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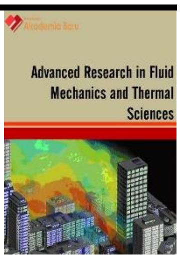

Heat Transfer Analysis of Sodium Carboxymethyl Cellulose Based Nanofluid with Titania Nanoparticles

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ABSTRACT

In this paper, an analysis is made for heat transfer unsteady flow of nanofluid over semi-infinite vertical plate with leading edge accretion/ablation. The impact of viscous dissipation in energy equation with Newtonian heating condition is also considered. Tiwari-Das model is used to incorporates the effects of nanoparticles volumetric fraction. Sodium carboxymethyl cellulose (SCMC) is considered as based fluid containing titania (TiO_2) nanoparticles. Similarity transformations are employed to transform the unsteady partial differential equations into a system of ordinary differential equations. The transformed equations along with relevant boundary conditions are solved numerically by Runge Kutta Fehlberg fourth-fifth order (RKF45) method in MAPLE software. The analysis shows that velocity and temperature field in the respective boundary layers depend on different physical parameters, namely Prandtl number, Eckert number, Casson parameter, Newtonian heating parameter, accretion/ablation parameter and nanoparticle volume fraction. Temerature shows higher value for Blasius flat plate, while for Rayleigh-Stokes is the lowest.

Keywords:
Nanofluid, Sodium carboxymethyl cellulose, Titania nanoparticles, Viscous dissipation, Newtonian heating.

1. Introduction

Conventional heat transfer fluids, namely; water, ethylene glycol and oils are used in many thermal systems have relatively poor thermal conductivities as compare to solids. Nanofluids, an enterprising and demonstrably more efficient type of working fluids, are obtained by suspension of ultrafine size (1-100 nm) particles in conventional heat transfer fluids. The term nanofluid was first put forward by Choi [1] in a useful study to refer the fluids with suspended nanoparticles. Nowadays, nanofluids play an important role in many industrial applications. After the fundamental work of Choi [1], many researchers have used nanofluids in their heat transfer flow problem and found that with a small amount of nanoparticles in the based fluids, the thermal conductivity increased significantly.

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Water based nanofluid flow containing Ag and Cu over a stretching sheet with convective boundary conditions in the presence of internal heat generation or absorption was discussed by Vajravelu *et al*., [2]. Thermophysical and magnetic field effects on Ag and Cu suspended nanofluid in a semi porous channel were investigated by Sheikholeslami *et al*., [3]. Natural convection flow of a viscous Cu, Al_2O_3 and TiO_2 water nanofluid past an accelerated vertical plate was investigated by Hussanan *et al*., [4]. Aly and Ebaid [5] studied the effect of heat transfer on MHD and radiation Marangoni boundary layer water based nanofluid flow with Cu and TiO_2 nanoparticles past a surface embedded in a porous medium. Khan *et al.,* [6] studied the effect of slip on CNT water based nanofluid flow in a channel with non-parallel walls. Hussanan *et al.,* [7] reported the analytical study of unsteady heat transfer of a micropolar nanofluid flow over a vertical plate with oxide nanoparticles in water, kerosene and engine oil. Azmi *et al.,* [8] used Al_2O_3 nanofluid with ethylene glycol as host fluid and found that heat transfer coefficient is enhanced with concentration. Swalmeh *et al.,* [9] analyzed the natural convection heat transfer flow of Cu-water and Al_2O_3-water micropolar nanofluids about a solid sphere. Recently, Hussanan *et al.,* [10] discussed Fe_3O_4 water based microploar ferrofluid over a stretching/shrinking sheet using effective thermal conductivity model. Some other comprehensive studies on nanofluids along with their applications are found in [11-15].

The effect of viscous dissipation changes the temperature distribution by playing a central role like an energy source, which affects the rate of heat transfer. Viscous dissipation plays a significant role in various applications such as such as cooling of nuclear reactors, oil exploration, bioengineering, chemical and food processing. Qasim and Noreen [16] investigated the effect of viscous dissipation on Casson fluid flow over permeable shrinking sheet. Laminar forced convection flow of Cu nanofluid in a trapezoidal microchannel heat-sink under the effects of Brownian motion and viscous dissipation have been proposed by Fani *et al.,* [17]. Heat and mass transfer flow due to nanofluid thin film caused by unsteady stretching sheet with viscous dissipation were analyzed by Qasim *et al.,* [18]. The model used for the nanofluid film incorporates the effects of Brownian motion and thermophoresis. Hussanan *et al.,* [19] studied the heat transfer characteristics in viscoplastic Casson fluid over a stretching sheet with viscous dissipation. In another paper, Hussanan *et al.,* [20] considered the viscous dissipation effect on sodium alginate viscoplastic Casson based nanofluid flow over a vertical plate. Entropy generation and heat transfer analysis in a viscous flow induced by a horizontally moving Riga plate in the in existence of viscous dissipation was demonstrated by Afridi *et al.,* [21].

In recent decade, various researchers used the constant or variable wall condition for temperature in their published articles. However, there are several problems of physical interest where the heat is transported to the fluid via a bounding surface with a finite heat capacity and the above conditions fail to work and the Newtonian heating condition is incorporated. This idea was first introduced by Merkin [22]. Considering the importance of Newtonian heating condition, many authors have used it in their convective heat-transfer problems and obtained the solutions either numerically [23-25] or analytically [26-30]. The above literature review reveals that no study exists to conduct on heat transfer analysis of sodium carboxymethyl cellulose (SCMC) based nanofluid flow over a vertical plate with leading edge accretion/ablation using titania (TiO_2) nanoparticles. Therefore, present study investigates the behavior of SCMC based nanofluid containing TiO_2 nanoparticles and a comparison between Newtonian heating and constant wall temperature is conducted. Tiwari-Das model is used to incorporate the effects of nanoparticles volumetric fraction [31]. The governing unsteady partial differential equations are converted into a system of nonlinear ordinary differential equations by introducing suitable similarity transformations. These reduced nonlinear differential equations are then solved numerically by Runge Kutta Fehlberg fourth-fifth order (RKF45) method.

2. Problem Formulation

Considered the unsteady incompressible TiO₂/SCMC based nanofluid flow past a semi-infinite vertical plate with leading edge accretion/ablation. Let the uniform free stream velocity be U and free stream temperature be denoted by T_∞. The x-axis is taken vertically up in direction of free stream, while y is the coordinate measured normal to it. The equations governing the flow of nanofluid are under viscous dissipation effects are

$$\vec{\nabla}\cdot\vec{V}=0, \tag{1}$$

$$\rho_{nf}\left(\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V}\right)=-\vec{\nabla}p+\operatorname{div}\left(\tau_{ij}\right), \tag{2}$$

$$\left(\rho C_p\right)_{nf}\left(\frac{\partial T}{\partial t}+\left(\vec{V}\cdot\vec{\nabla}\right)T\right)=K_{nf}\nabla^2T+\varphi, \tag{3}$$

where μ_{nf}, ρ_{nf}, K_{nf} and $\left(\rho C_p\right)_{nf}$ are dynamic viscosity, density, thermal conductivity and heat capacitance of TiO₂/SCMC based nanofluid, respectively, φ is the viscous dissipation, p is the pressure. The velocity vector for two-dimensional flow is

$$\vec{V}=\begin{cases}u=u\left(x,y\right)\\ v=v\left(x,y\right)\\ w=0\end{cases} \tag{4}$$

The constitutive relationship for viscoplastic fluid [32] is

$$\tau_{ij}=\begin{cases}2\left(\mu_B+\dfrac{\sigma_y}{\sqrt{2\pi}}\right)e_{ij}, & \pi>\pi_c\\ 2\left(\mu_B+\dfrac{\sigma_y}{\sqrt{2\pi_c}}\right)e_{ij}, & \pi<\pi_c\end{cases}, \tag{5}$$

where σ_y is the yield stress, μ_B is the plastic dynamic viscosity, $\pi=e_{ij}e_{ij}$ and e_{ij} is the (i,j)th component of deformation rate is

$$e_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right). \tag{6}$$

By removing the pressure gradient from Eq. (2), then rewritten Eq. (2) and (3), the final equations can be obtained as follows

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\left(1+\frac{1}{\beta}\right)\frac{\partial^2u}{\partial y^2}, \tag{7}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K_{nf}}{\left(\rho C_p\right)_{nf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{\left(\rho C_p\right)_{nf}}\left(1+\frac{1}{\beta}\right)\left(\frac{\partial u}{\partial y}\right)^2, \tag{8}$$

where $\beta = \mu_B\sqrt{2\pi_c}/\sigma_y$ is the Casson parameter. The appropriate initial and boundary conditions in case of Newtonian heating and constant wall temperature are

$$t<0: u=v=0,\ T=T_\infty \text{ for all } x, y, \tag{9}$$

$$\begin{aligned} t\geq 0:\ & u=v=0,\ \frac{\partial T}{\partial y}=h_s T \text{ or } T=T_w \text{ at } y=0, \\ & u\to U,\ T\to T_\infty \text{ as } y\to\infty. \end{aligned} \tag{10}$$

For present problem, we considered the following relations between sodium carboxymethyl cellulose and titania nanoparticles based nanofluid

$$\begin{aligned} &\rho_{nf}=(1-\phi)\rho_{bf}+\phi\rho_{np},\ \left(\rho C_p\right)_{nf}=(1-\phi)\left(\rho C_p\right)_{bf}+\phi\left(\rho C_p\right)_{np}, \\ &\mu_{nf}=\frac{\mu_{bf}}{(1-\phi)^{2.5}},\ \frac{K_{nf}}{K_{bf}}=\frac{\left(K_{np}+2K_{bf}\right)-2\phi\left(K_{bf}-K_{np}\right)}{\left(K_{np}+2K_{bf}\right)+\phi\left(K_{bf}-K_{np}\right)}. \end{aligned} \tag{11}$$

In order to proceed to the numerical solutions, the following similarity variables are introduced

$$\begin{aligned} &\psi(x,y,t)=U\sqrt{\left(\nu_{bf}t\right)\cos(\alpha)+\left(\nu_{bf}x/U\right)\sin(\alpha)}F(\xi), \\ &\xi=\frac{y}{\sqrt{\left(\nu_{bf}t\right)\cos(\alpha)+\left(\nu_{bf}x/U\right)\sin(\alpha)}},\ \theta(\xi)=\frac{T_\infty}{T-T_\infty} \text{ or } \theta(\xi)=\frac{T-T_\infty}{T_w-T_\infty}. \end{aligned} \tag{12}$$

The free stream function Ψ defines the velocity components as

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}. \tag{13}$$

Using free stream function Ψ into above equation, components of velocity u and v take the form

$$u=UF'(\xi),\ v=\left(\xi F'(\xi)-F(\xi)\right)\left[\frac{\nu_{bf}\sin(\alpha)}{2\sqrt{\left(\nu_{bf}t\right)\cos(\alpha)+\left(\nu_{bf}x/U\right)\sin(\alpha)}}\right]. \tag{14}$$

With the help of Eq. (12) to (14), Eq. (7) and (8) transform into the ordinary differential equations

$$\left(1+\frac{1}{\beta}\right)F'''(\xi)+\frac{1}{2}\left(\xi\cos(\alpha)+f\sin(\alpha)\right)(1-\phi)^{2.5}\left((1-\phi)+\phi\frac{\rho_{np}}{\rho_{bf}}\right)F''(\xi)=0, \tag{15}$$

$$\begin{aligned}&\left(\frac{K_{np}}{K_{bf}}\right)\theta''(\xi)+\frac{\Pr}{2}\left((1-\phi)+\phi\frac{(\rho C_p)_{np}}{(\rho C_p)_{bf}}\right)\left(\xi\cos(\alpha)+f\sin(\alpha)\right)\theta'(\xi)\\&+\Pr Ec(1-\phi)^{-2.5}\left(1+\frac{1}{\beta}\right)\left(F''(\xi)\right)^2=0.\end{aligned}\tag{16}$$

The transformed boundary conditions are

$$\begin{aligned}&F(\xi)=0,\ F'(\xi)=0,\ \theta'(\xi)=-\gamma\left(1+\theta(\xi)\right)\text{ or }\theta(\xi)=1\text{ at }\xi=0,\\&F'(\xi)\to1,\ \theta(\xi)\to0,\ \text{ as }\xi\to\infty,\end{aligned}\tag{17}$$

where

$$\Pr=\frac{\mu_{bf}(C_p)_{bf}}{K_{bf}},\ Ec=\frac{U^2}{(C_p)_{bf}(T_w-T_\infty)},\ \gamma=h_s\sqrt{\frac{\nu_{bf}}{a}},$$

are the Prandtl number, Eckert number and Newtonian heating parameter.

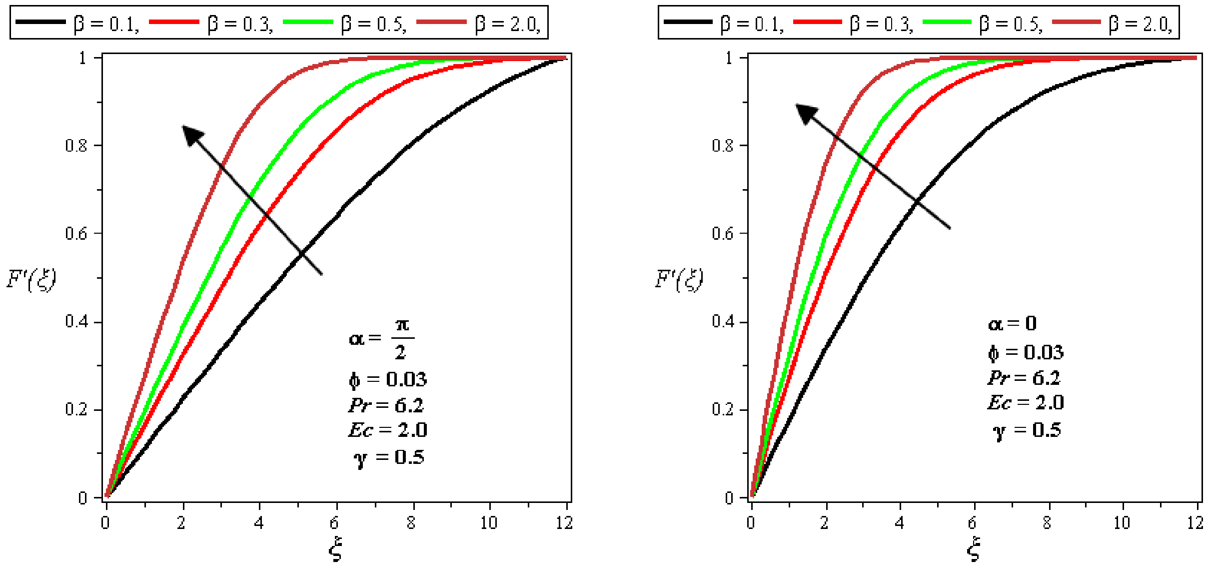
3. Results and Discussions

In this paper, effects of Newtonian heating on heat transfer unsteady flow of nanofluid over semi-infinite vertical plate with leading edge accretion/ablation are investigated. The impact of viscous dissipation in energy equation is also considered. The effects of different parameters such as the Prandtl number Pr, Eckert number Ec, Casson parameter β, Newtonian heating parameter γ, accretion/ablation parameter α and nanoparticle volume fraction $\emptyset$ on velocity $F'(\xi)$ and temperature $\theta(\xi)$ fields are investigated for both Blasius flat plate problem $\alpha=\pi/2$ and Rayleigh-Stokes problem $(\alpha=0)$ cases, separately. Thermo-physical properties of sodium carboxymethyl cellulose and titania nanoparticles are provided in Table 1.

Figures 1(a) and 1(b) show the behaviour of Casson fluid parameter β on velocity profiles $F'(\xi)$ for both Blasius and Rayleigh-Stokes problems. It can easily be observed that for both cases, increasing values of Casson parameter β gives a quite significantly increasing velocity profiles $F'(\xi)$. Figures 2(a) and 2(b) illustrate the influence of different values of Casson parameter β on the temperature profiles $\theta(\xi)$. From these figures, it can easily be seen that Casson parameter β affects the temperature profiles $\theta(\xi)$ starting near the plate $\eta<10$ and then it becomes uniform for both cases as $\eta\to\infty$. However, Rayleigh-Stokes flow problem have lower temperature near the plate as compared to Blasius flat plate problem.

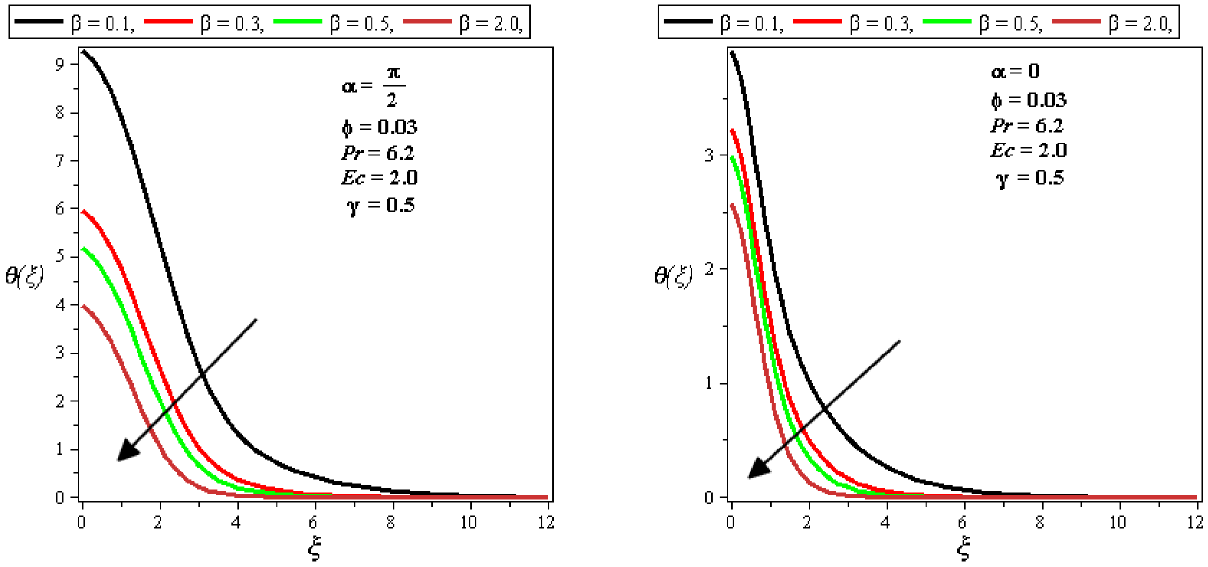
Table 1
Thermo-physical properties of sodium carboxymethyl cellulose and titania nanoparticles

Material	Symbol	ρ(kg.m^{-3})	C$_P$(J/kg.k)	K(W/m.k)
Titania	TiO_2	4230	642	11.7
Sodium Carboxymethyl Cellulose	SCMC	988	4178	0.6474



(a) (b)

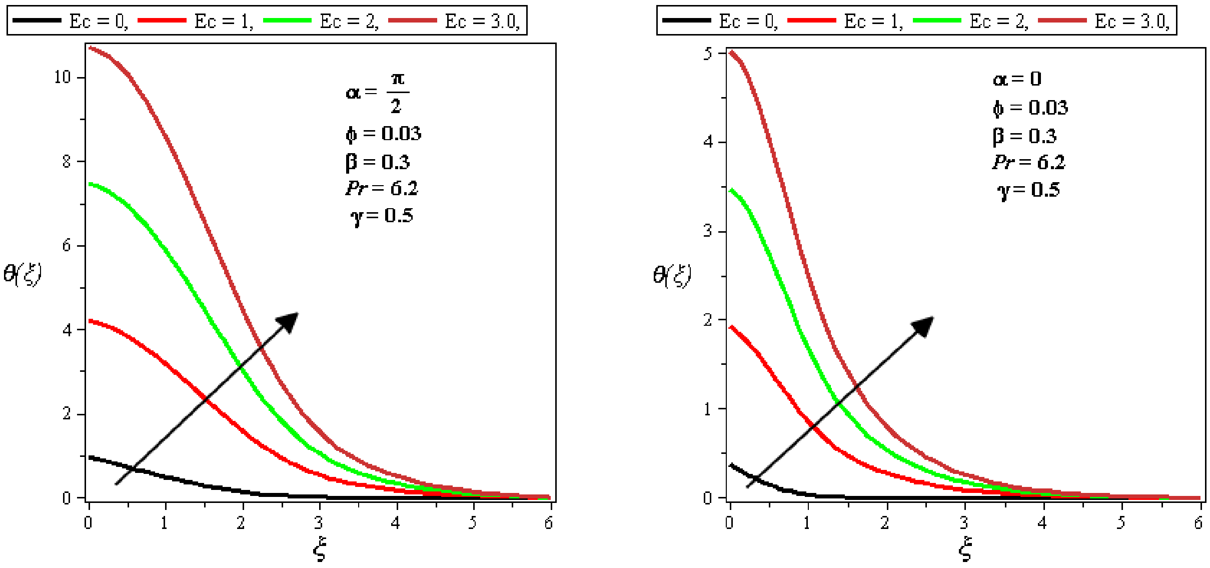
Fig. 1. Velocity profiles for different β, (a) Blasius flat plate (b) Rayleigh-Stokes problem.



(a) (b)

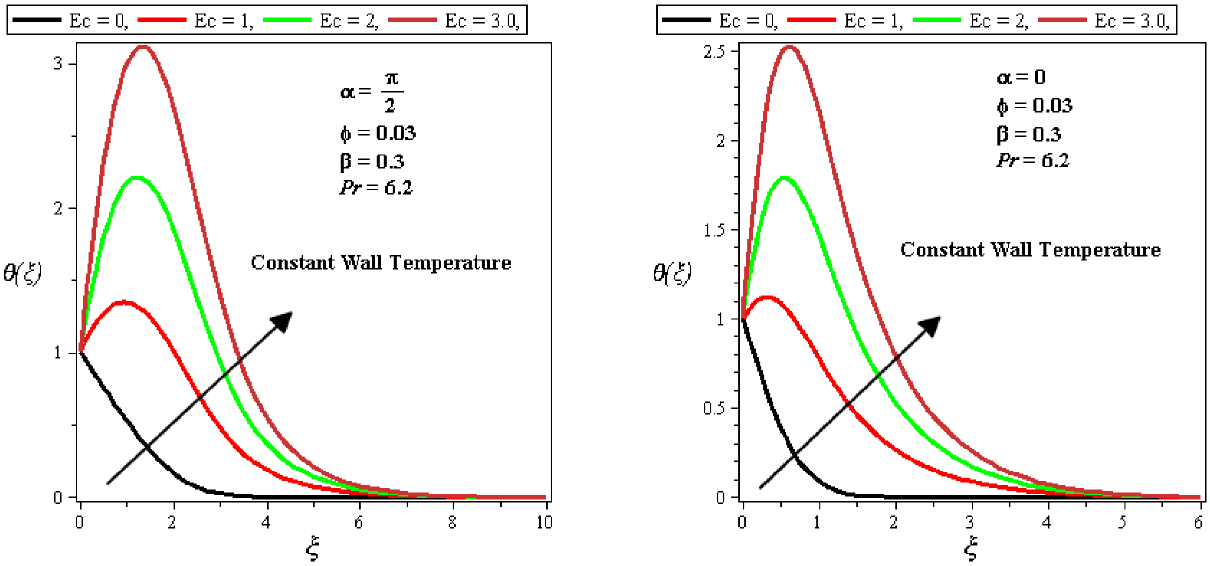
Fig. 2. Velocity profiles for different β, (a) Blasius flat plate (b) Rayleigh-Stokes problem

The effect of the Eckert number Ec on the variation of temperature $\theta(\xi)$ for both Blasius flat plate problem $(\alpha = \pi/2)$ and Rayleigh-Stokes problem $\alpha = 0$ are displayed in Figures 3(a) and 3(b). These figures show that fluid temperature $\theta(\xi)$ increase with the increase in the Eckert number Ec in both cases. Based on the definition of Eckert number (relationship between a kinetic energy flow and the enthalpy), the increase in its value suggests a progressive increase in temperature $\theta(\xi)$. Temperature in case of Blasius flat plate problem $(\alpha = \pi/2)$ is higher than Rayleigh-Stokes problem $\alpha = 0$. Figures 4(a) and 4(b) illustrate the effects of Eckert number Ec in case of Blasius flat plate problem $(\alpha = \pi/2)$ and Rayleigh-Stokes problem $\alpha = 0$ on the temperature $\theta(\xi)$, when the wall temperature is constant. The results show that temperature $\theta(\xi)$ field increases with increase in Eckert number Ec. Furhter, these figures show that in the presence of Newtonian heating parameter, temperature increases near the wall, which shows that thermal boundary layer becomes thicker with an increase in the values of Newtonian heating parameter γ.



(a) (b)

Fig. 3. Temperature profiles for different Ec, (a) Blasius flat plate (b) Rayleigh-Stokes problem



(a) (b)

Fig. 4. Temperature profiles for different Ec, (a) Blasius flat plate (b) Rayleigh-Stokes problem

4. Conclusions

In the present study, we investigated the heat transfer unsteady flow of sodium carboxymethyl cellulose based nanofluid using titania nanoparticles over semi-infinite vertical plate with leading edge accretion/ablation under Newtonian heating and constant wall temperature boundary conditions. Following are the key outcomes of the present study

i. Remarkable change occurs to velocity filed for Rayleigh-Stokes and Blasius flat plate problems.
ii. In the absence of viscous dissipation, the fluid has lower temperature in case of Newtonian heating.

iii. The heat transfer shows higher value for Blasius flat plate, while for Rayleigh-Stokes is the lowest.
iv. Thermal boundary layer thicknesses for $\alpha = 0$ is thinner than $(\alpha = \pi/2)$.

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