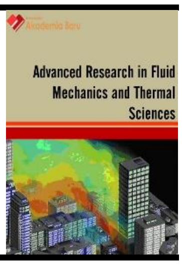
Journal of Advanced Research in Fluid Mechanics and Thermal Sciences 59, Issue 2 (2019) 220-231

Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879



MHD Flow of Fourth Grade Fluid Solve by Perturbation-Iteration Algorithm

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Assma Jassim Harfash [1,2,*], Abdul-Sattar Jaber Al-Saif [1]

[1] Department of Mathematics, College of Education for Pure Science, University of Basrah, Basrah, Iraq
[2] Department of Mathematics, College of Education, University of Misan, Misan, Iraq

ARTICLE INFO

Article history:
Received 9 March 2019
Received in revised form 25 May 2019
Accepted 12 June 2019
Available online 19 July 2019

Keywords:
MHD fluid; Fourth grade fluid; Perturbation-iteration algorithm

ABSTRACT

In this paper, a perturbation-iteration algorithm is used to solve nonlinear differential equations which represent the steady incompressible flow problem of a fourth grade non-Newtonian fluid between two stationary parallel plates in the presence of magnetic field. The governing partial differential equations have been transformed into an ordinary differential equation, before being solved. The numerical results have been computed and compared with the results of other methods. Moreover, the convergence of the numerical results has been tested for various values of physical parameters. The influence of some physical parameters such as the slip parameter, non-Newtonian parameter and magnetic field parameter on velocity is explained. The results show the reliability and efficiency of this analytical method.

1. Introduction

In recent years, non-Newtonian fluids have received a great interest in scientific research because of their great importance in technical and industrial applications. Non-Newtonian fluids are fluids that do not follow Newton's law and change their viscosity according to the forces that influence them, including shampoo, blood, tomato ketchup, honey, mud, paints, plastics and polymer melts. The non-Newtonian fluids may be classified as fluids for which the shear stress depends on the shear rate only or shear rate and time fluids which possess both elastic and viscous properties. It cannot be described as a single model because they are complex fluids. Therefore, several models describe the proposed behaviour of non-Newtonian fluids. Amongst these the models of non-Newtonian fluids are the second grade, third grade and fourth grade. It is worth to mention that the second grade fluid is used to predict the normal stress differences, but it does not distinguish between thick and thin shear because the viscosity of the shear is fixed. So some experiments may be described through fluids of three or four grade.

Many researchers have dealt with various cases of second and third grade fluid flow successfully

* *Corresponding author.*
E-mail address: assmaj1974@yahoo.com (Assma Jassim Harfash)

using analytical and numerical methods [1-9]. The researchers were also interested in analyzing and studying the problems of the fourth grade fluid flow as a general class of second and third grade fluids. Hayat *et al.,* [10], the finite difference approximate was used to solve the steady flow of an incompressible fourth grade fluid in the presence of magnetic field between parallel plates with one plate at rest and the other moving parallel to it at constant speed with a suction velocity normal to the plates. The homotopy analysis method (HAM) was used by Sajid *et al.,* [11] to investigate the steady flow of fourth grade fluid past a porous plate. Siddiqui *et al.,* [12], the homotopy perturbation method (HPM) and the traditional perturbation method were used to solve the non-linear equations modelling thin film flow of fourth grade fluid falling on the outer surface of an infinitely long vertical cylinder. Hayat and Sajid [13], a comparison between the result of HAM and HPM was made for solving thin film flow of a fourth grade fluid down a vertical cylinder, and the results revealed that HAM is very simple and effective and provides a simple way to control and adjust the convergence region. The finite difference approximation, Wanga and Wu [14], was also used for solving the unsteady incompressible flow of fourth-grade fluid which induced by a periodically oscillating two dimensional in a semi-infinite porous plate with suction/blowing within a uniform magnetic field.

Moreover, the optimal homotopy asymptotic method (OHAM) was proposed by Marinca [15] to find the approximate analytical solution of the steady flow problem of a fourth-grade fluid past a porous plate. HPM and OHAM were also utilised by Islam *et al.,* [16] to solve the steady flow of fourth-grade fluid when slippage between the plate. Forward and backward wave-front type travelling wave solutions had been constructed by Aziz and Mahomed [17] to study the problem of unsteady unidirectional flow of fourth grade fluid bounded by a suddenly moving the rigid plate. The results of the variational iteration method (VIM) and HAM were compared with the results of Adomians decomposition method (ADM) by Abbasi *et al.,* [18] for solving the steady flow of fourth grade fluid between two stationary parallel plates. The study of the incompressible fully developed flow of fourth grade fluid in a flat channel under an externally applied magnetic field with taking slip conditions at the wall of the channel was presented by Moakher *et al.,* [19, 20] where the collocation method (CM) and least square method (LSM) respectively, had been applied to solve this problem. The optimal auxiliary functions method (OAFM) was suggested to investigate the problem of thin film flow of a fourth grade fluid dawn a long vertical cylinder by Marinca [21]. The problem of peristaltic transport of an incompressible non-Newtonian, fourth grade fluid in a tapered asymmetric channel was discussed by Kothandapani *et al.,* [22] under long-wavelength and low-Reynolds number assumptions. Regular perturbation technique was used to find the series solutions for stream function, axial velocity and pressure gradient in this problem. *Khan et al.,* [23], the boundary layer flow problem of a fourth grade fluid and heat transfer over an exponential stretching sheet was investigated by using HAM and the Keller-box technique. Ali *et al.,* [24], the combined effects of heat and mass transfer were studied the unsteady flow of a fourth-grade fluid over an oscillating vertical porous plate in the presence of uniform magnetic field by using HAM.

The main objective of this study is to solve the steady flow problem of fourth grade fluid between two stationary parallel plates with magnetic field effect. The differential equations that is generated by the fourth grade fluid flow are considered which is the generalised formula of the second and third grade fluid flow. This problem includes highly nonlinear equations and their order is higher than that of the classical Navier-Stokes equations. In addition, the non-Newtonian fluid flow problems of the fourth grade require additional conditions at the boundary. For these reasons, finding exact solutions to these problems is very difficult, so we utilised in this study a semi analytical technique which is the perturbation-iteration algorithm (PIA) [25-39]. Although many semi analytical methods are presented in the literature for solving this problem, but here we are applied PIA for some reasons. Firstly, it can be applied to the problem directly without using linearization, discretization or any transformation. Secondly, it does not need to install a small auxiliary parameter in the equations, to reverse other perturbation methods which their solutions are restricted by the validity range of physical parameters. Moreover, this method was not previously used to solve this problem. In view of this we have established the approximate analytic solution based on the PIA.

The remaining parts of this study are as follows: In Section 2, the flow problem of fourth grade fluid between two stationary parallel plates in a magnetic field is formulated, and nonlinear ordinary differential equation of this problem is derived. The Mathematical properties of PIA are presented and applied to solve our problem and then we discuss the convergence of the resulted solutions, in Section 3. The numerical results are presented and discussed in Section 4. Finally, in Section 5, some conclusions were presented.

2. Mathematical Formulation

In this paper, we consider the incompressible flow of a fourth grade fluid between two stationary parallel plates in a magnetic field. The fluid motion between the two plates is towards x, driven by the constant pressure gradient and the distance between the two plates is $2d$, as shown in Figure 1.

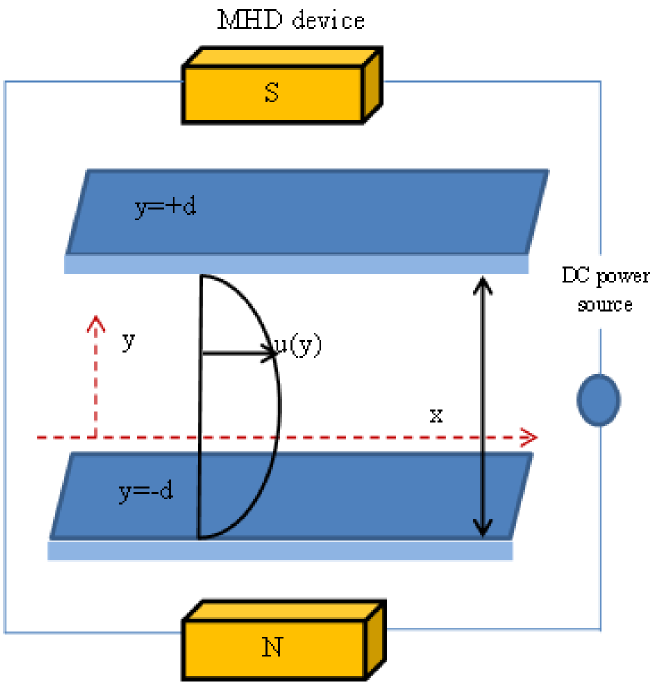


Fig. 1. Diagram of the physical system of flow problem

The governing equations of the incompressible flow of n-th grade fluid between two stationary parallel plates in a magnetic field are

$$\left.\begin{aligned} &\nabla \cdot V = 0, \\ &\rho \frac{DV}{Dt} = \nabla \cdot T + \boldsymbol{J} \times \boldsymbol{B}, \end{aligned}\right\} \tag{1}$$

where V is the velocity vector, ρ is the constant density, ∇ is the Nabla operator, T is the stress tensor, $\boldsymbol{J}$ is the electric current density and $\boldsymbol{B}$ is the total magnetic field such that $\boldsymbol{J} \times \boldsymbol{B} = \sigma\,(V \times \boldsymbol{B}) \times \boldsymbol{B}$, σ is the electrical conductivity of the fluid, $\boldsymbol{B} = (0, B_0, 0)$, and $\frac{D()}{Dt} = \frac{d()}{dt} + (V \cdot \nabla)(\,)$ denotes the material derivative. The stress tensor T defining a n-grade fluid is given by

$$T = -p\, I + \sum_{i=1}^{n} S_i, \tag{2}$$

if $n = 4$ then the system Eq. (1) give the fourth grade fluid as ,

$$\left.\begin{aligned}
S_1 &= \mu\, A_1,\\
S_2 &= \alpha_1\, A_2 + \alpha_2\, A_1^2,\\
S_3 &= \beta_1\, A_3 + \beta_2\, (A_1\, A_2 + A_2\, A_1) + \beta_3\, (tr\,(A_1)^2)\, A_1,\\
S_4 &= \gamma_1\, A_4 + \gamma_2\, (A_3\, A_1 + A_1\, A_3) + \gamma_3\, A_2^2 + \gamma_4 (A_2\, A_1^2 + A_1^2 A_2) + \gamma_5 (tr A_2) A_2\\
&\quad + \gamma_6\, (tr\, A_2)\, A_1^2 + (\gamma_7\, (tr\, A_3) + \gamma_8\, (tr\,(A_2\, A_1)))\, A_1,
\end{aligned}\right\} \tag{3}$$

where p is the pressure, I is the identity tensor, μ, α_1, α_2, β_1, β_2, β_3, γ_1, γ_2, γ_3, γ_4, γ_5, γ_6, γ_7 and γ_8 are material constants, and

$$\left.\begin{aligned}
A_0 &= I,\\
A_n &= \frac{d\, A_{n-1}}{d\, t} + A_{n-1}\, (\nabla\, V) + (\nabla\, V)^t\, A_{n-1}, \qquad n \geqslant 1.
\end{aligned}\right\} \tag{4}$$

We note that the third grade fluid can be inferred when $n = 3$ in Eq. (2), and this model reduces to a second grade fluid when $n = 2$ in Eq. (2), while the classical Navier-Stokes fluid can be gotten when $n = 1$ in Eq. (2).

We consider the velocity field of the form $V = (u(y),0,0)$ and the components of T are

$$\left.\begin{aligned}
T_{xx} &= -p + \alpha_2\, (\tfrac{du}{dy})^2 + 2\, \gamma_6\, (\tfrac{du}{dy})^4,\\
T_{xy} &= \mu\, \frac{du}{dy} + 2\, (\beta_2 + \beta_3)\, (\tfrac{du}{dy})^3,\\
T_{yx} &= \mu\, \frac{du}{dy} + 2\, (\beta_2 + \beta_3)\, (\tfrac{du}{dy})^3,\\
T_{yy} &= -p + (2\, \alpha_1 + \alpha_2)\, (\tfrac{du}{dy})^2 + 4\, (\gamma_3 + \gamma_4 + \gamma_5 + 0.5\, \gamma_6)\, (\tfrac{du}{dy})^4,\\
T_{zz} &= -p,\ T_{xz} = T_{yz} = T_{zx} = T_{zy} = 0.
\end{aligned}\right\} \tag{5}$$

The x-component of the momentum Eq. (1) takes the form

$$-\frac{dp}{dx} + \mu\, \frac{d^2u}{dy^2} + 6(\beta_2 + \beta_3)\left(\frac{du}{dy}\right)^2 \frac{d^2u}{dy^2} - \sigma\, B_0^2\, u(y) = 0, \tag{6}$$

the y-component has the form

$$-\frac{dp}{dy} + (2\, \alpha_1 + \alpha_2)\frac{d}{dy}\left(\frac{du}{dy}\right)^2 + 4\, (\gamma_3 + \gamma_4 + \gamma_5 + 0.5\gamma_6)\frac{d}{dy}\left(\frac{du}{dy}\right)^4 = 0, \tag{7}$$

and z-component has the form

$$-\frac{dp}{dz} = 0. \tag{8}$$

By integrating Eq. (7) with respect to y, we arrive at

$$p^{*} = -p + (2\,\alpha_1 + \alpha_2)\left(\frac{du}{dy}\right)^2 + 4\,(\gamma_3 + \gamma_4 + \gamma_5 + 0.5\,\gamma_6)\left(\frac{du}{dy}\right)^4 \tag{9}$$

Since $\frac{dp^*}{dy} = 0$, and $\frac{dp^*}{dz} = 0$, then $p^* = p^*(x)$. Then, the Eq. (6), (7) and (8) can reduce to single equation

$$\mu\left(\frac{d^2u}{dy^2}\right) + 6\,\beta\left(\frac{du}{dy}\right)^2\left(\frac{d^2u}{dy^2}\right) - \sigma\,B_0^2\,u(y) = A\,, \tag{10}$$

where $\beta = \beta_2 + \beta_3$ and $A = \frac{dp^*}{dx}$. Therefore, the problem is reduced to solve the second-order nonlinear ordinary differential Eq. (10) with following boundary conditions

$$\frac{du}{dy}|_{y=0} = 0 \quad \text{and} \quad \frac{du}{dy}|_{y=d} = -\lambda\,u(d) \tag{11}$$

The above equations are conveniently non-dimensional with the scales

$$\eta = \frac{y}{d}, \qquad U(\eta) = \frac{\mu\,u(y)}{A\,d^2}, \qquad N_f = \frac{A^2\,d^2\,\beta}{\mu^3}, \qquad Ha = B_0\,d\sqrt{\frac{\sigma}{\mu}}, \tag{12}$$

Eq. (10) and (11) in non-dimensional form become,

$$\frac{d^2U}{d\eta^2} + 6\,N_f\left(\frac{dU}{d\eta}\right)^2\frac{d^2U}{d\eta^2} - Ha^2\,U - 1 = 0, \tag{13}$$

with

$$\frac{dU}{d\eta}|_{\eta=0} = 0 \quad \text{and} \quad \frac{dU}{d\eta}|_{\eta=1} = -\lambda\,U(1) \tag{14}$$

3. Perturbation-Iteration Algorithm (PIA)

PIA is a combination of perturbation expansions and Taylor series expansions to construct an iteration scheme. It is obtained by taking different numbers of terms in the perturbation expansions and different orders of correction terms in the Taylor series expansions. Therefore, the perturbation iteration algorithm is called PIA(m,n) where the m is the number of the correction terms in the perturbation expansion, and n is the highest order derivative term in the Taylor series such that m should always be less than or equal to n.

In order to obtain approximate analytical solutions for the incompressible fluid flow of a fourth grade fluid through the channel with slip condition, PIA is applied to system (13) and (14). Thus, we consider the following equation

$$G(U,U',U'',\epsilon) = U'' + 6\,\epsilon\,N_f\,(U')^2\,U'' - Ha^2\,U - 1 = 0, \tag{15}$$

where ϵ is a small perturbation parameter, $U' = \frac{dU}{d\eta}$ and $U'' = \frac{d^2U}{d\eta^2}$. We also define the following perturbation expansions with only one correction term

$$U_{n+1} = U_n + \epsilon (U_c)_n \, , \tag{16}$$

where n represents the nth iteration and U_c is the correction terms in the perturbation expansion. By substituting (16) into (15) and expanding the resulted equations in a Taylor series with first order derivative terms for apply PIA (1,1) about $\epsilon = 0$, yields

$$G(U_n, U_n', U_n'', 0) + \epsilon \left[G_{U_{n+1}} (U_c)_n + G_{U_{n+1}'} (U_c)_n' + G_{U_{n+1}''} (U_c)_n'' + G_\epsilon \right] = 0, \tag{17}$$

such that

$$\left.\begin{aligned} G(U_n, U_n', U_n'', 0) &= U_n'' - Ha^2\, U_n - 1, \\ G_{U_{n+1}} &= -Ha^2, \\ G_{U_{n+1}'} &= 12\, \epsilon\, N_f\, U_{n+1}'\, U_{n+1}'' \, , \\ G_{U_{n+1}''} &= 1 + 6\, \epsilon\, N_f\, (U_{n+1}')^2, \\ G_\epsilon &= 6\, N_f\, (U_{n+1}')^2\, U_{n+1}'' \, . \end{aligned}\right\} \tag{18}$$

By calculating all derivatives at $\epsilon = 0$ and substituting the results into (17) yields the following linear ordinary differential equations

$$(U_c)_n'' - Ha^2\, (U_c)_n = -\frac{1}{\epsilon} (U_n'' - Ha^2\, U_n - 1) - 6\, N_f\, (U_n')^2\, U_n'' \tag{19}$$

We used an initial guesse $U(\eta) = 0$ which satisfies the boundary conditions (14) to find iterative solutions for this equation at $\epsilon = 1$. The iterative solutions by PIA (1,1) are

$$U_1(\eta) = \frac{\lambda \cosh(Ha\, \eta)}{Ha^2\, (Ha \sinh(Ha) + \lambda \cosh(Ha))} - \frac{1}{Ha^2},$$

$$\begin{aligned} U_2(\eta) = {} & U_1(\eta) + \tfrac{3}{8}\, \lambda^3\, N_f \, [(-2\, (Ha\, sinh(Ha) + \lambda\, cosh(Ha))\, (-2\, cosh^4(Ha\, \eta) - \\ & 2\, sinh(Ha\, \eta)\, cosh^3(Ha\, \eta) + 4\, cosh^2(Ha\, \eta) + \tfrac{1}{2}\, sinh(2\, Ha\, \eta)) + (-2\, \lambda\, cosh^4(Ha) - \\ & 6\, sinh(Ha)\, (Ha + \tfrac{1}{3} \lambda)\, cosh^3(Ha) + (-6\, Ha\, sinh^2(Ha) - 3\, Ha + 4\, \lambda)\, cosh^2(Ha) + \tfrac{7}{2}\, (Ha + \\ & \tfrac{1}{7}\, \lambda)\, sinh(2\, Ha) + Ha\, (sinh^2(Ha) + (\eta - 1)\, (Ha - \lambda) + 1))\, e^{-Ha} - ((\tfrac{1}{4}\, (3\, Ha - \\ & \lambda))\, (sinh(4\, Ha) - cosh(4\, Ha)) - (Ha + \lambda)\, cosh(2\, Ha) - 2\, Ha\, sinh(2\, Ha) + (\eta + 1)\, (Ha + \\ & \lambda)\, Ha - \tfrac{1}{4}\, Ha - \tfrac{5}{4}\, \lambda)\, e^{Ha})\, e^{-Ha\, \eta} - (2\, (Ha\, sinh(Ha) + \lambda\, cosh(Ha)))\, (cosh(2\, Ha\, \eta) + \\ & \tfrac{1}{4}\, (sinh(4\, Ha\, \eta) - cosh(4\, Ha\, \eta))) + e^{-Ha}\, (-2\lambda\, cosh^4(Ha) - 6\, sinh(Ha)\, (Ha + \\ & \tfrac{1}{3}\, \lambda)\, cosh(Ha)^3 + (-6\, Ha\, sinh^2(Ha) - 3\, Ha + 4\, \lambda)\, cosh^2(Ha) + \tfrac{7}{2}\, (Ha + \tfrac{1}{7}\, \lambda)\, sinh(2\, Ha) + \\ & Ha\, sinh^2(Ha) + (\tfrac{9}{4} - (x + 1)\, (Ha - \lambda))\, Ha - \tfrac{5}{4}\, \lambda) + e^{Ha}\, (-\tfrac{1}{4}\, (3\, Ha - \lambda)\, (sinh(4\, Ha) - \\ & cosh(4\, Ha)) + (Ha + \lambda)\, cosh(2\, Ha) + (2\, sinh(2\, Ha) + (\eta - 1)\, (Ha + \lambda) - 1)\, Ha)\, e^{Ha\, \eta}] \div \\ & [((3\, Ha^2\, \lambda + \lambda^3)\, cosh^3(Ha) + Ha\, sinh(Ha)\, (Ha^2 + 3\, \lambda^2)\, cosh^2(Ha) - 3\, Ha^2\, \lambda\, cosh(Ha) - \\ & Ha^3\, sinh(Ha))\, (2\, (Ha\, sinh(Ha) + \lambda\, cosh(Ha)))\, Ha^4] \end{aligned} \tag{20}$$

To test the convergence of the PIA, we suppose that

$$\left.\begin{array}{l} U_0(\eta) = \zeta_0, \\ U_1(\eta) = U_0(\eta) + \epsilon\,(U_c)_0(\eta) = \zeta_0 + \zeta_1, \\ U_2(\eta) = U_1(\eta) + \epsilon\,(U_c)_1(\eta) = \zeta_0 + \zeta_1 + \zeta_2, \\ \vdots \\ U_n(\eta) = U_{n-1}(\eta) + \epsilon\,(U_c)_{n-1}(\eta) = \zeta_0 + \zeta_1 + \zeta_2 + \ldots + \zeta_n = \sum_{k=0}^{n} \zeta_k. \end{array}\right\} \tag{21}$$

Now, by following the same method in [37] which can be summarized by the following definition

Definition 3.1 *Let*

$$\delta_n = \begin{cases} \frac{\| \zeta_{n+1} \|}{\| \zeta_n \|}, & \| \zeta_n \| \neq 0, \\ 0, & \| \zeta_n \| = 0, \end{cases} \tag{22}$$

the series solutions $\sum_{k=0}^{\infty} \zeta_k$ by perturbation iteration method converges to the exact solution if there is δ_n such that $0 < \delta_n < 1$ for all $n \in \mathbb{N}$.

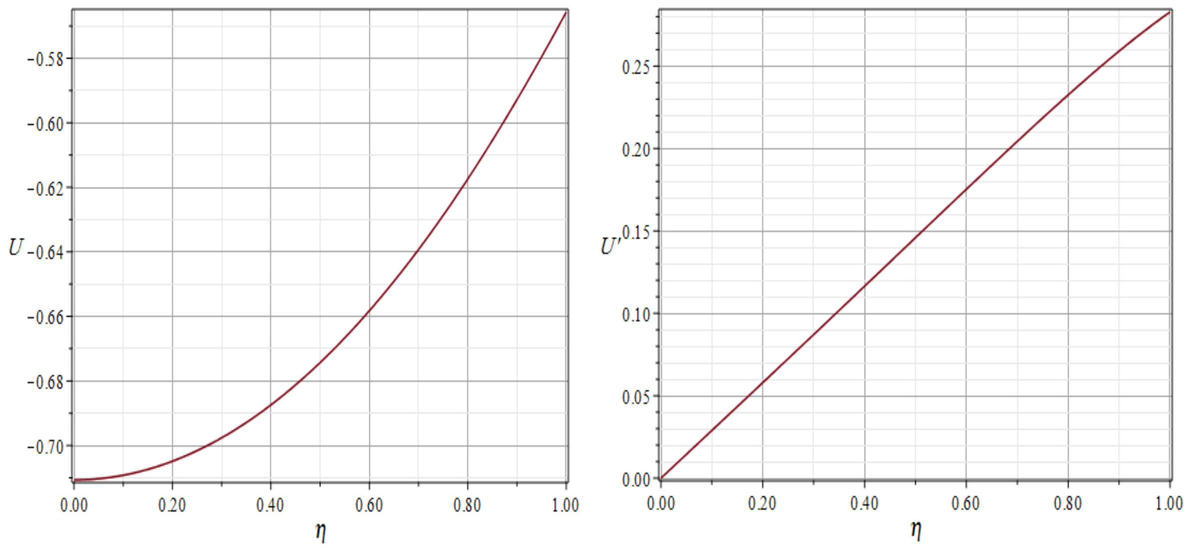
For example,
if $\lambda = 0.4$, $Ha = 1$ and $N_f = 1$ for all η in the domain $[0,1]$
then $\delta_0 = 0$, $\delta_1 = 0.03426861579$,
if $\lambda = 0.9$, $Ha = 1.5$ and $N_f = 0.3$
then $\delta_0 = 0$, $\delta_1 = 0.007529515923$,
if $\lambda = 0.5$, $Ha = 2$ and $N_f = 0.1$
then $\delta_0 = 0$, $\delta_1 = 0.000191508932$,
and if $\lambda = 0.5$, $Ha = 1$ and $N_f = 0.9$,
then $\delta_0 = 0$, $\delta_1 = 0.04642964414$.
We note that the convergence condition is valid for different values of physical parameters.

4. Results and Discussion

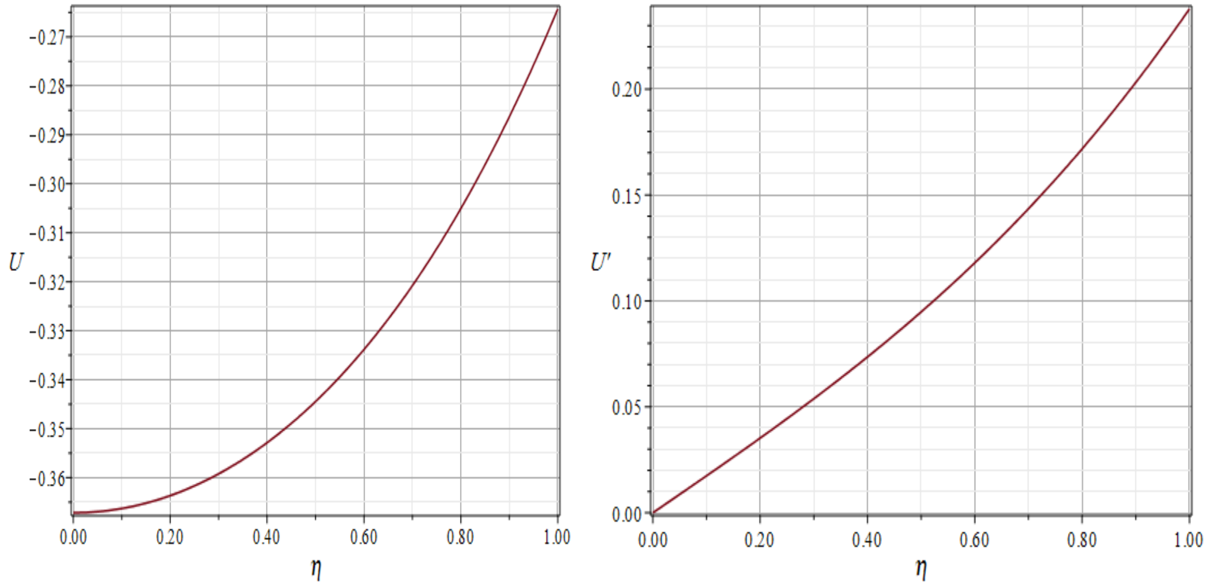
In this section, we focus on comparing the results of the analytical approximate solution obtained by a perturbation-iteration algorithm with the results of other studies for the problem of the steady incompressible flow of fourth grade fluid between two stationary parallel plates in the presence of magnetic field effect. We show the effect of some physical parameters on the computed results in the domain $[0,1]$. In Table 1, the values of computing velocity $U(\eta)$ by PIA are compared with the results that obtained by Moakher *et al.,* [19, 20] by applying the collocation method and least square method, respectively. The calculation time which is required to generate these solutions was 0.046s. We note that PIA gives results close to other results for the present problem. The results which are obtained from applying PIA are illustrated in Figure 2 for different values of active parameters. We note that, the values of $U(\eta)$ decrease and the values of $U'(\eta)$ increase from $\eta = 0$ to $\eta = 1$. Figure 3 shows the inverse relationship between the velocity and the magnetic parameter Ha, where the value of velocity decreases by increasing the value of the magnetic parameter, because the applied magnetic field affects in the form of Lorentz force thereby reduces the velocity value. In Figure 4, the variation of non-Newtonian parameter N_f on the velocity is described at $\lambda = 1$ and $Ha = 1$. The effect of the slip parameter λ on the velocity $U(\eta)$ is depicted in Figure 5 at $N_f = 0.5$ and $Ha = 1.5$. Also it is observed from this figure that the velocity decrease with increasing the slip parameter. The main reason for this is the increasing of slip parameter some part of fluid molecules strike solid surface and reflected diffusely increases then velocity decreases.

Table 1
Comparison of the present results of PIA with results of CM [19] and LSM [20] for $U(\eta)$ at $\lambda = 0.4$, $Ha = 1$ and $N_f = 1$

$\boldsymbol{\eta}$	PIA	CM [19]	LSM [20]
0	-0.7541296359	-0.755435424	-0.756148161
0.05	-0.7538222689	-0.755135498	-
0.10	-0.7528998171	-0.754233549	-0.754928597
0.15	-0.7513612403	-0.755135498	
0.20	-0.7492048374	-0.750610552	-0.751266500
0.25	-0.7464282942	-0.747882989	
0.30	-0.7430287564	-0.744540374	-0.745151678
0.35	-0.7390029224	-0.740579449	-0.741169625
0.40	-0.7343471653	-0.735996956	-0.736567863
0.45	-0.7290576822	-0.730789639	
0.50	-0.7231306774	-0.724954241	-0.725493618
0.55	-0.7165625833	-0.718487503	-0.719014744
0.60	-0.7093503270	-0.711386168	-0.711903466
0.65	-0.7014916454	-0.703646980	-0.704156173
0.70	-0.6929854610	-0.695266681	-0.704156173
0.75	-0.6838323255	-0.686242013	-0.686738689
0.80	-0.6740349439	-0.676569720	-0.677060964
0.85	-0.6635987899	-0.666246544	-0.666732216
0.90	-0.6525328333	-0.6552692277	-0.655748735
0.95	-0.6408503887	-0.6436345127	-0.644106858
1.00	-0.6285701164	-0.631339143	-0.631802927



(a) Ha= 1, N_f = 1, , λ= 0.5



(b) Ha= 1.5, N_f = 0.3, λ= 0.9

Fig. 2. The result obtained by PIA

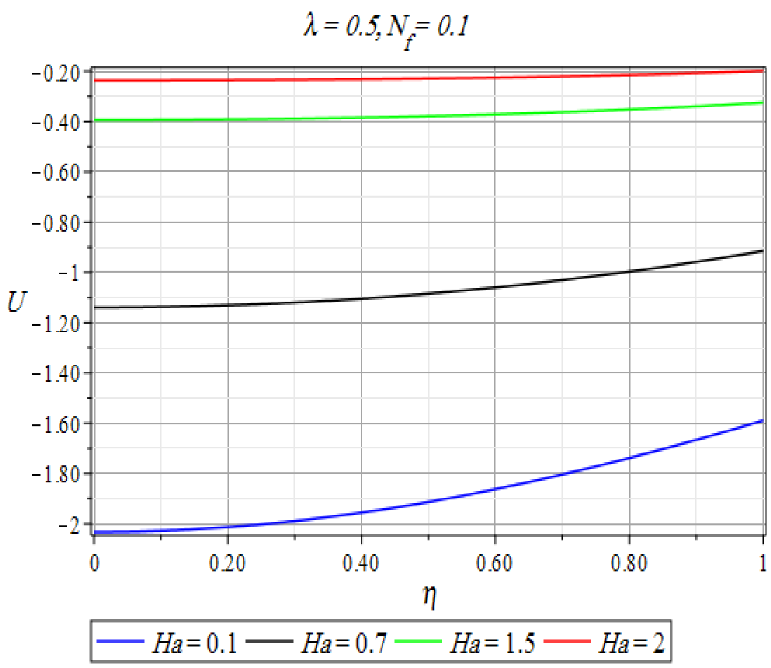


Fig. 3. The approximate analytical solution of velocities in different Ha number

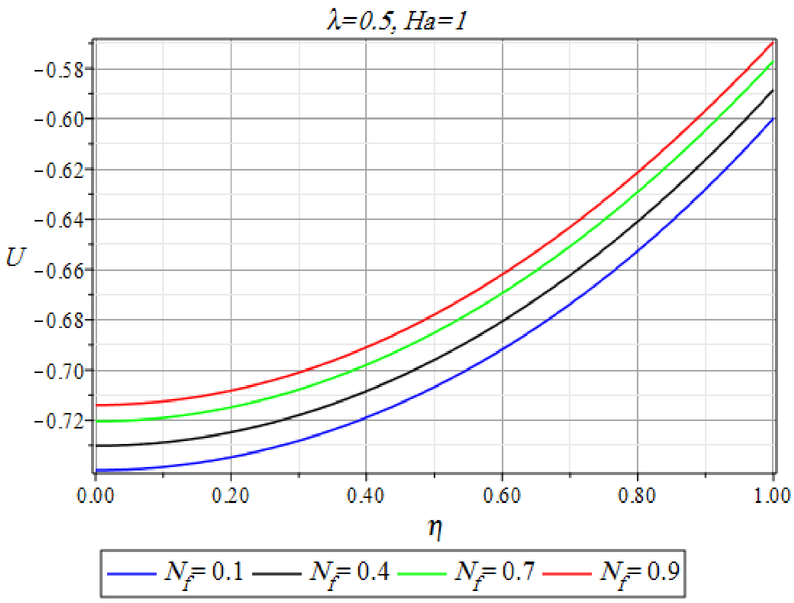


Fig. 4. The approximate analytical solution of velocities in different N_f number

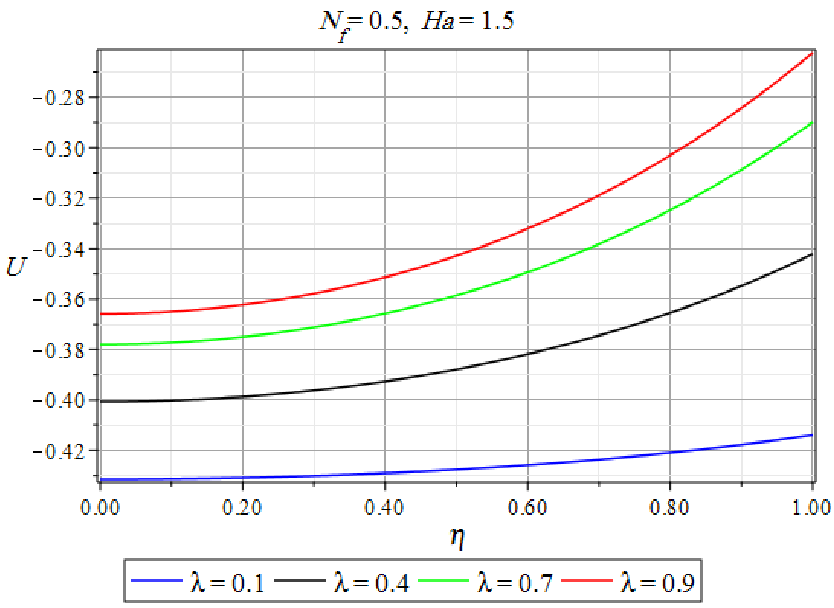


Fig. 5. The approximate analytical solution of velocities in different λ number

4. Conclusions

In this paper, perturbation-iteration algorithm has been used as semi analytical technique to solve the problem of steady incompressible flow of a fourth grade fluid between two stationary parallel plates in the presence of magnetic field. The numerical results of the proposed method are calculated and compared with the results of previous work. The effects of the slip parameter, the non-Newtonian parameter and the magnetic field parameter on the velocity have been investigated. The main conclusions are as follows.

i. The results of PIA are close to the results of the other methods.
ii. PIA gives a series of convergent solutions for all values of physical parameters which are used in this problem. Also, this method is fast in calculating results. Therefore, this method can be considered as good and effective technique for solving the problems of the flow of non-Newtonian fluid.
iii. The values of $U(\eta)$ are decreasing and the values of $U'(\eta)$ are increasing in the domain [0,1].
iv. The increase in the value of the slip parameter, the non-Newtonian parameter and the magnetic field parameter has an opposite effect on the velocity $U(\eta)$.

References

[1] Vajravelu, K., and David Rollins. "Hydromagnetic flow of a second grade fluid over a stretching sheet." *Applied Mathematics and Computation* 148, no. 3 (2004): 783-791.
[2] Hayat, Tasawar, Naveed Ahmed, M. Sajid, and Saleem Asghar. "On the MHD flow of a second grade fluid in a porous channel." *Computers and Mathematics with Applications* 54, no. 3 (2007): 407-414.
[3] Abbas, Zaheer, Tasawar Hayat, M. Sajid, and Saleem Asghar. "Unsteady flow of a second grade fluid film over an unsteady stretching sheet." *Mathematical and Computer Modelling* 48, no. 3-4 (2008): 518-526.
[4] Nadeem, S., Anwar Hussain, M. Y. Malik, and T. Hayat. "Series solutions for the stagnation flow of a second-grade fluid over a shrinking sheet." *Applied Mathematics and Mechanics* 30, no. 10 (2009): 1255-1262.
[5] Siddiqui, A. M., M. Hameed, B. M. Siddiqui, and Q. K. Ghori. "Use of Adomian decomposition method in the study of parallel plate flow of a third grade fluid." *Communications in Nonlinear Science and Numerical Simulation* 15, no. 9 (2010): 2388-2399.
[6] Keimanesh, M., Mohammad Mehdi Rashidi, Ali J. Chamkha, and R. Jafari. "Study of a third grade non-Newtonian fluid flow between two parallel plates using the multi-step differential transform method." *Computers and Mathematics with Applications* 62, no. 8 (2011): 2871-2891.
[7] Hatami, M., J. Hatami, and Davood Domiri Ganji. "Computer simulation of MHD blood conveying gold nanoparticles as a third grade non-Newtonian nanofluid in a hollow porous vessel." *Computer methods and programs in biomedicine* 113, no. 2 (2014): 632-641.
[8] Siddiqui, A. M., N. Z. Khan, M. A. Rana, and T. Haroon. "Flow of a second grade fluid through constricted tube using integral method." *Journal of Applied Fluid Mechanics* 9, no. 6 (2016): 2803-2812.
[9] Rashidi, M. M., S. Bagheri, E. Momoniat, and N. Freidoonimehr. "Entropy analysis of convective MHD flow of third grade non-Newtonian fluid over a stretching sheet." *Ain Shams Engineering Journal* 8, no. 1 (2017): 77-85.
[10] Hayat, T., Y. Wang, and K. Hutter. "Flow of a fourth grade fluid." *Mathematical Models and Methods in Applied Sciences* 12, no. 6 (2002): 797-811.
[11] Sajid, M., T. Hayat, and S. Asghar. "On the analytic solution of the steady flow of a fourth grade fluid." *Physics Letters A* 355, no. 1 (2006): 18-26.
[12] Siddiqui, A. M., R. Mahmood, and Q. K. Ghori. "Homotopy perturbation method for thin film flow of a fourth grade fluid down a vertical cylinder." *Physics Letters A* 352, no. 4-5 (2006): 404-410.
[13] Hayat, T., and M. Sajid. "On analytic solution for thin film flow of a fourth grade fluid down a vertical cylinder." *Physics Letters A* 361, no. 4-5 (2007): 316-322.
[14] Wang, Yongqi, and Wei Wu. "Unsteady flow of a fourth-grade fluid due to an oscillating plate." *International Journal of Non-Linear Mechanics* 42, no. 3 (2007): 432-441.
[15] Marinca, Vasile, Nicolae Herişanu, Constantin Bota, and Bogdan Marinca. "An optimal homotopy asymptotic method applied to the steady flow of a fourth-grade fluid past a porous plate." *Applied Mathematics Letters* 22, no. 2 (2009): 245-251.
[16] Islam, Sirajul, Z. Bano, I. Siddique, and Abdul Majeed Siddiqui. "The optimal solution for the flow of a fourth-grade fluid with partial slip." *Computers and Mathematics with Applications* 61, no. 6 (2011): 1507-1516.
[17] Aziz, Taha, and Fazal Mahmood Mahomed. "Closed-form solutions for a nonlinear partial differential equation arising in the study of a fourth grade fluid model." Journal of Applied Mathematics 2012 (2012): 1-16.
[18] Abbasi, M., A. Ahmadian CHashmi, I. Rahimipetroudi, and K. H. Hosseinzadeh. "Analysis of a fourth grade fluid flow in a channel by application of VIM and HAM." *Indian Journal of Scientific Research* 1, no. 2 (2014): 389-395.
[19] Moakher, P. Ghasemi, M. Abbasi, and M. Khaki. "New analytical solution of MHD fluid flow of fourth grade fluid through the channel with slip condition via collocation method." *International Journal of Advances in Applied Mathematics and Mechanics* 2, no. 3 (2015): 87-94.
[20] Moakher, P. Ghasemi, M. Abbasi, and M. Khaki. "Fully developed flow of fourth grade fluid through the channel with slip condition in the presence of a magnetic field." *Journal of Applied Fluid Mechanics* 9, no. 5 (2016): 2239-

2245.

[21] Marinca, Bogdan, and Vasile Marinca. "Approximate analytical solutions for thin film flow of a fourth grade fluid down a vertical cylinder." *Proceed Romanian Academy, Series A* 19, no. 1 (2018): 69-76.

[22] Kothandapani, M., V. Pushparaj, and J. Prakash. "Effect of magnetic field on peristaltic flow of a fourth grade fluid in a tapered asymmetric channel." *Journal of King Saud University-Engineering Sciences* 30, no. 1 (2018): 86-95.

[23] Khan, Najeeb Alam, Umair Bin Saeed, Faqiha Sultan, Saif Ullah, and Abdul Rehman. "Study of velocity and temperature distributions in boundary layer flow of fourth grade fluid over an exponential stretching sheet." *AIP Advances* 8, no. 2 (2018): 025011.

[24] Ali, Aamir, Surayya Saba, Saleem Asghar, and Salman Saleem. "Thermal and concentration effects of unsteady flow of non-Newtonian fluid over an oscillating plate." Comptes rendus de l'Académie bulgare des Sciences 71, no. 4 (2018): 467-475.

[25] Pakdemirli, Mehmet, and Hakan Boyacı. "Generation of root finding algorithms via perturbation theory and some formulas." *Applied Mathematics and Computation* 184, no. 2 (2007): 783-788.

[26] Aksoy, Yiğit, and Mehmet Pakdemirli. "New perturbation–iteration solutions for Bratu-type equations." *Computers and Mathematics with Applications* 59, no. 8 (2010): 2802-2808.

[27] Pakdemirli, Mehmet, Yiğit Aksoy, and Hakan Boyacı. "A new perturbation-iteration approach for first order differential equations." *Mathematical and Computational Applications* 16, no. 4 (2011): 890-899.

[28] Aksoy, Yiğit, Mehmet Pakdemirli, Saeid Abbasbandy, and Hakan Boyacı. "New perturbation-iteration solutions for nonlinear heat transfer equations." *International journal of Numerical methods for Heat and fluid flow* 22, no. 7 (2012): 814-828.

[29] Şenol, Mehmet, İhsan Timuçin Dolapçı, Yiğit Aksoy, and Mehmet Pakdemirli. "Perturbation-iteration method for first-order differential equations and systems." In *Abstract and Applied Analysis* 2013, (2013): 1-6.

[30] Timuçin Dolapçı, İhsan, Mehmet Şenol, and Mehmet Pakdemirli. "New perturbation iteration solutions for Fredholm and Volterra integral equations." *Journal of Applied Mathematics* 2013, (2013): 1-5.

[31] Khalid, M., Mariam Sultana, Faheem Zaidi, and Uroosa Arshad. "An effective perturbation iteration algorithm for solving Riccati differential equations." *International Journal of Computer Applications* 111, no. 10 (2015): 1-5.

[32] Aksoy, Yiğit, Ünal Göktaş, Mehmet Pakdemirli, and İhsan Timuçin Dolapçı. "Application of perturbation–iteration method to Lotka–Volterra equations." *Alexandria Engineering Journal* 55, no. 2 (2016): 1661-1666.

[33] Khalid, M., Fareeha Sami Khan, and Azhar Iqbal. "Perturbation-iteration algorithm to solve fractional giving up smoking mathematical model." *International Journal of Computer Applications* 142, no. 9 (2016): 1-6.

[34] Pakdemirli, Mehmet. "Perturbation–iteration method for strongly nonlinear vibrations." *Journal of Vibration and Control* 23, no. 6 (2017): 959-969.

[35] Şenol, M., and Hamed Daei Kasmaei. "Perturbation-Iteration Algorithm for Systems of Fractional Differential Equations and Convergence Analysis." *Progress in Fractional Differentiation and Applications* 3, no. 4 (2017): 271-279.

[36] Akinlabi, Grace O., and Sunday O. Edeki. "Perturbation iteration transform method for the solution of Newell-Whitehead-Segel model equations." *Journal of Mathematics and Statistics* 13, no. 1 (2017): 24-29.

[37] Bildik, Necdet. "General convergence analysis for the perturbation iteration technique." *Turkish Journal of Mathematics and Computer Science* 6 (2017): 1-9.

[38] Şenol, Mehmet, Marwan Alquran, and Hamed Daei Kasmaei. "On the comparison of perturbation-iteration algorithm and residual power series method to solve fractional Zakharov-Kuznetsov equation." *Results in Physics* 9 (2018): 321-327.

[39] Tasbozan, Orkun, Mehmet Şenol, Ali Kurt, and Ozan Özkan. "New solutions of fractional Drinfeld-Sokolov-Wilson system in shallow water waves." *Ocean Engineering* 161 (2018): 62-68.