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Comparative Study of Quasi Steady and Unsteady Damping Derivatives for Delta Wings in Hypersonic Flow for Half Sine Wave

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ABSTRACT

Correlation investigation of quasi-steady and unsteady damping derivatives of delta wing is analyzed and contemplated, with leading edges bent in the current paper. The current paper realizes that the quasi-steady damping derivative considers only the rate of pitch, not the rate of the angle of attack. In contrast, unsteady damping derivatives account for pitch rate and the rate of the angle of attack. Results show a reformist reduction in the damping derivatives as the Mach number increases in quasi-steady and unsteady cases. Outcomes are computed for a varied range of Mach Numbers and deflection angles due to the pitch rate of different amplitudes of half-sine waves. The outcomes for the damping derivative for a fixed pivot position show different behaviour for different values of the flow deflection angles observed here. As in straight leading edge, even in the curved leading edge, steady-state is attained earlier in quasi-steady than unsteady.

1. Introduction

An attempt to derive the calculation of damping derivatives with bent leading edges is carried out [1]. A similar investigation is conducted for a high-velocity stream wing with a curved leading edge for a delta wing. The wing having bent leading edges has countless benefits over the wing having a top straight edge. The delta wing with a straight leading edge has a linear distribution of the wing surface area than the curved leading edges [2]. Nonetheless, the wings have bent leading edges, supplanting a straight driving edge by a half-sine wave in the current case. Theoretical calculation of the stability derivative gives a thought about the performance of aviation vehicles. Subsequently, stability calculation is more significant before going for a model plan. There is a requirement for computing straightforward yet sensible precise techniques to felicitate the design process. The

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similitude of mathematical, kinematic, and dynamic is needed before the wind tunnel tests and after finalizing the design.

The hypothesis of oscillating airfoils at high Mach numbers for pitching oscillation is created by light hills, a piston analogy [3]. Ghosh and Mistry [4] likewise created a 2-D hypersonic similitude at significant incidence along with piston theory. Ghosh and Mistry [4] apply the hypothesis for the attached shock and the plan approximation for the windward surface [5]. Similitude and two similarity parameters were obtained by Ghosh [6] for oscillating delta wings with attached shock at significant incidence. This similitude has been extended to shock attached delta wings with bent leading edges at significant incidence in the current analysis. The lee surface is assumed to be zero.

Crasta and Khan [7,9-11] and Khan and Crasta [8] have assessed the viability of the flow deflection angle for wings with different sweep angles. Likewise, they contemplated the impact of incident angle on pitching and roll damping derivatives of a delta wing with curved leading edges. Aerodynamic variables are computed for a wing with variable-sweep angles by Crasta and Khan [7] by evaluating the effectiveness of the flow deflection angle. However, because of the quasi-steady nature, the hypothesis does not take into account the unsteady effect.

Monis *et al.,* [12-16] assessed the damping derivative for extraordinary Mach M and the stability derivatives in pitch and damping. Many researchers utilized analytical and numerical strategies to evaluate flow over a wedge at supersonic Mach numbers using the finite element method and developed its simulation [17-19]. Hence, it is crucial to address the computation of the angle of attack alone by first finding the unsteady damping derivative and then the quasi-steady one and its difference using the expression $-C_{m\dot{\alpha}} = (-C_{m\dot{\theta}}) - (-C_{mq})$. Therefore, this work has investigated quasi-steady and unsteady damping derivatives for curved leading edges for different Mach number and Pivot positions.

2. Methodology

About the pivot $x = x_0$ the pitching moment due to the lower surface only is given by Eq. (1), and for the windward side is shown in Eq. (2). The stream turns through a Prandtl-Meyer extension at the edge, leading to getting parallel to the upper surface on the expansion side of the flat plate. First Mach number, M_θ downstream of the expansion is computed first.

$$\overline{M} = \int_0^L (x-kL)\bar{P}_p\,dx - \frac{\partial\bar{M}}{\partial\dot{\theta}} = \int_0^L (x-kL)\frac{\partial\bar{P}_p}{\partial\dot{\theta}}dx \;-\frac{\partial\bar{M}}{\partial\dot{\theta}} = \int_0^L (x-kL)\rho_2\,a_2 u_\infty\left(\frac{c_2 x}{u_\infty}+\frac{c_3 L}{u_\infty}\right)dx \tag{1}$$

$$-\frac{\partial M}{\partial\dot{\theta}} = \rho_2 a_2 L^3\left[c_2\left(\frac{1}{3}-\frac{k}{2}\right)+c_3\left(\frac{1}{2}-k\right)\right] \tag{2}$$

When the angle of attack is zero, flat plate oscillating in a stream of Mach number, M_θ pressure perturbation is assumed to be the same. The oscillation of the expansion fan due to the flat plate oscillation isn't accounted for because of methodology. Significantly, the upper surface exhibits lower pressure when compared to the windward surface, as shown in Eq. (3).

$$V_p = \frac{u_\theta}{cos\,\mu_\theta} + \frac{(x-kL)\dot{\theta}}{cos\,\mu_\theta} \tag{3}$$

Piston Mach number is given by Eq. (4), where M_θ is the Mach number downstream of the expansion fan, a_θ is the sonic velocity, and μ_θ is the Mach angle downstream of Prandtl- Mayer expansion. The pressure ratio for the isentropic process is given by Eq. (5), and as θ and $\dot{\theta}$ tend to zero, the equation for the acoustic expression is given in Eq. (6).

$$M_p = \frac{v_p}{a_\theta} = \frac{M_\theta \theta}{\cos\mu_\theta} + \frac{(x-kL)\dot{\theta}}{a_\theta \cos\mu_\theta} \tag{4}$$

$$\frac{p_p}{P_\theta} = (1 - \frac{\gamma-1}{2} M_p)^{\frac{2\gamma}{\gamma-1}} \tag{5}$$

$$\frac{P_p}{P_\theta} = (1 - \gamma M_P) \tag{6}$$

The pitching moment can be defined using Eq. (7), where from Eq. (4), the pitching moment coefficient can be estimated from Eq. (8) and Eq. (9) using Eq. (10). When A_F = 0, Eq. (10) can be defined as Eq. (11) below. As per Crasta and Khan theory (i.e., Quasi-steady), taking only the windward surface $-C_{mq}$ is given by Eq. (12).

$$\bar{M} = \int_0^L (x-kL) P_p\, dx = \int_0^L (x-kL) P_\theta\, (1-\gamma M_P) dx - \frac{\partial \bar{M}}{\partial \dot{\theta}} = \int_0^L (x-kL) P_\theta\, (-\gamma) \frac{\partial M_p}{\partial \dot{\theta}} dx \tag{7}$$

$$-\frac{\partial \bar{M}}{\partial \dot{\theta}} = -\int_0^L (x-kL) P_\theta\, (-\gamma) \frac{(x-kL)}{a_\theta \cos\mu_\theta} dx \tag{8}$$

$$-C_{m_{\dot{\theta}}} = \frac{4}{\rho_\infty u_\infty b L^3} \left[-\frac{\partial \bar{M}}{\partial \dot{\theta}} \right] \tag{9}$$

$$C_{m_{\dot{\theta}}} = \frac{4\rho_2 a_2}{\rho_\infty u_\infty} \frac{1}{\left(\cot\varepsilon - \frac{4A_H}{\pi}\right)} \left[C_2 \left[\left(\frac{1}{8} - \frac{k}{6}\right) \cot\varepsilon - \frac{A_F}{4\pi} - \frac{A_H}{2\pi} - \frac{2A_H}{\pi^3} + \frac{k\left(A_H + \frac{A_F}{2}\right)}{\pi} \right] + C_4 \left[\left(\frac{k}{2} - \frac{k^2}{2} - \frac{1}{8}\right) \cot\varepsilon - \frac{3A_F}{4\pi} + \frac{A_H}{2\pi} - \frac{6A_H}{\pi^3} + \frac{k}{\pi}\left(\frac{3A_F}{2} - A_H\right) + 2k^2 \frac{A_H}{\pi} \right] \right] + \frac{p_\theta}{p_\infty} \frac{a_\infty}{a_\theta} \frac{M_\theta}{M_\infty} \frac{1}{\sqrt{M^2{}_\theta - 1}} \left[\left(1 - \frac{8k}{3} + 2k^2\right) \cot\varepsilon + \frac{2A_F}{\pi} - \frac{4A_H}{\pi} + \frac{16A_H}{\pi^3} + \frac{4k}{\pi}(2A_H - A_F) - \frac{8A_H k^2}{\pi} \right] \tag{10}$$

$$-C_{m_{\dot{\theta}}} = \frac{4\,Sin\,\psi}{M} \frac{1}{(\cot\varepsilon - \frac{4A_H}{\pi})} \left[C_2 \left[\left(\frac{1}{8} - \frac{k}{6}\right) \cot\varepsilon - \frac{A_H}{2\pi} - \frac{2A_H}{\pi^3} + \frac{k(A_H)}{\pi} \right] + C_4 [(\frac{k}{2} - \frac{k^2}{2} - \frac{1}{8}) \cot\varepsilon + \frac{A_H}{2\pi} - \frac{6A_H}{\pi^3} + \frac{k}{\pi}(-A_H) + 2k^2 \frac{A_H}{\pi}] \right] \tag{11}$$

$$-C_{mq} = \frac{\sin\alpha_0 F(s_1)}{\cos^2\phi (\cot\varepsilon - \frac{4A_H}{\pi})} \left[(k^2 - \frac{4}{3}k + \frac{1}{2}) \cot\varepsilon - \frac{1}{\pi}(2k-1) A_F + 2A_H (2k^2 - 2k + 1 - \frac{4}{\pi^2}) \right] \tag{12}$$

When A_F = 0, Eq. (12) can be defined as Eq. (13) below. Using the above analytical expressions (Eq. (10) and Eq. (12)), the damping derivatives' results in sporadic and quasi-steady cases can be obtained as given in Eq. (13).

$$-C_{mq} = \frac{\sin\alpha_0 F(s_1)}{\cos^2\phi (\cot\varepsilon - \frac{4A_H}{\pi})} \left[(k^2 - \frac{4}{3}k + \frac{1}{2}) \cot\varepsilon + 2A_H (2k^2 - 2k + 1 - \frac{4}{\pi^2}) \right] \tag{13}$$

Here the expansion term in both cases has been ignored. Also, for $A_F = 0$ and varying, A_H, the $-C_{m\dot{\alpha}}$ can calculate by using the expression, $-C_{m\dot{\alpha}} = (-C_{m\dot{\theta}}) - (-C_{mq})$ for various Mach numbers. The schematic representation of the delta wing is shown in Figure 1 below:

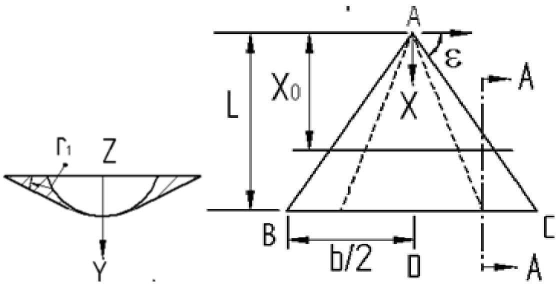

Fig. 1. Delta Wing geometry with parameters

3. Results and Discussions

The above expressions are derived for quasi-steady and the unsteady case for curved leading edges. This section shows the graphical representation of damping derivatives due to quasi-steady and unsteady cases and the angle of attack alone. Obtained are the results for the wing with bent leading edges for half-sine wave for higher Mach number and two angles of attack δ = 5 and 10.

Figure 2 represents changes of derivative in damping versus Pivot position for M = 5, 9, 15, 20 & δ = 5. These outcomes show that at the pivot location h=0, the magnitude of damping derivatives is relatively high. Here, the variation can be observed in the damping derivative between unsteady and quasi-steady ranges from sixty-seven percentage, seventy-two percentage, eighty percentage, fifty-three percentage, twenty-one percentage, and five percentage. The numerical value of the damping derivative is comparatively much higher in the case of unsteady flow. Initially, the damping derivative is relatively high at the pivot position h = o, which gradually decreases. This decline may be due to the centre of pressure position. Both the speculations match at the end of the trailing edge, i.e., 80% and more of pivot position.

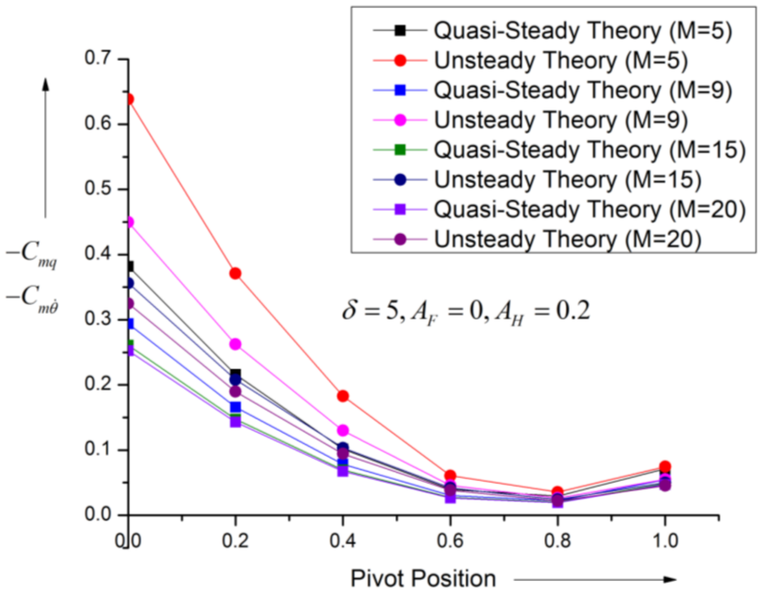

Fig. 2. Unsteady and Quasi-steady damping derivative vs. Pivot position M_∞ = 5, 9, 15, 20 and δ = 5

At Mach number 9, it can be observed that the percentage increase of flow deflection angle increases for the pivot position from 0 to 0.4 to fifty-three percentage to sixty-six percentage and decreases from pivot position 0.6 to 1. Thus, damping derivatives for unsteady and quasi-steady cases decline as the Mach number increases even though the deflection angle is constant. Moreover, when the Mach number moves from 5 to 9, the difference between the two theories is reduced.

Observation predicts that the change in the magnitudes of damping derivative between unsteady and quasi-steady ranges from 37%, 41%, 48%, 55%, and21when pivot varies from 0 to 1 at M = 15. Thus, it can be seen that the value of the damping derivative is comparatively higher in the case of unsteady flow at pivot 0. Initially, the value of the damping derivative is relatively high at the pivot position h = o. Then it gradually decreases, and when the pivot position crosses 75%, the derivative magnitudes match well. Further, as the Mach number increases, the differences in quasi and unsteady theories seem to lessen a lot.

Observation shows that the value of the derivative in damping is slightly higher in the case of unsteady flow at pivot 0 at M = 20. Initially, the value of the derivative in damping is slightly high at the pivot position h = o. Then it gradually decreases, and when the pivot position crosses 75%, the derivative magnitudes match well. Further, as the Mach number increases, the differences in quasi and unsteady theories seem to lessen a lot. Likewise, when the Mach number moves from 15 to 20, the percentage differences in the two theories is only 5%. Otherwise, the trend is similar to the above graphs.

Figure 3 portrays outcomes for a higher worth of the flow deflection (i.e., $\delta = 20$) for the inertia levels 5,9,15 &20. This situation variation in the damping derivative is fourteen, seventeen, twenty-one, zero, and minus eleven percentage for the pivot position 0, 0.2, 0.4, 0.6, 0.8, and 1. At the initial pivot position, the numerical value is marginally higher in the case of unsteady flow at pivot 0. Initially, the numerical value of the derivative in damping is slightly high at the pivot position h = o, and then it, steps by step, diminishes. When the pivot position crosses 0.5, the derivative magnitudes match well. The trend is like the above diagrams, yet the two theories' distinctions have continuously diminished when the angle of attack increments from 5 to 20. Perceptions show that higher the angle of attack, the two theories intently match with one another.

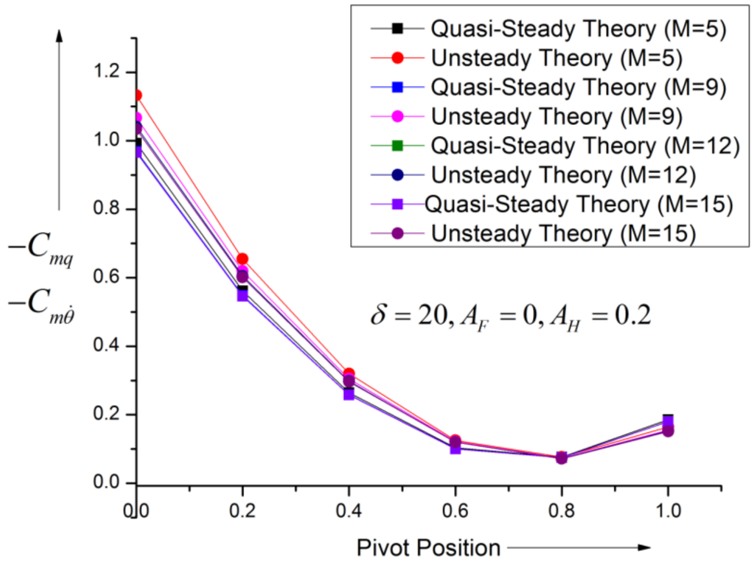

Fig. 3. Unsteady and Quasi-steady damping derivative vs. Pivot position M_∞ = 5, 9, 15, 20 and δ = 20

For M = 9, variations in the damping derivative are 10%, 13%, 18%, 20%, -3%, and -14% for the pivot position 0, 0.2, 0.4, 0.6, 0.8, 1. A similar trend is also observed in the previous case, but both theories show a much closer match than when the Mach number increases from 5 to 9. That is a clear indication that higher the inertia level and angle of attack, there is no difference in quasi and unsteady theories. The trend for both quasi-steady and unsteady theory is the same for M=15 and M=20. Further, the differences are also insignificant.

The change in damping derivative with Mach number k = 0, 0.6, 1 and δ = 5 degrees is addressed in Figure 4. The trend for both quasi-steady and unsteady theory is the same. However, there is a relatively significant difference in the numerical value of derivatives in damping concerning both theories. The discrepancies in a numerical value are 67%, 53%, 37%, and 29% when the Mach number moves from 5 to 20. For k = 0.6, the pattern for both quasi-steady and unsteady theories is the same. The derivative in damping reduces with the increase in pivot position in the case of both the hypothesis. However, there is a relatively significant difference in the numerical value of derivatives in damping concerning both theories. The discrepancies in a numerical value are 53%, 50%, 55%, and 46% when the Mach number moves from 5 to 20. At k = 1, the Damping derivative value lessens with the increase in pivot position in the case of both the hypothesis. There is a relatively small difference in the mathematical value of the damping derivative with respect to both theories despite a mismatch at Mach number 20. That may be due to the planform area at pivot position 1.

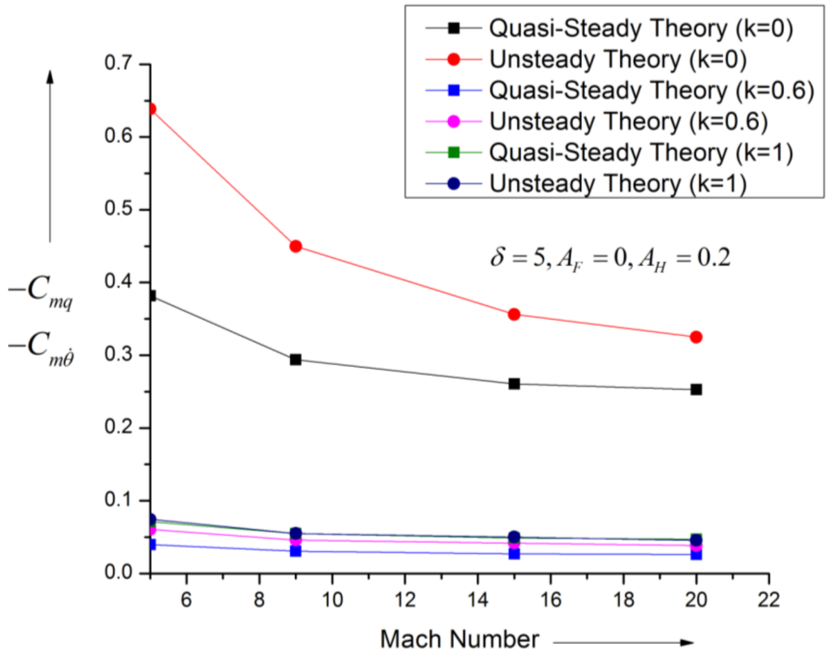

Fig. 4. Unsteady and Quasi-steady damping derivative vs. Pivot position k = 0, 0.6,1 and δ = 5

The change in damping derivative versus Mach number for a fixed Pivot k= 0, 0.6, 1 and δ = 20 is seen in Figure 5. The trend for both quasi-steady and unsteady theory is the same. For the high angle of attack, it's visible that there is a negligible difference between both theories. The reason may be that the pivot position is closer to the aerodynamic centre. There is no change in damping derivative numerical values at higher Mach numbers indicating the existence of the Mach number independence principle. At fixed Pivot k = 1 and δ = 20, there is an inconsistent distinction seen in the two hypotheses. The reason may be the planform area at the trailing edge.

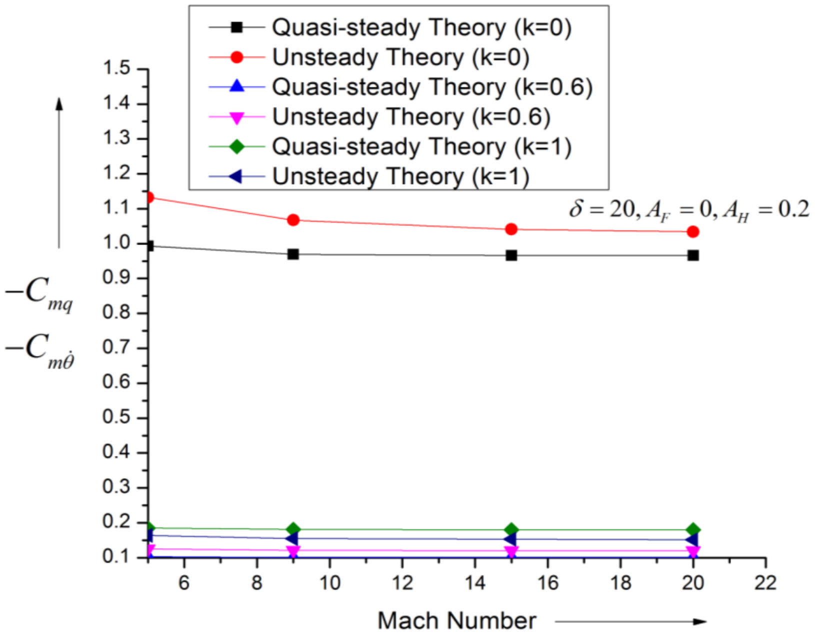

Fig. 5. Unsteady and Quasi-steady damping derivative vs Pivot position k = 0, 0.6, 1 and δ = 20

4. Conclusions

The following conclusions can be made on the above discussions:

i. There is a decrement in the damping derivative in quasi-steady and unsteady theories when there is an increment in Mach number.
ii. There is a reformist diminishing in the derivatives of damping for both the cases (i.e., unsteady and quasi-steady) with the increment in Mach number. The pattern stayed unaltered. Notwithstanding, with a further expansion in the Mach values, the extent of reduction has lessened.
iii. For Mach number 15 and above, there are no variations in the damping derivatives, and steady-state stability is achieved. At this point, it confirms the Mach number independence principle.
iv. For pivot positions, k = 0.4 to 0.6 reversal in the trend occurs when the increment is seen in Mach number, which is likewise seen in straight leading edges.

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