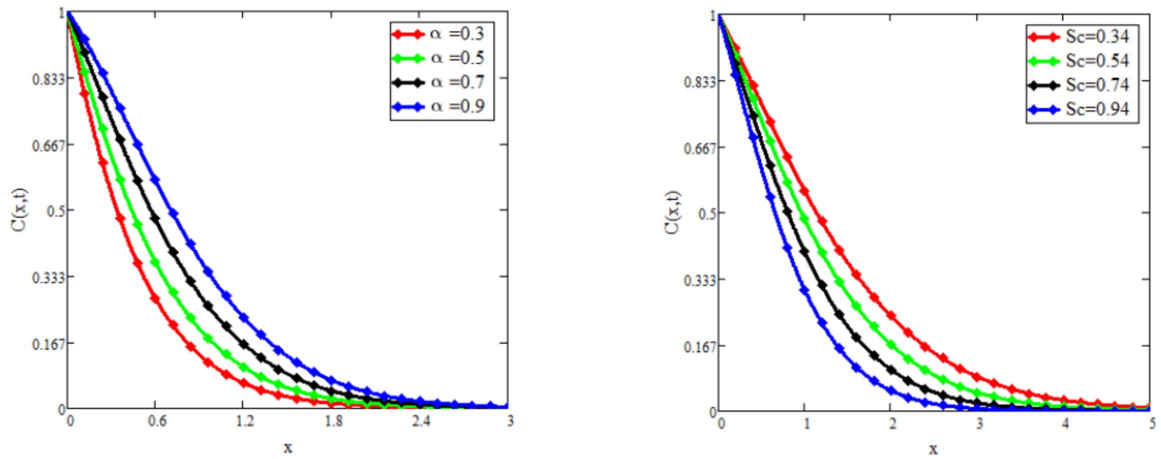

Fig. 7. Concentration profile $C(x,t)$ for various values of parameter α and Sc

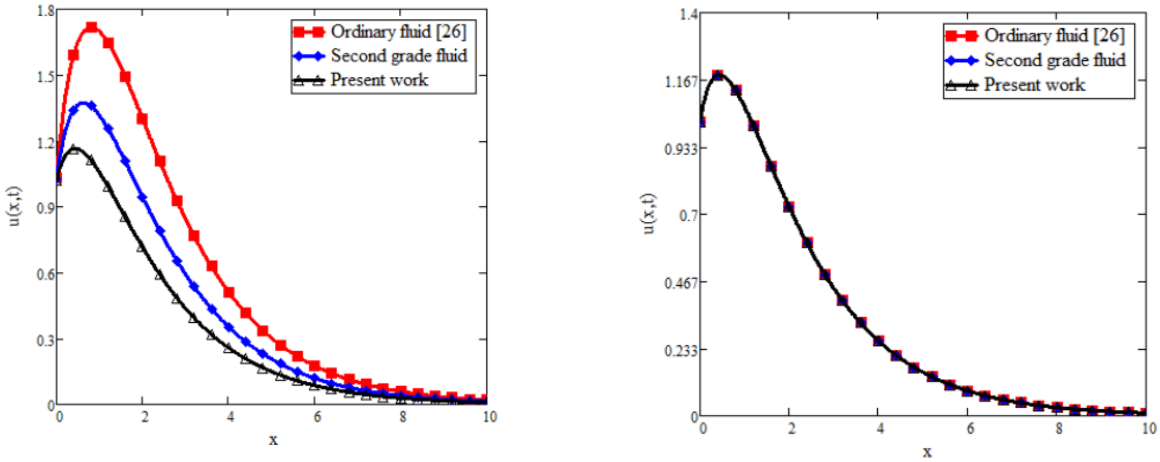

Fig. 8. Comparison of velocity profile for different values of α

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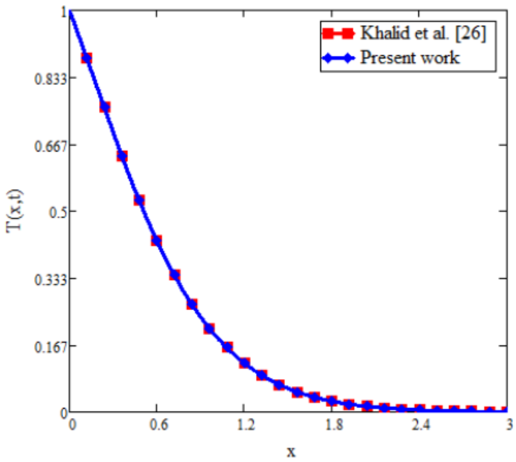

Fig. 9. Comparison of Temperature profile

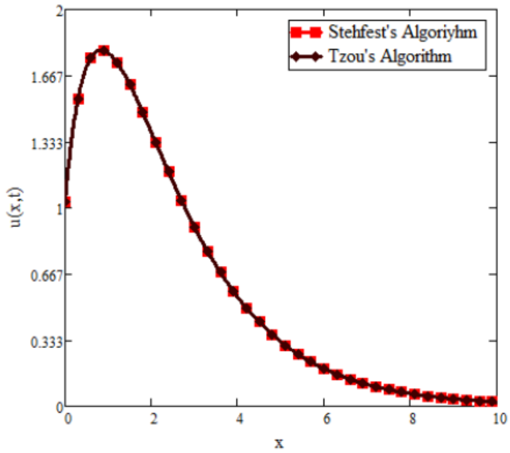

Fig. 10. Velocity profile obtained by Stehfest's and Tzou's algorithm

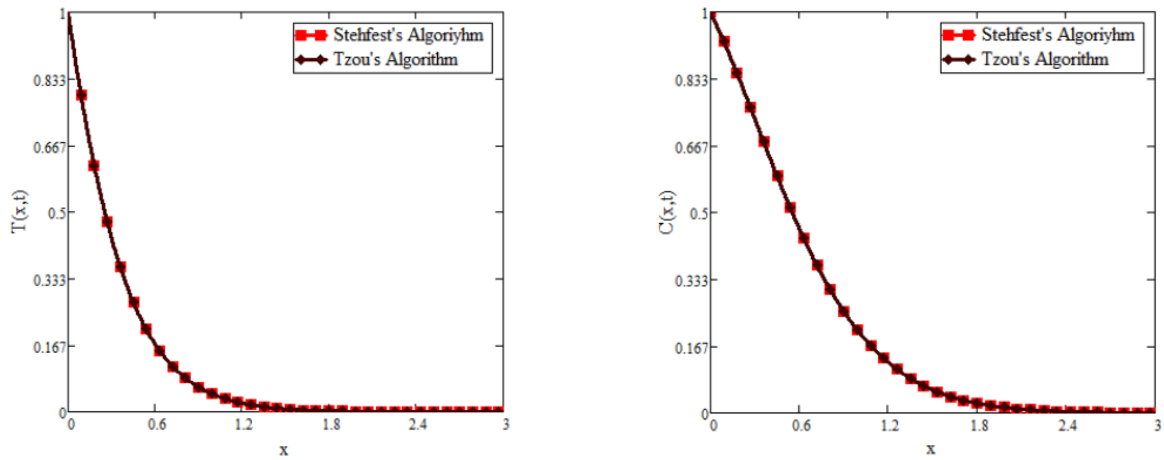

Fig. 11. Temperature and concentration profile obtained by Stehfest's and Tzou's algorithm

6. Conclusion

The flow of fractional second grade nanofluid(Ag-Cu) has been taken and solved using Laplace transform with solution. The conditions of flow problem are satisfied by the results. Different graphs have been plotted for flow parameters and then discussed. The key points of this flow model are

i. With higher Magnetic values, the velocity distribution slows down.
ii. With decreasing values of α, the velocity distribution slows down.
iii. Thermal buoyancy forces accelerate the fluid velocity.
iv. The fluid velocity decreased for increasing values of S.
v. The fluid velocity decreased for increasing values of Pr.
vi. The fluid velocity decreased for increasing values of ϕ.
vii. The Temperature of fluid decays down for larger values of Sc.
viii. The Temperature of fluid rises up for larger values of ϕ.
ix. The concentration level is an increasing function of fractional parameter.
x. The concentration level is a decreasing function of Sc.

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