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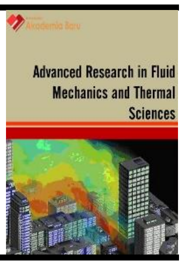

Analysis on Computational Efficiency of Convection Discretisation Schemes in SIMPLE Algorithm

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ABSTRACT

Computational Fluid Dynamics (CFD) is widely used to investigate heat transfer, fluid flow, chemical reaction and mass transfer phenomenon. While solving the Navier-Stokes equations, the convection term is always prone to numerical instability and therefore the discretisation of the convection term requires special attention. The performance of various convection schemes had been previously performed on one-dimensional convection-diffusion problem. Nevertheless, the numerical errors of these convection schemes are more pronounced in higher-dimensional problems especially those involving pressure term and flow recirculation. In this paper, the performances of convection schemes such as first order upwind differencing, second order upwind differencing, Quadratic Upstream Interpolation for Convective Kinematics (QUICK) and power-law schemes are investigated on the two-dimensional lid-driven flow problem in a square cavity. By using commercial CFD software ANSYS Fluent, the consistency, efficiency and accuracy of the results due to different convection schemes are compared. It is found that although the power-law scheme is the best in terms of iterative convergence rate, it is not accurate especially for high Re-flow. Higher order scheme such as QUICK is very accurate; however, its convergence rate is the lowest.

Keywords:
ANSYS Fluent, Discretisation of convection term, First order upwind, Second order upwind, Power-law, QUICK

1. Introduction

Continuity and Navier-Stokes equations are regarded as the most rudimental governing equations in CFD analysis [1]. The non-dimensional governing equations for the steady incompressible flow are listed in Eq. (1) to Eq. (2) in tensor form. In fact, the non-linear convective term in Eq. (2) is prone to numerical instability [2-5]. For example, Hatton and Turton [6], Nield [7], Coelho and Pinho [8] and Foroushani *et al.,* [9] experienced difficulties in investigating the Nusselt number correlation at different wall temperatures when strong convection occurs. Improper

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discretisation of the convection term will lead to unphysical spurious oscillation and inaccurate solution. Therefore, formulating an efficient way to discretize the convection term remains as the main challenge in modern CFD [10,11].

There are three important criteria [5] to be considered while discretizing the convective term: conservativeness, boundedness and transportiveness. Conservativeness represents the property in which the field value between the control volume should be the same; boundedness signifies the criterion for convergence by forming a diagonally dominant matrix; while transportiveness is the comparative property between the diffusion and convection effects.

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial(u_i u_j)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\frac{1}{Re}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right] \tag{2}$$

where $\partial(u_i u_j)/\partial x_j$ is the convection term, $-\partial P/\partial x_i$ is the pressure term, $\partial/\partial x_j\left[1/Re\left(\partial u_i/\partial x_j + \partial u_j/\partial x_i\right)\right]$ is the viscous term, while Re is Reynolds number.

The central differencing scheme is indeed an unbounded scheme whereby it produces unphysical noise in the numerical solution when the convection outweighs the diffusion effects [5]. Selection of an appropriate convection discretisation scheme is therefore important in achieving a consensus between computational efficiency and accuracy. There are some advanced discretisation methods such as total variation diminishing [12,13] and stability-controllable second-order difference scheme [14], etc.; however, these schemes are not discussed in the current work.

The performances of convection schemes have been mostly studied for one-dimensional convection-diffusion problem [2,5] in which the pressure term and possible counter-flow effect have been omitted. The computational efficiencies of the convection schemes could be different for multi-dimensional problem [15].

In the current work, we will compare the properties such as computational stability, convergence speed and accuracy of the convection schemes such as first order upwind (FOU) scheme, second order upwind (SOU) scheme, Quadratic Upstream Interpolation for Convective Kinematics (QUICK) scheme and power-law scheme are evaluated on two-dimensional lid-driven flow problem in a square cavity. The Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) algorithm implemented in commercial CFD software ANSYS Fluent is used in the current study.

2. Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) Revisited

The Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) algorithm was proposed by Patankar and Spalding [16] as a tool for coupling pressure and velocity terms. Instead of solving Eq. (1) and Eq. (2) by forming the Pressure Poisson equation when using fractional step method [17], SIMPLE algorithm deploys the guessing-and-correction concept for *x*-velocity, *y*-velocity and pressure. In ANSYS Fluent, Eq. (1) and Eq. (2) are discretized using finite volume method such that

$$\int \frac{\partial(u_i u_j)}{\partial x_j} dA = \int \left\{-\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\frac{1}{Re}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]\right\} dA \tag{3}$$

By re-arranging the terms upon discretisation, Eq. (3) becomes

$$au_i = \Delta P + \sum (au_i)_{nb} \tag{4}$$

where a and a_{nb} are the coefficients of the targeted and neighboring grids. Eq. (4) is computed based on a set of guessed values initially which requires further re-calculation. Meanwhile, in order to satisfy the continuity equation, Eq. (4) should be further modified in such a way that:

$$au_i' = \Delta P' + \sum(au_i)_{nb}' \tag{5}$$

$$u_i^c = u_i + u_i' \tag{6}$$

$$P^c = P + P' \tag{7}$$

where u_i' is the velocity error, P' is the pressure error, u_i^c is the corrected velocity while P^c is the corrected pressure; combining Eq. (4) to Eq. (7) will form

$$a(u_i^c - u_i) = \Delta P' + \sum a(u_i^c - u_i)_{nb}' \tag{8}$$

In SIMPLE algorithm, the velocity error of the neighboring grids as in Eq. (5) and Eq. (6) are ignored. Then, Eq. (8) is applied into the discretised equation of Eq. (1) in order to obtain the pressure error. The process is iterated until convergence is achieved, fulfilling both continuity and momentum principles. In order to stabilise the computation, the under-relaxation factor α is included into Eq. (7) to form Eq. (10). Hence, upon obtaining the pressure error, both corrected velocity and pressure can be now obtained in Eq. (9) and Eq. (10), respectively.

$$u_i^c = a^{-1}\Delta P' + u_i \tag{9}$$

$$P^c = P + \alpha P' \tag{10}$$

By using the corrected velocity and pressure fields, the computation of Eq. (4) to Eq. (10) is repeated until the continuity residual approaches zero. In SIMPLE algorithm, the selection of under-relaxation factor is important for the numerical stability [18]. The detailed description of SIMPLE algorithm can be found in many resources [5,19,20].

3. Numerical Insights on Discretisation Schemes

In this section, four different convection schemes: FOU, SOU, QUICK and power-law scheme are discussed here. The discretisation is done based on the staggered grid as illustrated in Figure 1. Note that the indices i and j here refer to the spatial locations instead of tensor form expression.

The general way of the discretising of convection term in Eq. (3) in two-dimensional domain are outlined in Eq. (11.1) – Eq. (11.4)

$$\int \left(u_x \frac{\partial u_x}{\partial x}\right) dxdy = (u_x)_{i,j}[(u_x)_e - (u_x)_w]\Delta y \tag{11.1}$$

$$\int \left(u_y \frac{\partial u_x}{\partial y}\right) dxdy = (u_x)_{i,j}\left[(u_x)_{i+1,j} - (u_x)_{i,j}\right]\Delta x \tag{11.2}$$

$$\int \left(u_x \frac{\partial u_y}{\partial x}\right) dxdy = (u_x)_{i,j}\left[\left(u_y\right)_{i+1,j} - (u_x)_{i,j}\right]\Delta y \tag{11.3}$$

$$\int \left(u_y \frac{\partial u_y}{\partial y}\right) dxdy = \left(u_y\right)_{i,j} \left[\left(u_y\right)_n - \left(u_y\right)_s\right] \Delta x \tag{11.4}$$

where e, w, n and s represent the imaginary velocity value at the east, west, north and south "faces" of the control volume respectively. The centroid of the control volume is to store pressure variables such as $P_{i+1,j}$, $P_{i-1,j}$, $P_{i,j+1}$ and $P_{i,j-1}$ respectively.

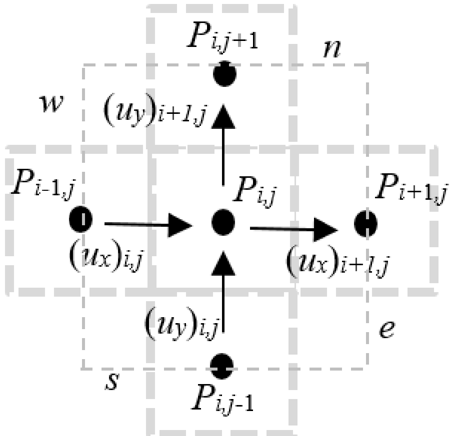


Fig. 1. Grid formation using staggered grid

3.1 First Order Upwind (FOU) Differencing Scheme

FOU differencing scheme is one of the oldest convection discretisation, proposed by Courant *et al.,* [21], Torrance [22] and Runchal *et al.,* [23]. The face velocities in Eq. (11.1) and Eq. (11.4) are calculated in accordance with Eq. (12.1) to Eq. (12.4).

$$\left(u_x\right)_e = \begin{cases} \left(u_x\right)_{i,j} & \left(u_x\right)_{i+1,j} - \left(u_x\right)_{i,j} > 0 \\ \left(u_x\right)_{i+1,j} & \left(u_x\right)_{i+1,j} - \left(u_x\right)_{i,j} < 0 \end{cases} \tag{12.1}$$

$$\left(u_x\right)_w = \begin{cases} \left(u_x\right)_{i-1,j} & \left(u_x\right)_{i,j} - \left(u_x\right)_{i-1,j} > 0 \\ \left(u_x\right)_{i,j} & \left(u_x\right)_{i,j} - \left(u_x\right)_{i-1,j} < 0 \end{cases} \tag{12.2}$$

$$\left(u_y\right)_n = \begin{cases} \left(u_y\right)_{i,j} & \left(u_y\right)_{i,j+1} - \left(u_y\right)_{i,j} > 0 \\ \left(u_y\right)_{i+1,j} & \left(u_y\right)_{i,j+1} - \left(u_x\right)_{i,j} < 0 \end{cases} \tag{12.3}$$

$$\left(u_y\right)_s = \begin{cases} \left(u_y\right)_{i,j-1} & \left(u_y\right)_{i,j} - \left(u_y\right)_{i,j-1} > 0 \\ \left(u_y\right)_{i,j} & \left(u_y\right)_{i,j} - \left(u_x\right)_{i,j-1} < 0 \end{cases} \tag{12.4}$$

Although FOU is simple, it is prone to false diffusion [20,23] when multi-dimensional problem is considered. Therefore, more grid points are necessary for reasonable accuracy.

3.2 Second Order Upwind (SOU) Differencing Scheme

SOU differencing scheme was proposed by Warming and Beam [24] and Hodge *et al.,* [25]. The face velocity as from Eq. (12.1) to Eq. (12.4) are calculated based on Eq. (13.1) to Eq. (13.4).

$$(u_x)_e = \begin{cases} \dfrac{3(u_x)_{i,j} - (u_x)_{i-1,j}}{2} & (u_x)_{i+1,j} - (u_x)_{i,j} > 0 \\ \dfrac{3(u_x)_{i+1,j} - (u_x)_{i+2,j}}{2} & (u_x)_{i+1,j} - (u_x)_{i,j} < 0 \end{cases} \tag{13.1}$$

$$(u_x)_w = \begin{cases} \dfrac{3(u_x)_{i-1,j} - (u_x)_{i-2,j}}{2} & (u_x)_{i,j} - (u_x)_{i-1,j} > 0 \\ \dfrac{3(u_x)_{i,j} - (u_x)_{i+1,j}}{2} & (u_x)_{i,j} - (u_x)_{i-1,j} < 0 \end{cases} \tag{13.2}$$

$$(u_y)_n = \begin{cases} \dfrac{3(u_y)_{i,j} - (u_y)_{i,j-1}}{2} & (u_y)_{i,j+1} - (u_y)_{i,j} > 0 \\ \dfrac{3(u_y)_{i,j+1} - (u_y)_{i,j+2}}{2} & (u_y)_{i,j+1} - (u_x)_{i,j} < 0 \end{cases} \tag{13.3}$$

$$(u_y)_s = \begin{cases} \dfrac{3(u_y)_{i,j-1} - (u_y)_{i,j-2}}{2} & (u_y)_{i,j} - (u_y)_{i,j-1} > 0 \\ \dfrac{3(u_y)_{i,j} - (u_y)_{i,j+1}}{2} & (u_y)_{i,j} - (u_x)_{i,j-1} < 0 \end{cases} \tag{13.4}$$

Monotonic SOU [26] is the variant of SOU designed to reduce the potential oscillation. However, the method is not available in ANSYS Fluent and therefore it is not discussed here. Due to its second order accuracy which requires more computational stencils, SOU scheme is more accurate than FOU scheme [27] at the expense of more complex implementation. This issue has been addressed by Kajishima and Taira [17].

3.3 Quadratic Upstream Interpolation for Convective Kinematics (QUICK) Scheme

QUICK scheme is one of the third-order upwind differencing schemes, put forward by Leonard [28]. This scheme involves three consecutive adjacent nodes. Although some variants of QUICK scheme had been reported by Leschziner [29], Han *et al.,* [30] and Pollard and Siu [31], these schemes are not discussed here. The face values approximated via QUICK scheme can be found using Eq. (14.1) to Eq. (14.4). Due to its boundedness, spurious oscillation may still occur. Flux-limiter schemes [32,33] can be used to address this problem.

$$(u_x)_e = \begin{cases} \dfrac{3(u_x)_{i+1,j} + 6(u_x)_{i,j} - (u_x)_{i-1,j}}{8} & (u_x)_{i+1,j} - (u_x)_{i,j} > 0 \\ \dfrac{3(u_x)_{i,j} + 6(u_x)_{i+1,j} - (u_x)_{i+2,j}}{8} & (u_x)_{i+1,j} - (u_x)_{i,j} < 0 \end{cases} \tag{14.1}$$

$$(u_x)_w = \begin{cases} \dfrac{3(u_x)_{i,j} + 6(u_x)_{i-1,j} - (u_x)_{i-2,j}}{8} & (u_x)_{i,j} - (u_x)_{i-1,j} > 0 \\ \dfrac{3(u_x)_{i-1,j} + 6(u_x)_{i,j} - (u_x)_{i+1,j}}{8} & (u_x)_{i,j} - (u_x)_{i-1,j} < 0 \end{cases} \tag{14.2}$$

$$(u_y)_n = \begin{cases} \dfrac{3(u_y)_{i,j+1} + 6(u_y)_{i,j} - (u_y)_{i,j-1}}{8} & (u_y)_{i,j+1} - (u_y)_{i,j} > 0 \\ \dfrac{3(u_y)_{i,j} + 6(u_y)_{i,j+1} - (u_y)_{i,j+2}}{8} & (u_y)_{i,j+1} - (u_x)_{i,j} < 0 \end{cases} \tag{14.3}$$

$$(u_y)_s = \begin{cases} \dfrac{3(u_y)_{i,j} + 6(u_y)_{i,j-1} - (u_y)_{i,j-2}}{8} & (u_y)_{i,j} - (u_y)_{i,j-1} > 0 \\ \dfrac{3(u_y)_{i,j-1} + 6(u_y)_{i,j} - (u_y)_{i,j+1}}{8} & (u_y)_{i,j} - (u_x)_{i,j-1} < 0 \end{cases} \tag{14.4}$$

3.4 Power-Law Scheme

In power-law scheme [2], the effect of Peclet number (the ratio between convection and diffusion effects) is considered. Peclet number Pe for different dimensions are defined as in Eq. (15.1) and Eq. (15.2). The subscripts x and y represent the Pe in x- and y- momentum equations, respectively.

$$Pe_x = \frac{\rho u_x \Delta x}{\mu} \tag{15.1}$$

$$Pe_y = \frac{\rho u_y \Delta y}{\mu} \tag{15.2}$$

where μ is the dynamic viscosity of the fluid.

By using Peclet number, the face velocity can now be computed from Eq. (16.1) to Eq. (16.4).

$$(u_x)_e = \begin{cases} (u_x)_{i,j} + \left[(u_x)_{i+1,j} - (u_x)_{i,j}\right]\left[\dfrac{\sqrt{\chi_x} - 1}{\chi_x}\right] & 0 < Pe_x < 10 \\ (u_x)_{i+1,j} + \left[(u_x)_{i,j} - (u_x)_{i+1,j}\right]\left[\dfrac{\sqrt{\chi_x} - 1}{\chi_x}\right] & -10 < Pe_x < 0 \end{cases} \tag{16.1}$$

$$(u_x)_w = \begin{cases} (u_x)_{i-1,j} + \left[(u_x)_{i,j} - (u_x)_{i-1,j}\right]\left[\dfrac{\sqrt{\chi_x}-1}{\chi_x}\right] & 0 < Pe_x < 10 \\ (u_x)_{i,j} + \left[(u_x)_{i-1,j} - (u_x)_{i,j}\right]\left[\dfrac{\sqrt{\chi_x}-1}{\chi_x}\right] & -10 < Pe_x < 0 \end{cases} \tag{16.2}$$

$$(u_y)_n = \begin{cases} (u_y)_{i,j} + \left[(u_y)_{i,j+1} - (u_y)_{i,j}\right]\left[\dfrac{\sqrt{\chi_y}-1}{\chi_y}\right] & 0 < Pe_y < 10 \\ (u_y)_{i,j+1} + \left[(u_y)_{i,j} - (u_y)_{i,j+1}\right]\left[\dfrac{\sqrt{\chi_y}-1}{\chi_y}\right] & -10 < Pe_y < 0 \end{cases} \tag{16.3}$$

$$(u_y)_s = \begin{cases} (u_y)_{i,j-1} + \left[(u_y)_{i,j} - (u_y)_{i,j-1}\right]\left[\dfrac{\sqrt{\chi_y}-1}{\chi_y}\right] & 0 > Pe_y > 10 \\ (u_y)_{i,j} + \left[(u_y)_{i,j-1} - (u_y)_{i,j}\right]\left[\dfrac{\sqrt{\chi_y}-1}{\chi_y}\right] & -10 < Pe_y < 0 \end{cases} \tag{16.4}$$

where $x_x = |Pe_x|/(1 - 0.1|Pe_x|)^5$ and $x_y = |Pe_y|/(1 - 0.1|Pe_y|)^5$.

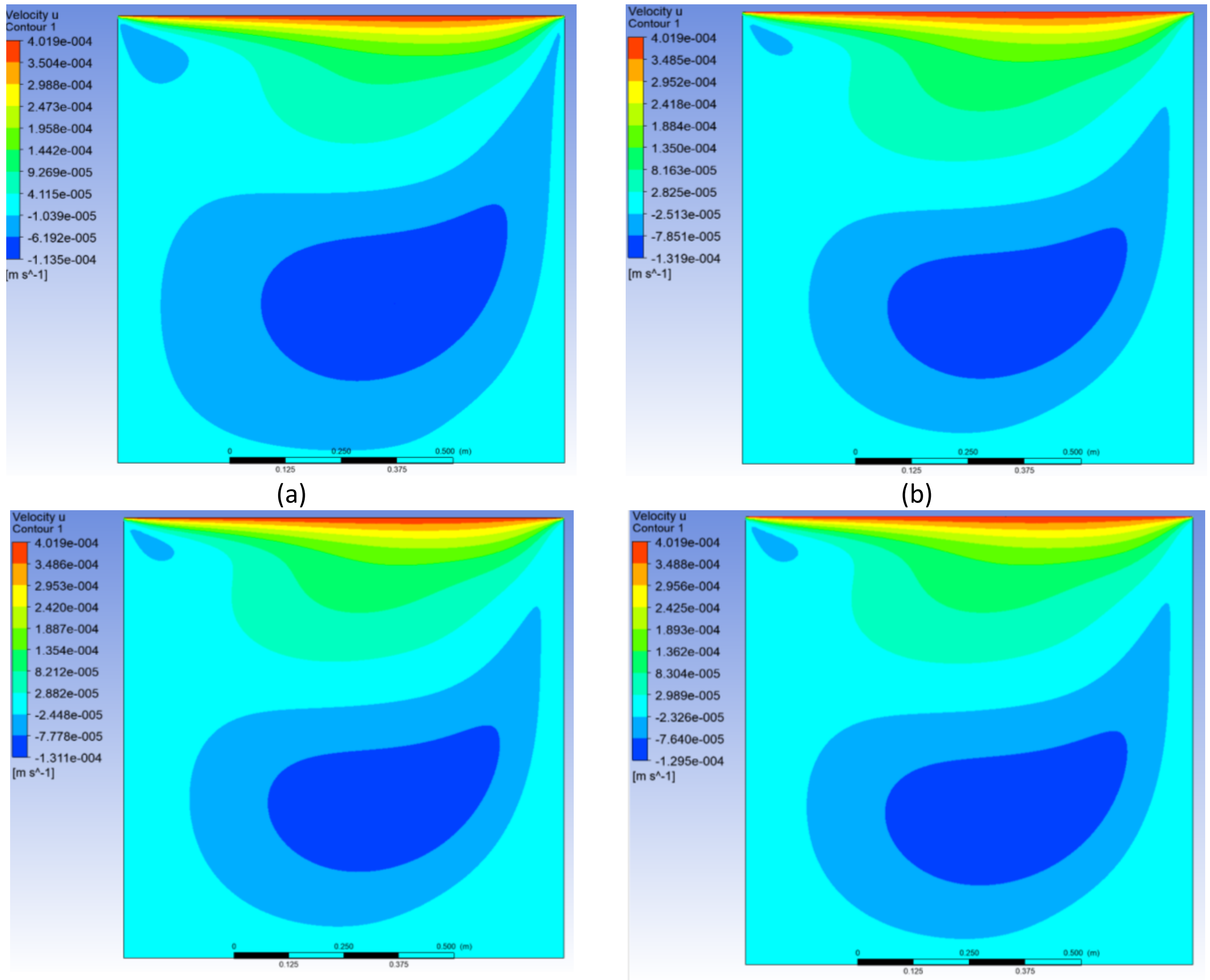
Power-law scheme is similar to the SOU method, except that the adjacent grid values are interpolated exponentially in former scheme. It is important to note that when the Peclet number grows larger, power-law is equivalent to FOU since $1/\infty \rightarrow 0$. Meanwhile at zero Peclet number, the convection term is negated.

4. Numerical Implementation on Lid-Driven Cavity

The lid-driven flow in a square cavity is simulated here. By using the grid resolution 129 × 129, the flow cases of Reynolds numbers 400 and 3200 are simulated. The grid resolution applied is in accordance with grid size of the benchmark results computed by Ghia *et al.,* [34]. The velocity and pressure field are obtained from different convection schemes are discussed in Section 4.1. Their computational performances are discussed in Section 4.2. Dirichlet and Neumann boundary conditions are applied for the velocity and pressure fields, respectively. The under-relaxation factor is set to 0.3. The maximum iteration set is 10000 to ensure sufficient allowable room prior to convergence.

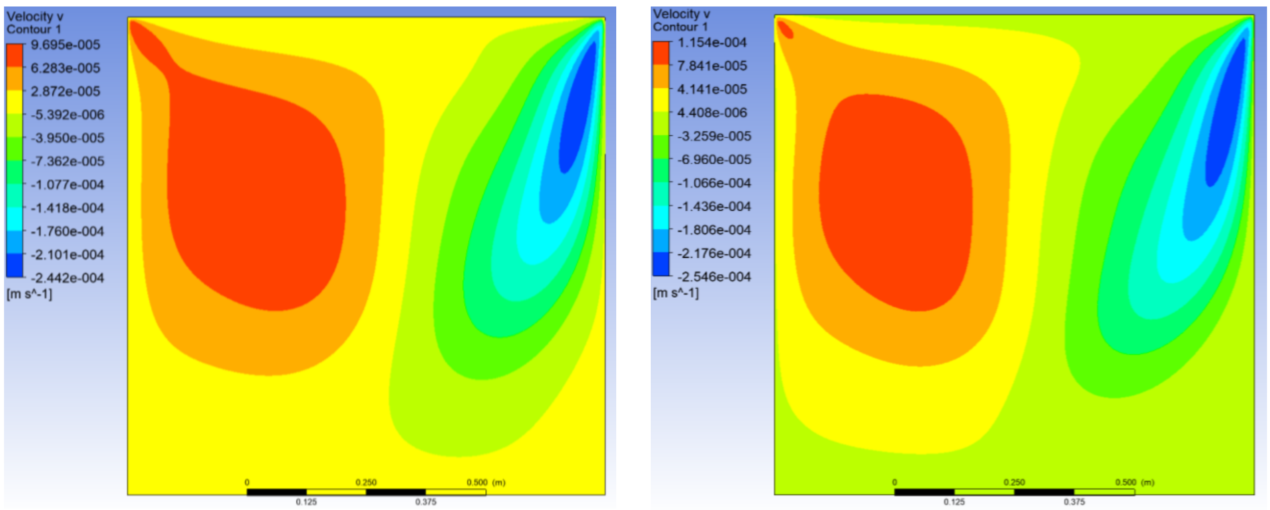
4.1 Flow Fields on Lid-Driven Cavity

At Re = 400, *x*-velocity is quite low at the centre of the cavity. Meanwhile the *y*-velocity is high at the upper left and upper right regions of the cavity. The pressure is indeed slightly higher at the top right corner of the cavity. The flow patterns computed using SOU, QUICK and power-law scheme are almost similar in general. When FOU is applied, both the velocity and pressure fields are slightly over-estimated as shown from Figures 2 - 4.

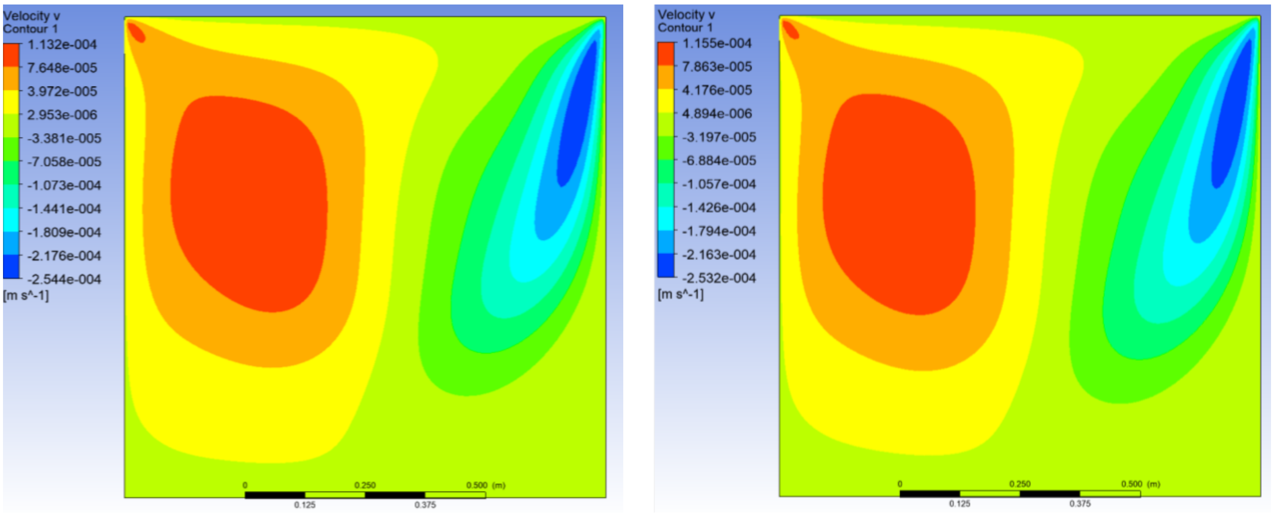


(c) (d)

Fig. 2. Computed u_x field at Re = 400 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129

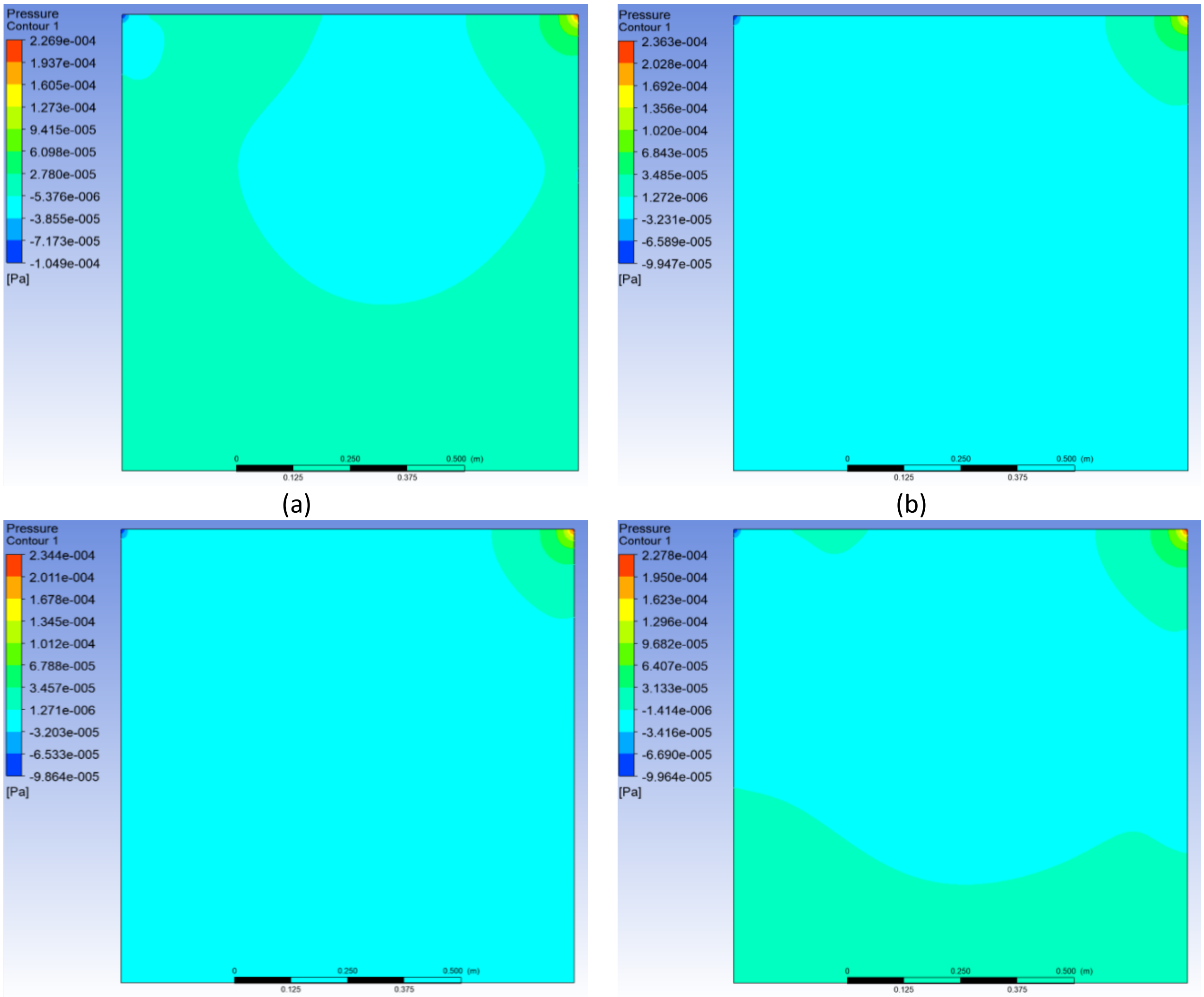


(a) (b)



(c) (d)

Fig. 3. Computed u_y field at Re = 400 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129



(c) (d)

Fig. 4. Computed pressure field at Re = 400 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129

However, when Re is increased to 3200, the flow patterns computed using FOU and power-law schemes are very similar. The observation is applicable for SOU and QUICK scheme. Figures 5 - 7 show the u_x, u_y and pressure fields at Re = 3200. The computational performance of various schemes is discussed in the next subtopic.

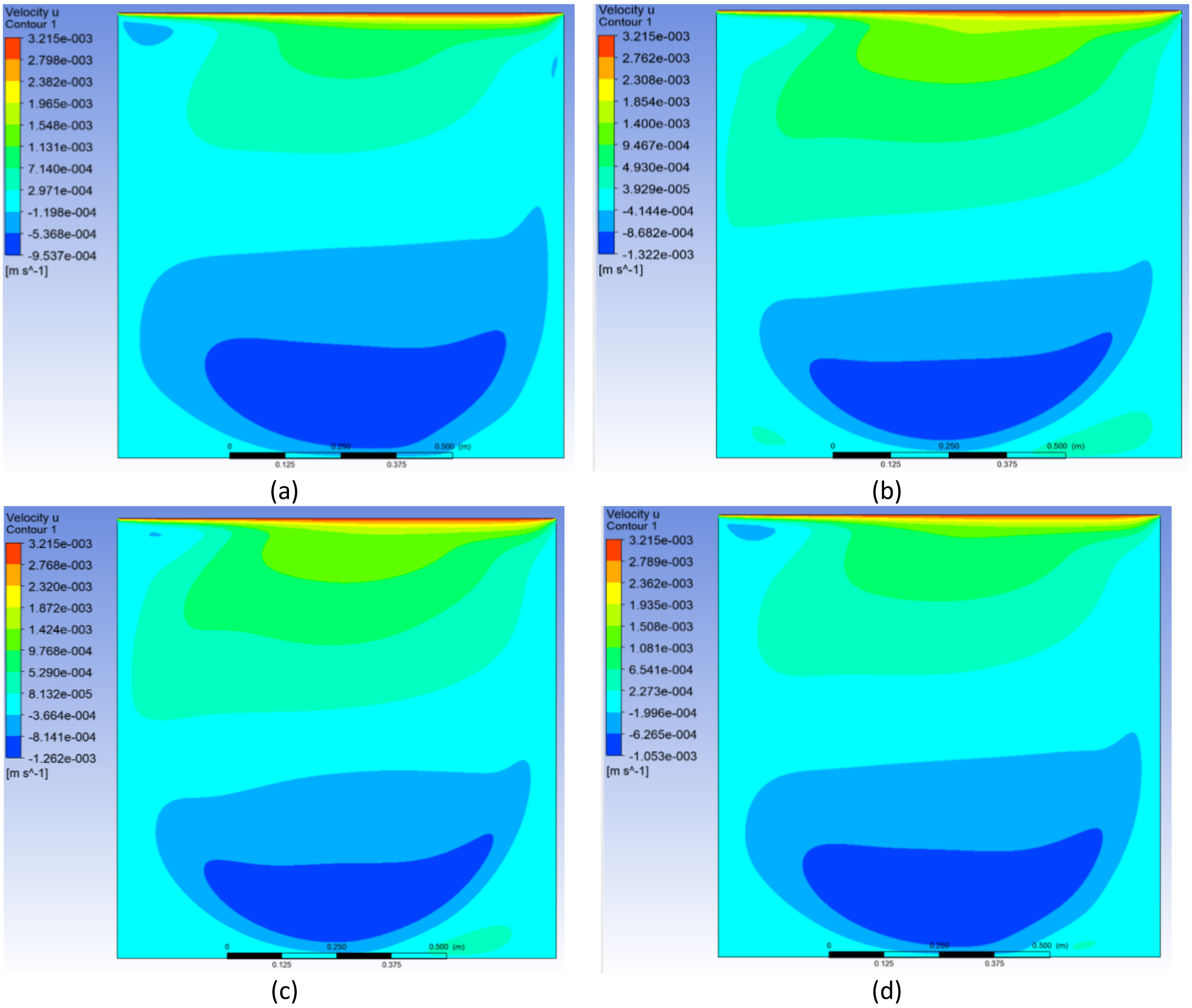


Fig. 5. Computed u_x field at Re = 3200 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129

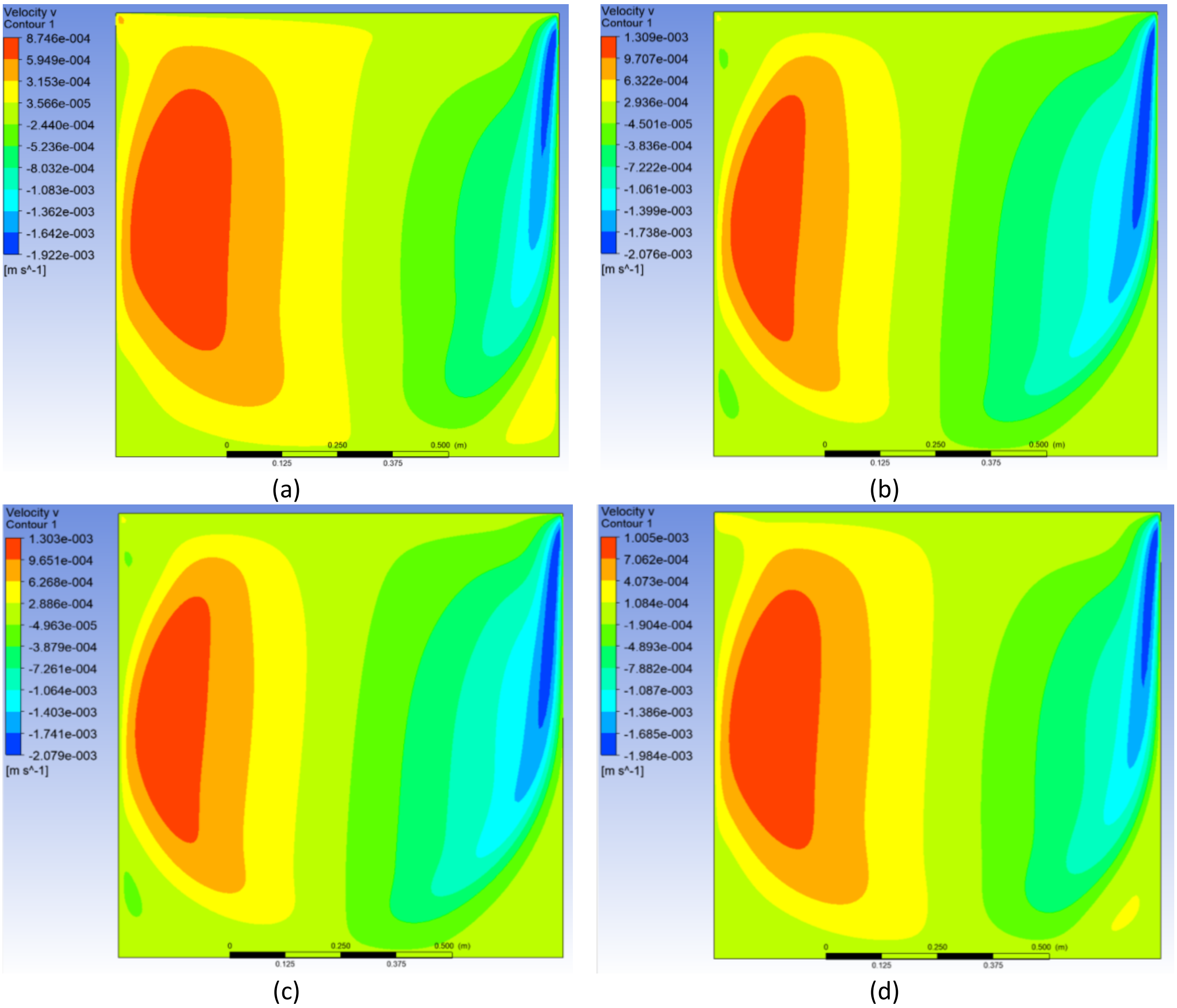
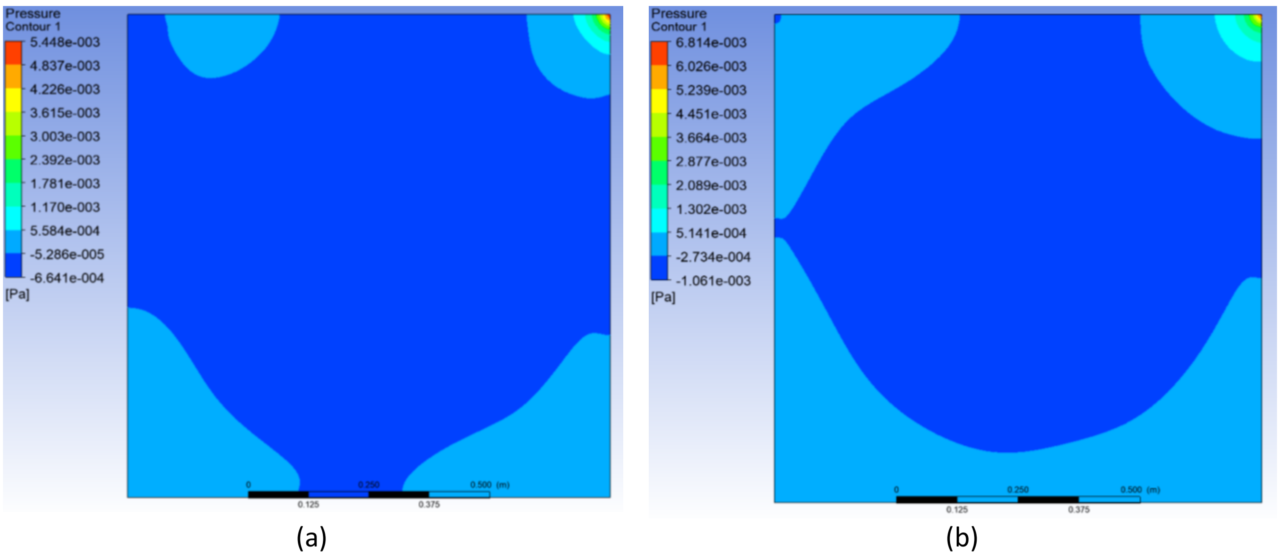
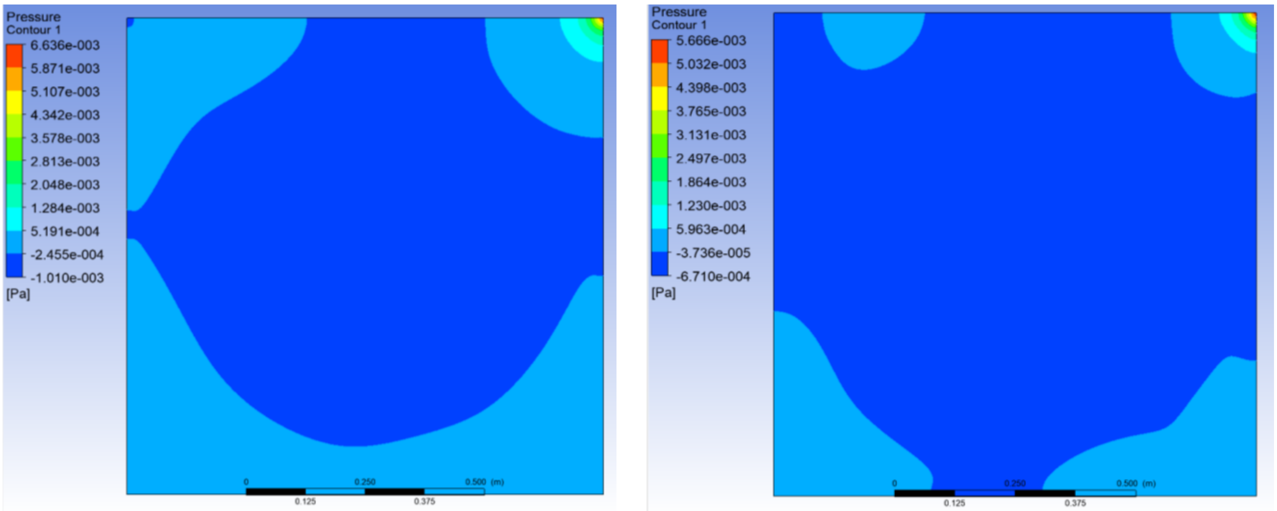


Fig. 6. Computed u_y field at Re = 3200 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129





(c) (d)

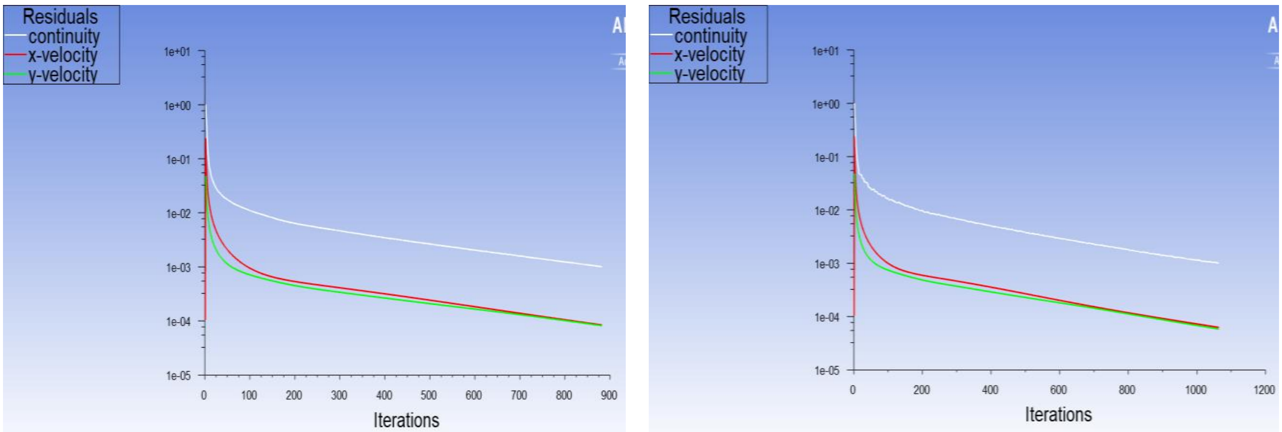
Fig. 7. Computed pressure field at Re = 3200 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129

4.2 Comparison on Computational Performances

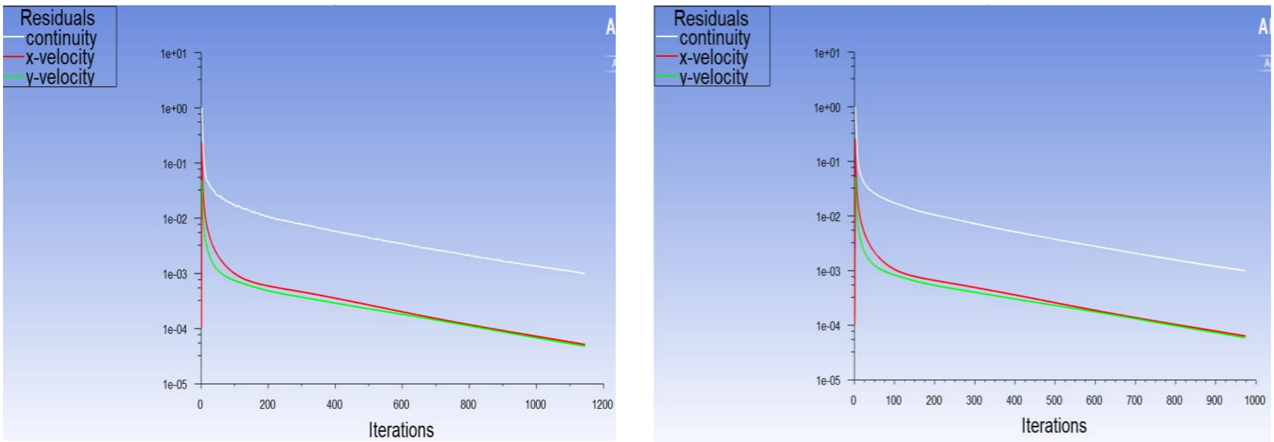
The computational performances in terms of convergence speed, stability and accuracy of various convection schemes are compared. The convergence curve at Re = 400 and Re = 3200 are illustrated respectively in Figure 8 and Figure 9, respectively. Table 1 summarises the number of iterations required to achieve convergence.

From Table 1, at Re = 400, the convergence of FOU and power-law scheme are more rapid than those of SOU and QUICK schemes. At Re = 3200, power-law scheme outperforms FOU in terms of convergence speed. In fact, in terms of convergence speed, power-law scheme is the best among the investigated convection schemes.

However, at Re = 3200, numerical instability can be observed when SOU and QUICK scheme is applied. Chen and Falconer [35] and Li and Tao [36] addressed this issue by introducing a modified QUICK scheme.



(a) (b)



(c) (d)

Fig. 8. Convergence plot at Re = 400 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129

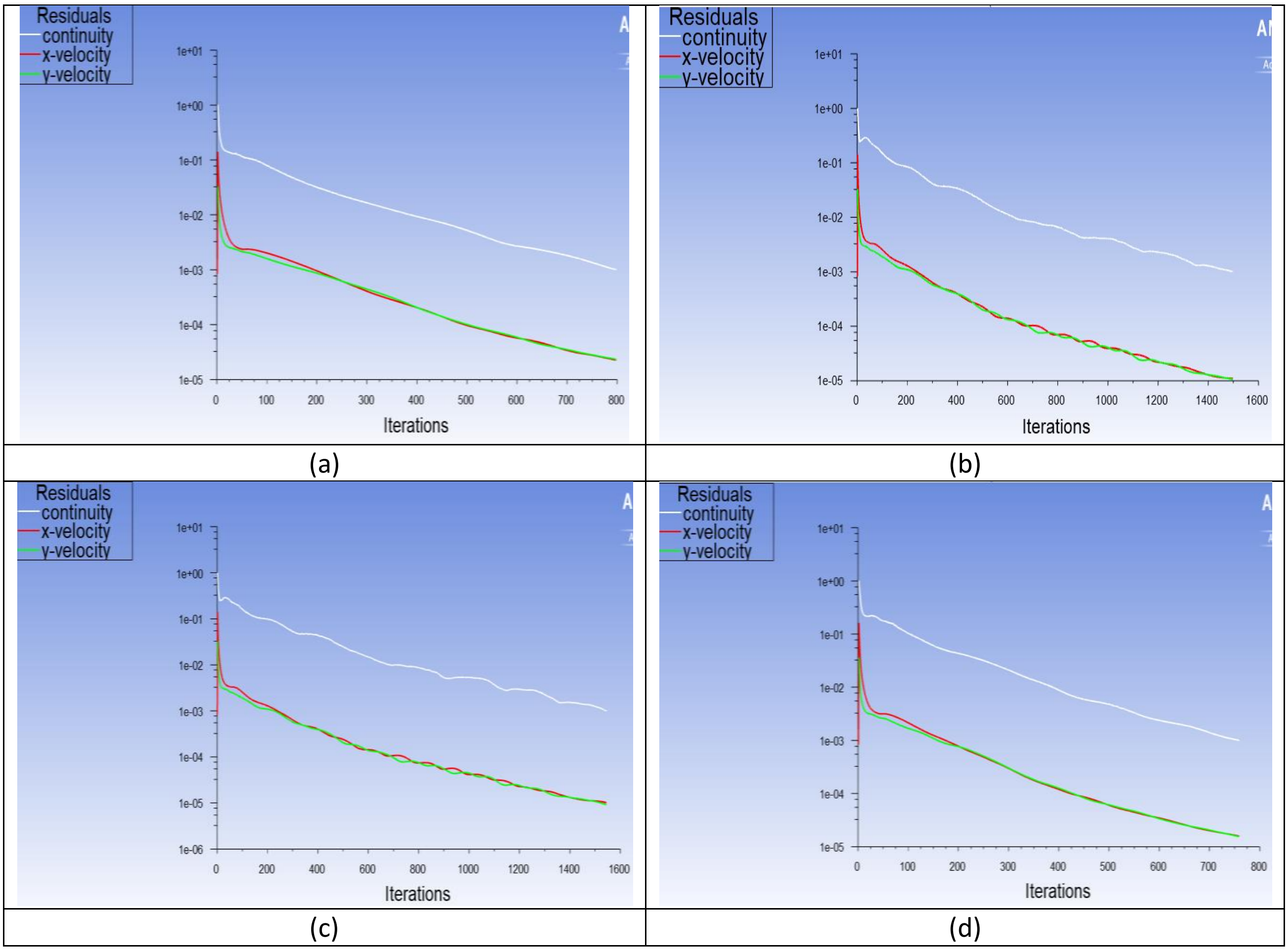


Fig. 9. Convergence plot at Re = 3200 computed by: (a) FOU; (b) SOU; (c) QUICK; (d) power-law scheme with grid number 129 × 129

Table 1
Iteration required for convergence due to different convection schemes at different Re

	FOU	SOU	QUICK	Power-Law
Re = 400	882	1064	1145	974
Re = 3200	798	1498	1545	759

The numerical accuracy is studied by validating our results computed from 129 × 129 grids with the benchmark data reported by Ghia *et al.,* [34], which was obtained stream-vorticity scheme [37,38]. Since our results are computed based on the dimensional equations, the obtained velocities are normalised before the analysis is made. The numerical accuracy is measured using the standard deviation defined as

$$\text{Standard deviation} = \sqrt{\frac{\sum\left(u_{computed} - u_{benchmark}\right)^2}{n}} \tag{17}$$

where n is the number of grid points.

At both Re = 400, the standard deviation of higher order schemes such as SOU and QUICK are relatively small for both *x*- and *y*- velocities, as shown in Table 2. FOU is the most inaccurate scheme, while the accuracy of power-law scheme is between those of FOU and higher order scheme.

At Re = 3200, similar performances can be observed, except that SOU scheme outperforms QUICK scheme slightly which could be due to the unboundedness of QUICK scheme (numerical wiggles) [35].

From the illustrations shown in Figures 10 - 13 and Table 2, higher order upwind schemes (SOU and QUICK) could produce more accurate result at high Re as more neighbouring points are involved during the computation. Power-law is slightly more accurate than FOU; however, its performance in capturing flow discontinuity may not be as good as those of higher-order schemes. At very high Re, the accuracy of power-law scheme approaches FOU. This finding is in accordance with the finding of Leonard and Drummond [39] that at multi-dimensional flow problem with high Pe, cross-wind artificial diffusion may appear if QUICK scheme is applied, and this will seriously deteriorate the flow accuracy.

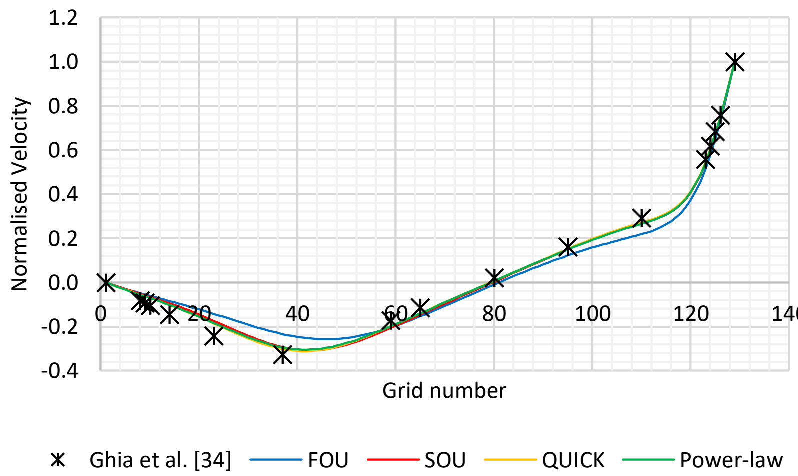


Fig. 10. Comparison of velocity field of u_x at the middle line of the *x*-axis of the lid-driven cavity at Re = 400

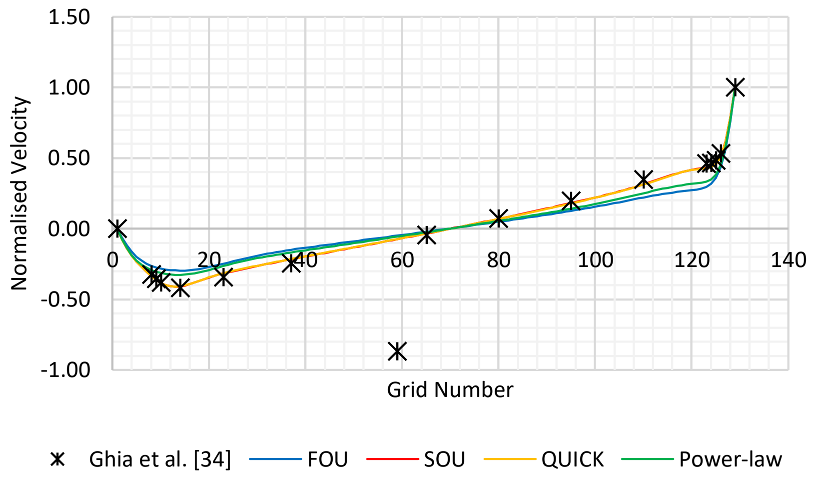


Fig. 11. Comparison of velocity field of u_x at the middle line of the *x*-axis of the lid-driven cavity at Re = 3200

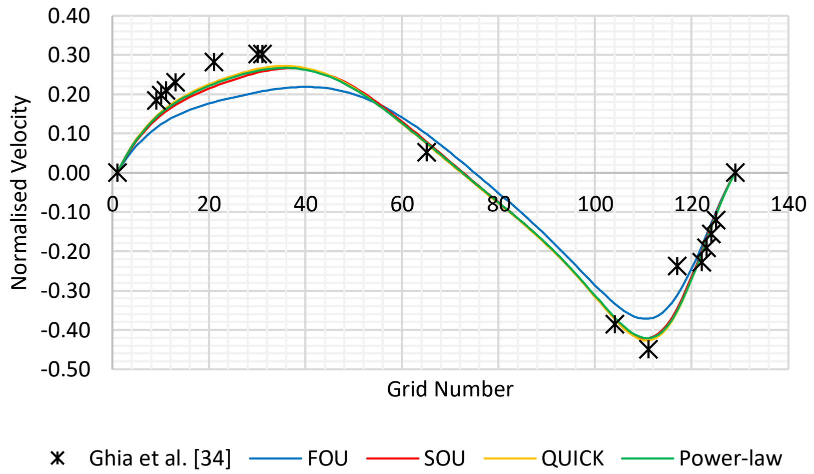


Fig. 12. Comparison of velocity field of u_y at the middle line of the *y*-axis of the lid-driven cavity at Re = 400

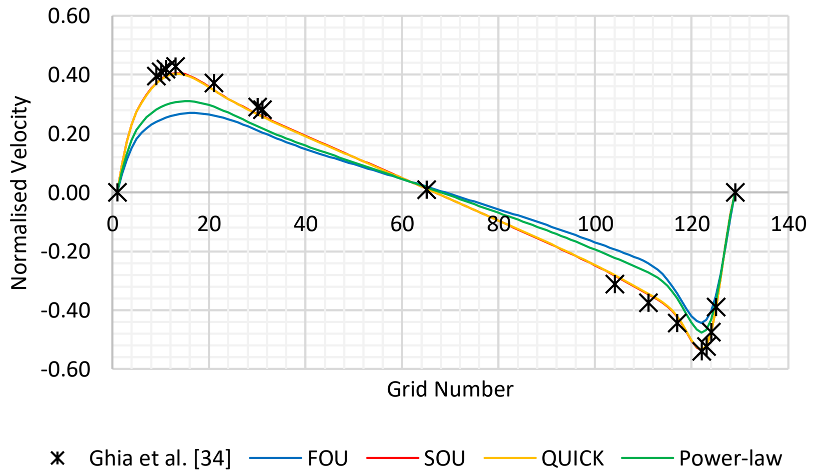


Fig. 13. Comparison of velocity field of u_y at the middle line of the *y*-axis of the lid-driven cavity at Re = 3200

Table 2
Variance for different convection schemes at different Re with comparison with the data of Ghia *et al.,* [34]

	Re	FOU	SOU	QUICK	Power-Law
u_x	400	0.050674	0.031427	0.025919	0.027278
	3200	0.219507	0.193073	0.193439	0.209298
u_y	400	0.065517	0.045027	0.041246	0.042926
	3200	0.107170	0.018951	0.020092	0.079247

5. Concluding Remarks

The computational performances of FOU, SOU, QUICK and power-law schemes have been analysed on two-dimensional lid-driven flow problem in a square cavity at Re = 400 and Re = 3200. The advantages and disadvantages of four investigated convection schemes can be summarised as follows.

I. FOU is the simplest in terms of implementation, which converges faster than higher order upwind schemes such as SOU and QUICK. It gives the most rapid convergence when Re is high. However, it is accurate only when the Re is small.

II. Higher order schemes (SOU and QUICK) are more complex in terms of implementation and they need more iterations to reach convergence. Their convergence curves may fluctuate as well. However, they are accurate when the flow is highly convective. Also, they are more capable in capturing flow with high gradient.

III. Power-law is basically a “smoothed” and improved version of FOU by considering the Peclet number. However, it inherits most of the features of FOU. It shows good accuracy and convergence at low Re flow; however, its accuracy is inferior to those of higher order schemes at high Re.

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