

Flow Characteristics of Transition Region Between Laminar and Turbulent Gas Flows Through Micro-Tubes

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ABSTRACT

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Flow characteristics in the transition region between laminar and turbulent gas flows through micro-tubes were experimentally investigated. The experiments were performed for nitrogen gas flowing through two stainless steel micro-tubes of $D = 124$ and $162 \mu\text{m}$ and two fused silica micro-tubes of $D = 100$ and $151 \mu\text{m}$. The Mach numbers and gas bulk temperatures at the inlet and locations near the exit were also obtained by measuring stagnation temperatures, stagnation pressures, pressures at locations near exit and mass flow rates. The average Fanning friction factors also were obtained. The mass flow rate levels off in the transition flow region for micro-tubes as the stagnation pressure increases. The Mach number increases with an increasing Reynolds number in the laminar flow region. However Mach number decreases when Reynolds number goes higher in the transition flow region since the mass flow rate stays nearly constant with an increase of the stagnation pressure in the transition flow region. On the other hand, the bulk temperature increases in the transition flow region. This was validated by measuring wall temperature of micro-tubes whose outer walls are thermally insulated. The decrease in Mach number and the increase in the bulk temperatures in the transition flow region are the dominant scaling effect for the smallest diameter micro-tubes.

Keywords:

Adiabatic wall temperature, Mach number, friction factor

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1. Introduction

An increasing number of applications using micro-electro mechanical systems (MEMS) has created a serious need to understand the flow characteristics in micro-geometries. Since the experimental work on microchannel gas flows of Choi *et al.*, [1], many experimental and numerical studies have been undertaken to understand the flow and heat transfer characteristics of gas flow in microchannels. In the case of microchannel gas flows, it is found that the flow accelerates due to the gas expansion and thermal energy conversion into kinetic energy. This results in a static temperature decrease of the gas [2]. Flow characteristics of micro gas flow in laminar and turbulent flow regions were numerically and experimentally investigated in literature [3-11]. In most of the studies, the

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values of Reynolds number of the transitional regime were reported and compared with the conventional value ($2300 < Re < 4000$). However, there also seems to be no study to investigate flow characteristics in the transition region between laminar and turbulent gas flow through micro-tubes. This is the motivation of the present experimental study with micro-tubes in which the diameters range from 100 to 163 μm .

2. Experimental Setup

2.1 Micro-Tube

In the present study, the experiments were carried out using two stainless steel micro-tubes and two fused silica micro-tubes. The tubes inner diameters were measured by flowing water in the tubes. The details about the diameter measurement are documented in our previous paper by Asako *et al.*, [2]. The diameters were measured as 124 μm (SST1) and 162 μm (SST2) for the two stainless steel micro-tubes and 100 μm (FST1) and 151 μm (FST2) for the two fused silica micro-tubes. In the case of the stainless steel micro-tubes, in order to measure local pressures two static pressure tap holes on the micro-tube wall at intervals of 5 and 10 mm from the outlet were fabricated by Electrical Discharge Machining (EDM). The interval between the two pressure holes, measured with an Universal Measuring Microscope (Mitutoyo, MF-UD505B), is 5 mm. The diameters of the static pressure holes measured with a microscope (Keyence, VK-8500) are about 50 μm as shown in Figure 1. In the case of the fused silica tube of $D = 151 \mu\text{m}$ and $L = 61 \text{ mm}$, in order to measure the wall temperature, two thermocouples (bare wire type-K) are attached to the micro-tube wall at the location of 51 and 56 mm along the length with a high conductivity epoxy. The tube dimensions are also listed in Table 1.

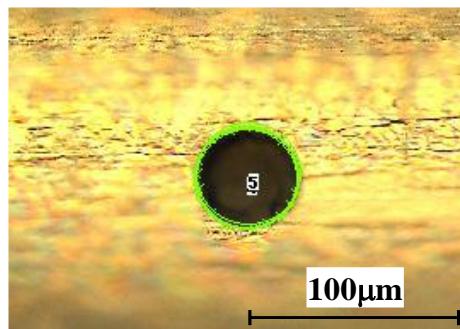


Fig. 1. Microscopic image of a static pressure hole ($D = 124 \mu\text{m}$)

Table 1
 Micro-tube dimensions

Micro-tube	D [μm]	Outer Diameter [μm]	L [mm]	Pressure holes		Wall temperatures	
				x_1 [mm]	x_2 [mm]	x_1 [mm]	x_2 [mm]
SST1	124	340	50	40	45	-	-
SST2	162	370	100	85	95	-	-
FST1	100	350	28	-	-	-	-
FST2	151	350	61	-	-	51	56

2.2 Configuration of Experimental Setup

The schematic diagram of the experimental setup is shown in Figure 2. Nitrogen is used as the test gas in the present study. Compressed nitrogen gas flows into a micro-tube through a single stage regulator, and a desiccant tube, into a flow meter (Kofloc 3100, 0~1 ℓ/min for $D = 124 \mu\text{m}$ and 0~2ℓ/min for $D = 162 \mu\text{m}$, $100 \mu\text{m}$ and $151 \mu\text{m}$), and the mass flow rate is measured at the upstream section of the micro-tube. A gauge pressure transducer (Krone KDM30, 0~1 MPa) and a thermocouple (K sheathed type) were inserted into the chamber at the upstream section of the micro-tube and the gas pressure and temperature in the chamber are measured.

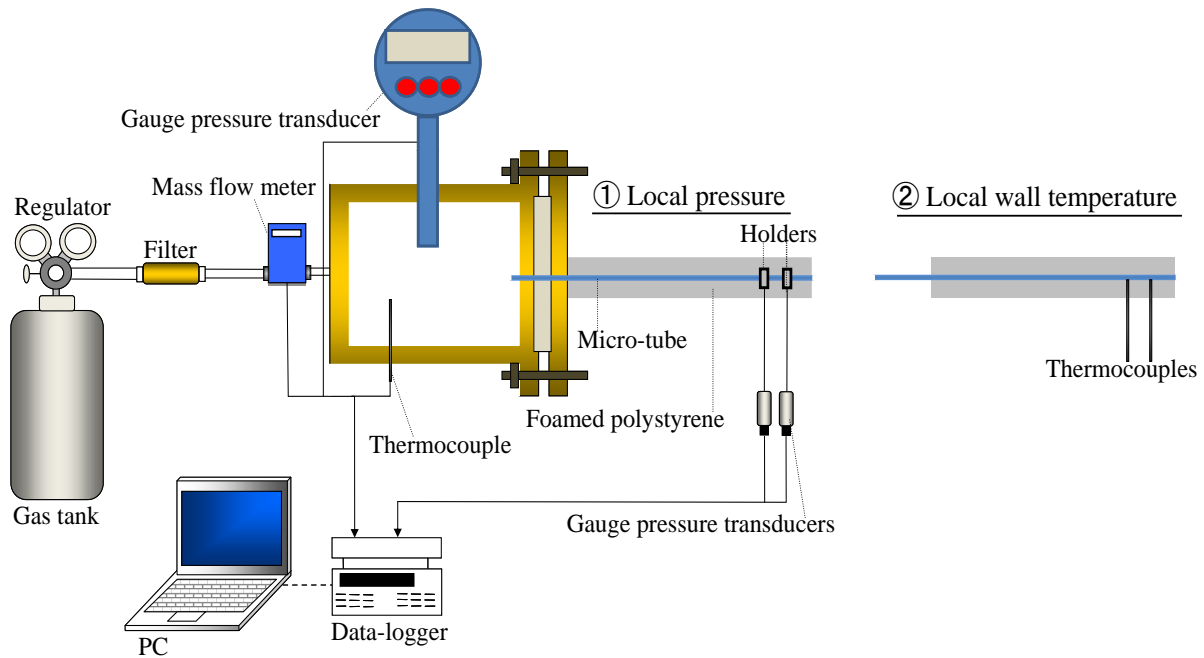


Fig. 2. Schematic diagram of experimental setup

The measured gas temperature and pressure are considered to be the stagnation temperature, T_{stg} and the stagnation pressure, p_{stg} since the gas flowed into the chamber stagnates. The gas flowed through the micro-tube was discharged into the atmosphere. The static pressures at two locations near the outlet were measured with the pressure transducers (Valcom VESX, 0~500 kPa) connected to the pressure tap holes via pressure tap holders for the stainless steel micro-tubes (① in Figure 2.). The wall temperature were measured with thermocouples (K bare wire type) attached to the micro-tube wall at two locations near the outlet with a high conductivity epoxy (② in Figure 2.). The micro-tube exterior is covered with foamed polystyrene to avoid heat gain or loss from the surrounding environment. The signals from the pressure transducers and the flow meter are collected by a PC through a data acquisition system (Eto Denki, CADAC21).

3. Data Reduction

3.1 Reynolds Number and Mach Number

Reynolds number and Mach number are defined as

$$Re = \frac{\rho u D}{\mu} = \frac{4\dot{m}}{\pi \mu D} \quad (1)$$

$$Ma = \frac{u}{a} = \frac{u}{\sqrt{\gamma RT}} \quad (2)$$

where, ρ is the density, μ is the viscosity, \dot{m} is the mass flow rate, a is speed of sound, γ is the heat capacity ratio and R is the gas constant.

In order to estimate flow regions of laminar, transitional and turbulent flow by measuring mass flow rates and wall temperatures, an average Fanning friction factor between the inlet and outlet considering the effect of a decrease in temperature is employed in the present study. Kawashima and Asako [12] defined the four multiples of the Fanning friction factor for an adiabatic wall (Fanno flow) as

$$f_f = \frac{4\tau_w}{\frac{1}{2}\rho u^2} = \frac{2D}{p} \left(\frac{dp}{dx} \right) - \frac{2Dp}{\rho^2 u^2 RT} \left(\frac{dp}{dx} \right) - \frac{2D}{T} \left(\frac{dT}{dx} \right) \quad (3)$$

where, τ_w is shear stress on a wall.

The temperature in Eq. (3) can be determined by solving the following quadratic equation obtained from the total temperature balance between given two points (inlet and x) [12]

$$\alpha \frac{\rho_{in}^2 u_{in}^2 R^2}{2c_p p^2} T^2 + T - \left(T_{in} + \frac{u_{in}^2}{2c_p} \right) = 0 \quad (4)$$

where, c_p is the specific heat at constant pressure and the inlet values of velocity, density and temperature, ρ_{in} , u_{in} and T_{in} are obtained with isentropic process between the inlet and stagnation area under the assumption of ideal gas [13]. α is the kinetic energy loss coefficient which is proposed to be 2 for laminar and 1 for turbulent flows respectively.

The temperature at x is a function of the pressure at x for an adiabatic wall. Substituting the temperature, T obtained by Eq. (4) into Eq. (3) and integrating Eq. (3) between two pressure ports (x_1 and x_2), the following average Fanning friction factor can be obtained as [14]:

$$f_{f,ave} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} f_f dx = \frac{D}{x_2 - x_1} \left\{ \int_{x_1}^{x_2} \left(\frac{2}{p} dp \right) - \int_{x_1}^{x_2} \left(\frac{2p}{\rho^2 u^2 RT} dp \right) - \int_{x_1}^{x_2} \left(\frac{2}{T} dT \right) \right\} \quad (5)$$

$$= \frac{D}{x_2 - x_1} \left[-2 \ln \frac{p_1}{p_2} + 2 \ln \frac{T_1}{T_2} - \frac{1}{\left(\rho_{in}^2 u_{in}^2 R \times \left(T_{in} + \frac{u_{in}^2}{2c_p} \right) \right)} \right. \\ \left. \times \left\{ \frac{p_2^2 - p_1^2}{2} + \frac{B^2}{2} \ln \frac{p_2 + \sqrt{p_2^2 + B^2}}{p_1 + \sqrt{p_1^2 + B^2}} + \frac{1}{2} \left(p_2 \sqrt{p_2^2 + B^2} - p_1 \sqrt{p_1^2 + B^2} \right) \right\} \right]$$

where,

$$B^2 = 4 \times \alpha \frac{\rho_{in}^2 u_{in}^2 R^2}{2c_p} \times \left(T_{in} + \frac{u_{in}^2}{2c_p} \right) \quad (6)$$

4. Result and Discussion

The experiments on nitrogen gas flow were performed with two stainless steel micro-tubes, SST1 and SST2, and two fused silica micro-tubes, FST1 and FST2 as tabulated in Table 1. The stagnation pressure ranged from 120 to 1500 kPa with 10 ~ 50 kPa intervals. The obtained mass flow rate, Reynolds number and inlet Mach number are tabulated in Table 2.

Table 2
 Experimental results

Micro-tube	D [μm]	p_{stg} [kPa]	\dot{m} [$\times 10^{-6}$ kg/s]	Re	Ma_1	Ma_2
SST1	124	120-800	0.20-6.65	113-3822	0.04-0.33	0.04-0.42
SST2	162	120-800	0.29-9.52	129-4188	0.04-0.35	0.04-0.50
FST1	100	150-1550	0.33-12.4	236-8866		
FST2	151	120-800	0.29-10.9	136-5131		

4.1 Measured Data: Mass Flow Rate, Pressure and Wall Temperature

The measured mass flow rates for all micro-tubes are plotted in Figure 3 as a function of the stagnation pressure. The mass flow rate increases with the increase in the stagnation pressure since the gas at the micro-tube outlet is discharged into the atmosphere. It increases with a low slope in the range " $A \leq p_{\text{stg}} \leq B$ " in the figure which is a region predicted to be a transitional flow by their Reynolds number. In the range of $p_{\text{stg}} > B$ which is predicted to be a turbulent flow by its Reynolds number, the mass flow rate increases with a lower slope than that of $p_{\text{stg}} < A$, and a higher slope than that of " $A \leq p_{\text{stg}} \leq B$ ".

The local pressures at two locations near the outlet of SST1 and SST2 were measured and pressure gradients over the length are plotted in Figure 4 as a function of Reynolds number. The pressure gradient is large with an increase in Reynolds number. Note that the pressure gradient of incompressible flow remains constant. The similar phenomenon can be observed for the smaller and shorter tube since the compressibility effect is more dominant for the smaller diameter and shorter length tube. In the range of " $A \leq Re \leq B$ " which is a region predicted to be transitional flow by their Reynolds number, the pressure gradient increases steeply since the mass flow rate in that range slightly increases with an increase in the stagnation pressure as shown in Figure 3.

The measured wall temperatures at two locations near the outlet of FST2 are plotted in Figure 5 as a function of the stagnation pressure. The inlet Mach numbers obtained by Eq. (2) are also plotted in the figure. In the case of gas flow through micro-tube with an adiabatic wall, the gas bulk temperature strongly depends on gas velocity since thermal energy converts into kinetic energy. The adiabatic wall temperature qualitatively has the same trend as that of the gas bulk temperature [15]. Therefore, the measured wall temperature distributions have an opposite tendency to the inlet Mach number as shown in Figure 5. In the range of " $A \leq p_{\text{stg}} \leq B$ " ($2404 \leq Re \leq 3022$) which is a region predicted to be a transitional flow by their Reynolds number, the measured wall temperatures increase since the kinetic energy converts into thermal energy due to a decrease in Mach number. And then, in the range of $p_{\text{stg}} > B$ ($Re > 3022$) which is a region changed to turbulent flow and choked

flow simultaneously, both the inlet Mach number and wall temperature remain nearly constant. As a result of that, the phenomenon of flow transition and choking of micro-tubes can be estimated by measuring adiabatic wall temperature.

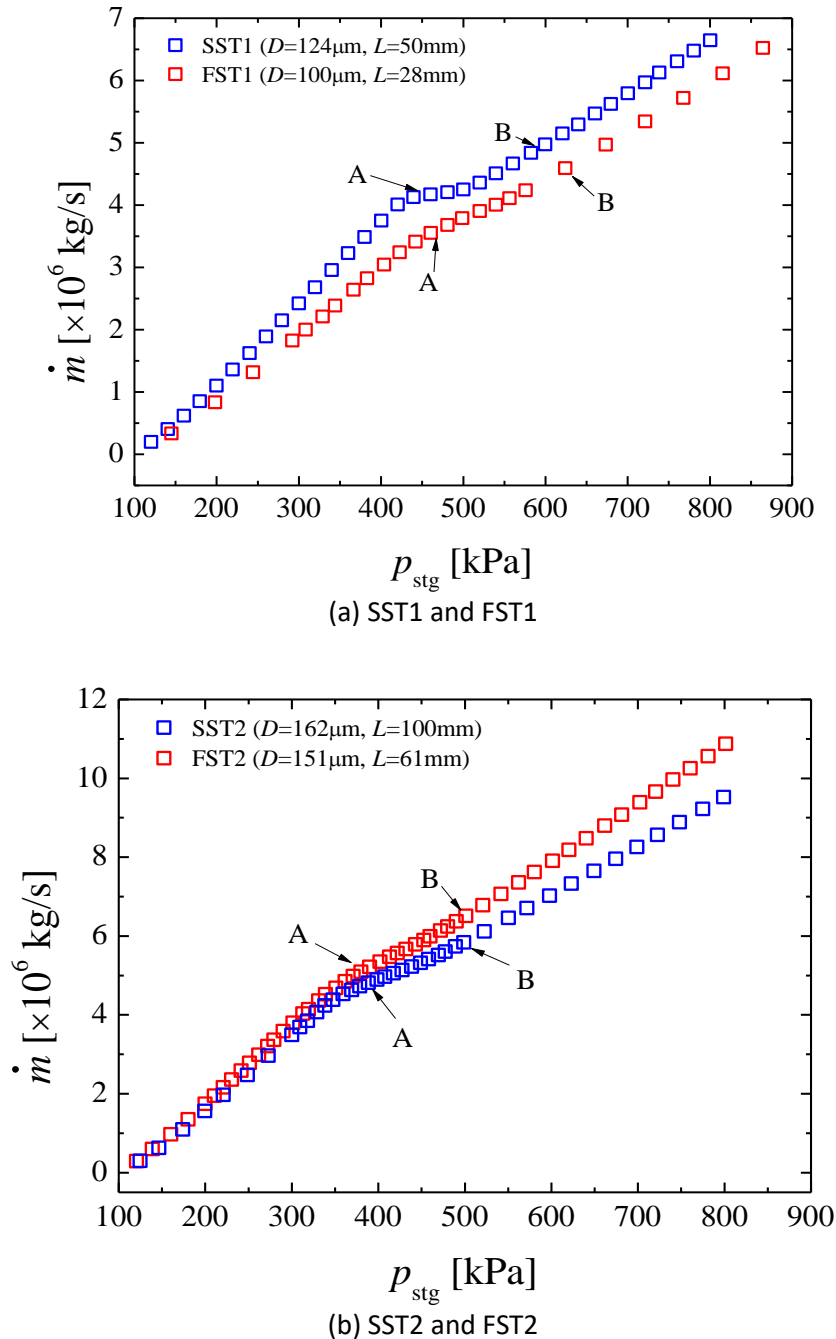


Fig. 3. Mass flow rate vs p_{stg} for (a) SST1 and FST1; (b) SST2 and FST2

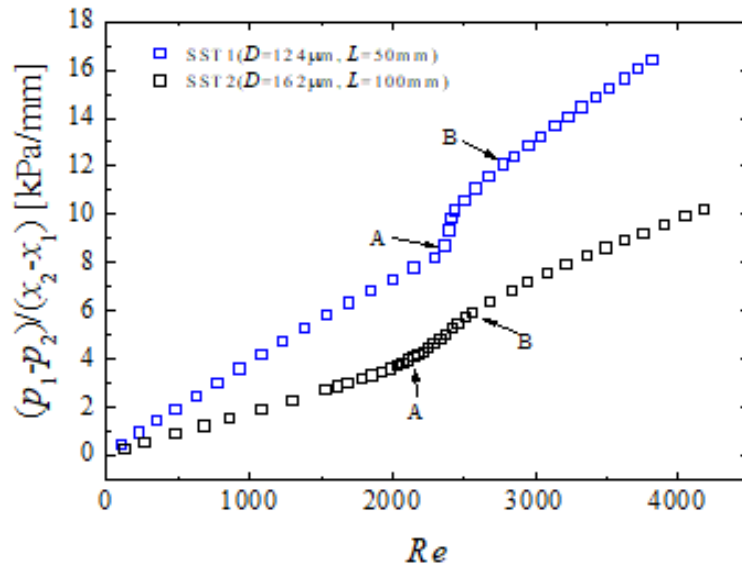


Fig. 4. Pressure gradient along the length vs Re

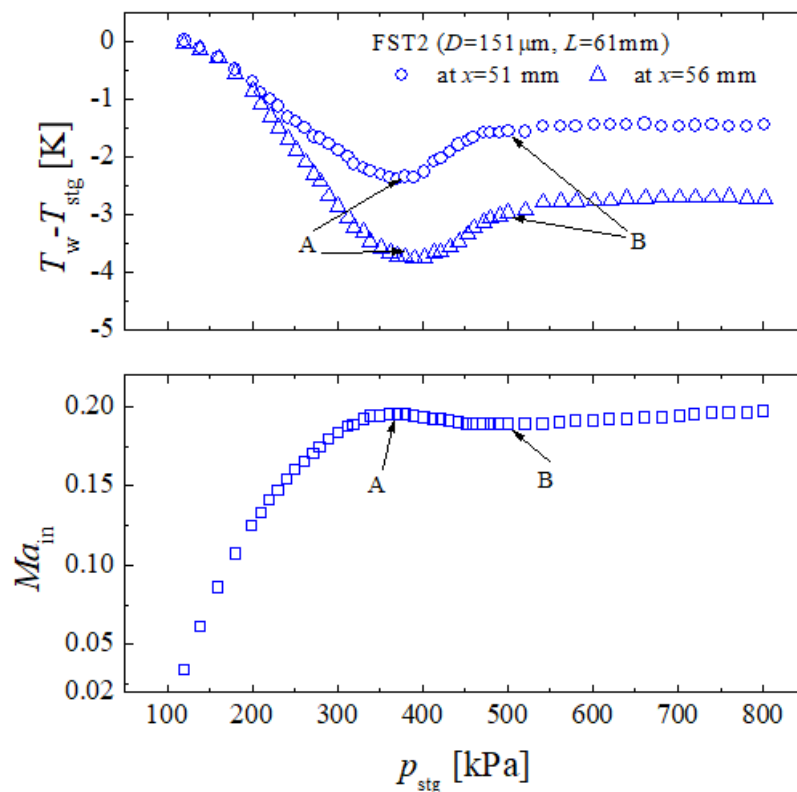


Fig. 5. Wall temperatures and inlet Mach numbers vs Re

4.2 Obtained Values: Mach Number, Gas Bulk Temperature and Average Fanning Friction Factor

The local Mach numbers obtained for SST1 and SST2 are plotted in Figure 6 as a function of Re . Their corresponding gas bulk temperatures, obtained from Eq. (4) are also plotted in Figure 7. In the range of $Re < "A"$ which is a region predicted to be a laminar flow by its Reynolds number, the Mach numbers increase with an increase in the Reynolds number. And in the range of $"A" \leq Re \leq "B"$ which is a region predicted to be a transitional flow, in the case of SST2 it increases with a lower slope than

that of $Re < "A"$. And in the case of SST1 it decreases. The reason is that the mass flow rate slightly increases or levels off with the increase in the stagnation pressure. In the range of $Re > "B"$ which is a region predicted to be turbulent flow, the flow changes to turbulent flow while the flow becomes choked. The Mach number in that range remains nearly unchanged due to flow choking.

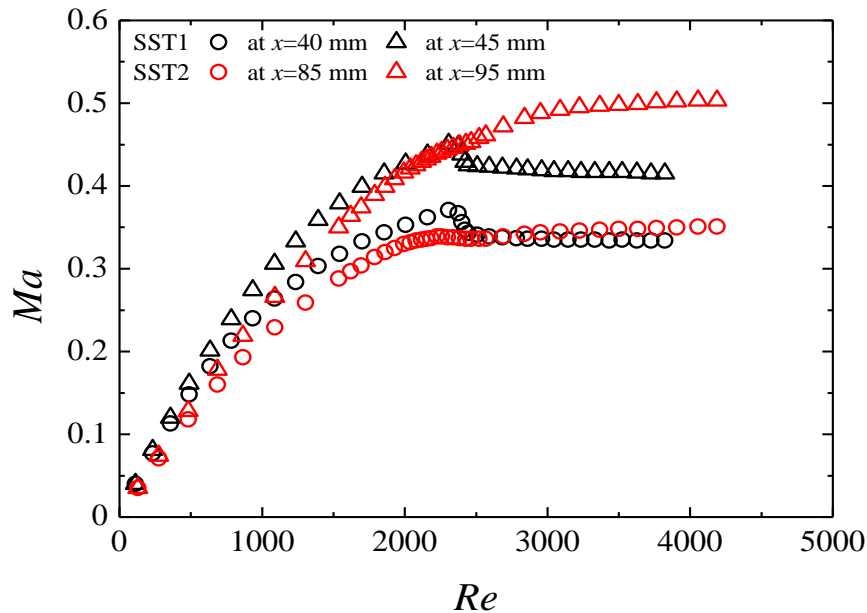


Fig. 6. Mach numbers vs Re

On the other hand, as can be seen in Figure 7, the bulk temperature decreases with an increase in the Reynolds number since kinetic energy converts into thermal energy. In the range of $"A" \leq Re \leq "B"$, it increases since the mass flow rate change of a tube with the smaller diameter is not large considering a constant pressure change. And in the range of $Re > "B"$, the pressure of the micro-tube outlet is higher than the back pressure and the flow is choking (under-expanded) as the stagnation pressure increases. This is a reason why the bulk temperature remains nearly unchanged the same as the Mach number. A decrease in Mach number and an increase in bulk temperature in the transitional flow region are more dominant for smaller diameter and shorten length tubes. For the smaller tube diameter, the flow choking phenomenon occurs at the smaller Reynolds number.

The average Fanning friction factors between the inlet and outlet, $f_{f,ave}$ for all tubes were obtained by Eq. (5). The values of $f_{f,ave}$ are plotted on a Moody chart in Figure 8. The solid line and dotted line in the figures represent the values obtained by the theoretical formula ($f=64/Re$) and $f=0.3164/Re^{0.25}$ (*Blasius* equation) for incompressible flow theory, respectively. As can be seen in the figures the flow transits from laminar flow to turbulent flow in the range of $"A" \leq Re \leq "B"$ the same as conventional tubes. In the laminar flow regime on the figure, the values of $f_{f,ave}$ deviate more and more from that of an incompressible flow with an increasing Mach number because of the compressibility effect. In the turbulent flow region on the figure, the values of $f_{f,ave}$ of SST1 and SST2 are higher than the *Blasius* equation. In that range, Mach number nearly remains constant as shown in Figure 6. Therefore, the effect of surface roughness may be more dominant than that of compressibility. In the case of FST2, the values of $f_{f,ave}$ coincide with the *Blasius* equation. However, in the case of FST1, the values of $f_{f,ave}$ are lower than the *Blasius* equation since the flow becomes choked before changing to turbulent flow.

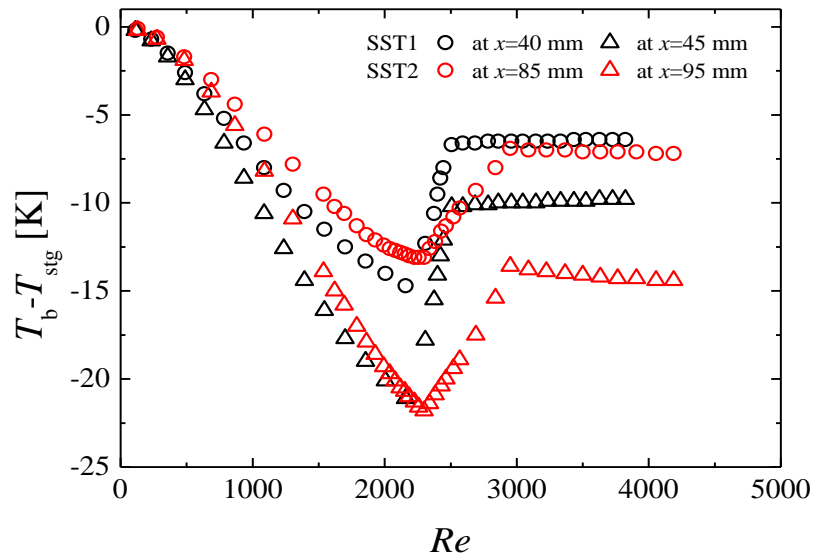


Fig. 7. Gas bulk temperatures vs Re

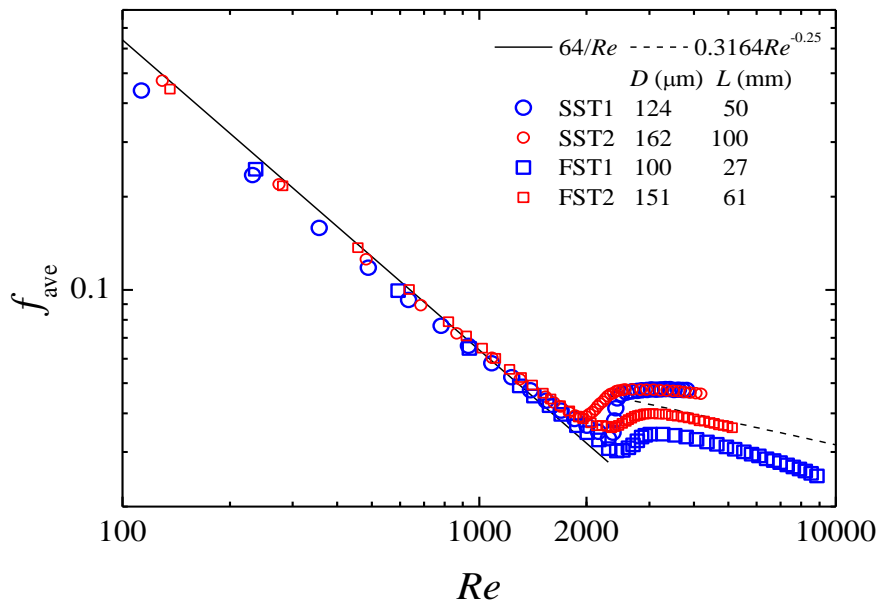


Fig. 8. Average Fanning friction factor vs Re

5. Conclusions

The experimental study to obtain flow characteristics of transition region between laminar and turbulent gas flow through micro-tubes was performed. The following conclusions were reached.

- i. The value of Reynolds number to transit to turbulent from laminar flow ranged from 2000 to 4000, the same as that of a conventional channel.
- ii. In the transitional flow region, the obtained Mach number decreases but the obtained bulk temperature and the measured wall temperature increase with the increase in Reynolds number (stagnation pressure).

- iii. In the case of FST1 ($D = 100 \mu\text{m}$ and $L = 28 \text{ mm}$) with smooth inner wall, the values of $f_{f, \text{ave}}$ are lower than the *Blasius* equation since the flow becomes choked before changing to turbulent flow.

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