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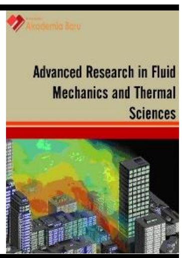

Influence of Velocity and Thermal Slip on the Peristaltic Transport of a Herschel-Bulkley Fluid Through an Inclined Porous Tube

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ABSTRACT

The present paper investigates the impact of velocity slip and thermal slip on the peristaltic transport of a Herschel-Bulkley fluid, flowing through a uniform two-dimensional porous tube under the assumptions of long wavelength and low Reynolds number. The mathematical representations of temperature and velocity fields, pressure gradient, and stream function have been found through the closed-form solutions of the energy and momentum equations. Numerical integration has been employed to compute the frictional force and pressure rise. The influence of relevant parameters in the problem have been discussed and presented graphically. The results reveal the increasing effects of thermal and velocity slip on pressure rise and temperature. Also, trapping phenomena of the Herschel-Bulkley fluid is discussed. The volume of the bolus is observed to increase along with the velocity slip parameter.

1. Introduction

Peristalsis is the fluid flow mechanism in which the fluid is transported through a distensible tube by means of progressive waves of expansion and contraction. This mechanism has attracted many researchers, owing to its important applications in the fields of engineering science and medicine. The motion of chyme through the gastro-intestinal tract, the transport of urine to the urinary bladder, the activity of swallowing of food through the esophagus, mixing of the contents in the bile duct, blood pump in dialysis are just a few of the examples which use the principles of peristalsis. Latham [1] carried out the first investigations on peristaltic transport of urine flow through the ureter. Since then, several researchers have carried out numerical, analytical and experimental studies on peristalsis. Burns and Parkes [2] used the low Reynolds number assumptions along with the linearized boundary conditions to carry out studies on the peristaltic motion in a two-dimensional as well as

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axisymmetric case. Fung and Yih [3] developed a model to help the characterization of the basics of fluid mechanics involved in the process of pumping, which had restrictions on the wave amplitude of peristalsis to be small but had no such limitations on the Reynolds number. Shapiro *et al.,* [4] studied the fluid mechanics of peristaltic pumping in a flexible tube under the assumptions of low Reynolds number and long wavelength. These studies helped throw light on the ureteral reflux and bacterial transport from the bladder to the kidney.

Though all the above investigations aim at understanding the peristaltic motion involved in blood flow and other physiological problems, the studies have been conducted by treating the fluids as Newtonian. Raju and Devanathan [5] initiated the studies on non-Newtonian fluids by considering the power-law model. Inspired by this, several researchers have worked in this direction. Srivastava *et al.,* [6] considered the two-layered model of blood, where the core layer was considered to be a Casson fluid, consisting of a suspension of erythrocytes, and the peripheral plasma layer as a Newtonian fluid. Pandey *et al.,* [7] studied the peristaltic motion of a three-layered power law fluid with different viscosities. Several researchers have investigated the fluid flow using various non-Newtonian models with differing geometries and configurations [8-10]. Blair and Spanner [11] observed that at moderate shear rates, blood flow closely obeys Casson's model. They also reported that Casson's model and the Herschel-Bulkley model for blood does not have much difference over the range in which the Casson model is valid (0-1,00,000 sec^{-1}). However, Herschel-Bulkley model is valid even in the range over which the Casson model ceases to be valid. Moreover, blood is considered to behave both in Newtonian and non-Newtonian ways. Hence, Herschel-Bulkley model describes the blood flow more accurately, as Newtonian model as well as shear-thinning and shear-thickening behavior of blood can also be explained through the model. Vajravelu *et al.,* [12] analysed the peristaltic transport of a Herschel-Bulkley fluid through an inclined tube. Manjunatha *et al.,* [13] researched the peristaltic transport of a three-layered model with different viscosities. Similar studies were carried out by Rajashekhar *et al.,* [14] that explored the two-layered peristaltic flow of blood by means of a Herschel-Bulkley fluid flow model.

While studying the flow of physiological fluids, many researchers have focused on the porous medium of transmission over the recent years. This is because the human lungs, stones in the gallbladder, blood vessels of small radius, etc. act as natural porous media. Also, within the lumen of the coronary artery, fatty cholesterol and blood clots act as a porous medium. This concept was first taken into account by Lukashey [15] who considered porous capillary walls in his study on peristaltic transport of liquid motion. The impact of porous boundaries of the medium on the peristaltic movement of fluid was investigated by Shehawey and Husseny [16]. Their work was extended by Nadeem and Akram [17] for a non-Newtonian model by considering a linear Maxwell model. Ramachandra and Mishra [18] incorporated the Beavers-Joseph and Saffman boundary conditions to investigate the peristaltic flow of a power-law fluid within a porous medium and analysed the results for pseudoplastic and dilatant fluids. Hayat *et al.,* [19], in their studies on Newtonian fluids exhibiting peristaltic movement within a porous medium under the influence of partial slip, observed that the trapped bolus expands with the slip parameter. Khan *et al.,* [20] considered the slip conditions at the boundary of a porous medium to study the peristaltic transport of a non-Newtonian fluid having variable viscous properties. The MHD flow of a Jeffrey fluid over a porous layer was analysed by Sreenadh *et al.,* [21]. Recently, several researchers have studied the impact of boundary conditions on different types of fluids [22-25].

Regulation of body temperature, an essential function of the human circulatory system, is an important factor to be considered while studying the peristaltic motion of physiological fluids. The heat in the body which is produced by the skeletal muscles is removed mainly by the convective heat transfer of blood. The principles of heat transfer have been employed by several researchers to explore information on how the human body transfers heat. Srinivas and Kothandapani [26] carried out studies on Newtonian fluids to analyse the effects of heat transfer during the peristaltic transport, through an asymmetric channel. Further, their work was extended by Srinivas and Gayathri [27] to study the effects of transfer of heat on the peristaltic motion of a Newtonian fluid through a vertical asymmetric channel acting as a porous medium. Studies were carried out by Nadeem and Akbar [28] for a Herschel-Bulkley fluid to analyse the heat transfer effects of peristaltic transport through an inclined tube. They considered the no-slip thermal boundary conditions and obtained the results for Power-law fluids, Newtonian fluids and Bingham fluids as a special case of their model. A Newtonian nanofluid flowing past a Darcy-Brinkman porous medium under the effect of a uniform external Magnetic field was examined by Wakif *et al.,* [29]. The effects of thermal radiation on the convective flow of a fluid in the region of fully developed flow were examined by Prasad *et al.,* [30]. Recently, several authors studied the impact of slip, heat transfer and convective boundary conditions on classical and biological fluids in different geometries and assumptions [31-41].

Considering the above discussions, the current paper aims to analyse the effects of heat transfer, thermal and velocity slips on the peristaltic motion of a Herschel-Bulkley fluid model through an inclined porous tube. Closed form solutions of the energy and momentum equations have been obtained. The frictional force and pressure rise have been computed through numerical integration. The influence of shear stress, Darcy number, angle of inclination, Brinkman number, thermal and velocity slip parameters on pressure rise, pressure gradient, frictional force and temperature profile have been analysed graphically. The results reveal that, velocity and thermal slip have an increasing effect on the pressure rise and temperature. Also, the trapping phenomenon for the Herschel-Bulkley fluid is discussed. It is observed that the volume of the bolus increases with the velocity slip parameter.

2. Mathematical formulation and Closed form solutions

Let us consider a two-dimensional flow of Herschel-Bulkley fluid through a porous tube of radius a inclined at an angle γ with the horizontal surface (Figure 1). The flow of fluid is considered to be axisymmetric. (R, Θ, Z) is the chosen cylindrical coordinate system. A sinusoidal wave train of wavelength λ and amplitude b is taken at both upper and lower walls of the tube. At any axial location, the instantaneous radius of the tube is given by

$$R = H(Z,t) = a + b\sin\left[\frac{2\pi}{\lambda}(Z-ct)\right] \tag{1}$$

We assume the length of the tube to be an integral multiple of the wavelength λ and constant pressure difference across the ends of the tube. Due to unsteady flow in the laboratory frame (R, Θ, Z), we consider a wave frame (r, θ, z) moving away from the fixed frame at a constant velocity *c*. The corresponding transformations between these two frames are defined as

$$r = R,\ \ z = Z - ct,\ \ \psi = \Psi - \frac{R^2}{2},\ \ p(Z,t) = P(z) \tag{2}$$

where *P* and *p* are the pressures and $\Psi$and Ψ are the streamlines in the fixed and wave frames respectively. Pressure is considered to be uniform at any location around the axis of the tube under the assumption of long wavelength approximation.

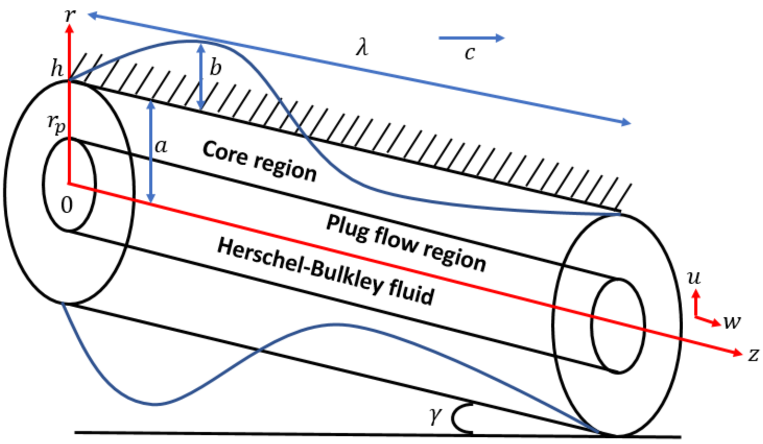

Fig. 1. Geometry of an inclined peristaltic tube with porous wall

Consider the non-dimensional variables as given below [28]

$$\bar{r}=\frac{r}{a},\bar{z}=\frac{z}{\lambda},\bar{t}=\frac{ct}{\lambda},\bar{F}=\frac{F}{\lambda\mu c},\bar{\tau_o}=\frac{\tau_0}{\mu\left(\frac{c}{a}\right)^n},\bar{\tau_{rz}}=\frac{\tau_{rz}}{\mu\left(\frac{c}{a}\right)^n},\mathrm{Pr}=\frac{\mu c_p}{k},\delta=\frac{a}{\lambda},\varepsilon=\frac{b}{a},\bar{r_p}=\frac{r_p}{c},$$
$$\mathrm{Re}=\frac{\rho ac}{\mu},\bar{p}=\frac{pa^{n+1}}{\lambda c^n\mu},\bar{Q}=\frac{Q}{\pi a^2 c},\bar{\Psi}=\frac{\psi}{\pi a^2 c},u=-\frac{1}{r}\frac{\partial\Psi}{\partial z},w=\frac{1}{r}\frac{\partial\Psi}{\partial r},\theta=\frac{T-T_1}{T_1-T_0},Ec=\frac{c^2}{c_p(T_0-T_1)}. \quad (3)$$

where $\overline{\tau_0}$ is the yield shear stress and $\overline{w}$ and $\overline{u}$ are the axial and radial velocities.

By lubrication approach (neglecting the frictional forces), the governing momentum and energy equations can be written in the simplified form as [28]:

$$\mathrm{Re}\,\delta\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)w=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)+\delta\frac{\partial}{\partial r}\left(\tau_{zz}\right) \quad (4)$$

$$\mathrm{Re}\,\delta^3\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)u=-\frac{\partial p}{\partial r}+\frac{\delta}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)+\delta^2\frac{\partial}{\partial r}\left(\tau_{rz}\right) \quad (5)$$

$$\mathrm{Re}\,\delta\,\mathrm{Pr}\left(u\frac{\partial}{\partial r}+w\frac{\partial}{\partial z}\right)\theta=Ec\,\mathrm{Pr}\left(\delta\frac{\partial u}{\partial r}\tau_{rr}+\frac{\partial w}{\partial r}\tau_{rz}+\delta^2\frac{\partial u}{\partial z}\tau_{zr}+\tau_{zz}\frac{\partial w}{\partial r}\delta\right)+\frac{\partial^2\theta}{\partial r^2}+\frac{1}{r}\frac{\partial\theta}{\partial r}+\delta^2\frac{\partial^2\theta}{\partial z^2} \quad (6)$$

Neglecting the inertial terms ($Re = 0$) and considering the long wavelength assumption ($\delta \ll 1$), Eq. (4)-(6) take the form as below

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)=-\frac{\partial p}{\partial z}+\frac{\sin\gamma}{F_1} \quad (7)$$

$$0=\frac{\partial p}{\partial r} \quad (8)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)=Br\left(-\frac{\partial w}{\partial r}\tau_{rz}\right) \tag{9}$$

where Brinkman number ($Br = EcPr$) and $F_1 = \frac{\mu c^n}{\rho g a^{n+1}}$.

The non-dimensional constitutive equation for Herschel-Bulkley fluid is given as follows

$$-\frac{\partial w}{\partial r}=\begin{cases}(\tau_{rz}-\tau_0)^{\frac{1}{n}}, & \tau_{rz}\geq\tau_0 \text{ (core region)},\\ 0, & \tau_{rz}\leq\tau_0 \text{ (plug flow region)}\end{cases} \tag{10}$$

The corresponding boundary conditions are [10]

τ_{rz} is finite at $r = 0$ (11a)

$$\frac{\partial w}{\partial r}=-\frac{\alpha w}{\sqrt{Da}} \text{ at } r=h=1+\varepsilon\sin(2\pi z) \tag{11b}$$

$$\frac{\partial\theta}{\partial r}=0 \text{ at } r=0 \tag{11c}$$

$$\theta+\beta\frac{\partial\theta}{\partial r}=0 \text{ at } r=h=1+\varepsilon\sin(2\pi z) \tag{11d}$$

The expression for velocity obtained, on solving Eq. (7), subject to the boundary conditions 11(a) and 11(b), is

$$w=\left(\frac{P+f}{2}\right)^k\left[\frac{\left(h-r_p\right)^{k+1}-\left(r-r_p\right)^{k+1}}{k+1}+\left(h-r_p\right)^k\frac{\sqrt{Da}}{\alpha}\right] \tag{12}$$

where $P=-\frac{\partial p}{\partial z}$, $f=\frac{\sin\gamma}{F_1}$, $k=\frac{1}{n}$.

The upper limit for the plug flow region is attained from the condition $\frac{\partial u}{\partial r} = 0$ at $r = r_p$. Using this, we get

$$r_p=\frac{2\tau_0}{P+f} \tag{13}$$

Moreover, $\tau_{rz} = \tau_h$ at $r = h$ [42] gives $P + f = \frac{\tau_h}{h}$. Therefore,

$$\frac{r_0}{h}=\frac{\tau_0}{\tau_h}=\tau, \quad 0<\tau<1 \tag{14}$$

The plug flow velocity is obtained by taking $r = r_p$in Eq. (12)

$$w_p = \left[\frac{(P+f)(h-r_p)}{2}\right]^k \left[\frac{(h-r_p)}{k+1} + \frac{\sqrt{Da}}{\alpha}\right] \tag{15}$$

Volume flux at any cross-section of the tube is represented by

$$q = \int_0^{r_p} w_p r dr + \int_{r_p}^{h} wrdr$$

$$q = \left(\frac{(P+f)(h-r_p)}{2}\right)^k \left[\frac{h^2}{2}\left(\frac{h-r_p}{k+1} + \frac{\sqrt{Da}}{\alpha}\right) - \frac{(h-r_p)^2((k+2)h+r_p)}{(k+1)(k+2)(k+3)}\right] \tag{16}$$

It is worth noting that the results of Nadeem and Akbar [28] can be obtained as a special case of the present model by substituting $Da = 0$ in Eq. (16). The dimensionless time-averaged flux is given by

$$\overline{Q} = \int_0^1 \int_0^h r(\mathrm{w}-1) dr\, dz = q + \int_0^1 h^2 dz = q + 1 + \frac{\varepsilon^2}{2} \tag{17}$$

We have

$$w = \frac{1}{r}\frac{\partial \psi}{\partial r} \tag{18}$$

And

$$\Psi_p = 0 \text{ at } r = 0, \Psi_1 = \Psi_p \text{ at } r = r_p \tag{19}$$

Integrating Eq. (18) and using the conditions given by Eq. (19), we obtain the stream function as

$$\psi = \left[\frac{(P+f)(h-r_p)}{2}\right]^k \left[\frac{(h-r_p)}{k+1} + \frac{\sqrt{Da}}{\alpha}\right] - \left[\frac{(P+f)(r-r_p)}{2}\right]^k \left[\frac{(r-r_p)^2((k+2)r+r_p)}{(k+1)(k+2)(k+3)}\right] \tag{20}$$

Using Eq. (10) in Eq. (9) and considering the conditions given by Eqs. 11(c) and 11(d), we obtain the expression for temperature as

$$\theta = Br\left(\frac{P}{2}\right)^{k+1}\left[\begin{array}{l}\frac{\left(r^{k+3}-h^{k+3}\right)}{(k+3)^2}+\frac{kr_p\left(h^{k+2}-r^{k+2}\right)}{(k+2)^2}+\frac{k(k-1)r_p^{\,2}\left(r^{k+1}-h^{k+1}\right)}{2(k+1)^2}\\ +\frac{(k-1)(k-2)r_p^{\,3}\left(h^k-r^k\right)}{6k}+\frac{2(-r_p)^{k+3}\left(\frac{\beta}{h}-\log r+\log h\right)}{(k+1)(k+2)(k+3)}\\ -\frac{\beta\left(h-r_p\right)^{k+1}}{h}\left(\frac{\left(h-r_p\right)^2}{k+3}+\frac{2r_p\left(h-r_p\right)}{k+2}+\frac{r_p^{\,2}}{k+1}\right)\end{array}\right] \tag{21}$$

3. Pumping Characteristics

The pressure rise (ΔP) over one cycle of the wave is given by

$$\Delta P = \int_0^1 \frac{\partial p}{\partial z}dz\,. \tag{22}$$

The dimensionless frictional force *F* along the wall over one wave length is

$$F = \int_0^1 h^2\left(-\frac{\partial p}{\partial z}\right)dz. \tag{23}$$

4. Results and Discussion

Parametric analysis is carried out to study the peristaltic transport of Herschel-Bulkley fluid flowing through a porous tube with inclination. The flow is assumed to be incompressible, laminar and steady; and incorporates the effects of heat transfer and thermal slip. The advantage of using this model is that the corresponding results can also be obtained and analysed for power-law, Newtonian and Bingham fluids. In the present study, the pressure rise (ΔP) and frictional force(*F)* are obtained from Eqs. (22) and (23) by using Weddle's rule through MATLAB. The various pertinent parameters involved in the problem are shear stress (τ), Darcy number (Da), velocity slip parameter (α), fluid behaviour index (n), angle of inclination (γ), thermal slip parameter(β) and Brinkman number (Br). The influence of each of these parameters is analysed on time-averaged flow rate $\overline{Q}$, pressure rise, pressure gradient (P), frictional force, temperature(θ) and streamlines (Ψ) through the graphs plotted in Figures 2-10. The values of the parameters kept constant in our analysis are $\tau = 0.2, Da = 0.0002, \alpha = 0.2, \varepsilon = 0.2, n = 3, \beta = 0.2, \gamma = \frac{\pi}{4}$ and $Br = 0.25$.

The variation in ΔP versus $\overline{Q}$ are graphed in Figures 2(a)-2(e). A rise in the values of τ, n and γ have an increasing effect on ΔP. Also, it is evident that in comparison to a power law fluid, the peristaltic pumping for Herschel-Bulkley fluid over the walls of the tube occurs against a greater pressure rise; the reason being the plug flow region present in a Herschel-Bulkley fluid. The pumping curves for Herschel-Bulkley fluid and Newtonian fluid ($\tau_0 = 0$ and $n = 1$) intersect with each other when $\overline{Q} = 1$. This information helps in equalizing the pumping rate of Newtonian and Herschel-Bulkley fluid for a given value of $\overline{Q}$ by adjusting the peristalsis velocity. Moreover, increase in Da

means an increase in the porosity of the walls, which decreases $\overline{Q}$. This behavior is observed in Figure 2(d).

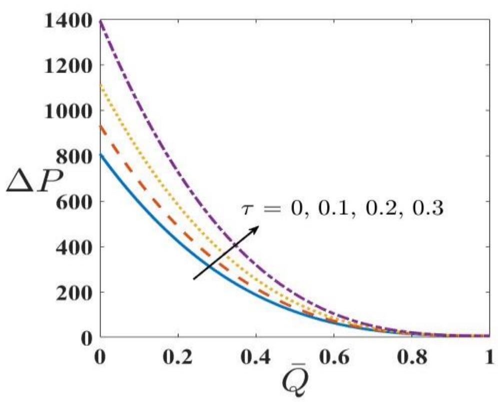

Fig. 2(a). $\Delta\boldsymbol{P}$ versus $\overline{Q}$ for varying τ

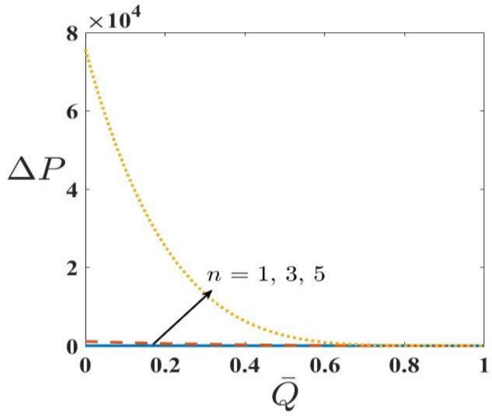

Fig. 2(b). $\Delta\boldsymbol{P}$ versus $\overline{Q}$ for varying n

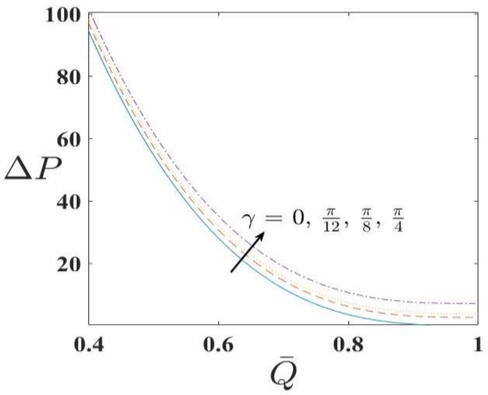

Fig. 2(c). $\Delta\boldsymbol{P}$ versus $\overline{Q}$ for varying γ

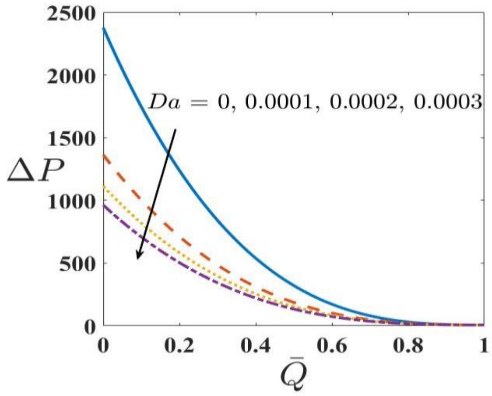

Fig. 2(d). $\Delta\boldsymbol{P}$ versus $\overline{Q}$ for varying Da

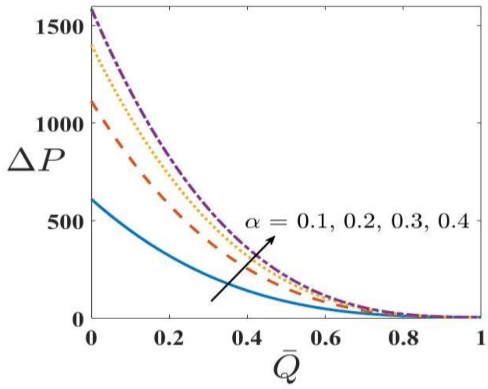

Fig. 2(e). $\Delta\boldsymbol{P}$ versus $\overline{Q}$ for varying α

The effect of α on ΔP is opposite to that of Da. The comparison between the results obtained from the present model and Nadeem and Akbar [28] model is shown in Figure 3, where it is clear that the results are in good agreement. Figures 4(a)-4(e) show the impact of different parameters on *F*. It can be seen that the behaviour of *F* is opposite to that of ΔP. Figures 5(a)-5(e) are plotted to see the effect on *P*. As expected, the pressure gradient *P* is maximum at the narrowest part of the tube, that is, $z = 0.75$. Further, for positive values of *P*, an adverse pressure gradient is registered (which opposes the flow) in the range $z \in [0.2, 1.2]$. The magnitude of *P* increases with an increase in τ and *n*. This behavior is due to the fact that as the shear stress and the shear thickening of the fluid increases, higher pressure gradient is needed across the tube to enable the flow of the fluid. A similar behavior is observed for an increase in the values of α, whereas the opposite trend is observed for Da and γ.

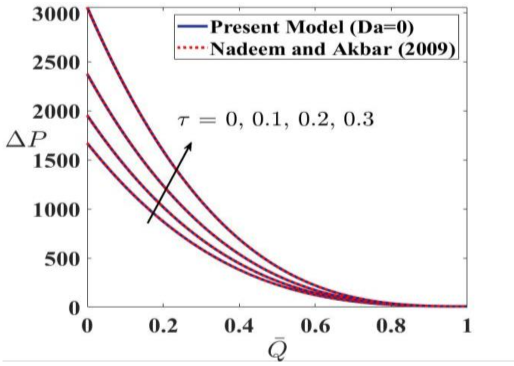

Fig. 3. Validation of the model

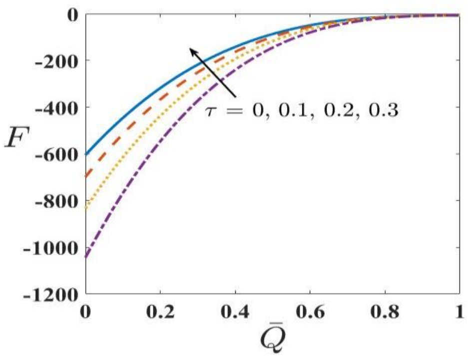

Fig. 4(a). F versus $\overline{Q}$ for varying τ

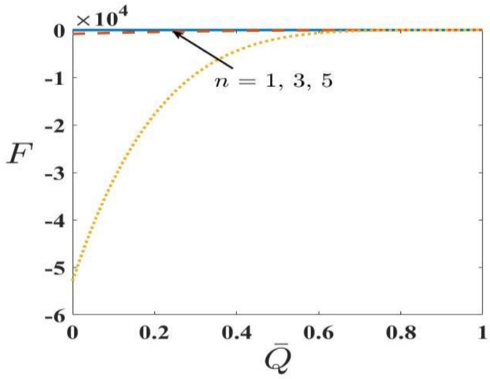

Fig. 4(b). F versus $\overline{Q}$ for varying n

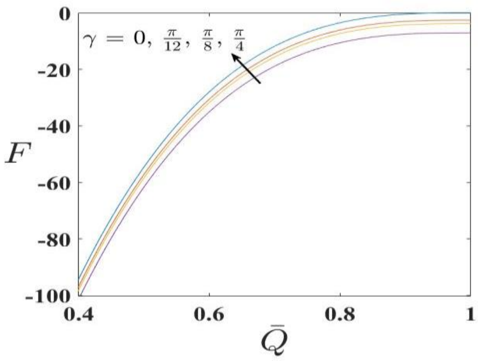

Fig. 4(c). F versus $\overline{Q}$ for varying γ

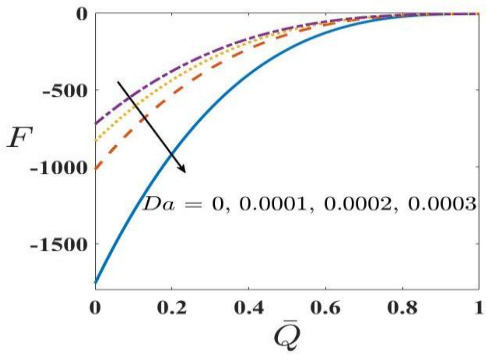

Fig. 4(d). F versus $\overline{Q}$ for varying Da

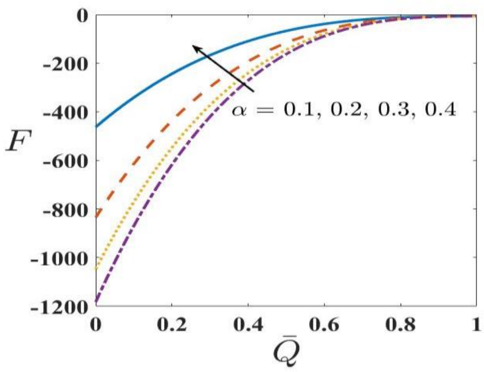

Fig. 4(e). F versus $\overline{Q}$ for varying α

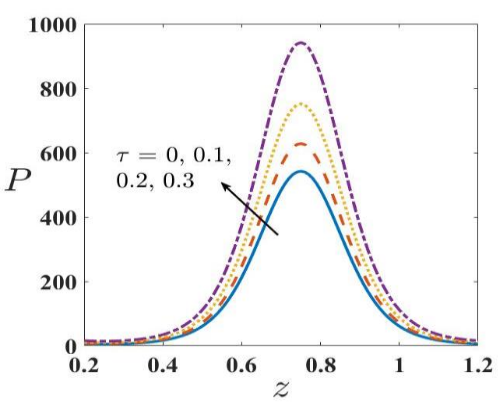

Fig. 5(a). P for different values of τ

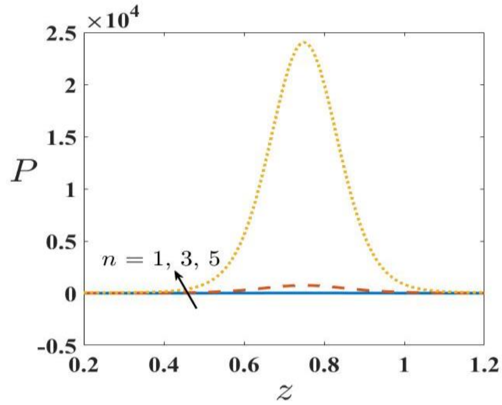

Fig. 5(b). P for different values of n

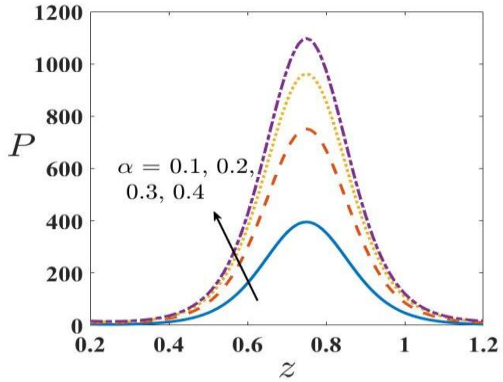

Fig. 5(c). P for different values of α

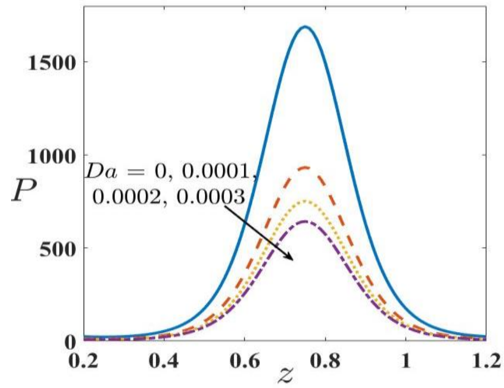

Fig. 5(d). P for different values of Da

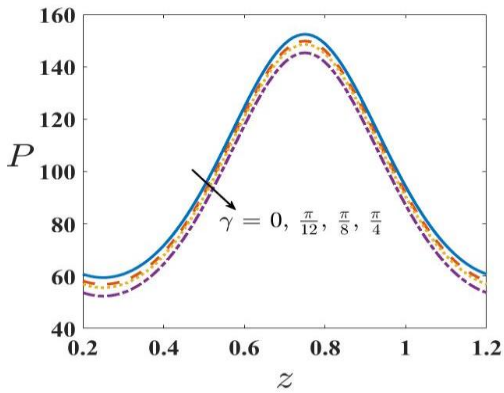

Fig. 5(e). P for different values of γ

The effects of τ, Da, α, n, γ, β and Br on the temperature profile are sketched in Figures 6(a)-6(g). Figure 6(a) illustrates the effect of α on temperature. Here the magnitude of temperature increases with increase in the value of α. Figure 6(b) portrays the variation of β on temperature. Here an increase in temperature is observed near the axis of the tube and the effect is negligible towards the boundary walls. Figure 6(c) reveals that an increase in the magnitude of temperature is because of the higher values of Br. This is because, Ec occurs due to the viscous dissipation effects and it therefore enhances the temperature. Further, an increase in the value of Pr decreases the value of thermal conductivity and thereby increasing the temperature. However, the opposite behaviour is noticed near the walls. Figure 6(d) portrays the variation of Da on temperature. Here the decay in temperature is observed near the axis of the tube and opposite behavior is noticed near the walls. Figure 6(e) is graphed to illustrate the effect of γ on temperature. An increment in γ results in an increase in the temperature. Figure 6(f) depicts the variation of temperature due to the influence of n. An increase in the value of n results in an increase in the temperature. Figure 6(g) shows the effect of τ on temperature. Here an increment in τ enhances the temperature.

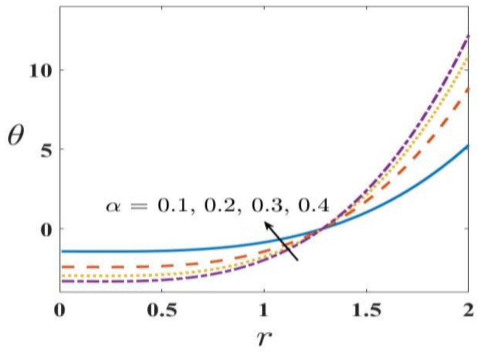

Fig. 6(a). θ for different values of α

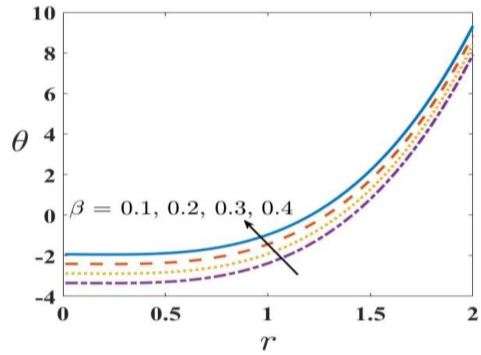

Fig. 6(b). θ for different values of β

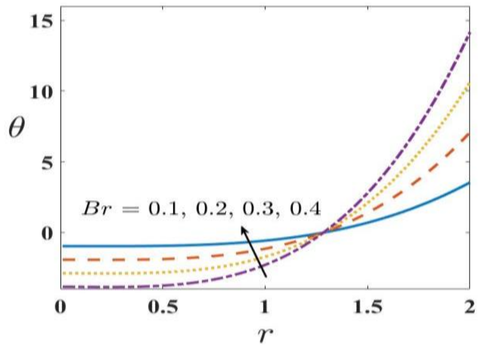

Fig. 6(c). θ for different values of Br

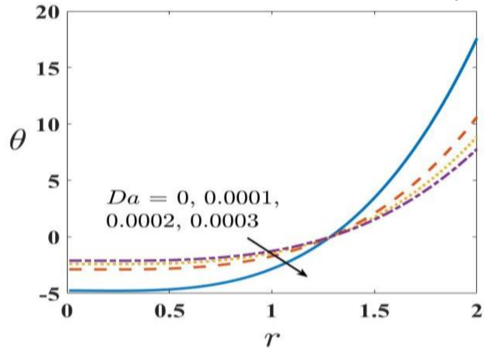

Fig. 6(d). θ for different values of Da

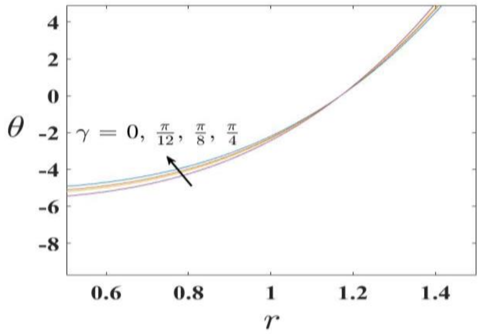

Fig. 6(e). θ for different values of γ

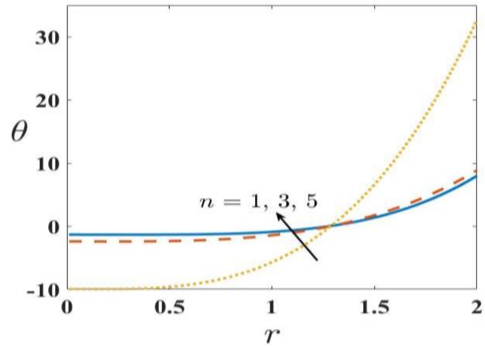

Fig. 6(f). θ for different values of n

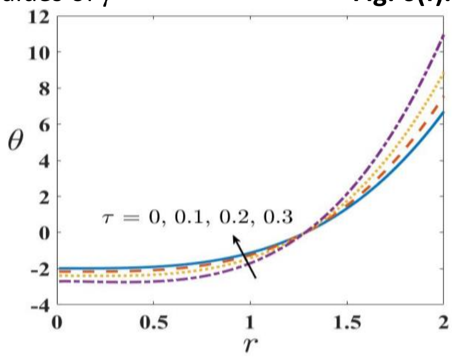

Fig. 6(g). θ for different values of τ

An important phenomenon in peristalsis is trapping. It involves the formation of an inside flowing bolus, which is then pushed forward in the tube by the sinusoidal motion of the peristaltic waves.

The effects of α, ε, τ, and Da on the trapped bolus is depicted in Figures 7-10. The volume of the trapped bolus was seen to increase with increase in α, ε and τ due to which the number of bolus formed increases; whereas, the volume of the trapped bolus saw a reduction with increase in Da, thus decreasing the number of bolus formed.

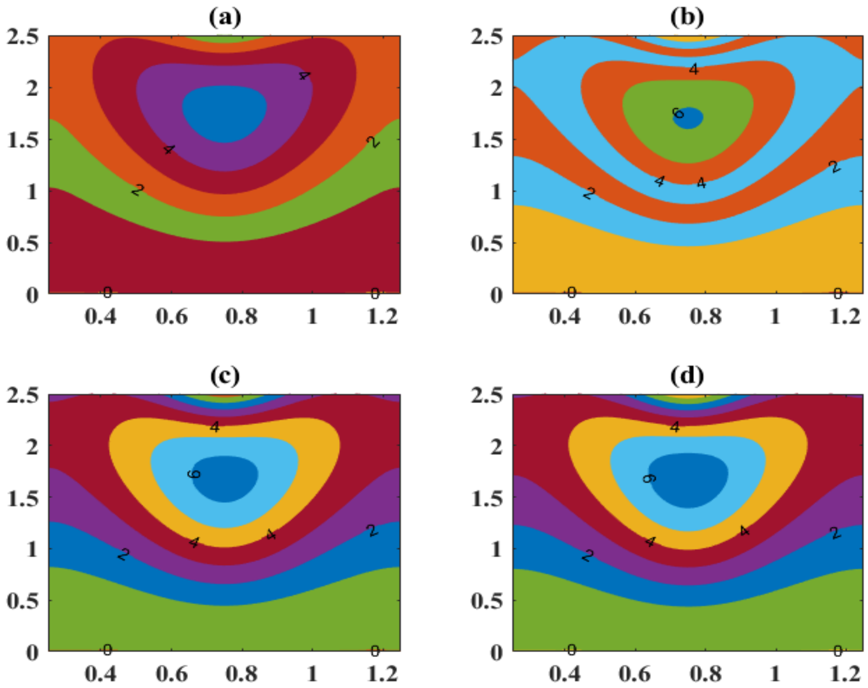

Fig. 7. Streamlines for varying (a) $\alpha = 0.1$, (b) $\alpha = 0.2$, (c) $\alpha = 0.3$ and (d) $\alpha = 0.4$.

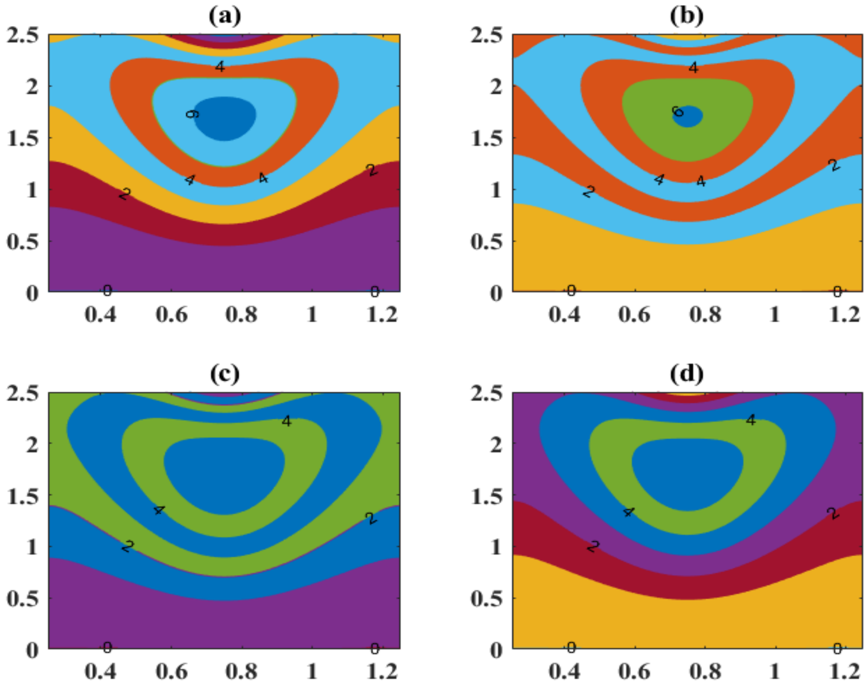

Fig. 8. Streamlines for varying (a) $Da = 0.0001$, (b) $Da = 0.0002$, (c) $Da = 0.0003$ and (d) $Da = 0.0004$.

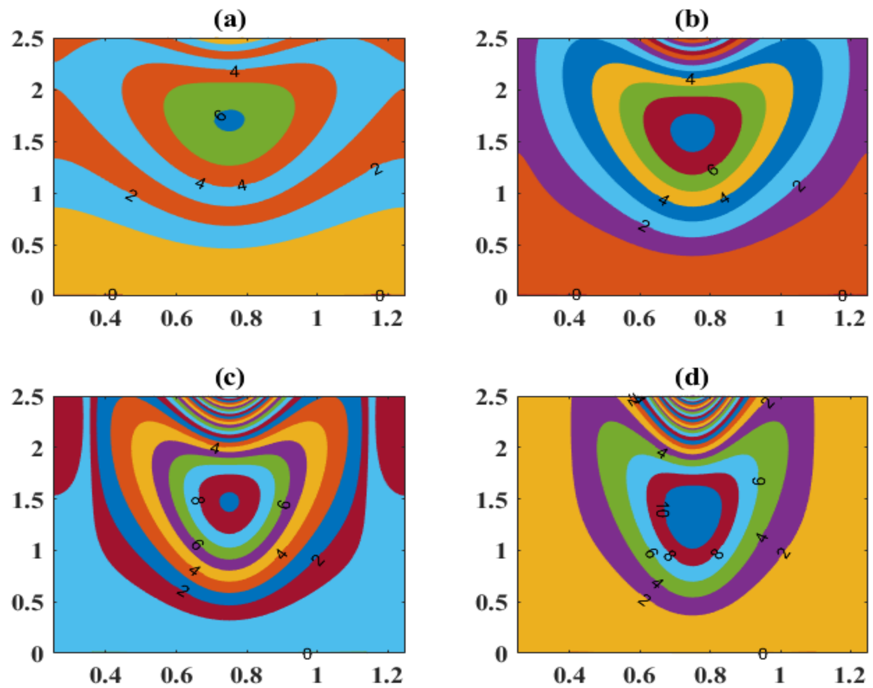

Fig. 9. Streamlines for varying (a) $\varepsilon = 0.3$, (b) $\varepsilon = 0.4$, (c) $\varepsilon = 0.5$ and (d) $\varepsilon = 0.6$.

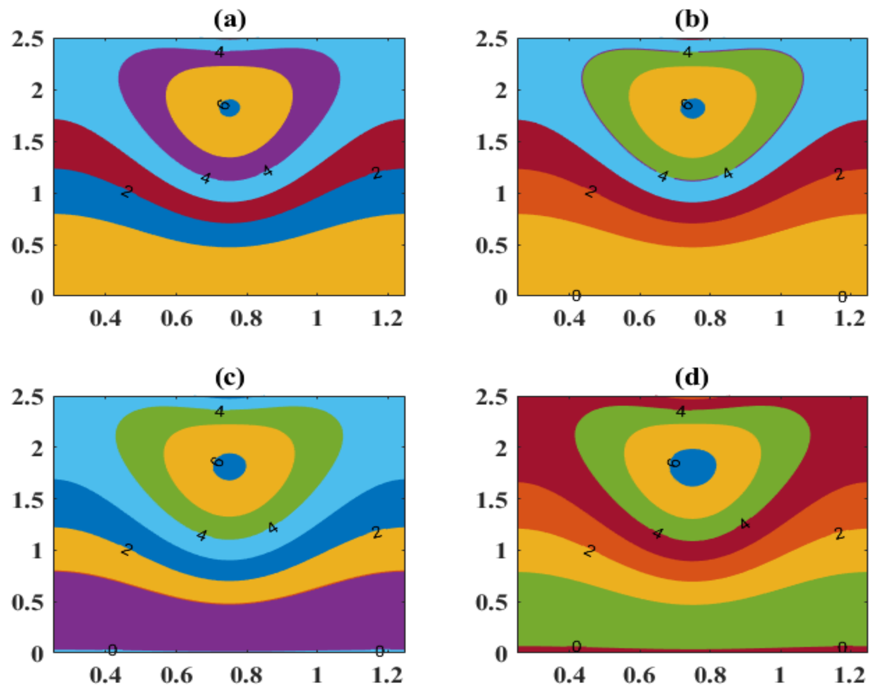

Fig. 10. Streamlines for varying (a) $\tau = 0.1$, (b) $\tau = 0.2$, (c) $\tau = 0.3$ and (d) $\tau = 0.4$.

5. Conclusions

The present paper deals with the investigation of peristaltic transport of a Herschel-Bulkley fluid through an inclined porous tube, taking into account the heat transfer characteristics. The effects of thermal and velocity slip conditions have been incorporated in the model. The results of the present model have wide applications in the field of medicine and engineering. The conclusions can be summarized as follows

i. In comparison to a power law fluid, the peristaltic pumping for Herschel-Bulkley fluid over the walls of the tube occurs against a greater pressure rise.
ii. An increase in the velocity slip parameter, angle of inclination, shear stress and shear thickening of the fluid have an increasing effect on the pressure gradient as well as pressure rise.
iii. The frictional forces behave in an opposite way as compared to the behavior of pressure rise.
iv. The magnitude of temperature increases with an increase in the values of velocity slip parameter, thermal slip parameter, Brinkman number, angle of inclination, shear stress and shear thickening of the Herschel-Bulkley fluid.
v. The volume of the trapped bolus increases for larger values of the velocity slip parameter, amplitude ratio and shear stress, and smaller values of Darcy number.

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References
[1] Latham, Thomas Walker. "Fluid motions in a peristaltic pump." PhD diss., Massachusetts Institute of Technology, 1966.
[2] Burns, J. C., and T. Parkes. "Peristaltic motion." *Journal of Fluid Mechanics* 29, no. 4 (1967): 731-743.
[3] Fung, Y.C. and Yih, C.S. "Peristaltic transport." *J. App. Mech.* 35, (1968): 669-675.
[4] Shapiro, A. H., Jaffrin, M. Y. and Weinberg, S. L. "Peristaltic pumping with long wavelengths at low Reynolds number." *Journal of Fluid Mechanics* 37, (1969): 799-825.
[5] Raju, K.K. and Devanathan, R. "Peristaltic motion of a non-Newtonian fluid." *Rheologia Acta* 11, (1972): 170-178.
[6] Srivastava, L.M. and Srivastava, V.P. "Peristaltic transport of blood: Casson model-II." *Journal of Biomechanics* 17, (1984): 821-829.
[7] Pandey, S.K., Chaube, M.K. and Tripathi, D. "Peristaltic transport of multi layered power-law fluids with distinct viscosities: a mathematical model for intestinal flows." *Journal of Theoretical Biology* 278, (2011): 11-19.
[8] Vajravelu, K., Sreenadh, S., Devaki, P. and Prasad, K.V. "Peristaltic pumping of a Casson fluid in an elastic tube." *Journal of Applied Fluid Mechanics* 9, (2016): 1897-1905.
[9] Prasad, K.V., Vajravelu, K., Vaidya, H., Basha, N.Z. and Umesh, V. "Thermal and species concentration of MHD Casson fluid at a vertical sheet in the presence variable fluid properties." *Ain Shams Engineering Journal*, (2017).
[10] Manjunatha, G. and Rajashekhar, C. "Slip effects on peristaltic transport of Casson fluid in an inclined elastic tube with porous walls." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 43, (2018): 67-80.
[11] Blair, G.W.S. and Spanner, D.C. "An Introduction to Biorheology." *Elsevier Scientific Publ. Co., Amsterdam*, (1974).
[12] Vajravelu, K., Sreenadh, S. and Ramesh, B. "Peristaltic transport of a Herschel-Bulkley fluid in an inclined tube." *International Journal of Non-linear Mechanics* 40, (2005): 83-90.
[13] Manjunatha, G., Basavarajappa, K.S., Thippeswamy, G. and Hanumesh, V. "Peristaltic transport of three layered viscous incompressible fluid." *Global Journal of Pure and Applied Mathematics* 9, (2013): 93-107.
[14] Rajashekhar, C., Manjunatha, G., Prasad, K.V., Divya, B.B. and Vaidya, H. "Peristaltic transport of two-layered blood flow using Herschel–Bulkley Model." *Cogent Engineering* 5, (2018): 1-16.
[15] Lukashey, A.E. "Mathematical model of the peristaltic transport of liquid initiated by the autowave process of mass transport through the porous capillary wall." *J. Kaledney* 55, (1993): 109-113.
[16] Shehawey, E.F. and Husseny, S.Z.A. "Effect of porous boundaries on peristaltic transport through a porous medium." *Acta Mech*. 143, (2000): 165-177.
[17] Nadeem, S., and Safia Akram. "Peristaltic flow of a Maxwell model through porous boundaries in a porous medium." *Transport in Porous Media* 86, no. 3 (2011): 895-909.
[18] Ramachandra, R.A. and Mishra, M. "Peristaltic transport of a power-law fluid in a porous tube." *Journal of non-Newtonian Fluid Mechanics* 121, (2004): 163-174.

[19] Hayat, T., Husssain, Q. and Ali, N. "Influence of partial slip on the peristaltic flow in a porous medium." *Physica A: Statistical Mechanics and its Applications* 387, (2008): 3399-3409.

[20] Khan, A.A., Ellahi, R. and Usman, M. "The effects of variable viscosity on the peristaltic flow of non-Newtonian fluid through a porous medium in an inclined channel with slip boundary conditions." *Journal of Porous media* 16, (2013).

[21] Sreenadh, S., Prasad, K.V., Vaidya, H., Sudhakara, E., Krishna, G.G. and Krishnamurthy, M. "MHD Couette flow of a Jeffrey fluid over a deformable porous layer." *International Journal of Applied and Computational Mathematics* 3, (2017): 2125-2138.

[22] Rajashekhar, C., Manjunatha, G. and Naveen, C. "Analytical Solutions on the Flow of blood with the Effects of Hematocrit, Slip and TPMA in a porous tube." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 47, (2018): 201-208.

[23] Sampath Kumar, V. S. and Pai, N. P. "Analysis of Porous Eliptical Slider through Semi-Analytical Technique." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 48, no. 1 (2018): 80-90.

[24] Rajashekhar, C., Manjunatha, G., Hanumesh Vaidya, Prasad, K. V. "Peristaltic flow of Herschel-Bulkley fluid in an elastic tub with slip at porous walls." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 52, (2018): 63-75.

[25] Vasanth, K. Rajendrappa, Hanumagowda, B. Naganagowda, Jogyalla, S. Kumar and Raju, B. Thimmaiah. "Combined effects of Piezo-Viscous dependency non-Newtonian Couple stress in porous squeeze-film cirular plate." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 51, no. 2 (2018): 158-168.

[26] Srinivas, S. and Kothandapani, M. "Peristaltic transport in an asymmetric channel with heat transfer – a note." *International Communication in Heat and Mass Transfer* 35, (2008): 514-522.

[27] Srinivas, S. and Gayathri, R. "Peristaltic transport of a Newtonian fluid in a vertical asymmetric channel with heat transfer and porous medium." *Applied Mathematics and Computation* 215, (2009): 185-196.

[28] Nadeem, S. and Akbar, N.S. "Influence of heat transfer on a peristaltic transport of Herschel-Bulkley fluid in a non-uniform inclined tube." *Communication in Nonlinear Science and Numerical Simulation* 14, (2009): 4100-4113.

[29] Wakif, Abderrahim, Zoubair Boulahia, and Rachid Sehaqui. "Numerical analysis of the onset of longitudinal convective rolls in a porous medium saturated by an electrically conducting nanofluid in the presence of an external magnetic field." *Results in physics* 7 (2017): 2134-2152.

[30] Prasad, K.V., Mallikarjun, P. and Vaidya, H. "Mixed convective fully developed flow in a vertical channel in the presence of thermal radiation and viscous dissipation." *International Journal of Applied Mechanics and Engineering* 22, (2017): 123-144.

[31] Mebarek-oudina, F. and Bessaïh, R. "Numerical modeling of MHD stability in a cylindrical configuration." *Journal of the Franklin Institute* 351, (2014): 667-681.

[32] Mebarek-Oudina, F. and Bessaih, R. "Oscillatory magnetohydrodynamic natural convection of liquid metal between vertical coaxial cylinders." *Journal of Applied Fluid Mechanics* 9, (2016): 1655-1665.

[33] Mebarek-Oudina, F. "Numerical modelling of the hydrodynamic stability in vertical annulus with heat source of different lengths." *Engineering Science and Technology, an International Journal* 7, (2017): 1324-1665.

[34] Wakif, Abderrahim, Zoubair Boulahia, Farhad Ali, Mohamed R. Eid, and Rachid Sehaqui. "Numerical Analysis of the Unsteady Natural Convection MHD Couette Nanofluid Flow in the Presence of Thermal Radiation Using Single and Two-Phase Nanofluid Models for Cu–Water Nanofluids." *International Journal of Applied and Computational Mathematics* 4, no. 3 (2018): 81.

[35] Wakif, Abderrahim, Zoubair Boulahia, S. R. Mishra, Mohammad Mehdi Rashidi, and Rachid Sehaqui. "Influence of a uniform transverse magnetic field on the thermo-hydrodynamic stability in water-based nanofluids with metallic nanoparticles using the generalized Buongiorno's mathematical model." *The European Physical Journal Plus*133, no. 5 (2018): 181.

[36] Wakif, Abderrahim, Zoubair Boulahia, and Rachid Sehaqui. "A semi-analytical analysis of electro-thermo-hydrodynamic stability in dielectric nanofluids using Buongiorno's mathematical model together with more realistic boundary conditions." *Results in Physics* 9, (2018): 1438-1454.

[37] Wakif, A., Z. Boulahia, A. Amine, I. L. Animasaun, M. I. Afridi, M. Qasimd, and R. Sehaqui. "Magneto-convection of Alumina-water nanofluid within thin horizontal layers using the revised generalized Buongiorno's model." *Frontiers in Heat and Mass Transfer*, no. 12 (2018): 1-15.

[38] C. Rajashekhar, G. Manjunatha, H. Vaidya, B.B. Divya and K.V. Prasad, "Peristaltic flow of Casson liquid in an inclined porous tube with convective boundary conditions and variable liquid properties." *Frontiers in Heat and Mass Transfer*, no. 11 (2018): 35.

[39] H. Vaidya, G. Manjunatha, C. Rajashekhar and K.V Prasad, "Role of slip and heat transfer on peristaltic transport of Herschel-Bulkley fluid through an elastic tube." *Multidiscipline Modelling in Materials and Structures*, no. 14 (2018): 940-959.

[40] H. Vaidya, C. Rajashekhar, G. Manjunatha and K.V. Prasad, "Peristaltic mechanism of a Rabinowitsch fluid in an inclined channel with compliant wall and variable liquid properties." *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, no. 41 (2019): 52.
[41] H. Vaidya, C. Rajashekhar, G. Manjunatha and K.V. Prasad, "Rheological properties and peristalsis of Rabinowitsch fluid through compliant porous walls in an inclined channel." *Journal of Nanofluids*, no. 8 (2019): 970-979.
[42] Bird, R. B., Stewart, W. E. and Lightfoot, E. N. (1976). *Transport Phenomena,* New York: Wiley.