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Flow Analysis of Second Grade Fluid with Wall Suction/Injection and Convective Boundary Condition

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ABSTRACT

Analysis of unsteady heat and mass transfer in second grade fluid over a flat plate with wall suction and injection is considered. Mixed convection, convective boundary conditions and oscillating wall starting from rest are deliberated. Some suitable variable transformations are introduced for the reduction of partial differential equations to ordinary differential equations. Then the model is tackled using a combination of Laplace transform, the perturbation technique and numerical inversion of Laplace transform is implemented to obtain the semi analytical solutions for momentum, heat and concentration. Graphical presentation is illustrated for the ingrained parameters to shed light on the physical insight of the flow, heat and mass transfer of the fluid. Velocity of the fluid maximizes with increasing values of Grashof number due to dominant buoyancy effects. Finally, comparison has been made between the results of two algorithms (Tzou and Stehfest's algorithm).

Keywords:
Second grade fluid; suction and injection; oscillating wall; semi analytical and numerical laplace transform

1. Introduction

The behavior of non-Newtonian fluids in industry and engineering has been predicted by a range of constitutive equations. Differential type fluid is one of simplest type amongst them, also known as second grade fluid [1-6]. Veerakrishna *et al*.,[7] studied Hall effects on second grade fluid driven by oscillating pressure gradient between two vertical plates.

The flow of non-Newtonian fluids due to oscillating plate has a huge theoretical and applied research implementation in engineering and industry. The exact solution for second grade fluid past oscillating plate was proposed by Rajagopal [8] for the first time. Ramesh and Devakar [9] examined second grade fluid peristaltic flow in an inclined channel. They considered four different types of wave forms. They concluded that velocity and trapping bolus increases with maximizing inclination angle.

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Ismail *et al.,* [10] tackled exact solution for the flow of second grade fluid past infinite inclined plate. The flow of second grade fluid corresponding to cosine and sine oscillations of an infinite plate are examined by Fetecau and Fetecau [11] via Fourier sine transforms. Besides the exact solution the literature has an interesting research on numerical study of second grade fluid problems. Dehghan and Abbaszadeh [12] investigated flow of a second grade fluid and applied finite element method to acquire the numerical solution of fractional order model for Rayleigh-Stokes problem. Cortell [13] investigated numerically flow of second grade fluid over a stretching sheet with chemically reactive species for higher order reactions. They came up with a conclusion that concentration boundary layer gets diminished with due to the effect of destructive chemical reactions.

The literature survey indicated that there is a wide range of literature available on the flow of heat transfer free convection flow having extensive range of applications in industry, chemical manufacturing and nuclear reactors. Such studies include different fluid models, magneto hydrodynamic cases, with and without heat transfer. Exact solution for non-Newtonian fluid was studied by Aman *et al.,* [14,15]. Analytical solution for non-Newtonian nanofluid with CNTs along a vertical channel was examined by Aman *et al.,* [16]. Numerical analysis on natural convection heat transfer of a heat sink with cylindrical pin Fin was investigated by Qin *et al.,* [17]. Natural convection of sodium alginate nanofluid flow over a vertical plate was studied for numerical solution by Hussanan *et al.,* [18]. The exact solution of a non-Newtonian hybrid nanofluid was examined by Aman *et al.,* [19]. They considered alumina and copper nanoparticles for the hybrid concept of nanofluids along with the non-integer derivative model to express the flow model of the problem. They spotted a decrease in the flow of nanofluid due to magnetic effect and a rise due to buoyancy forces. Convection flow of some fluids with exact and numerical studies can be found in [20-22] and the references therein.

Stehfest [23] in 1970, proposed an algorithm to obtain numerical inversion of Laplace transform. This algorithm is applicable in acquiring solutions where the ordinary differential equations are complicated to obtain the inverse Laplace transform. Tzou and Puri [24] presented another algorithm for the same purpose. Furthermore, researchers implemented these algorithms in their semi-analytical or numerical solutions. Recently, Aman *et al.,* [25] investigated second order slip effect on MHD flow of fractional Maxwell fluid and acquired their inverse Laplace transform using Stehfest's algorithm [23]. Imran *et al.,* [26] obtained their solution via Stehfest's and Tzou's algorithms for the boundary layer flow of Maxwell fluid over an infinite surface with slip effects.

The above literature motivates us to study the heat transfer analysis of second grade fluid with wall suction/injection. To the best of author this problem has not been done yet. The problem is tackled using Laplace transform, Perturbation method and numerical inverse Laplace transform method Tzou's Algorithm [24].The results are mapped graphically for important parameters and analyzed thoroughly. For the sake of validation, the present results are compared with another numerical method "Stehfest's algorithm" [23].

2. Mathematical Formulation

An unsteady flow of a second grade fluid is considered where the plate oscillation velocity at $t>0$ is $H(t^*)e^{iwt}$ with wall suction and injection. The non-dimensional governing equations are

$$\rho\left(\frac{\partial u^*}{\partial t^*}-V_w\frac{\partial u^*}{\partial z^*}\right)=\mu\frac{\partial^2 u^*}{\partial z^{*2}}+\alpha_1\frac{\partial^3 u^*}{\partial z^{*2}\partial t^*}-\alpha_1 V_w\frac{\partial^3 u^*}{\partial z^3}+\rho g\beta_T\left(T^*-T_\infty^*\right)+\rho g\beta_C\left(C^*-C_\infty^*\right),\quad z^*>0,\;\; t^*>0, \tag{1}$$

$$\rho c_p\left(\frac{\partial T^*}{\partial t^*}-V_0\frac{\partial T^*}{\partial z^*}\right)=k\frac{\partial^2 T^*}{\partial z^{*2}}, \tag{2}$$

$$\left(\frac{\partial C^*}{\partial t^*}-V_0\frac{\partial C^*}{\partial z^*}\right)=D_m\frac{\partial^2 C^*}{\partial z^{*2}}, \tag{3}$$

where ρ is the density of the fluid, μ is the kinematic viscosity of the fluid, α_1 is the viscoelastic parameter for second grade fluid, V_w is the transpiration velocity and $u^*=u^*\left(z,t^*\right), v=V_w$, where u and v are velocity components in the $x-$ and $z-$coordinate directions, respectively. The transpiration velocity $V_w<0$ denotes injection and $V_w>0$ denotes suction. The bottom surface of the plate is heated by convection from a hot fluid of temperature T^* which provides a heat transfer coefficient h_f. The subjected initial and boundary conditions are

$$\begin{array}{llll}
u^*(z^*,0)=0 & T^*(z^*,0)=T^*_\infty & C^*(z^*,0)=C^*_\infty & z>0, \\
u^*=U_0H(t^*)\exp(i\omega^*t^*), at\ z^*=0, t^*>0, & -k\dfrac{\partial T^*}{\partial z^*}=h_f(T^*_w-T^*)\ at\ z^*=0, & C(0,t)=C_w & t^*>0, \\
u^*(\infty,t^*)=0 & \mathrm{T}^*(\infty,t^*)=T^*_\infty & C^*(\infty,t^*)=C^*_\infty & as\ z^*\to\infty,\ t^*>0.
\end{array} \tag{4}$$

where ω^* is the oscillating frequency, $H(t^*)$ is the unit step function. U_0 is the amplitude of wall. Introducing the following non-dimensional variables.

$$u=\frac{u^*}{U_0},\ z=\frac{U_0z^*}{\nu},\ t=\frac{U_0^2t^*}{\nu},\ \theta=\frac{T^*-T^*_\infty}{T^*_w-T^*_\infty},\ \omega=\frac{\omega^*\nu}{U_0^{\ 2}},\ C=\frac{C^*-C^*_\infty}{C^*_w-C^*_\infty}, \tag{5}$$

The non dimensionalized system in the form of ODEs is

$$\left(\frac{\partial u}{\partial t}-V_0\frac{\partial u}{\partial z}\right)=\frac{\partial^2 u}{\partial z^2}+\alpha\frac{\partial^3 u}{\partial z^2\partial t}-\alpha V_0\frac{\partial^3 u}{\partial z^3}+Gr\theta+GmC,\quad z>0,\ \ t>0, \tag{6}$$

$$\left(\frac{\partial \theta}{\partial t}-V_0\frac{\partial \theta}{\partial z}\right)=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial z^2}, \tag{7}$$

$$\left(\frac{\partial C}{\partial t}-V_0\frac{\partial C}{\partial z}\right)=\frac{1}{Sc}\frac{\partial^2 C}{\partial z^2}, \tag{8}$$

with boundary conditions

$$\begin{array}{l}
u(z,0)=0,\theta(z,0)=1,C(z,0)=0\ \text{at}\ z>0,u(\infty,t)=0,\theta(\infty,t)=0,\ C(\infty,t)=0\ \text{ at }t>0,\\
u(0,t)=H(t^*)e^{iwt},\ \dfrac{\partial\theta(0,t)}{\partial z}=-\dfrac{Br}{\mathrm{Re}}\left[1-\theta(0)\right],\ C(0,t)=1\ \text{at}\ t>0,
\end{array} \tag{9}$$

where $\alpha, Gr, Gm,$ Pr, Sc, Br and Re denotes Second grade fluid parameter, Grashof number, Solutal Grashof number, Prandtl number, Schmidt number, Brinkman number and Reynolds number.

Laplace transform is employed to Eq. (6)-(8) and then solved via Inverse Laplace transform given by Tzou and Puri [24]. We used numerical Inverse Laplace here due to complexity of the resulting equations. The solution for Tzou's algorithm is given as

$$u(z,t)=\frac{e^{4.7}}{2}\left(\frac{\bar{u}\left(z,\frac{4.7}{t}\right)}{2}+\operatorname{Re}\left(\sum_{q=1}^{N1}(-1)^{q}\bar{u}\left(z,\frac{4.7+n\pi i}{t}\right)\right)\right), \tag{10}$$

where i is the imaginary unit, N_1 is a natural number [24] $N_1 >> 1$. The solution for Stehfest's algorithm [23] is employed here to acquire results in order to compare with those obtained via Tzou's algorithm. Stehfest's solution is given by

$$u(y,t)=\frac{\ln(2)}{t}\sum_{j=1}^{2n}\left[\left((-1)^{j+n}\sum_{l=floor\left(\frac{j+1}{2}\right)}^{\frac{(j+n)-|j-n|}{2}}\left[\frac{i^{n}(2l)}{(n-1)l(l-1)(j-1)(2l-j)}\right]\bar{u}\left(y,\frac{j\ln(2)}{t}\right)\right)\right], \tag{11}$$

where n the "Stehfest number" should be even.

3. Results

A parametric study has been analysed for the flow of second grade fluid over a plate with wall suction/injection. The semi-analytical solution is acquired via Laplace transform and numerical Inverse Laplace transform Tzou's Algorithm [24] due to complexity of the equations. The results are plotted for various embedded parameters and compared with another numerical method.

Figure 1 shows the variation of fluid concentration with suction parameter, V_0. Concentration of the fluid is expressively a decreasing function of V_0. This phenomenon is the reason of suction effect occurring at the surface to draw the fluid on the surface. Thus by maximizing the suction parameter, the species boundary layer gets slowed down. The result is plotted for $0 \le V_0 \le 1.2$ and $t = 0.5$. Variation of temperature with different values of Reynolds number is depicted in Figure 2. The increasing values of Reynolds number $0.01 \le \text{Re} \le 0.1$ tends to decrease the fluid temperature for fixed values of $V_0 = 0.8, \Pr = 0.5$ and $Br = 0.3$. The temperature decreases due to dominancy of inertial forces subjected to the internal movement of layers of the fluid.

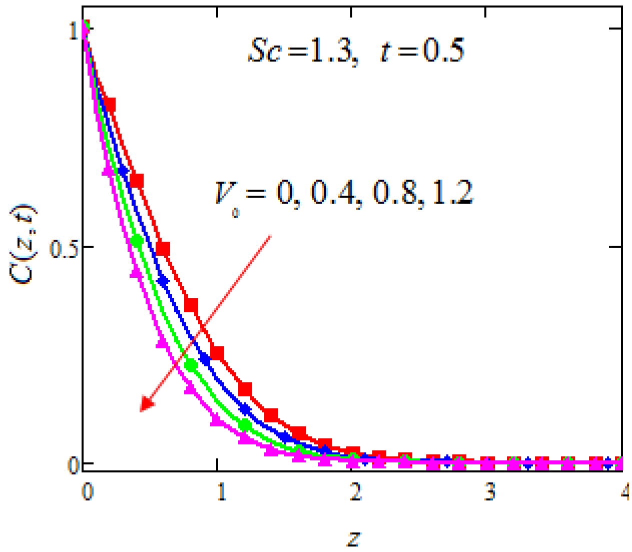

Fig. 1. Concentration profile variation with suction parameter, V_0

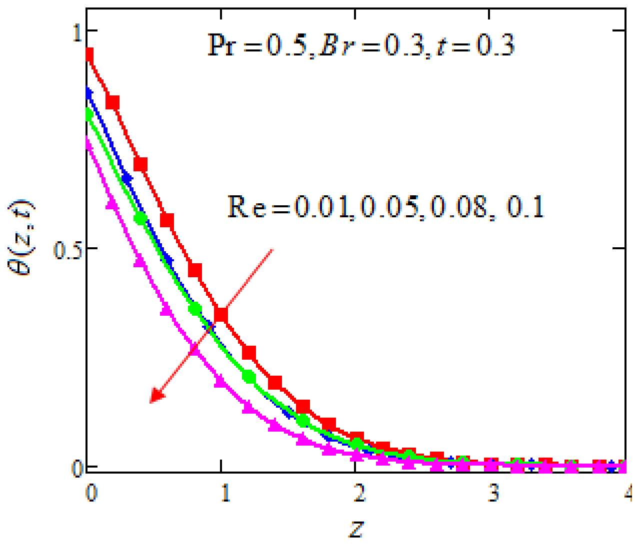

Fig. 2. Temperature profile variation with Reynolds number, Re

Figure 3 illustrates that temperature of the fluid gets decreased with increasing suction/injection parameter for $V_0 = 0,1,2,3$ and $V_0 = 0,-1,-2,-3$. The hot fluid injection occur through the wall and fluid becomes hot. The fluid temperature is maximum near the plate and reduces gradually for $z > 0$. Maximum variation occurs in the area $0.5 \le z \le 1.5$ above the plate. Figure 4 shows that temperature of the fluid increases with maximizing Brinkman number, Br on the temperature of the fluid. The increasing brinkman number leads to maximize heat conduction from the wall, thus temperature raises.

Figure 5 depicts the effect of Grashof number on the flow of fluid. It can be seen from the plot that velocity of the fluid accelerates with increasing values of $Gr = 0,4,6,8$ and $\alpha \ll 1, Gm = 1.2, V_0 = 0.8$. The flow has lowest velocity in the absence of Grashof number, $Gr = 0$. Figure 6 shows the effect of Solutal Grashof number on the flow of fluid. It can be seen from the plot that velocity of the fluid accelerates with increasing values of $Gm = 0,1,2,3$ and $\alpha \ll 1, Gr = 1.2, V_0 = 0.8$. The velocity is smallest in the absence of Solutal Grashof number, $Gm = 0$.

The influence of Reynolds number on the flow velocity is shown in Figure 7. The velocity profile tends to decrease as the Reynolds number goes higher. The results are plotted for Re= 0.0001, 0.1, 0.5, 1 and other parameters were taken constant, $\alpha \ll 1, Gr = 1.2, Gm = 1.2, Br = 0.8, V_0 = 0.8$. Figure 8 depicts the influence of suction/injection parameter on the flow velocity. The behaviour of velocity profile is entirely identical behaviour in the two cases: $V_0 < 0$, and $V_0 > 0$. For suction, $V_0 > 0$, the velocity profile decreases with decreasing suction parameter while in case of decreasing injection parameter the velocity of the fluid increases. The decrease in the fluid flow is due to the rise in fluid viscosity. The results are plotted for various values of suction parameter, $V_0 = 0,1,2,3$, injection parameter, $V_0 = 0,-0.3,-0.7,-1$ and other constant values: $\alpha \ll 1, Gr = 1.2, Gm = 1.2, \mathrm{Pr} = 2.5, Br = 0.8, Sc = 0.3$.

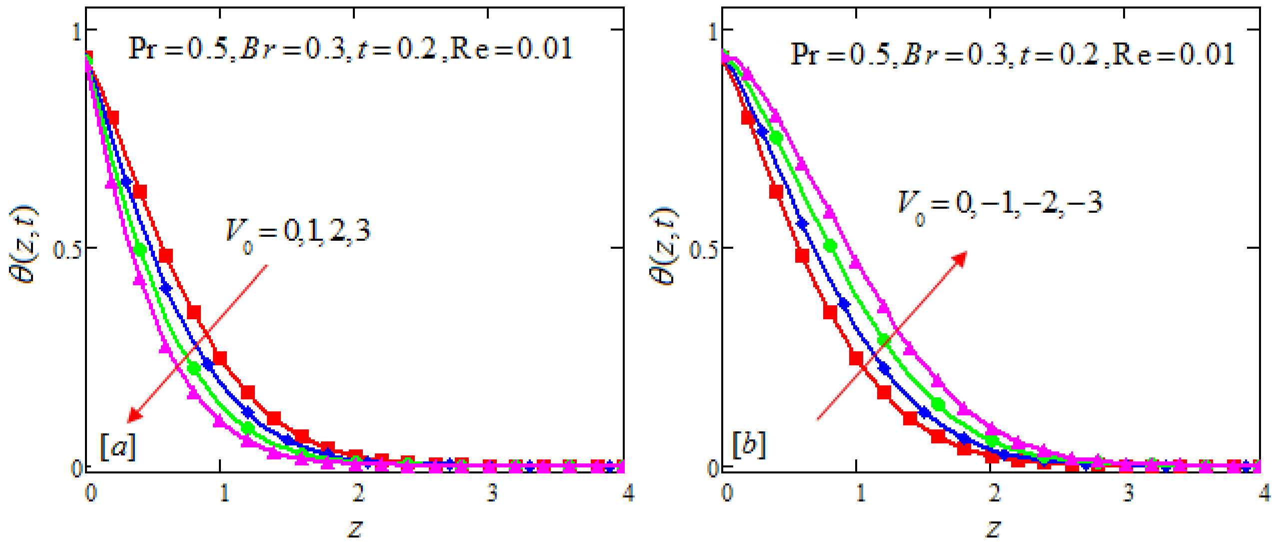

Fig. 3. Temperature profile variation with suction/injection parameter, V_0

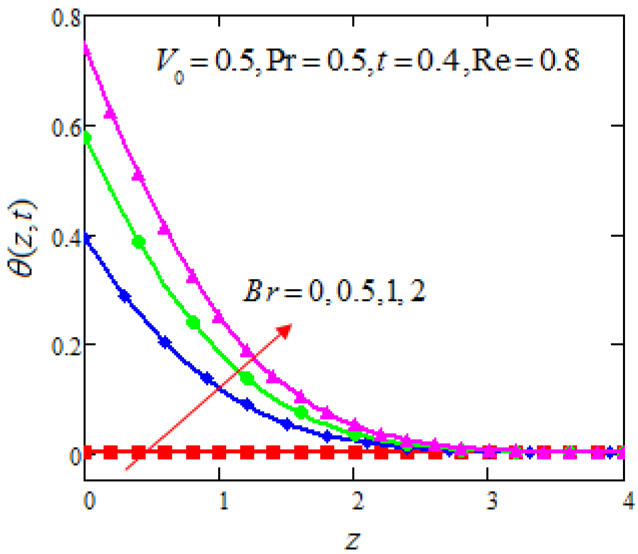

Fig. 4. Temperature profile variation with Brinkman number, Br

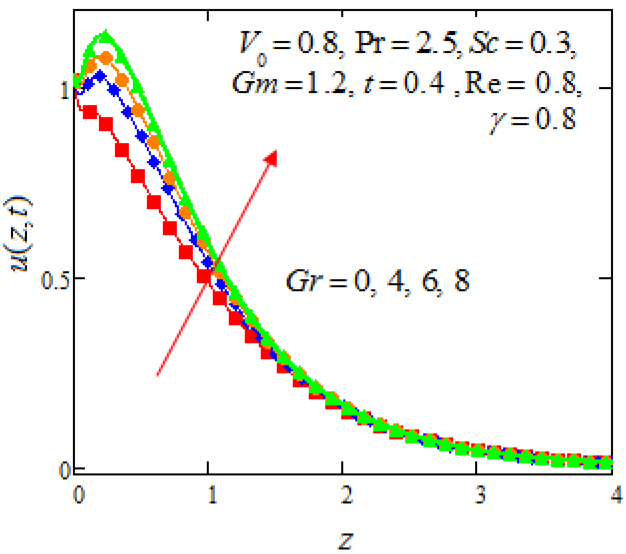

Fig. 5. Velocity profile variation with Grashof number, Gr

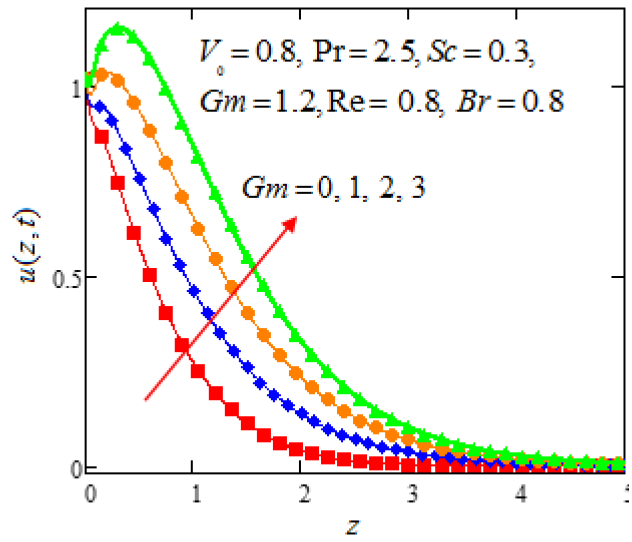

Fig. 6. Velocity profile variation with Solutal Grashof number, Gm

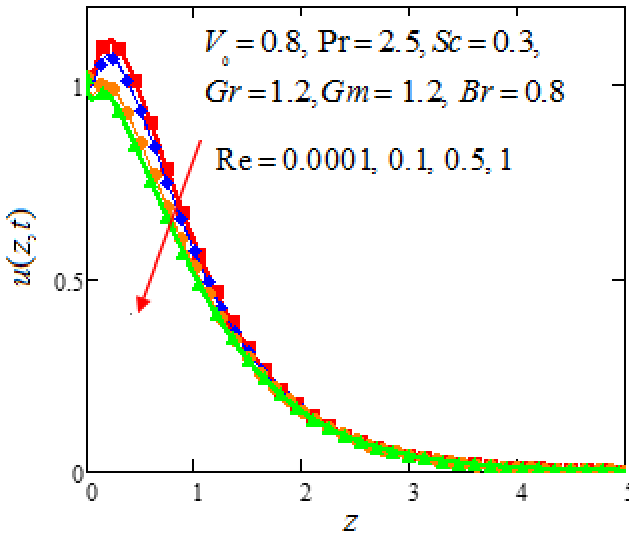

Fig. 7. Velocity profile variation with Reynolds number, Re

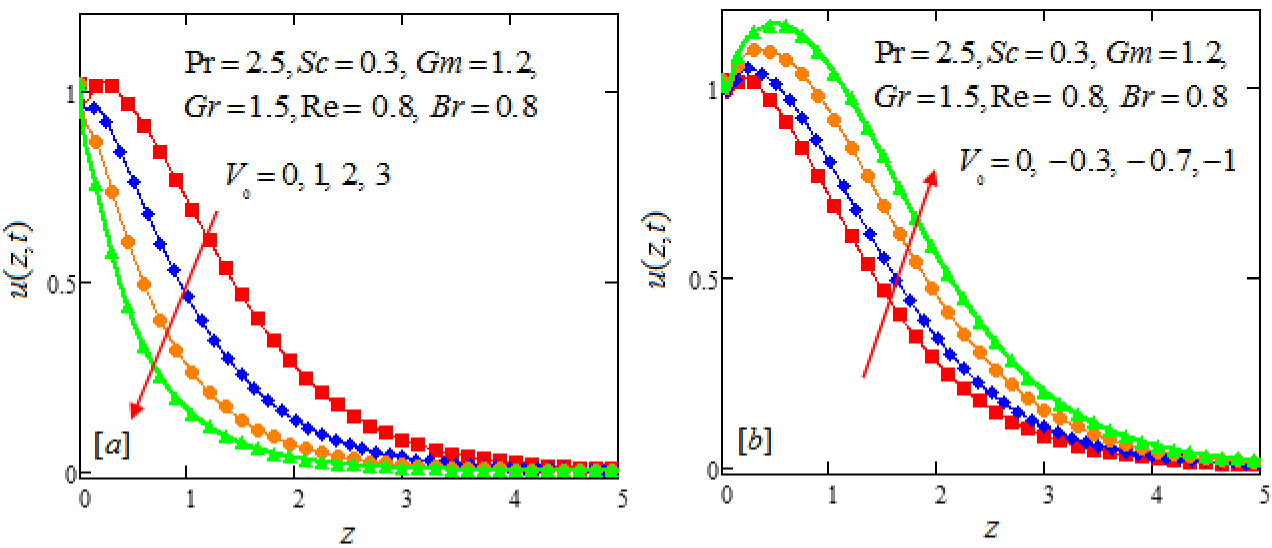

Fig. 8. Velocity profile variation with Suction/injection parameter, V_0

Figure 9 illustrates the validation of present results for Tzou's algorithm with that of Stehfest's algorithm [23]. The results acquired via two different techniques are in great consent for fixed values of parameters involved.

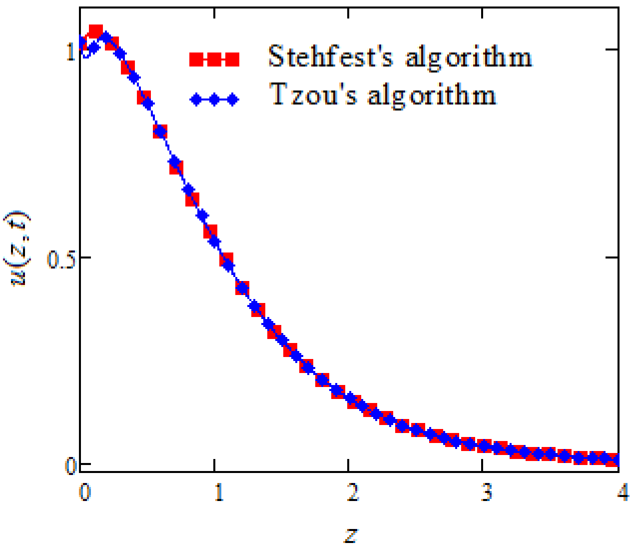

Fig. 9. Comparison of Velocity profile using Tzou's algorithm and Stehfest's algorithm

4. Conclusions

The theoretical data on semi analytical solution of second grade fluid flow problem with wall suction and injection is presented here. The solution is acquired using Laplace transform and numerical Inverse Laplace shedding light on the results in terms of the ingrained parameters. Velocity and temperature of fluid decreases with increasing suction/injection parameter. Grashof number accelerates the fluid velocity due to the dominant buoyancy forces and less viscous forces. The velocity decreases with increasing Reynolds number. The dominant inertial forces slower down the flow velocity. The results obtained are validated via comparison graph with another numerical technique Stehfest's algorithm.

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