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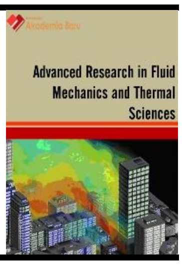

Analysis of MHD Jeffery Hamel Flow with Suction/Injection by Homotopy Analysis Method

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ABSTRACT

This article treats analytically, the magnetohydrodynamic flow through non-parallel porous walls or between two solid porous plates intersecting at an angle, that can be interpreted as a combination between the classical Jeffery Hamel flow added of one injection/suction. The governing equations of the problem are converted from traditional Navier-Stokes equations of fluid mechanics accompanied by those of Maxwell's electromagnetism to ordinary nonlinear differential equations for modeling. A semi-analytic solution is developed by using Homotopy Analysis Method (HAM) whereas the numerical solution is presented by Runge-Kutta scheme. A comparative study between the analytical and numerical solutions is made. The results confirm clearly that the two methods coincide closely for different angles (α), Reynolds numbers (Re), injection parameter (S) and Hartmann number (Ha).

1. Introduction

The incompressible viscous fluid flow between non-parallel walls or through convergent divergent channels, mathematically known as Jeffery-Hamel problem, is invented by George Barker Jeffery [1] and Georg Hamel [2]. The pioneers initially introduced this type of flow and managed to give it a celebrity occupying a primary place in the field of fluid dynamics, constituting a reliable mathematical formulation for many mechanical situations. Therefore, has encouraged many scientists to conduct extensive research in recent years, because of their use in many industrial and natural areas [3-6].

The main objective searched in this work is to discover the behavior of an unsteady Magnetohydrodynamic fluid flow in convergent-divergent channels; this interest is directly related to its practical and industrial utility. The several applications in MHD generators, MHD pumps, accelerators, nuclear reactors and flow meters in biomedical engineering, for example in the flow of blood in the capillaries [7], in the dialysis of blood in artificial kidney [8], and in many other

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engineering areas such as the design of filters, in transpiration cooling boundary layer control [9]. Gupta [10] studied the heat and mass transfer on a stretching sheet with blowing or suction. Sakiadis [11] investigated the boundary layer flow over a stretched surface moving with constant velocity. Erickson *et al.,* [12] based on the work of Sakiadis and included blowing or suction at the stretched sheet surface on a continuous moving surface with constant velocity and studied its effects on the heat and mass transfer in the boundary layer. Nadeem *et al.,* [13] studied the influence of heat and mass transfer on Newtonian bio-magnetic fluid of blood flow through a tapered porous artery with a stenosis. The simulation of variable viscosity and Jeffrey fluid model for blood flow through a tapered artery with a stenosis is presented by Akbar and Nadeem [14]. Akbar and Nadeem [15] presented the analytical and numerical analysis of Vogel's model of viscosity on the peristaltic flow of Jeffrey fluid. The effect of variable thermal conductivity and the inclined magnetic field on MHD plane poiseuille flow in a porous channel with non-uniform plate temperature is investigated by Chutia [16]. On the other hand, in this work, the magnetohydrodynamic (MHD) nature of the flow is considered, this term was first introduced by Bansal [17], this theory, concerns the inducing current in a moving conductive fluid in presence of magnetic field. The interest of the study of such flow, is due to its rapidly increasing and extensive applications in various areas of technology and engineering as MHD generators, accelerators, pumps, and flow meters, Damping and controlling of electrically conducting fluid can be achieved by means of an electromagnetic body force (Lorentz force). For these, the MHD is the subject of much research as [18-23]. The mechanics of the fluid through a divergent channel in presence of electromagnetic field are detailed by Ganji [24]. Harada *et al.,* [25] studied the fundamental characteristics of linear Faraday MHD. In 2005 Anwari *et al.,* [26] bring an improvement to the Haraday work numerically and theoretically and put their print in this field.

In addition, several methods of solving non-linear problems and the effects of MHD for different fluids and geometries are investigated by many researchers, such as [27-48].

In this paper, we have applied one of the most important methods for highly nonlinear problems, the well-known Homotopy Analysis Method (HAM) which was firstly employed by Liao [49-50]. Rashidi *et al.,* [51] employed HAM to investigate the free convective heat and mass transfer in a steady 2D magnetohydrodynamic fluid flow over a vertical stretching. Domairry *et al.,* [52] studied and solved a nonlinear ordinary differential equation through Homotopy Analysis Method (HAM). These analytical methods have already been successfully applied to solve the problem of nanofluid flow and heat transfer characteristics between two horizontal plates in a rotating system by Sheikholeslami *et al.,* [53]. Rashidi *et al.,* [54] used the Homotopy Analysis Method (HAM) to find the analytic solutions for the velocity and the temperature distributions, and to study the steady mixed convection in two-dimensional stagnation flows of a micro polar fluid around a vertical shrinking sheet. Differential equations governing the MHD Jeffery-Hamel flow, and a comparison between the results and the numerical solution is provided.

2. Governing Equations

The problem is described from the continuity and the Navier- Stokes equations written in polar coordinates format [52,54].

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r(u_r - vsin\theta)\right) + \frac{1}{r}\frac{\partial}{\partial \theta}(vcos\theta) = 0, \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r) = 2\frac{1}{r}vsin\theta \Rightarrow ru_r = 2rvsin\theta + f(\theta) \;\; \Rightarrow u_r = 2vsin\theta + \frac{f(\theta)}{r},$$

$$\frac{(u_r - vsin\theta)\,(\partial(u_r - vsin\theta))}{\partial r} + \frac{(vcos\theta)}{r}\frac{\partial(u_r - vsin\theta)}{\partial\theta} - \frac{(vcos\theta)^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + v\left(\frac{\partial^2(u_r - vsin\theta)}{\partial r^2} + \frac{1}{r}\frac{(u_r - vsin\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2(u_r - vsin\theta)}{\partial\theta^2} - \frac{(u_r - vsin\theta)}{r^2} - \frac{2\partial(vcos\theta)}{r^2\partial\theta}\right) - \frac{\sigma B_0^2}{\rho r^2}(u_r - vsin\theta), \tag{2}$$

$$(u_r - vsin\theta)\frac{\partial(vcos\theta)}{\partial r} + \frac{(vcos\theta)}{r}\frac{\partial(vcos\theta)}{\partial\theta} + \frac{(vcos\theta)}{r}(u_r - vsin\theta) = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta}$$
$$+v\left(\frac{1}{r^2}\frac{\partial^2(vcos\theta)}{\partial\theta^2} - \frac{vcos\theta}{r^2} + \frac{2}{r^2}\frac{\partial(u - vsin\theta)}{\partial\theta}\right),$$

The flow is assumed to be purely radial emerging from a line source that is if $u = u(r,\theta)$

From Eq. (1),

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) - \frac{vsin\theta}{r} - \frac{vsin\theta}{r} = 0 \Longrightarrow ru = 2vrsin\theta + f(\theta).$$

$$u_r = 2vsin\theta + \frac{f(\theta)}{r}, \tag{3}$$

Substituting (3) in the momentum Eq. (2) and after calculating all derivatives with respect to f we get

$$\left(\frac{f}{r} + vsin\theta\right)\left(-\frac{f}{r^2}\right) + \frac{(vcos\theta)}{r}\left(vcos\theta + \frac{f'}{r}\right) - \frac{(vcos\theta)^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$
$$+v\left(\frac{2f}{r^3} + \frac{1}{r}\left(-\frac{f}{r^2}\right) + \frac{1}{r^2}\left(\frac{f''}{r} - vsin\theta\right) - \left(\frac{f}{r^3} + \frac{vsin\theta}{r^2}\right) + \frac{2}{r^2}vsin\theta\right)$$
$$-\frac{\sigma B_0^2}{\rho r^2}\left(\frac{f'}{r} + vsin\theta\right), \tag{4}$$

$$-\frac{(vcos\theta)}{r}vsin\theta + \frac{(vcos\theta)}{r}\left(\frac{f}{r} + vsin\theta\right) = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + v\left(-\frac{1}{r^2}vcos\theta - \frac{vcos\theta}{r^2} + \frac{2f'}{r^3} + \frac{2vcos\theta}{r^2}\right),$$
$$-\frac{f^2}{r^3} - \frac{f}{r^2}\,vsin\theta + \frac{f'}{r^2}vcos\theta = -\frac{1}{\rho}\frac{\partial p}{\partial r} + v\frac{f''}{r^3} - \frac{\sigma B_0^2}{\rho r^2}\left(\frac{f'}{r} + vsin\theta\right),$$
$$\frac{f}{r^2}\,vcos\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + v\left(\frac{2f'}{r^3}\right), \tag{5}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = +\frac{f^2}{r^3} + \frac{f}{r^2}\,vsin\theta - \frac{f'}{r^2}vcos\theta + v\frac{f''}{r^3} - \frac{\sigma B_0^2}{\rho r^2}\left(\frac{f'}{r} + vsin\theta\right),$$

$$\frac{1}{\rho}\frac{\partial p}{\partial\theta} = -\frac{f}{r}\,vcos\theta + v\left(\frac{2f'}{r^2}\right), \tag{6}$$

By eliminating the pressure term between Eq. (6) and (3), after deriving the first equation with respect of θ and the second one with respect of *r*, we obtain [52]

$$\frac{1}{\rho}\frac{\partial^2 p}{\partial r\partial\theta} = +\frac{2ff'}{r^3} + \frac{f'}{r^2}\,vsin\theta + \frac{f}{r^2}\,vcos\theta - \frac{f''}{r^2}vcos\theta + \frac{f'}{r^2}vsin\theta + v\frac{f'''}{r^3} - \frac{\sigma B_0^2}{\rho r^2}\left(\frac{f'}{r} + vcos\theta\right),$$

$$\frac{1}{\rho}\frac{\partial^2 p}{\partial r \partial \theta} = +\frac{f}{r^2}\, v cos\theta - \, v\left(\frac{4f'}{r^3}\right), \tag{7}$$

$$0 = +\frac{2ff\prime}{r^3} + \frac{2f\prime}{r^2}\, v sin\theta - \frac{f\prime\prime}{r^2} v cos\theta + v\left(\frac{f'''}{r^3} + \frac{4f'}{r^3}\right) - \frac{\sigma B_0^2}{\rho r^2}\left(\frac{f'}{r} + v cos\theta\right), \tag{8}$$

$$0 = f''' + \frac{2ff'}{v} + \frac{2rf'}{v}\, v sin\theta - \frac{f''}{v} r v cos\theta + 4f' - \frac{\sigma B_0^2}{\rho v}(f' + v r cos\theta), \tag{9}$$

If Eq. (9) is normalized by the value of the velocity profile at $\theta = 0$:

$$F(\theta) = \frac{f(\theta)}{f(0)} \; and \; \eta = \frac{\theta}{\alpha}\,, V = \frac{v}{f(0)}\,,$$
$$\frac{1}{\alpha^3} f(0)F''' + \frac{2(f(0))^2 FF''}{v\alpha} + \frac{2f(0)rF'}{v\alpha}\, v sin(\alpha\eta)$$
$$-f(0)\frac{F''}{v\alpha^2} r v cos(\alpha\eta) + 4\frac{1}{\alpha} f(0)F' - \frac{\sigma B_0^2}{\rho v}\left(f(0)\frac{1}{\alpha}F' + v r cos(\alpha\eta)\right) = 0,$$
$$F''' + \frac{2\alpha^2 f(0)FF'}{v} + \frac{2\alpha^2 rF'}{v}\, v sin(\alpha\eta) - \frac{\alpha F''}{v} r v cos(\alpha\eta) + 4\alpha^2 F' - \frac{\sigma B_0^2}{\rho v}\Big(\alpha^2 F' +$$
$$\frac{1}{f(0)} v r \alpha^3 \, cos(\alpha\eta)\Big) = 0, \tag{10}$$

$$Re = \frac{f(0)\alpha}{v},$$
$$F''' + 2\alpha Re FF' + \; 2\alpha SF' sin(\alpha\eta) - S\alpha F'' cos(\alpha\eta) + 4\alpha^2 F'$$
$$-Ha\left(\alpha^2 F' + \frac{S}{Re}\alpha^3 \, cos(\alpha\eta)\right) = 0;$$
$$F''' - S\alpha F'' cos(\alpha\eta) + 2\alpha Re FF' + (2\alpha S sin(\alpha\eta) + 4\alpha^2 - Ha\alpha^2)F'$$
$$-\frac{HaS}{Re}\alpha^3 \, cos(\alpha\eta) = 0, \tag{11}$$

The boundary conditions are: $f(0) = 1 \;\; , f'(0) = 0\,, f(1) = 0;$ (12)

i. Divergent channel : $\alpha > 0\,, f_{max} > 0;$
ii. Convergent channel : $\alpha < 0\,, f_{max} < 0.$ (13)

The Hartmann number is

$$Ha = \frac{\sigma B_0^2}{\rho v}, \tag{14}$$

The Reynolds's number is

$$Re = \frac{f(0)\alpha}{v}\,, \tag{15}$$

The injection parameter is

$$S = \frac{VR\alpha}{v}. \tag{16}$$

3. Fundamentals of Homotopy Analysis Method (HAM)

3.1 Pressure Distribution

In this section,

$$N[\phi(\eta,q)]=\frac{\partial^3\phi(\eta,q)}{\partial\eta^3}+2\alpha Re\phi(\eta,q)\frac{\partial\emptyset(\eta,q)}{\partial\eta}+\left(4\alpha^2-H\alpha^2+2\alpha Ssin(\alpha\eta)\right)\frac{\partial\emptyset(\eta,q)}{\partial\eta}-\frac{H\alpha S}{Re}\alpha^3cos(\alpha\eta)-S\alpha cos(\alpha\eta)\frac{\partial^2\phi(\eta,q)}{\partial\eta^2}, \tag{17}$$

where $q\epsilon[0,1]$ the embedding parameter, $\hbar\neq 0$ is a nonzero auxiliary parameter. As the embedding parameter increases from 0 to 1, $\emptyset(\eta,q)$ varies from the initial guess $f_0(\eta)$ to the exact solution $f(\eta)$:

$$\emptyset(\eta,0)=f_0(\eta),\ \emptyset(\eta,1)=f(\eta), \tag{18}$$

Expanding $\phi(\eta,q)$ in Taylor series with respect to q, we have:

$$\emptyset(\eta,q)=f_0(\eta)+\textstyle\sum_{m=1}^{\infty}f_m(\eta)q^m\ , \tag{19}$$

where

$$f_m(\eta)=\frac{1}{m!}\frac{\partial^m\phi(\eta,q)}{\partial q^m}\Big]_{q=0}. \tag{20}$$

The auxiliary function $H(\eta)$, initial approximation $f_0(\eta)$ and the auxiliary linear operator L must be chosen in such a way that all solutions of the corresponding high-order deformation equations exist

$$L[\phi(\eta,q)]=\frac{\partial^3\phi(\eta,q)}{\partial\eta^3}, \tag{21}$$

That the L is

$$L[-0.5c_1\eta^2+c_2\eta+c_3]=0, \tag{22}$$

where c_1, c_2 and c_3 are constants. We must guess the initial value of $f(\eta)$ so that it to satisfy the boundary conditions. According to the discussed limitation and under the rule of solution expression and initial conditions, the initial guess is

$$c_1=2\ ,c_2=0\ ,c_3=1. \tag{23}$$

The zero order deformation equation is

$$(1-q)L[\phi(\eta,q)-f_0(\eta)]=q\hbar H(\eta)N[\phi(\eta,q)], \tag{24}$$

$$\phi(0,q)=1\ ,\phi(1,q)=0\ ,\frac{\partial\emptyset(0,q)}{\partial\eta}=0, \tag{25}$$

The auxiliary function is

$$H(\eta)=1. \tag{26}$$

4. Application

The Differentiating Eq. (19), m times with respect to the embedding parameter q and then setting q=0 and finally dividing them by m! and From Eq. (10), (17). We have the m[th]-ordre deformation equation form>1.

$$f_m(\eta) = \chi_m f_{m-1}(\eta) + \hbar \iiint_0^\eta H(\eta) R_m(f_{m-1}) d\eta + c_1\eta^2 + c_2\eta + c_3, \tag{27}$$

$$f_m(0) = 0\,, f_m(1) = 0\,, f'_m(0) = 0, \tag{28}$$

where

$$R_m(f_{m-1}) = f'''_{m-1}(\eta) + 2\alpha Re \sum_{n=0}^{m-1} f_n(\eta)\, f'_{m-1-n}(\eta) + ((4-H)\alpha^2 + 2\alpha S sin(\alpha\eta)) f'_{m-1}(\eta) - \frac{HaS}{Re}\alpha^3 cos(\alpha\eta) - S\alpha cos(\alpha\eta) f''(\eta), \tag{29}$$

$$\chi_m = \begin{cases} 0, m \leq 1; \\ 1, m > 1. \end{cases} \tag{30}$$

We now successively obtain

$$f_0(\eta) = 1 - \eta^2\,, \tag{31}$$

$$\text{f}(1) = -\frac{(\alpha^3 Ha + 8hRe - 2\alpha hrRe)S\eta}{\alpha^2 Re} - \frac{1}{3}\alpha^2 h\eta^4 + \frac{1}{12}\alpha^2 hHa\eta^4 - \frac{1}{6}\alpha hRe\eta^4 + \frac{1}{30}\alpha hRe\eta^6 - \frac{4hrS\eta Cos[\alpha\eta]}{\alpha^2} + \eta^2(\frac{\alpha^2 h}{3} - \frac{1}{12}\alpha^2 hHa + \frac{2\alpha hRe}{15} + \frac{8hrS}{\alpha^2} - \frac{2hrS}{\alpha} + \frac{\alpha HaS}{Re} + \frac{4hrScos[\alpha]}{\alpha^2} - \frac{12hrSsin[\alpha]}{\alpha^3} + \frac{2hrSsin[\alpha]}{\alpha^2} - \frac{HaSsin[\alpha]}{Re}) + \frac{12hrSsin[\alpha\eta]}{\alpha^3} - \frac{2hrSsin[\alpha\eta]}{\alpha^2} + \frac{HaSsin[\alpha\eta]}{Re}. \tag{32}$$

The injection parameter is the ratio of inertial to viscous forces. It's given by $S = \frac{V\alpha r}{\upsilon}$ where V is the injection velocity of the fluid, r is the characteristic length and υ is the kinematic viscosity.

5. Results and discussion

In this section, Figure 1 shows the MHD Jeffery-Hamel flow in convergent/divergent channel with angle 2α. In Figure 2-9 we discuss about the effect of the dimensionless velocity F(η) versus the dimensionless angle η with various value of parameters *Ha,* Re, α and S for both case, divergent and convergent channel using the analytical and numerical methods. Where the results of the HAM method are presented with a dashed line.

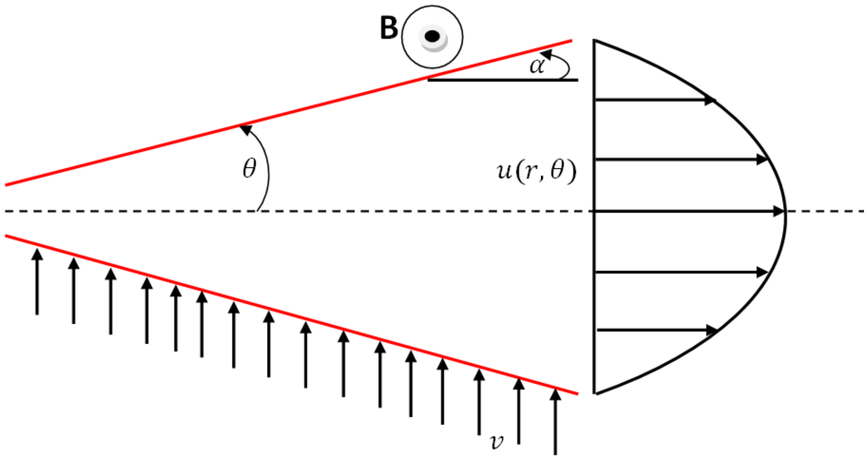

Fig. 1. Geometry of the problem

Figure 2 and 3 display that it is worth mentioning to point out that the velocity $F(\eta)$ increases for divergent and convergent channel, when Hartman number increases. The Lorentz force effect is in opposite of the momentum's direction that stabilizes the velocity profile.

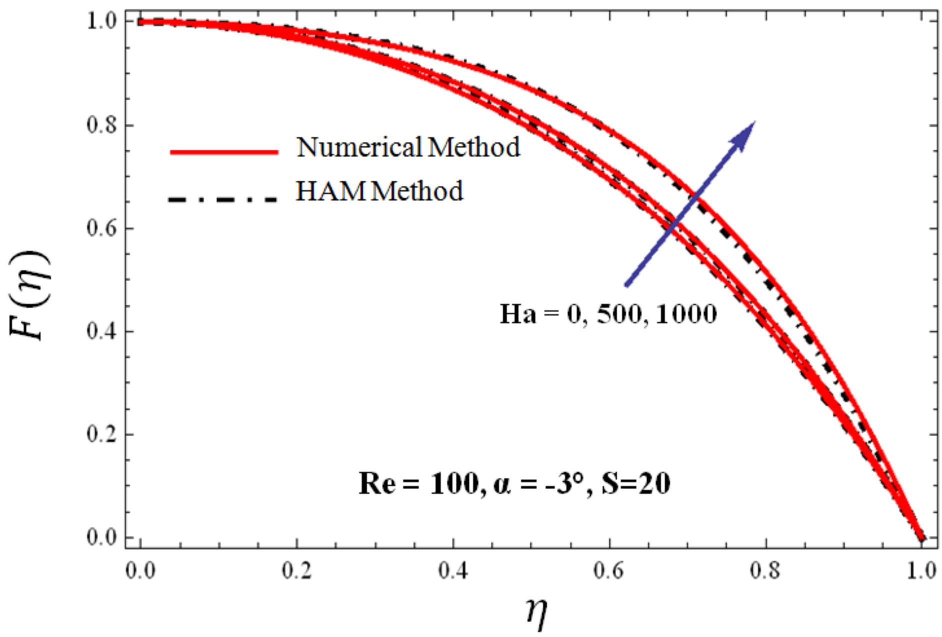

Fig. 2. Velocity profile variation for various Hartmann number (Ha) in converging channel case

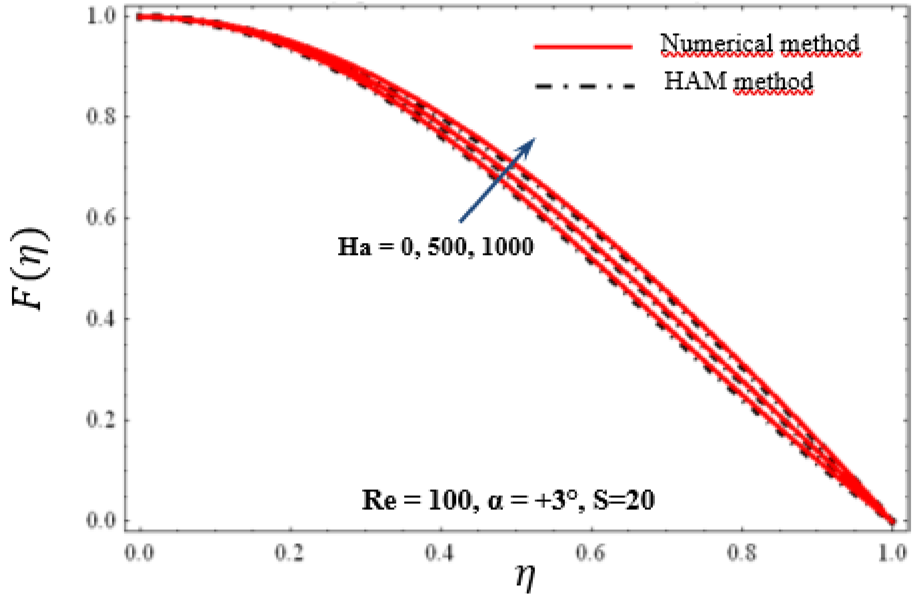

Fig. 3. Velocity variation profile for various Hartmann numbers in diverging channel case

The Figure 4 and 5 show the effects of injection/section parameter S, in the case of the divergent channel. The suction has an effect that favors the radial velocity, because the radial component of the V velocity takes the same direction of the radial velocity u, and we can see that the suction velocity behaves conversely in the case of convergent channel, due to direction of the radial component of V, which is opposite to the flow radial velocity, so it tend to decreases it.

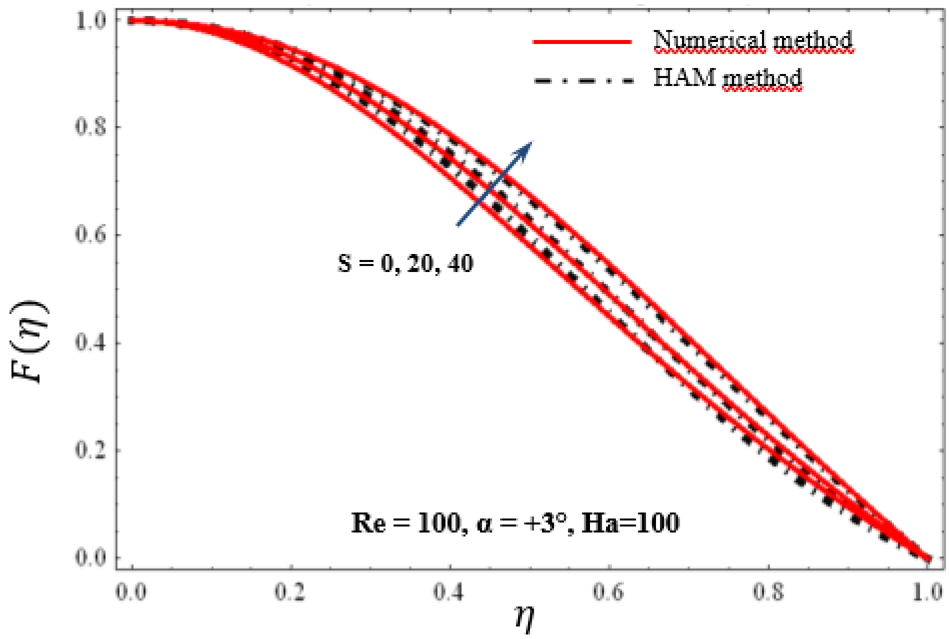

Fig. 4. Velocity variation profile for various S in diverging channel case

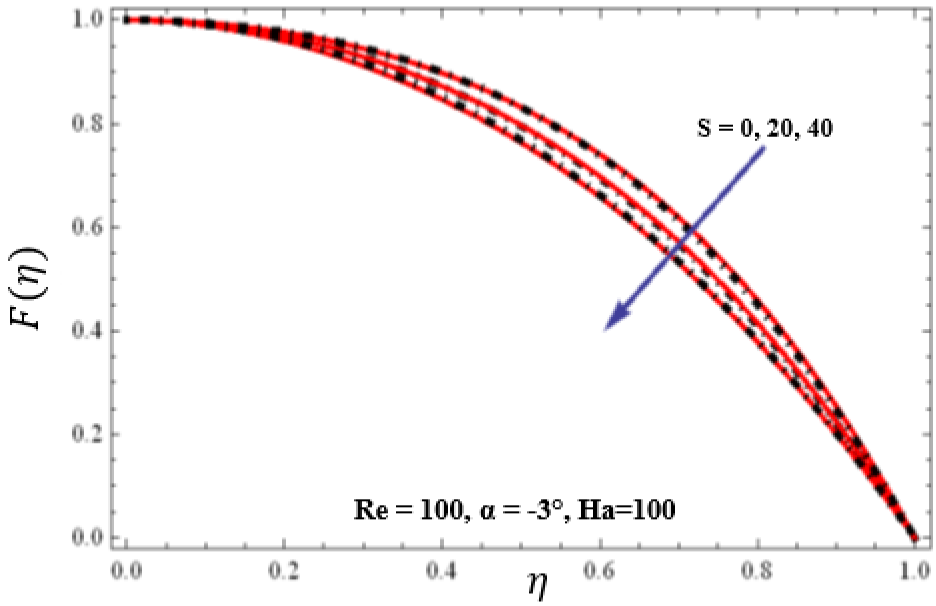

Fig. 5. Velocity variation profile for various S in converging channel case

In Figure 6 and 7, we discuss about the effect of the dimensionless velocity $F(\eta)$ versus the dimensionless angle η with various value of α angle for some given fixed values of Ha, Re and S. We can note that the velocity increases with the α angle in the convergent channel and decreases in the case of the divergent one.

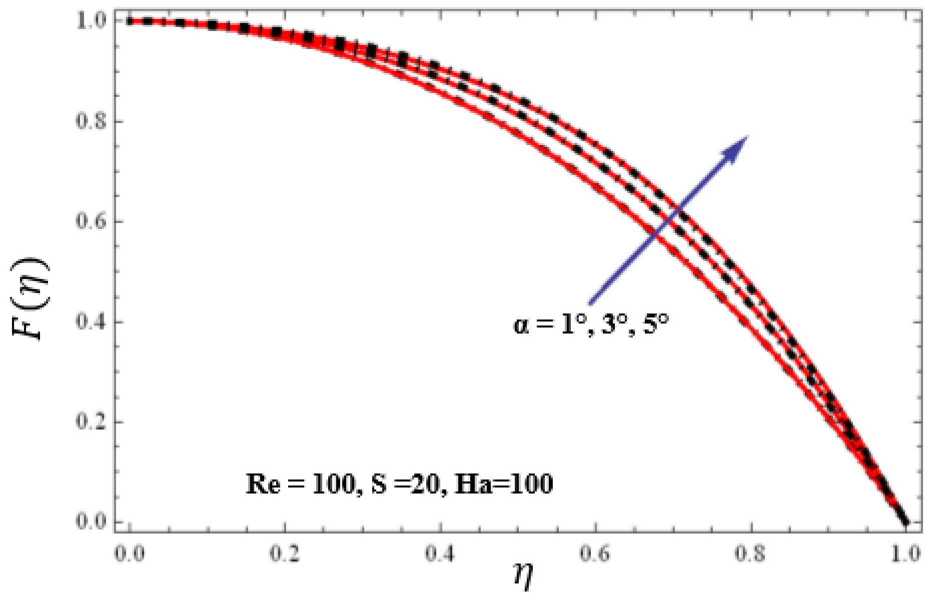

Fig. 6. Velocity variation profile for various inclination angles (α) in converging channel case

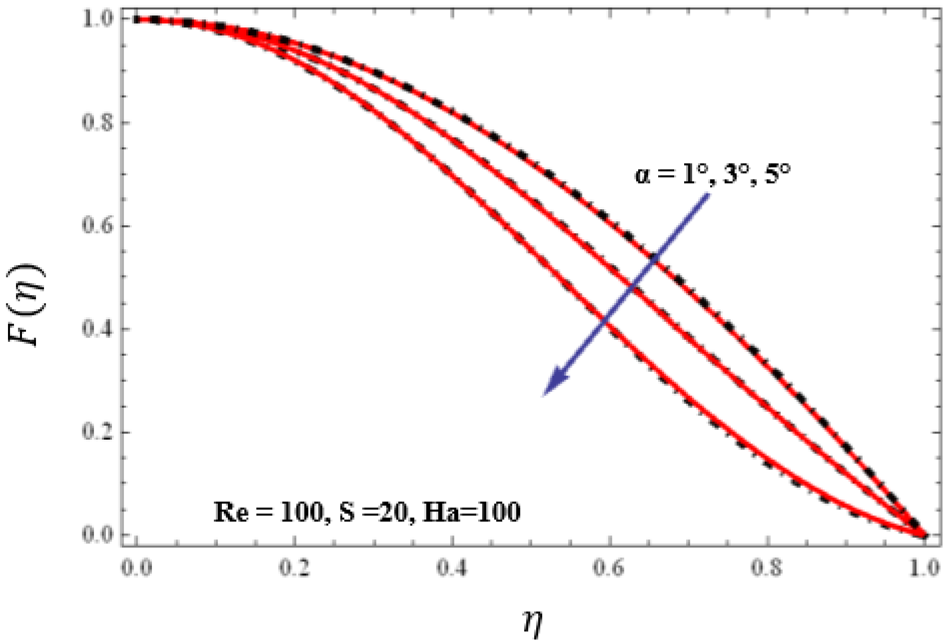

Fig. 7. Velocity variation profile for various inclination angles (α) in diverging channel case

In the Figure 8 and 9, it is worth to mention that the Reynolds number which indicates the relative significance of the inertia effect compared to the viscous effect has a contrary behavior in the case of a divergent in which velocity profile decrease as *Re* increase and the convergent case which behaves conversely, and it is obvious to advance such a judgment when we see that the velocity profile increases as *Re* increases.

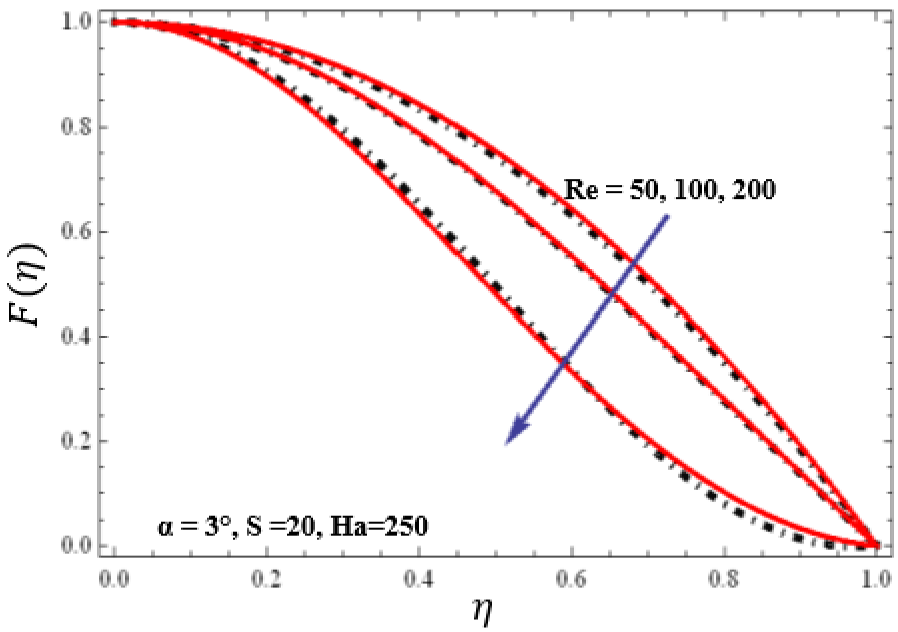

Fig. 8. Velocity variation profile for various Reynolds numbers in diverging channel case

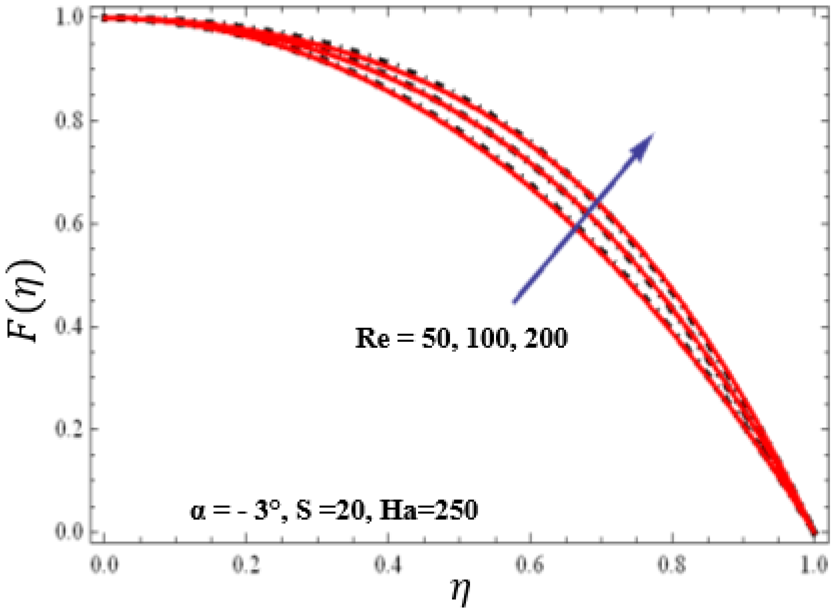

Fig. 9. Velocity variation profile for various Reynolds numbers in converging channel case

5. Conclusions

In this paper we have solved, the 3rd-order magnetohydrodynamic Jeffery-Hamel flow with injection/suction via an analytical method. To this effect, we chose Homotopy Analysis Method (HAM) and compare it with a numerical method (the Runge-Kutta method of order 4) using Mathematica software.

From this investigation, the major outcomes can be summerized as

i. Homotopy Analysis method is a powerful approach for solving MHD Jeffery-Hamel flow in high magnetic field, and it has been successfully applied and it can be observed that there is a good agreement between the analytical and numerical results.
ii. Increasing Reynolds numbers leads to adverse pressure gradient which causes velocity reduction near the walls. And an opposite behavior in the case of convergent channel.
iii. Increasing Hartmann number will lead to increasingly velocity in the both case (divergent or convergent channel).
iv. Increasing suction/injection parameter *S*, allow an increasing velocity in divergent channel case and an opposite behavior in convergent channel case and it is due to the fact that the vertical velocity has a radial component in same direction of fluid flow in the divergent channel and acts conversely in the case of convergent one.

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