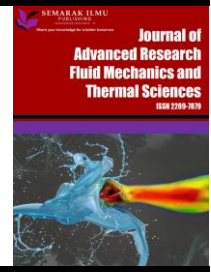




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Soret-Dufour Mechanisms on Unsteady Boundary Layer Flow of Tangent Hyperbolic and Walters-B Nanoliquid

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ABSTRACT

Thermal radiation, chemical reaction, and Soret-Dufour produce effects on the tangential hyperbolic and waters-B nanoliquid unsteady flow which are explored in this work due to their applications in oil and gas industry, microfluidics, aerospace and heat transfer enhancement. With the special interests in the combined Joule heating as well as viscous dissipation, the flow equations is developed to model the transport of species and thermal energy. By utilising the non-dimensional similarity variables, the equations are reduced to their dimensionless PDE form. Using the spectrum relaxation technique (SRM), the new PDES is numerically solved. The Gauss-Seidel SOR method is used to decouple and linearize equation systems. The study is carried out by investigating the graphs as flow parameters vary. It was found that the Brownian motion parameter enhances the rate of heat transfer. Furthermore, by reducing the hydrodynamic boundary layer, the introduction of magnetism was observed to decrease the speed of fluid motion.

1. Introduction

Nanometer-sized particles consisting of metals or carbide oxides are involved in the analysis of nanofluids. In order to prevent the breach of nanoparticles present in common base fluids including ethylene glycol, water, and oil, Choi *et al.*, [1] pioneered the abstraction of nanofluids. Microelectronics, fuel cells, and medicinal procedures are characteristics of several nanofluids. According to Uddin *et al.*, [2] materials such as oxide ceramics, carbide ceramics, metals, and different types of carbon are used to create nanoparticles. The excellent qualities of nanofluids, such as high thermal conductivity, long-term stability, and homogeneity, are due to the nanoparticles' small size and huge surface areas. As a result, nanofluids have uses in the delivery of medications, peristaltic pumps for the treatment of diabetes, solar collectors, and nuclear applications. Das [3] looked at the analysis of the Lie group's stagnation point flow of a nanofluid. According to the study, as magnetic

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field parameters increase, the rate of heat transfer slows, raising the temperature of the fluid in the process. It was concluded in the work of Krishnamurthy *et al.*, [4] that increasing values of thermophoresis and Brownian motion parameters increase the temperature profile. Also, heat transfer shows very little change due to the presence of Brownian motion and thermophoresis parameters. The research demonstrated that as viscosity rises, the shear tension between the fluid and the surrounding surface also rises. Ahmad Khan *et al.*, [5] examined the numerical analysis of the Cattaneo-Christov heat flux model for the flow of viscoelastic fluid. In the study of Afshar *et al.*, [6] it was noted that nanoparticles are dispersed randomly in the fluid due to drag, weight and Brownian forces that carry energy in every direction. By using Flex PDE software, Akbari *et al.*, [7] explained the examination of blood nanofluid in a 3rd-grade non-Newtonian fluid with a magnetic field. The stability convergence study of a chemically and hydromagnetically flowing Casson nanofluid was clarified by Mondal *et al.*, [8]. A mathematical analysis of MHD micropolar nanofluid flow influenced by buoyancy was carried out by Lund *et al.*, [9]. Areekara *et al.*, [10] furthered the study by taking nanofluid flow over the rotatory surface while Animasaun *et al.*, [11] took on the case of non-porous moving walls.

The significance of free convection non-Newtonian flow are largely found in mechanical forming processes, melt spinning, food processing, glass-fibre production processes, and so on have gained the attention of scholars in recent time. Analysing free convection together with thermal radiation involves the process of incompressible laminar flow of viscous fluids. Sravanthi [12] studied free convection MHD slip flow past an exponentially stretchable inclined sheet. Sreedevi *et al.*, [13] investigated the Soret and Dufour mechanism on MHD flow with laminar mixed convection flow for heat and mass transmission. Ramana Reddy [14] studied MHD non-Newtonian fluids flow with a non-uniform heat source. Mondal *et al.*, [15] elucidated MHD mixed convection mass transport past an inclined plate. Naser Ali *et al.*, [16] recently studied nanofluids based on synthesis, stability, and thermophysical properties. Researchers in fluid mechanics are currently very interested in the study of non-Newtonian fluid. Due to the difficulty of the non-Newtonian fluid problem, research has focused on its solution using various numerical methods. Due to the addition of the non-Newtonian element to the momentum equations, momentum expressions are improved. It strengthens the momentum equation and increases the force on fluid flow. Recently, Zaigham Zia *et al.*, [17] investigated the heated surface-induced three-dimensional Casson fluid flow. Their study's Figure 5 demonstrates how the Casson fluid parameter reduces the velocity field. Heat and mass transmission of Casson fluid over a vertical plate was examined by Ganesh Kumar *et al.*, [18]. The Shooting method and Runge-Kutta-Fehlberg's fourth-fifth order were their chosen method of solution. Their findings demonstrate that the velocity profile is slowed down by the Casson fluid parameter. Vijaya *et al.*, [19] explored the radiation effects on erratic Casson fluid flow. Mahanthesh and Gireesha's [20] research looked upon the Marangoni convective 2-phase flow of Casson fluid. The Runge-Kutta-Fehlberg method was adopted and the flow equations were resolved. The investigation revealed that the dusty fluid's velocity is dominated by the Casson fluid's velocity. MHD Casson fluid reactive flow through a porous plate was studied by Vijaya and Reddy [21]. Oyem [22] dealt with the MHD Blasius flow with Darcy-Soret effects. The study of Eyring-Powell non-Newtonian fluid has been modified in the studies of Oke [23] and Oke [24] to the MEP fluids. Oke *et al.*, [25] numerically analysed the effect of rotation on the flow of MEP fluid.

Khan and associates [26] investigated various non-Newtonian Carreau fluid motion solutions past an angled decreasing sheet. Williamson nanofluid flow was examined by Mair Khan *et al.*, [27] for heat and mass transport. Utilising similarity variables, the PDEs were converted into ODEs and solved utilising shooting methods. The Weighberg number was found to increase the velocity field, according to their analysis. In penetrable coaxial cylinders, Zubaidah and Seripah [28] described

continuous MHD 3rd-grade non-Newtonian nanofluid flow. MHD non-Newtonian nanofluid past an inclined plate with Soret-Dufour effects was clarified by Idowu and Falodun in [29]. Asad *et al.*, [30] took a deeper look into the MHD flow of nanofluids whose base fluids are non-Newtonian fluids and the surface was considered as a permeable medium. Rehman and Salleh [31] have looked into the approximate analytical study of non-Newtonian hybrid nanofluid in steady MHD mixed motion on a stretching surface. Bouslimi *et al.*, [32] extended the studies into the MHD Williamson nanofluid over a penetrable material. Thamarai Kannan *et al.*, [33] included radiation in such flows. Other recent studies on non-Newtonian nanofluids include the dissertation by Samuel [34] which is a comprehensive study of non-Newtonian nanofluids over a rotating nonuniform wall, the specific case of Casson nanofluid flow over the porous wall by Vyakaranam *et al.*, [35], a hybrid of dust and Titanium dioxide in the nanofluid [36] and chemical reactions and Cattaneo-Christov heat on nanofluid flow by Rani *et al.*, [37].

In this study, we consider the flow of tangent hyperbolic viscoelastic nanofluid with incompressibility assumption. The flow is seen to take place in the presence of a magnetic field and the Soret-Dufour mechanism is considered significant due to the presence of high concentration. The effects of various parameters that emerged from the equations are later studied on the flow.

2. Mathematical Formulation

We investigated tangent hyperbolic liquid and viscoelastic nanofluid flow. The assumed characteristics of the considered nanofluid include homogeneity, dilution, and incompressibility. According to this theory, the fluid flow is affected downward by gravitational acceleration. The y^* -coordinate is normal to the plate whereas the x^* -coordinate is along the half-infinite vertical plate (see Figure 1). Far from the leading edge, all fluid flow is self-similar. The following assumptions are also taken into consideration for the flow model, which represents a heat and mass transfer mechanism. The penetrable medium is homogeneous. A perpendicular magnetic field with a constant strength B_0 is imposed in the direction of fluid movement. Because of the high level of species concentration, the Soret-Dufour mechanisms are important.

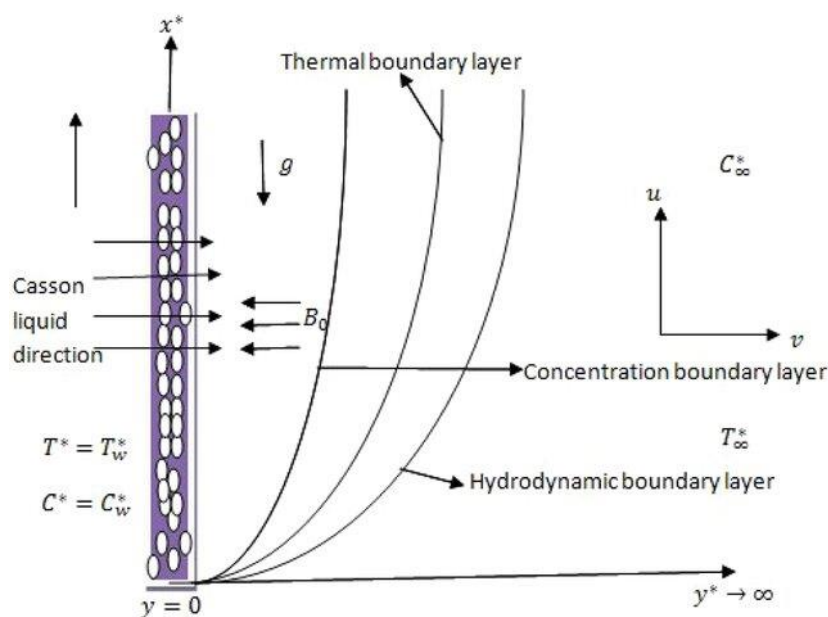


Fig. 1. Physical model of the problem

The boundary layer's nanoparticle distribution is thought to be uniform. The equations follow below since the penetrable medium and fluid characteristics are held constant and the Boussinesq's approximation is accurate:

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\begin{aligned} & \nu(1-n) \frac{\partial^2 u^*}{\partial y^{*2}} + \sqrt{2\nu n} \Gamma \frac{\partial u^*}{\partial y^*} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho} u^* + g\beta_t(T - T_\infty) \\ & + g\beta_c(C - C_\infty) - \frac{\nu}{K} u^* - \frac{K_0}{\rho} \left(\frac{\partial^3 u^*}{\partial t \partial y^2} + v^* \frac{\partial^3 u^*}{\partial y^3} \right) \end{aligned} \right] \tag{2}$$

$$\begin{aligned} \frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} &= \alpha \frac{\partial^2 T}{\partial y^{*2}} + \frac{\nu}{c_p} \left(\frac{\partial u^*}{\partial y^*} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y^*} + \frac{Q_0}{\rho c_p} (T - T_\infty) \\ &+ \frac{Dk_T}{c_s c_p} \frac{\partial^2 C}{\partial y^{*2}} + \frac{\sigma B_0^2}{\rho c_p} u^2 + \tau \left[D_B \frac{\partial C}{\partial y^*} \frac{\partial T}{\partial y^*} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y^*} \right)^2 \right] \end{aligned} \tag{3}$$

$$\frac{\partial C}{\partial t^*} + v^* \frac{\partial C}{\partial y^*} = D \frac{\partial^2 C}{\partial y^{*2}} + \frac{Dk_T}{T_m} \frac{\partial^2 T}{\partial y^{*2}} - K_r(C - C_\infty) \tag{4}$$

subject to the constraints:

$$\begin{aligned} u &= U_0, T = T_w + \psi(T_w - T_\infty)e^{n^*t^*}, C = C_w + \psi(C_w - C_\infty)e^{n^*t^*}, \text{ at } y^* = 0 \\ u^* &\rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } y^* \rightarrow \infty \end{aligned} \tag{5}$$

On Integrating Eq. (1), we can determine the normal suction velocity normal. According to Idowu and Falodun's [29] analysis, the wall suction velocity is a function of a constant and a time-dependent quantity given as

$$v^* = -v_0(1 + \delta A e^{n^*t^*}) \tag{6}$$

Considering the fact that $\frac{\partial q_r}{\partial y^*} \gg \frac{\partial q_r}{\partial x^*}$ as heat flux spreads only towards the y^* -axis. The difference in temperature is small in a way that T^4 is expressed as

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{7}$$

Utilizing Rosseland approximation (suggesting that the tangent hyperbolic liquid is optically thick),

$$q_r = -\frac{4\sigma_0}{3ke} \frac{\partial T^4}{\partial y^*} \quad (8)$$

here σ_0 is the Stefan-Boltzmann constant which ke represents the mean absorption coefficient. The energy equation becomes;

$$\frac{\partial T}{\partial t^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} + \frac{v}{c_p} \left(\frac{\partial u^*}{\partial y^*} \right)^2 + \frac{16\sigma_0 T_\infty^3}{3\rho c_p ke} \frac{\partial^2 T}{\partial y^{*2}} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{Dk_T}{c_s c_p} \frac{\partial^2 C}{\partial y^{*2}} + \frac{\sigma B_0^2}{\rho c_p} u^2 \quad (9)$$

To simplify the flow equations in a dimensionless form, we use the following

$$u = \frac{u^*}{u_0}, y = \frac{v_0^2 y^*}{\nu}, t = \frac{v_0^2 t^*}{\nu}, n = \frac{v n^*}{v_0^2}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C_\infty}{C_w - C_\infty} \quad (10)$$

Employing the Eq. (10) above to obtain the following equations:

$$\frac{\partial U}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial U}{\partial y} = \left\{ (1 - n) \frac{\partial^2 U}{\partial y^2} - M^2 U - \gamma \left(\frac{\partial^3 U}{\partial t \partial y^2} - (1 + \epsilon A e^{nt}) \frac{\partial^3 U}{\partial y^3} \right) \right. \\ \left. + Gr\theta + Gm\phi - \frac{1}{Pr} U \right\} \quad (11)$$

$$\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \left\{ \left(\frac{1+R}{Pr} \right) \frac{\partial^2 \theta}{\partial y^2} + Ec \left(\frac{\partial U}{\partial y} \right)^2 + Du \frac{\partial^2 \phi}{\partial y^2} + Q\theta \right\} \\ \left. + M^2 Ec U^2 + Nb \frac{\partial \phi}{\partial y} \frac{\partial \theta}{\partial y} + Nt \left(\frac{\partial \theta}{\partial y} \right)^2 \right\} \quad (12)$$

$$\frac{\partial \phi}{\partial t} - (1 + \epsilon A e^{nt}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - Kr\phi \quad (13)$$

Subject to:

$$U = 1, \theta = 1 + \epsilon e^{nt}, \phi = 1 + \epsilon e^{nt}, \text{ at } y = 0 \quad (14)$$

$$U \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0, \text{ at } y \rightarrow \infty \quad (15)$$

3. Spectral Relaxation Technique (SRM)

The formulated PDEs shall be solved numerically utilizing SRM. The SRM is an iterative approach which adopts the Gauss-seidel SOR type to simplify the equations into a linearised uncoupled system. The Chebyshev pseudo-spectral method (Motsa, 2012) is then used to discretise the linearised uncoupled system. Suppose we have a PDE of the form

$$\mathbb{L}[u(x)] = f(x), \quad (16)$$

where \mathbb{L} is a linear operator, $u(x)$ is the unknown function, and $f(x)$ is a known function. An approximate solution $u_N(x)$ is sought over a set of basic functions $\{\phi_k(x)\}_{k=1, \dots, N}$ such that

$$u_N(x) = \sum_{k=1}^N a_k \phi_k(x) \tag{17}$$

where a_k are the coefficients. Replacing u by u_N , we have

$$\mathbb{L}[\sum_{k=1}^N a_k \phi_k(x)] = f(x) \tag{18}$$

Evaluating the different points of x , we have

$$\sum_{k=1}^N L_{ik} a_k = f(x_i), \quad i = 1, 2, \dots, N \tag{19}$$

where L_{ik} are the elements of the linear operator \mathbb{L} . The resulting system of Eq. (19) is solved using the Gauss-Seidel Successive Over-Relaxation technique.

4. Results and Discussion

Using the spectral relaxation technique, the system of nonlinear PDEs Eq. (11) to Eq. (13) subject to Eq. (14) and Eq. (15) are numerically solved. The relevant fluid parameters are studied as they vary on dimensionless flow properties, including the viscoelastic parameter, Weissenberg number, magnetic parameter, thermal Grashof number, Dufour parameter, and others in the domain $[0,1]$.

Soret parameter (So) is studied on the flow as seen in Figure 2. Increasing the value of So improves the distributions of velocity and concentration. Soret parameter increase enhances the temperature difference between the fluid and the surface. The fluid diffuses from the heated zone to a cooled region near the plate as a result of increasing o .

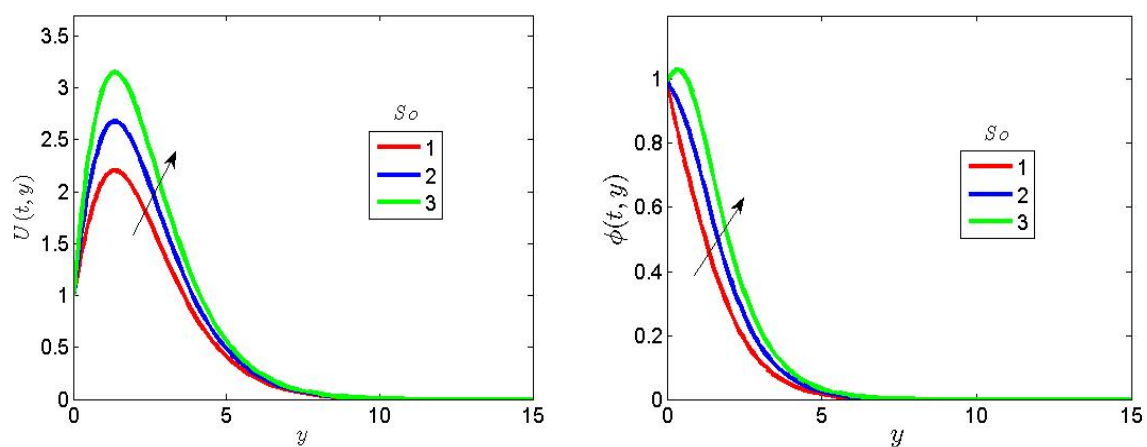


Fig. 2. Effect of Soret parameter on the velocity and concentration distributions

The impact of the Dufour parameter (Do) on the distributions of velocity and temperature is seen in Figure 3. Both the velocity and the concentration of nanofluid particles are shown to increase with a greater value of Do . This demonstrates that increasing Do causes the differential between fluid concentration and the surface to grow. It has been found that the Dufour parameter (Do) has the opposite effect as the Soret parameter.

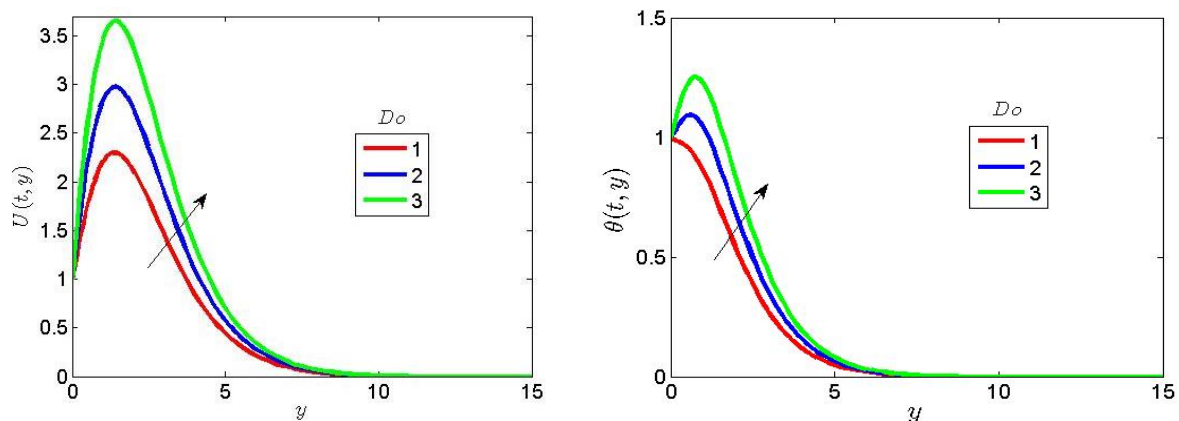


Fig. 3. Effect of Dufour parameter on the velocity and temperature distributions

Viscoelastic parameter (β) is studied on the distributions of velocity and temperature as in Figure 4. It has been found that a larger value increases fluid temperature and velocity. This demonstrates that the boundary layer fluid movement is not adversely affected by the induced magnetic field. Additionally, because the fluid's viscosity is so low, it has little impact on how much momentum the nanoparticles can gain. The effect of the normal stress coefficient on the motion of the fluid is depicted by the viscoelastic parameter (β). From Figure 4, the fluid velocity and temperature distributions dramatically increase at a particular location in the flow domain.

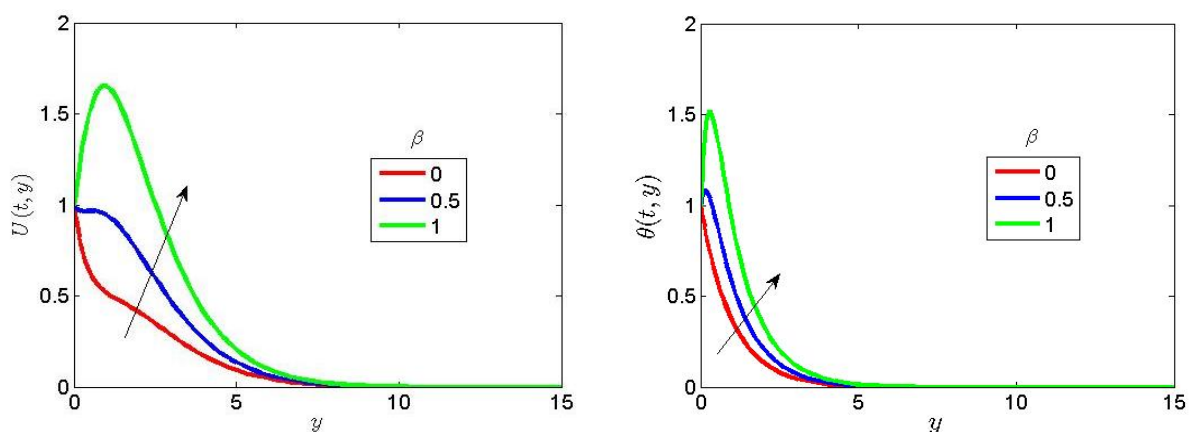


Fig. 4. Effect of viscoelastic parameter on the velocity and temperature distributions

The impact of the Weissenberg number (We) on the velocity distribution is shown in Figure 5. An increase in We is seen to cause a zigzag flow at the wall, which eventually changes to a normal flow distant from the plate.

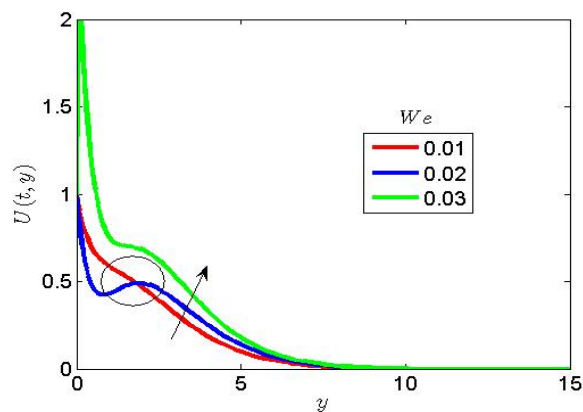


Fig. 5. Effect of Weissenberg number on the velocity distribution

Magnetic parameter (M) effects on the velocity distribution are depicted in Figure 6. We found that a rise in M causes the velocity distributions to fall. Physically, as M increases, the magnetic field dampens effects on the flow. The fluid's motion is slowed down by the damping effect on the flow, which also causes the nanoparticles to get heated. The applied magnetic field's transverse influence on velocity triggers the Lorentz force, which opposes the hydrodynamic boundary layer.

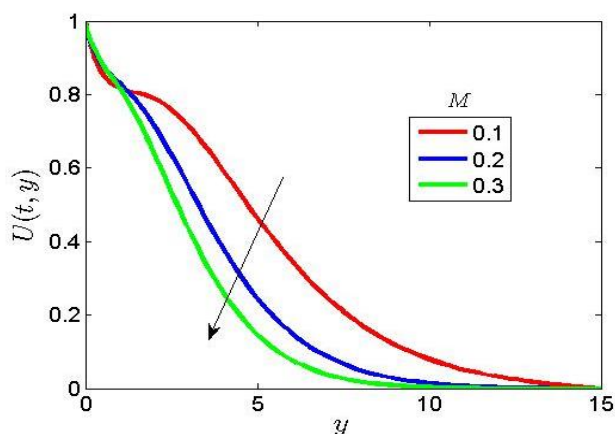


Fig. 6. Effect of Magnetic parameter on the velocity distribution

Thermal Grashof number (Gr) is investigated on the velocity distributions (Figure 7). Increasing Gr improves the hydrodynamic boundary layer and velocity. The flow of fluids within the boundary layer is propelled by the thermal Grashof number, which functions as a buoyancy force.

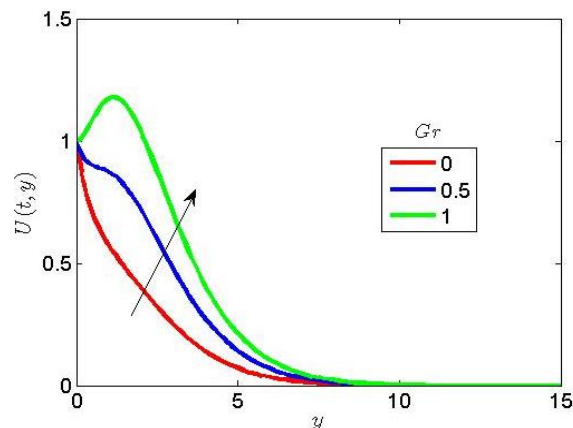


Fig. 7. Effect of thermal Grashof number on the velocity distribution

The thermal radiation parameter (R_r) on the distributions of velocity and temperature is seen in Figure 8. It is noted that both the velocity and temperature distributions are improved by a higher value of R_r . The main factor that contributes to the thermal radiation parameter is conduction heat transmission. Higher thermal radiation tends to increase the fluid's thermal state, warming the nanofluids' constituent particles. The findings are very similar to those of Alao *et al.*, (2016). Setting $R_r=0$ demonstrates that the Rosseland model is absent from this study.

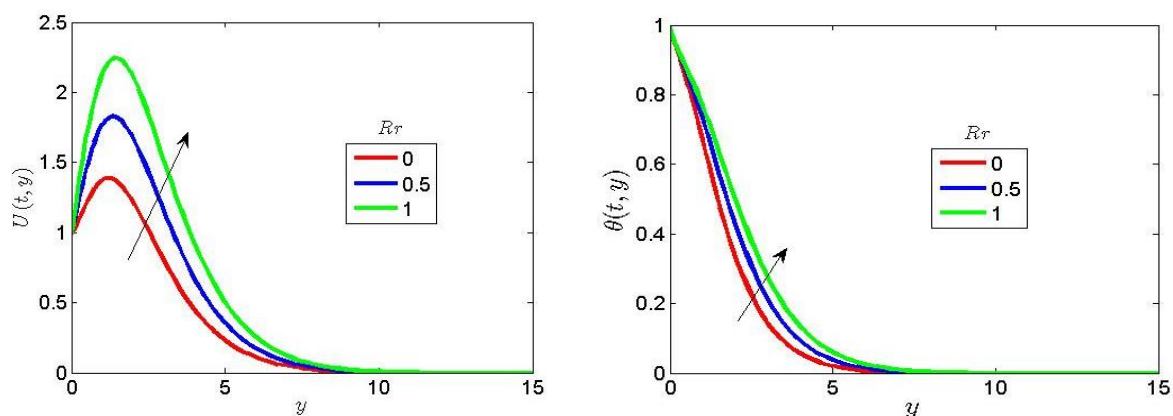


Fig. 8. Effect of thermal radiation parameter on the velocity and temperature distributions

The impact of the Prandtl number (Pr) on the temperature and velocity distributions is seen in Figure 9. The distributions of velocity and temperature are seen to decrease as Pr increases. Physically, a greater value of Pr reduces the thermal diffusivity, which results in a decline in the boundary layer's capacity to transmit heat. As a result, a higher value of Pr causes greater viscosities, which slow down an electrically conducting fluid's velocity.

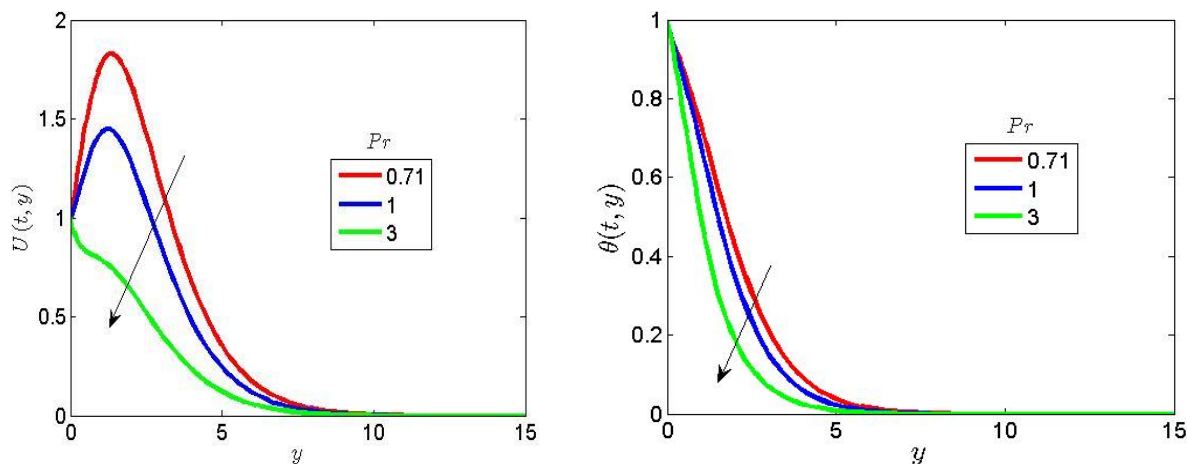


Fig. 9. Effect of Prandtl number on the velocity and temperature distributions

The relationship between the distributions of velocity and temperature and the Eckert number (Ec) is shown in Figure 10. The link between fluid kinetic energy and enthalpy is depicted by the Eckert number. Raising Ec elevates the temperature. This is plausible because elastic deformation stores energy within the fluid region and heat is dissipated through viscosity. As a result, frictional heating is the cause of the sources of heat storage in the fluid. The fluid temperature is more significantly impacted with a higher Eckert number when a magnetic field is present in the model.

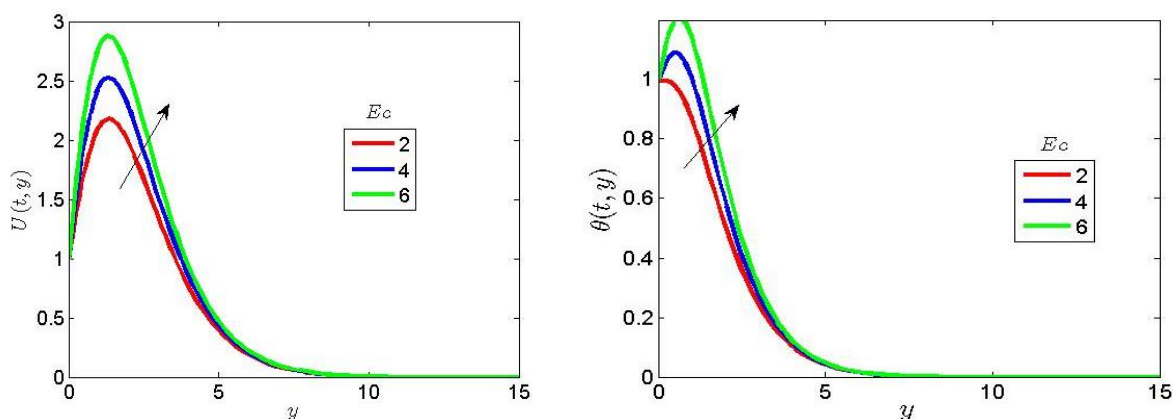


Fig. 10. Effect of Eckert number on the velocity and temperature distributions

The distributions of velocity and temperature are affected by the Brownian motion parameter (Nb), as seen in Figure 11. It has been found that a greater amount of Nb improves the temperature and fluid velocity distributions. Physically, as Nb rises, the temperature of the layer causes the nanoparticles to warm up. The fluid travels very quickly by thickening the hydrodynamic boundary layer due to a larger Brownian motion value.

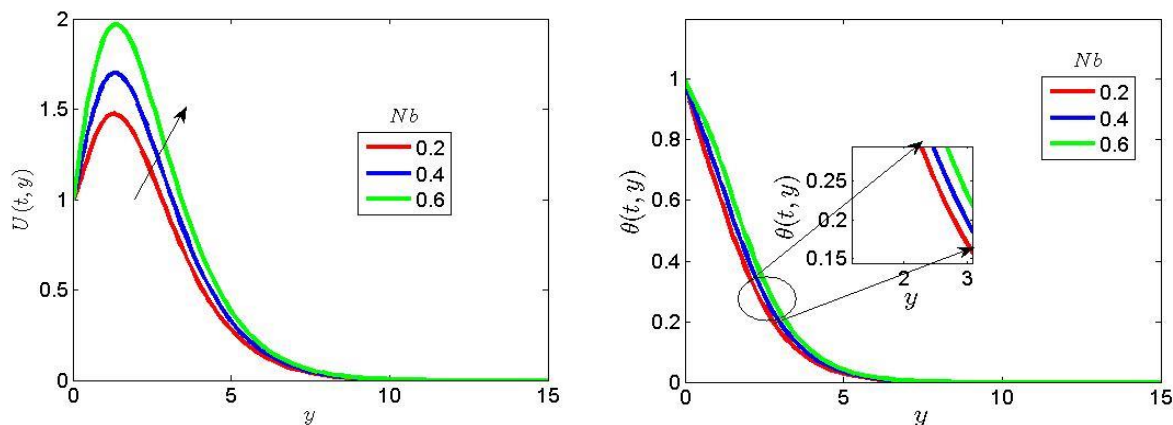


Fig. 11. Effect of Brownian motion parameter on the velocity and temperature distributions

Figure 12 represents the effect of the thermophoresis parameter (Nt) on velocity distributions. A higher value of Nt raises the velocity distributions. Thermophoresis refers to the movement of particles from a hot surface to a cold surface.

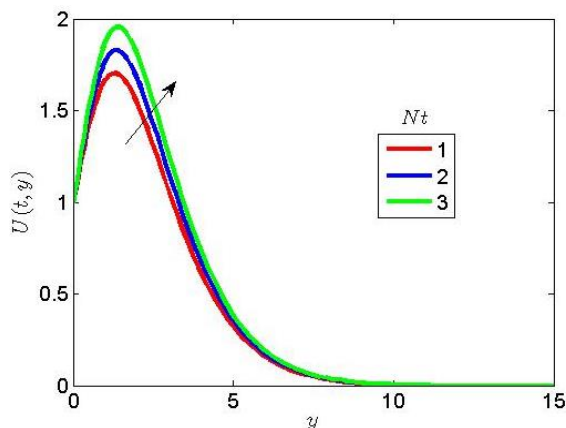


Fig. 12. Effect of thermophoresis parameter on the velocity distribution

Schmidt number (Sc) effects on the velocity and concentration distributions respectively are depicted in Figure 13. Increasing Sc inhibits the velocity and concentration distributions respectively. If $Sc < 1$, the specie diffusivity dominates but, if $Sc > 1$, the momentum diffusivity dominates. Physically, when $Sc=1$ specie and momentum diffusivity are at the same rate.

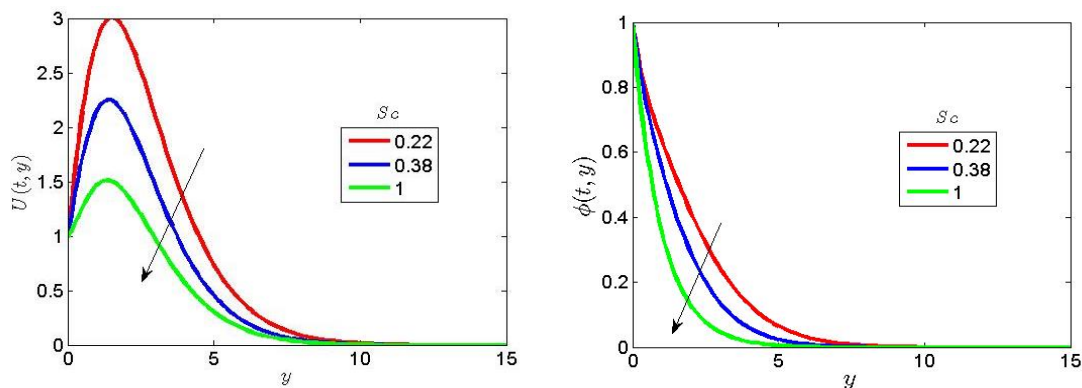


Fig. 13. Effect of Schmidt number on the velocity and concentration distributions

The chemical reaction parameter (kr) effects on the velocity and concentration distributions are depicted in Table 1. Increasing kr declines velocity alongside the concentration distributions.

Table 1
 Local skin friction, local Nusselt number and Sherwood number with varying parameters

β	We	M	Gr	Rr	Pr	Ec	Nb	Nt	Sc	Kr	U(0)	$\theta(0)$	$\phi(0)$
0											2.84731	0.95231	0.80001
0											1.92081	-1.1472	0.80001
1											0.37621	-4.4369	0.80001
	0										0.48201	0.81331	0.71441
	0										0.48211	0.81331	0.71441
	0										0.48771	0.81331	0.71441
		0									2.66501	0.60001	0.77831
		1									2.69631	0.60001	0.77831
		1									2.74661	0.60001	0.77831
			1								1.18271	0.33941	0.71931
			2								1.44191	0.33941	0.71931
			3								1.70111	0.33941	0.71931
				0							0.08681	0.34501	0.60281
				1							0.66441	0.33941	0.60281
				1							1.30751	0.34141	0.60281
					0.7						0.66441	0.33941	0.77831
					1						1.21911	0.34031	0.77831
					3						2.30631	0.41391	0.77831
						0					0.05521	0.04171	0.05921
						0					-0.5542	-0.2561	0.05921
						0					-1.1637	-0.5539	0.05921
							1				0.43661	0.28351	0.80001
							2				0.87331	0.39201	0.80001
							3				1.24191	0.48811	0.80001
								1			0.32371	0.33941	0.17951
								2			0.66441	0.33941	0.17951
								3			1.00501	0.33941	0.17951
									0.2		-1.015	0.41911	0.60161
									0.4		0.02461	0.41911	0.67831
									1		1.19341	0.41911	1.02971
										0	0.39201	0.14391	0.69741
										1	0.66441	0.14391	0.80001
										1	1.22891	0.14391	1.63841

6. Conclusions

This work presents a numerical solution for the analysis of constant viscosity and thermal conductivity on the tangent hyperbolic and viscoelastic liquid flows characterised by nanofluids. The boundary layer mixing of the tangent hyperbolic and viscoelastic liquids is investigated in Figure 1. The fluid boundary layer exhibits a variety of flow behaviour as a result of their combination. We looked at how relevant flow parameters affected the dimensionally invariant velocity, temperature, and concentration distributions. The main outcomes include;

- i. The Rosseland approximation shows that the fluid is optically thin.
- ii. Higher values of Brownian motion improve the velocity and temperature, and the imposed magnetic field strength is observed to invoke the Lorentz force on the fluid flow by slowing the motion of an electrically conducting fluid.
- iii. Higher thermal radiation improves the fluid velocity and temperature as well as the thickness of the thermal boundary layer.
- iv. The larger the Schmidt number and chemical reaction decreases the flow behaviour.

Due to the presence of thermal radiation and heat generation, the current work will be useful in the field of thermal engineering. Due to the usage of nanofluids in lowering harmful substances, it also has applications in biomedical engineering.

Acknowledgement

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