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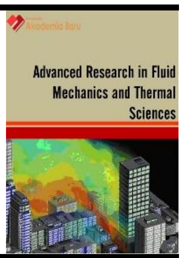

Effect of Coriolis Force and Magnetic Field on Thermal Convection in an Anisotropic Porous Medium

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ABSTRACT

Effect of Coriolis force and magnetic field on thermal convection in an anisotropic porous medium has been studied numerically. Linear stability analysis has been implemented to verify the presence of Coriolis force and magnetic field on the thermal convection in a horizontal anisotropic porous medium heated from below. The Darcy model is used for the momentum equation and Boussinesq approximation is considered for the density variation in the porous medium. The eigenvalue problems of the perturbed state were obtained from a normal mode analysis and solved using Chebyshev Tau method numerically with respect to upper free conducting and lower free conducting boundary condition. By using Fortran software, it is found that the thermal anisotropy parameter destabilized the system while the effect of Coriolis force and magnetic field help to stabilize the system and can delay the onset of convection and mechanical anisotropic parameter advanced the onset of convection in the system.

Keywords:
Anisotropic Porous Medium, Coriolis force, Magnetic field, Chebyshev Tau Method

1. Introduction

The studies of thermal convection in a porous medium have received much attention of the researchers due to its innumerable application in the field of engineering and geophysical. For instance, the storage of carbon dioxide in the underground water. Thermal convection refers to the transfer of heat by the movement of fluid due to the density difference when heat is applied to the system. Bénard [1] is the first who studied theoretically and experimentally the onset if thermal convection in a fluid layer. Later, Rayleigh [2] has investigated the onset in a horizontal fluid layer.

The studies of convection in an anisotropic porous medium have been considered widely by many scientists due to its validity application in the real physical field. Combarnous and Castinel [3] studied the threshold of thermal convection in a porous medium with thermal anisotropic properties.

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Epherre [4] investigated the effect of thermal and mechanical anisotropic properties on the onset of convection in a porous medium. Shiina and Hishida [5] studied the threshold of convection in a high porosity anisotropic porous medium in the presence of vertical circular thin wire stretched across a lower hot and upper cold layer. Capone and several researchers [6] examined the effect of internal heating on the threshold of penetrative convection in an anisotropic porous medium. In a study by Capone *et al.,* [7], the linear and nonlinear stability analysis are applied to study the thermal convection in a horizontal anisotropic porous medium in the presence of non-homogenous porosity and thermal diffusivity. Kim [8] considered the relaxed energy method to solve the thermal convection problem in a fluid saturated an anisotropic porous medium. Aly and Ahmed [9] explored the thermal and mixed convection in a cavity saturated an anisotropic porous medium by considering the non-Darcy model. They solved the problem using an incompressible smoothed particle hydrodynamics method. Paoli *et al.,* [10] studied the onset of convection in a porous medium with an anisotropic permeability in order to understand the carbon dioxide sequestration process.

Vadasz [11] studied both the linear and weak nonlinear stability analysis in order to observe the effect of Coriolis force on the onset of thermal convection in an anisotropic porous medium. Malashetty and Swamy [12] has considered using the linear theory and nonlinear theory which is weak on the onset of convection in horizontal porous layer in the presence of effect of rotation and anisotropy. Vanishree and Siddeshwar [13] investigated the effect of Coriolis force and temperature dependent viscosity on the onset of thermal convection in an anisotropic porous medium governed by Brinkman model. Malashetty *et al.,* [14] studied the effect of Coriolis force on the threshold of double diffusive convection in an anisotropic porous medium saturated with binary viscoelastic fluid. Bhadauria *et al.,* [15] explored the thermal convection problem in an anisotropic medium in the presence of Coriolis force and internal heat generation.

Alchaar *et al.,* [16] studied the convection problem in an isotropic porous medium in the presence of magnetic field. Further, the study of convection in an anisotropic porous medium saturated with binary fluid in the presence of magnetic field and Soret effect has been considered by Srivastava *et al.,* [17]. Sekar *et al.,* [18] investigated the effect of magnetic field and Soret effect on the convection problem in an anisotropic porous saturated with ferrofluid.

In addition, Jehad *et al.,* [19] have considered to do the research regarding the performance of three different turbulence models and they have predicted the region with separation and the reattachment behind the edge of step. While, Sidik *et al.,* [20] have made a combination of nanoparticles and refrigerant and become Nanorefrigerant. Nanofluids are thought by Choi [21] to be the next-generation heat transfer fluids and they offer exciting possibilities due to their enhanced heat transfer performance compared to ordinary fluids.

Thus, the aim of present study is to investigate the effects of Coriolis force and magnetic field on the threshold of thermal instability and on the steady flow patterns on critical Rayleigh number in a horizontal anisotropic porous medium with respect to upper free conducting boundary conditions. The resulting eigenvalue problem is solved using Chebyshev Tau method together with QZ algorithm.

2. Methodology

An infinite horizontal anisotropic porous medium with depth, d bounded vertically between the planes $z = 0$ and $z = 1$, which is heated from below and gravity $\vec{g} = (0,0,-g)$ acting vertically downward on it, is considered (Figure 1). The system is subjected to vertical magnetic field and uniformly adverse temperature difference, $\Delta T = T_l - T_u$, where T_l and T_u is the fixed value of lower and upper temperature respectively.

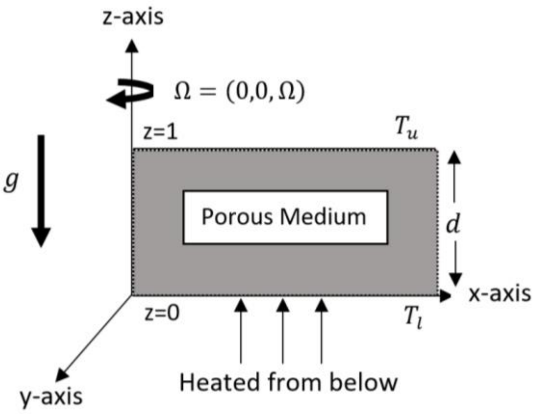

Fig. 1. Physical model and coordinate system

The system is rotated around vertical axis with a constant angular velocity, $\Omega = (0,0,\Omega)$. The mechanical and thermal properties is assumed to be isotropic in the horizontal direction. The density of the porous depend linearly on temperature which given by

$$\rho = \rho_0\big(1 - \alpha(T - T_0)\big), \tag{1}$$

where, ρ and T_0 is the reference density and temperature respectively and α is the thermal expansion coefficient.

The momentum equation is governed by using a Darcy model and the density variation is assumed to follow the Boussinesq approximation which are given by

$$\nabla.\vec{w} = 0, \tag{2}$$

$$\nabla.B = 0, \tag{3}$$

$$\frac{\rho_0}{\phi}\frac{\partial \vec{w}}{\partial t} + \frac{\mu}{\vec{K}}\cdot\vec{w} + 2(\vec{w}\times\Omega) + \nabla p - \rho\vec{g} - \mu_m B\cdot\nabla B = 0, \tag{4}$$

$$\gamma\frac{\partial \mathrm{T}}{\partial \mathrm{t}} + (\vec{w}\cdot\nabla)T = \kappa_{T_z}(\nabla^2 T), \tag{5}$$

$$\frac{\partial \mathrm{B}}{\partial \mathrm{t}} + (\vec{w}\cdot\nabla)B = (B\cdot\nabla\vec{w}) + \gamma_m(\nabla^2 B), \tag{6}$$

where, $\vec{w} = (u, v, w)$ is the velocity vector, p is the pressure, ϕ is the porosity, $\vec{K} = K_x(\vec{\imath\imath} + \vec{\jmath\jmath}) + K_z\big(\vec{kk}\big)$ is the is the permeability tensor, Ω is the angular velocity, μ is the dynamic viscosity, μ_m is the magnetic permeability, γ is the ratio of heat capacity, T is the temperature, κ_{T_z} is the vertical thermal diffusivity, γ_m is the magnetic viscosity.

2.1 Basic State

The basic state of the porous at rest is given by

$$\vec{\mathrm{w}}_b = (0,0,0), \quad p = p_b(z), \quad T = T_b(z), \\ B = B_b(z), \quad \rho = \rho_b(z), \tag{7}$$

where, subscript b represent the basic.

Substitute Eq. (7) into Eqs. (1)-(6) to obtain

$$\frac{dp_b}{dz} = -\rho g, \quad \frac{d^2T_b}{dz^2} = 0, \quad \rho_b = \rho_0[1 - \alpha(T_b - T_0)], \tag{8}$$

The conduction state temperature takes the form

$$T_b = \frac{-\Delta T}{d} z + T_l, \tag{9}$$

2.2 Perturbed State

The infinitesimal perturbation for the basic state are given by

$$\vec{w} = \vec{w}_b + \vec{w}', \quad \vec{T} = \vec{T}_b + T', \quad \vec{B} = \vec{B}_b + B',$$
$$\vec{p} = \vec{p}_b + p', \quad \vec{\rho} = \vec{\rho}_b + \rho', \tag{10}$$

where, primes represent the perturbation quantities. Using Eq. (10) into Eqs. (1)-(6) and eliminate the pressure term by applying the curl curl identity on the momentum equation of the porous medium. Then, transform the resulting equations using the following transformation

$$(x', y', z') = (x^* d, y^* d, z^* d), \quad t = \frac{\gamma d^2 t^*}{\kappa_{T_z}}, \quad T' = (\Delta T)T^*,$$
$$B' = B_b B^*, \quad (u', v', w') = \left(\frac{\kappa_{T_z} u^*}{d} \frac{\kappa_{T_z} v^*}{d} \frac{\kappa_{T_z} w^*}{d}\right) \tag{11}$$

to get

$$\left[\frac{Da}{\gamma Pr}\frac{\partial}{\partial t}\nabla^2 + \nabla_h^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^{*2}}\right] w^* - Ra\nabla_h^2 T^* + \sqrt{Ta}\frac{\partial \zeta}{\partial z} + B\frac{Pr}{Pm}\nabla_h^2 \frac{\partial B^*}{\partial z} = 0, \tag{12}$$

$$\left[\frac{\partial}{\partial t^*} - \eta\nabla_h^2 - \frac{\partial^2}{\partial z^{*2}} + \vec{u}^* \cdot \nabla\right] T^* - w^* = 0, \tag{13}$$

$$\left[\frac{\partial}{\partial t^*} - \nabla^2\right] B^* - \frac{Pm}{Pr}\frac{\partial w^*}{\partial z} = 0, \tag{14}$$

where, $Da = K_z/d^2$ is the Darcy number, $Pr = \mu/\rho_0\kappa_{T_z}$ is the Prandtl number, $Pm = \gamma_m/\kappa_{T_z}$ is the magnetic Prandtl number, $Ra = \alpha g \Delta T d K_z/\nu\kappa_{T_z}$ is the thermal Rayleigh number, $\nu = \mu/p_0$ is the kinematic viscosity, $Ta = 2\Omega K_z/\nu$ is the Taylor number, ζ is the vorticity, $B = \mu_m B_b^2 d^2/\mu\gamma_m$ is the Chandrasekhar number, $\xi = K_x/K_z$ and $\eta = \kappa_{T_x}/\kappa_{T_z}$ is the mechanical and thermal anisotropy parameter respectively and $\epsilon_n = \phi/\gamma$ is the normalized porosity.

2.3 Linear Stability Analysis

We applied the linear stability analysis on Eqs. (12)-(14) by eliminating the nonlinear term on the system and solved the resulting equations using normal mode expansion which defined as

$$(w,T,\zeta)=\big(W(z),\theta(z),\zeta(z)\big)exp\big[i\big(a_x+a_y\big)+\sigma t\big], \tag{15}$$

to get

$$\left[\frac{\sigma Da}{Pr}(D^2-a^2)+\left(\frac{D^2}{\xi}-a^2\right)+BD^2\right]W+a^2Ra\theta+\sqrt{Ta}D\zeta=0, \tag{16}$$

$$[\sigma-(D^2-\eta a^2)]\theta-W=0, \tag{17}$$

$$\frac{\sigma}{Pr}\zeta-\left(D^2-a^2+\frac{1}{\xi}\right)\zeta-\sqrt{Ta}DW=0, \tag{18}$$

where, a_x, a_y represent the horizontal wave number in x −direction and y −direction respectively, σ is the growth rate parameter, $D=d/d_z$ and $a^2=a_x^2+a_y^2$.

2.3 Numerical Solution

Equations (16)-(18) are solved numerically by Chebyshev Tau method with QZ algorithm subjected to upper free conducting boundary condition which are given by

$$W=\theta=\zeta=0 \text{ at } z=0,1. \tag{19}$$

The map has been transformed and we get $z\in[0,1]$ into $x\in[-1,1]$ using $x=2x-1$ and obtained

$$\frac{\partial}{\partial z}=2\frac{\partial}{\partial x}=D, \qquad x\in[-1,1]. \tag{20}$$

Given that

$$y_r(x)=\sum_{k=0}^{M-1}\alpha_{kr}T_k(x), \qquad 1\le r\le 6 \tag{21}$$

where, $T_k(x)$ is the first kind Chebyshev polynomials and variables y_r defined by

$$y_1=W, \quad y_2=DW, \quad y_3=\theta, \quad y_4=D\theta, \quad y_5=\zeta, \quad y_6=D\zeta. \tag{22}$$

Substitute (22) into Eq. (16)-(18) together with boundary condition (19) to obtain a system of six ordinary differential equations and boundary conditions. The eigenvalue problem can be rearranged as

$$\frac{dY}{dx}=HY+\sigma JY \tag{23}$$

where, H and J are real matrix of order 6×6. We can reduce (23) into $EX = \sigma FX$ where E and F are matrices in the block forms with boundary conditions be in the $1Mth, 2Mth, \ldots, 6Mth$ rows of E and F. This system can be solved using QZ algorithm in the FORTRAN programming.

3. Results and Discussion

In this paper, we have considered a Darcy model in order to study the convection problem in a horizontal anisotropic porous medium bounded between upper and lower free conducting plate in the presence of vertical magnetic field and Coriolis force. The linear stability is applied and the resulting eigenvalue obtained are solved numerically using Chebyshev Tau method with QZ algorithm. The natural stability curve for various parameter which included Taylor number, Chandrasekhar number, thermal and mechanical anisotropic parameter are plotted in order to observe its effect on Rayleigh number, Ra. The obtained results are presented graphically to show how the various parameters reacts on the Rayleigh number, Ra versus the wave number, a in Figure 2-5, respectively. In addition, the critical Rayleigh number, Ra_c which is the minimum value of the Rayleigh number will be calculated and displayed on Figure 6-8.

Figure 2 represents the graph of Ra versus a for various Taylor numbers, Ta with respect to fixed values of $B = 1, \xi = 0.5, \eta = 0.1, Pr = 1$. Increasing the value of Ta cause the value of Ra to increase too. Since Ta is directly proportional to the angular velocity, Ω, the higher the value of Ta, the higher the Coriolis force acting on the system. Thus, the effect of Coriolis force is to delay the onset of stationary convection. In the absence of magnetic field $(B = 0)$, the onset of convection is more advanced as compared with the presence of magnetic field $(B = 1)$ at $Ta = 20,50,500$. Therefore, the combined effect of magnetic field and Coriolis force can make the system become more stabilize.

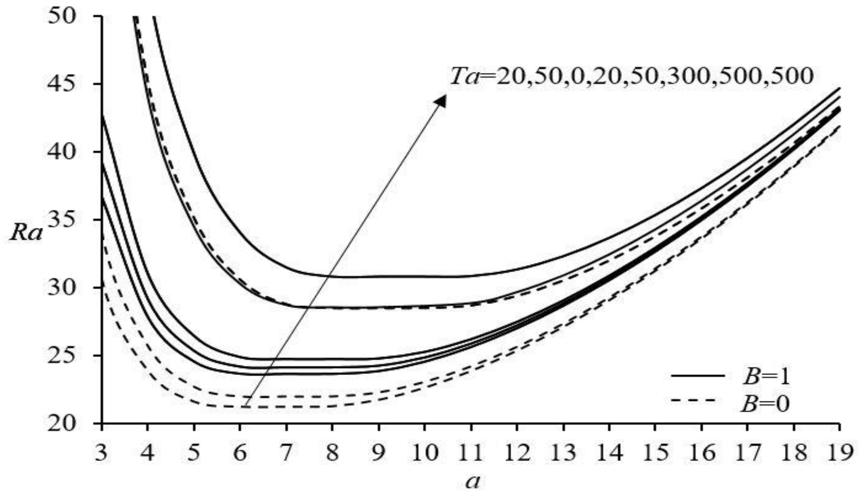

Fig. 2. The neutral stability curve Ra versus a for various values of Ta

The influenced of Chandrasekhar number, B towards Ra for a fixed value of $Ta = 100, \xi = 0.5, \eta = 0.1, Pr = 1$ is revealed in Figure 3. We observed that Ra decreasing with an increase in the value of Chandrasekhar number, B. It is also found that the increasing value of B will increase the value of Ra when wave number, $a = 10$ and therefore magnetic field parameter has the stabilizing effect on the stability of the system. This finding was also reported by Dhananjay *et al.,* [22]. In the absence of magnetic field $(B = 0)$, the stationary convection set up earlier.

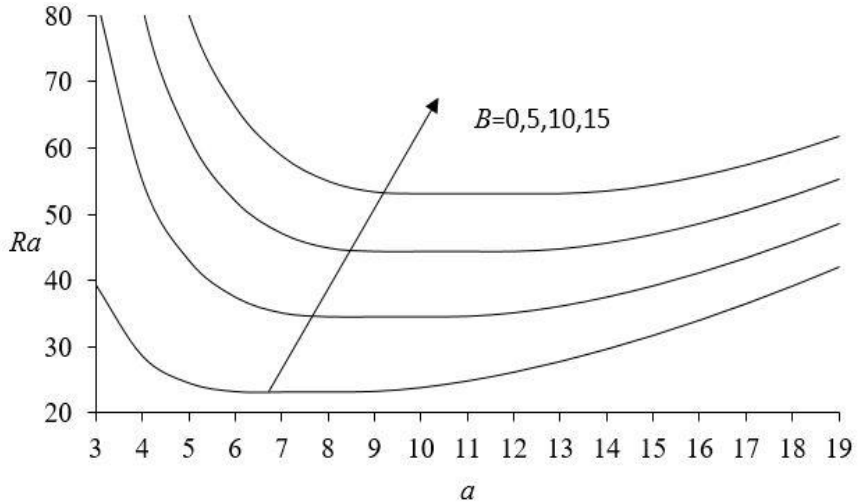

Fig. 3. The neutral stability curve Ra versus a for various values of B

Figure 4 shows the effect of increasing mechanical anisotropic parameter, ξ on the value of Ra at fixed value of $Ta = 100, B = 1, \eta = 1, Pr = 1$. It can be clearly seen that, with the increasing the value of ξ, the value of Ra will decreases. This is due to increasing ξ is correspond to increase in horizontal permeability of the porous medium and thus enhance the movement of heat vertically upward. As a result, steady state convection set up earlier. For the isotropic case, which is at $(\xi = 1, \eta = 1)$, the threshold of stationary convection is delay as compared at $\xi > 1$ and set up earlier compared at $\xi < 1$.

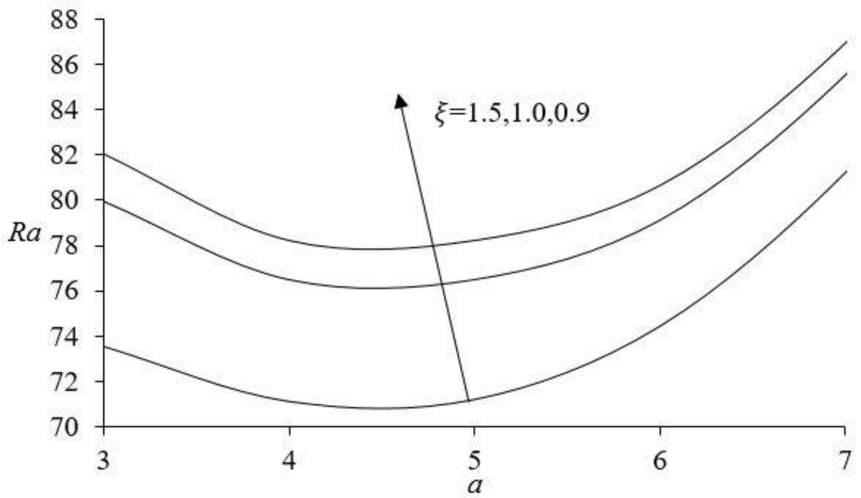

Fig. 4. The neutral stability curve Ra versus a for various values of ξ

Figure 5 displayed the neutral stability curve Ra versus a for various values of η at a fixed value of $Ta = 100, B = 1, \xi = 1, Pr = 1$. It is shown that, the effect of increasing η is to stabilize the system. Increasing the value of η will lead to increase in value of Ra. From the graph, the onset of stationary convection is delay with the presence of anisotropic porous medium when $\eta < 1$. It also have been proved by Malashetty and Swamy [12]. Hence, the anisotropic porous medium effect plays an important role in stabilize the system.

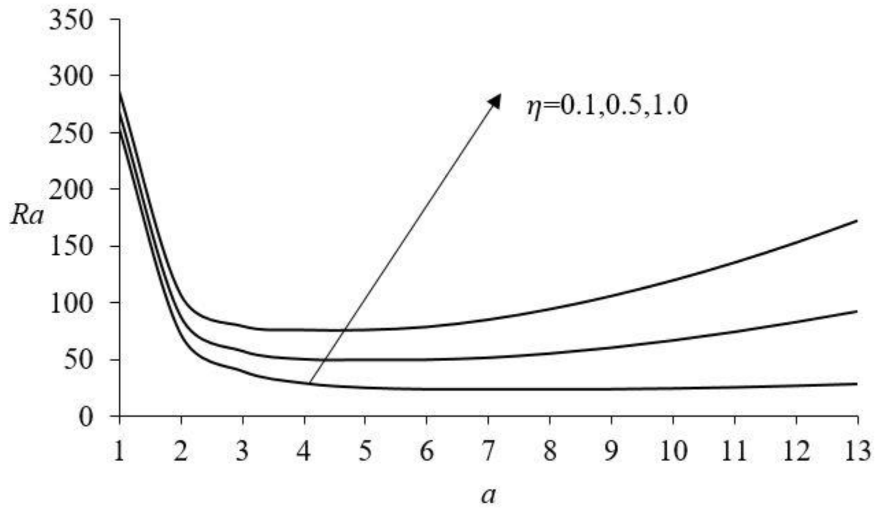

Fig. 5. The neutral stability curve Ra versus a for various values of η

Further, we also consider the critical Rayleigh number, Ra_c which has been obtained and are depicted in the Figure 6-8, respectively. The relation between ξ, η and B has been calculated and displayed in the figures shown. Figure 6 shows the plots of critical Rayleigh number, Ra_c in porous medium with Chandrasekhar number, B for different values of Ta. From Figure 6, we found that on increasing both B and Ta, the critical value of Rayleigh number, Ra_c increases. It can be seen that on the onset of the convective, the Coriolis force effect will stabilize in the sense that the value of the critical Rayleigh number, Ra_c increases with the existence of the magnetic field as reported by Muddamallappa *et al.,* [23].

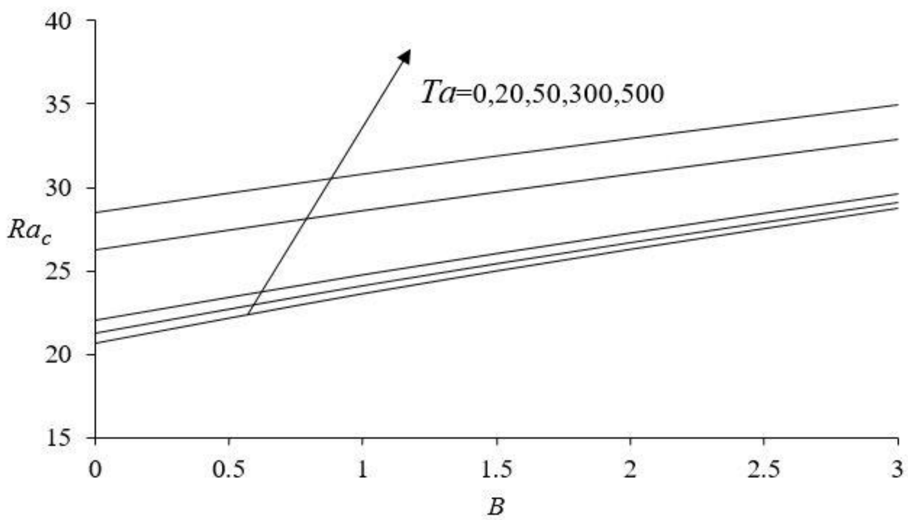

Fig. 6. The critical Rayleigh number, Ra_c versus B for various values of Ta

Figure 7 shows the critical Rayleigh number, Ra_c in porous medium versus mechanical anisotropy parameter, ξ for different values of Ta when $B = 1, \eta = 1, Pr = 1$. In the figure, the values of Ra_c decreases when Ta increases, respectively. This result can be compared with result from Epherre [4] that concerned about the reducing of the critical Rayleigh number when $Ta = 0$. However, in the present results, it is concerned that the critical Rayleigh number approach to minimum value with increasing of ξ. Thus, this figure also revealed the stabilizing effect of magnetic field and Coriolis force.

The graph of critical Rayleigh number, Ra_c in porous medium against the Taylor number, Ta with various values of thermal anisotropy parameter, η is shown in Figure 8. It is observed that Ra_c increases as η increases, respectively. The Figure 8 also shows the critical Rayleigh number increases when $\eta = 1.0$ compared to other values of η. In addition, the thermal anisotropy parameter, η increases with the increase in the Taylor number, Ta that have the effect of magnetic field and Coriolis force in order to stabilize the system. Besides, as verified by Shatha [24], these results are strengthening the fact that the linear instability analysis is exactly controlling the physics for the onset of convection.

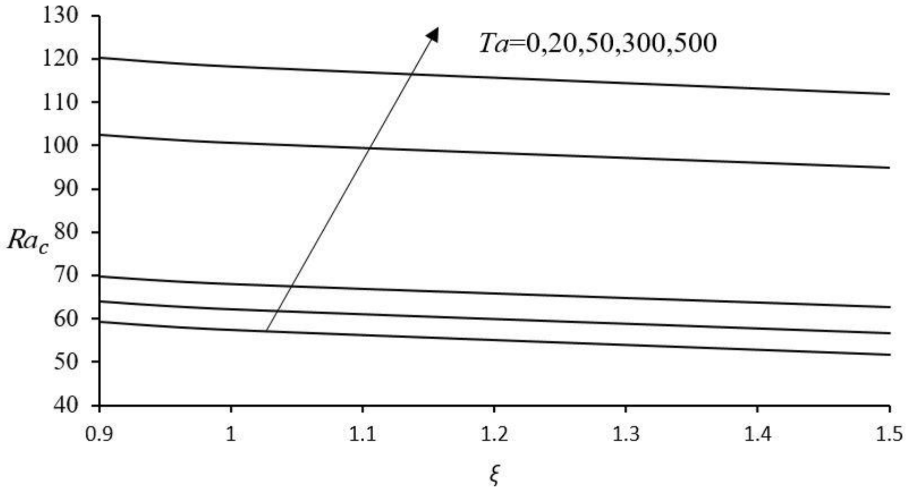

Fig. 7. The critical Rayleigh number, Ra_c versus ξ for various values of Ta

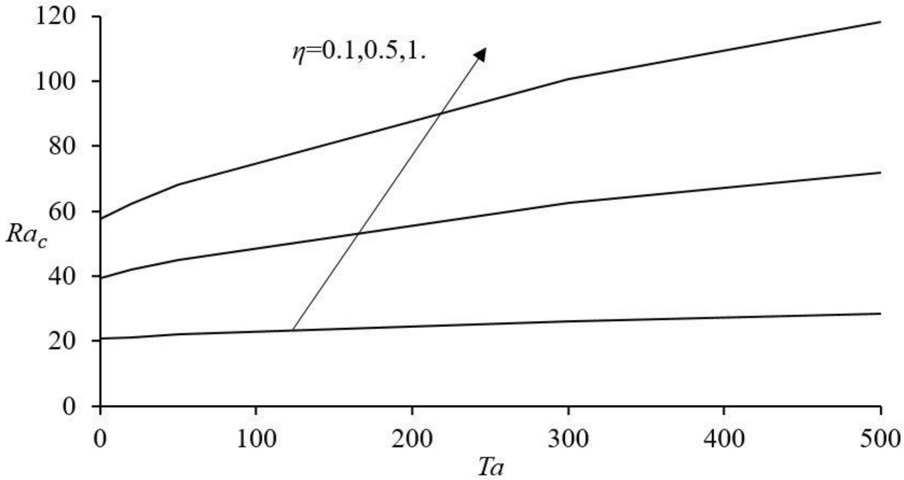

Fig. 8. The critical Rayleigh number, Ra_c versus Ta for various values of η

4. Conclusions

The threshold of stationary thermal convection in a horizontal anisotropic porous medium which is heated from below in the presence of Coriolis force and magnetic field is studied analytically by using linear stability analysis. The eigenvalue problem of the perturbed state obtained from a normal mode analysis are solved using Chebyshev Tau method with QZ algorithm. In this study, the effect of magnetic field is to inhibit the arrival of convection in the system. The combined effect of magnetic field and Coriolis force become more stabilize. The critical Rayleigh number, Ra_c increases when both effect of magnetic field and Coriolis force exist and it will delay the system of the onset of the convective. Based on the results obtained, mechanical anisotropic parameter, ξ act as stabilizer on the system while thermal anisotropy parameter, η act as destabilizer on the system.

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