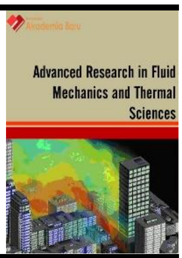
Journal of Advanced Research in Fluid Mechanics and Thermal Sciences 59, Issue 2 (2019) 232-242

Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879



Mixed Convection Flow and Heat Transfer of Carbon Nanotubes Over an Exponentially Stretching/Shrinking Sheet with Suction and Slip Effect

Open Access

Nur Syazana Anuar[1,*], Norfifah Bachok[1,2], Norihan Md Arifin[1,2], Haliza Rosali[1]

[1] Department of Mathematics, Faculty Science, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia
[2] Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM Serdang, Selangor, Malaysia

ARTICLE INFO

Article history:
Received 9 March 2019
Received in revised form 19 April 2019
Accepted 22 June 2019
Available online 19 July 2019

ABSTRACT

The steady two-dimensional mixed convection boundary layer flow and heat transfer of carbon nanotubes (CNTs) over an exponentially stretching/shrinking sheet with suction and slip is studied numerically. Water (base fluid) along with two types of CNTs, namely single and multi-walled CNTs are taken into consideration. Stretching/shrinking velocity and wall temperature are assumed to vary as prescribed exponential functions. The governing boundary layer equations of the problem are transformed into an ordinary differential equation via exponential similarity transformation. The resulting ordinary differential equations are solved numerically using the bvp4c package in Matlab software. The effects of governing parameters, namely, mixed convection parameter, exponentially stretching/shrinking parameter, nanoparticle volume fraction parameter, slip parameter and suction parameter on dimensionless velocity, temperature, skin friction coefficients and Nusselt numbers are discussed and presented graphically in detail. It has been found that dual solutions exist for an exponentially stretching and shrinking sheets.

Keywords:
Carbon nanotubes; dual solutions; exponential stretching/shrinking; heat transfer; mixed convection

1. Introduction

The study of mixed convection flow has a great important role in engineering, especially in atmospheric boundary-layer flows, heat exchangers, nuclear reactors, solar collectors and in electronic equipment. It is worth mentioning that area of boundary layer mixed convection flow over a stretching/shrinking surface has attracted the interest of many researchers because of its many applications. Therefore, the investigations of mixed convection near a stagnation point flow towards a vertical wall with suction or injection and prescribe surface heat flux was considered by Ishak *et al.,* [1,2]. A similar problem was also investigated by Ali *et al.,* [3] where they considered the problem in magnetohydrodynamic with induced magnetic field. Yasin *et al.,* [4] examined the mixed convection

* *Corresponding author.*
E-mail address: nursyazana931@gmail.com (Nur Syazana Anuar)

of vertical surface by considering in a porous medium saturated by nanofluid with suction or injection. Afterwards, the study of boundary layer flow against a vertical surface problem was considered by Rahman *et al.,* [5], Mustafa *et al.,* [6], Abdullah *et al.,* [7], Sandeep *et al.,* [8], Sivasankaran *et al.,* [9], Jamaludin *et al.,* [10] and Turkyilmazoglu [11] in various ways.

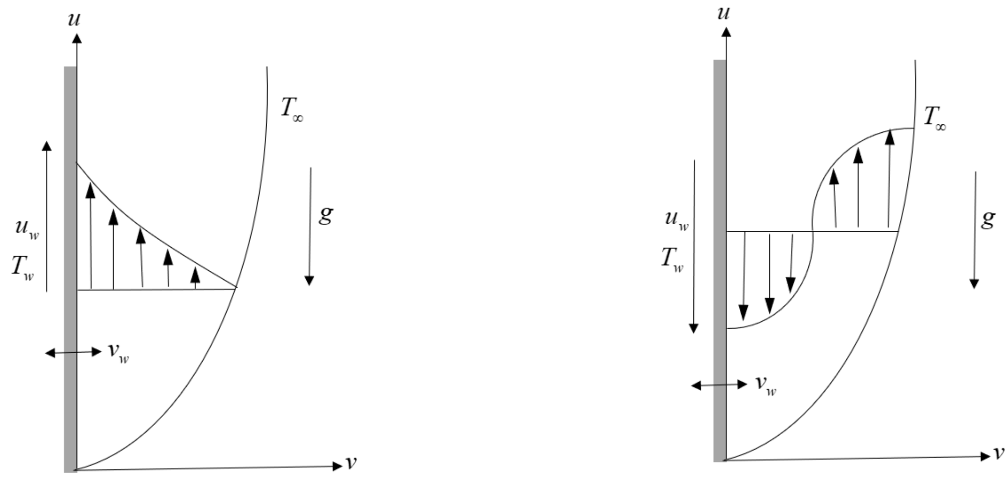
Currently, many authors have shown attention to the investigation of boundary layer flow and heat transfer over an exponentially stretching/shrinking sheet because of its practical significance in manufacturing and engineering processes such as extraction of polymer, glass–fiber production, wire drawing, paper production, crystal growing, hot rolling and many more. Bhattacharyya [12] examined the steady boundary layer flow and heat transfer over an exponentially shrinking sheet and reported dual solutions exist for a shrinking case. Later, Bhattacharyya and Vajravelu [13] investigated the heat transfer over an exponentially shrinking sheet in the stagnation region, while Bachok *et al.,* [14] considered the same problem in a nanofluid with stretching/shrinking sheet. Moreover, study of boundary layer flow and heat transfer past a permeable exponentially stretching/shrinking sheet with generalized slip velocity has been investigated by Hafidzuddin *et al.,* [15]. Therefore, a number of studies on exponentially stretching/shrinking surface with various effects [16-19] have been reviewed since then.

However, only a few researches have extensively studied the effect of buoyancy forces on the boundary layer exponentially stretching sheet. Partha *et al.,* [20] investigated the boundary layer flow of mixed convection and heat transfer from an exponentially stretching surface by considering viscous dissipation effect. Afterwards, Pal [21] studied the mixed convection heat transfer over an exponentially stretching surface with magnetic field. In addition, the problem of steady laminar two-dimensional flow and heat transfer over an exponentially shrinking vertical sheet with suction has been studied by Rohni *et al.,* [22]. Further, Patil *et al.,* [23] have considered Soret and Dufour effects on the steady double diffusive mixed convection boundary layer flows over an impermeable exponentially stretching sheet. Later, the double diffusive over an exponentially vertical stretching surface with viscous dissipation was discussed by Patil *et al.,* [24]. Very recently Patil and Kumbarwadi [25] discussed the effects of magnetohydrodynamic, heat source/sink and cross diffusion of mixed convection flow over exponentially stretching sheet. A comprehensive collection of physical features on induced motion and heat transfer can also be found in literatures [26-29].

All of the above-mentioned studies do not deal with carbon nanotube. It seems that Iigima [30], is the first who discovered carbon nanotubes (CNTs) in the beginning of the nineties of the last century. In recent developments, the interest in the applications of single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs) has significant increase in industry and medical fields due to its direct impacts on increasing the thermal conductivity of the base fluids. Since then, many authors [31-37] have investigated the flow and heat transfer in carbon nanotubes. To the best of author's knowledge, no one has ever sought to study the mixed convection flow and heat transfer over an exponentially stretching/shrinking sheet with suction and slip effect. The prime objective of the present attempt is to extend the study of Rohni *et al.,* [22] to carbon nanotubes with slip effect and to work out the range of the parameters for which the solution of the present problem is unique or multiple. Using model presented by Xue [38], the partial differential equations are transformed to a set of nonlinear ordinary differential equations using a similarity transformation and solved it numerically using the bvp4c function in Matlab software. The influence of the involved parameters is illustrated through graphs and table.

2. Methodology

A steady, incompressible and two-dimensional flow past a vertical stretching/shrinking plate in a water base fluid containing two types (SWCNTs and MWCNTs) of carbon nanotubes have been considered for investigation. The $x-$ axis is taken along the sheet and the $y-$ axis is perpendicular to the sheet. Further, we have assumed that the temperature of the sheet is $T_w(x) = T_\infty + T_o e^{x/2L}$, where T_∞ is the ambient temperature and T_o is a constant that measures the rate of temperature increase along the surface. It is also assumed that the surface is stretched/shrinked with the velocity $u_w(x) = ae^{x/L}$, where a is the positive constant and L is a characteristic length of a sheet. The physical model is depicted in Figure 1.



(a) Stretching sheet (b) Shrinking sheet

Fig. 1. Schematic diagram of the problem

The boundary layer equations may be written as [22]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} + \frac{g(\rho\beta)_{nf}}{\rho_{nf}}(T - T_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

The corresponding boundary conditions are

$$\begin{gathered} u = \varepsilon u_w(x) + L_1\left(\frac{\partial u}{\partial y}\right),\quad v = v_w(x),\quad T = T_w(x) \text{ at } y = 0 \\ u \to 0,\quad T \to T_\infty \text{ as } y \to \infty \end{gathered} \tag{4}$$

where u and v are corresponding velocity components in x and y directions, T is the temperature, g is the acceleration due to gravity, L_1 is the slip factor, $\mu_{nf}, \alpha_{nf}, \beta_{nf}$ and ρ_{nf} are the dynamic viscosity, thermal diffusivity, thermal expansion and density of the nanofluid, respectively. The slip factor is defined as $L_1 = L_s e^{-x/2L}$ where L_s is the initial length of slip factor. The mass flux velocity is $v_w(x) = -(av_f/2L)^{1/2} e^{x/2L} s$ where s is the constant mass flux velocity, respectively, with $s > 0$ for the suction, $s < 0$ for injection of the fluid and $s = 0$ for an impermeable surface. ε is the constant stretching/shrinking parameter with $\varepsilon > 0$ corresponding to the stretching sheet and $\varepsilon < 0$ corresponding to the shrinking sheet, respectively. The effective properties of nanofluids are given by

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \quad \mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}}, \quad \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_{CNT},$$
$$(\rho\beta)_{nf} = (1-\varphi)(\rho\beta)_f + \varphi(\rho\beta)_{CNT}, \quad (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_{CNT}, \tag{5}$$
$$\frac{k_{nf}}{k_f} = \frac{1-\varphi+2\varphi\dfrac{k_{CNT}}{k_{CNT}-k_f}\ln\dfrac{k_{CNT}+k_f}{2k_f}}{1-\varphi+2\varphi\dfrac{k_f}{k_{CNT}-k_f}\ln\dfrac{k_{CNT}+k_f}{2k_f}}.$$

Here, φ is the nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid, k_{nf}, k_f and k_{CNT} is the thermal conductivity of the nanofluid, fluid and carbon nanotubes, respectively, β_f and β_{CNT} are the thermal expansion coefficients of the fluid and carbon nanotubes, ρ_f and β_{CNT} are the densities of the fluid and carbon nanotubes, respectively. On the other hand, the use of the above expression for $k_{nf}/\, k_f$ are taken from Xue [38] where the model compensating the effects of the space distribution on CNTs and considering rotational elliptical nanotubes with very large axial ratio.

In order to solve Eq. (1)-(3) along with the boundary conditions (4), the following similarities variables are introduced

$$\psi = (2av_f L)^{1/2} e^{x/2L} f(\eta), \quad \eta = y\left(\frac{a}{2v_f L}\right)^{1/2} e^{x/2L}, \quad \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}, \tag{6}$$

where v_f is the kinematic viscosity of the fluid and ψ represents the stream function which identically satisfy Eq. (1) and defined as $u = \partial\psi/\,\partial y$ and $v = -\partial\psi/\,\partial x$. Using (6), Eq. (2) and (3) take the following form

$$\frac{1}{(1-\varphi)^{2.5}\left(1-\varphi+\varphi\rho_{CNT}/\rho_f\right)} f''' + ff'' - 2f'^2 + 2\frac{\left(1-\varphi+\varphi(\rho\beta)_{CNT}/(\rho\beta)_f\right)}{\left(1-\varphi+\varphi\rho_{CNT}/\rho_f\right)}\lambda\theta = 0, \tag{7}$$
$$\frac{1}{\Pr}\frac{k_{nf}/k_f}{\left[1-\varphi+\varphi(\rho C_p)_{CNT}/(\rho C_p)_f\right]}\theta'' + f\theta' - f'\theta = 0. \tag{8}$$

The transformed boundary conditions are

$$\begin{aligned} &f(0) = s, \quad f'(0) = \varepsilon + \sigma f''(0), \quad \theta(0) = 1, \\ &\quad f'(\eta) \to 0, \quad \theta(\eta) \to 0 \text{ as } \eta \to \infty \end{aligned} \tag{9}$$

where primes denote the differentiation with respect to η. λ is the mixed convection parameter, $\Pr$ is the Prandtl number and σ is the slip parameter defined respectively as

$$\lambda = \frac{Gr}{\mathrm{Re}_x^2}, \quad \Pr = \frac{\nu}{\alpha}, \quad \sigma = L_s\left(\frac{a}{2\nu_f L}\right)^{1/2}, \tag{10}$$

with $Gr = g\beta_f(T_w - T_\infty)L^3/\nu_f^2$ being Grashof number and $Re_x = u_w L/\nu_f$ is the Reynolds number. Further, $\lambda > 0$ corresponds to assisting flow, $\lambda < 0$ corresponds to opposing flow and $\lambda = 0$ corresponds to force convection flow (non-buoyant case).

3. Results and Discussion

The system of coupled nonlinear differential Eq. (7) and (8) under the boundary condition (9) have been solved numerically using the function bvp4c in Matlab software. The thermophysical properties of the base fluid (water) and carbon nanotubes are illustrated in Table 1. Following Bachok *et al.,* [14], the range of nanoparticle volume fraction φ considered in this study are in between 0 and 0.2 $(0 \le \varphi \le 0.2)$. In order to discuss the behavior of fluid flow and its heat characteristics, the effect of mixed convection parameter λ, slip parameter σ, nanoparticle volume fraction parameter φ as well as exponentially stretching/shrinking sheet ε parameter on skin friction, heat transfer, velocity and temperature profiles are illustrated graphically.

Table 1
Thermophysical properties of water and carbon nanotube (Khan *et al.,* [27])

Physical Properties	Base fluid	Nanoparticle	
	Water	SWCNT	MWCNT
$\rho(kg/m^3)$	997	2 600	1 600
$c_p\,(J/kgK)$	4 179	425	796
$k\,(W/mK)$	0.613	6 600	3 000

To verify the accuracy of the results obtained in this study, the numerical values of the reduced skin friction coefficient $f''(0)$ and the reduced heat transfer $-\theta'(0)$ are compared with those reported by Elbashbeshy [39] and Hafidzuddin *et al.,* [15]. The comparisons, which are shown in Table 2, are found to be in excellent agreement, and thus we are confident that the present method is accurate. The numerical values for skin friction $f''(0)$ in the presence of both carbon nanotubes for various emerging parameters are presented in Table 3 for a future reference.

Table 2
The comparison values of $-f''(0)$ and $-\theta'(0)$ for different s when $\varphi = \lambda = 0, \varepsilon = 1$ and $\Pr = 0.72$

s	Elbashbeshy [39]		Hafidzuddin *et al.,* [15]		Present study	
	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$	$-f''(0)$	$-\theta'(0)$
0	1.28181	0.767778	1.28182	0.767669	1.28181	0.767338
0.6	1.59824	1.014517	1.59824	1.014570	1.59824	1.014521

Table 3
Values of $f''(0)$ with λ and σ when $\varphi = 0.1, \varepsilon = -0.4, s = 3.2$ and $Pr = 6.2$

λ	σ	water-SWCNT		water-MWCNT	
		First solution	Second solution	First solution	Second solution
0	0	1.061771	-2.370614	0.962573	-1.900389
	0.2	0.706017	-1.605684	0.662467	-1.326615
	0.4	0.524955	-1.223574	0.501096	-1.025874
-0.4	0	0.941291	-2.508976	0.850443	-2.024689
	0.2	0.628723	-1.703452	0.587874	-1.417003
	0.4	0.468785	-1.297616	0.445966	-1.095985
-0.6	0	0.880839	-2.578298	0.794206	-2.086976
	0.2	0.589703	-1.752252	0.550239	-1.462265
	0.4	0.440327	-1.334134	0.418047	-1.130819

The illustration of skin friction and heat transfer are shown in Figures 2 and 3 for different values of slip parameter in water-SWCNT. It can be seen that the slip parameter decreases the skin friction but increases the heat transfer rate for the case of shrinking sheet. In the presence of the slip parameter, flow velocity near the shrinking wall is no longer equal to the shrinking velocity of the wall. With the increasing values in slip, such slip velocity increases and consequently the fluid velocity will decrease. This is due to the fact that the generation of vorticity for shrinking surface can be partially transmitted to the fluid under the slip condition. However, reverse trend is observed for the case of stretching sheet. Hence, it is observed that slip has a substantial effect on the solutions. For exponentially stretching sheet, dual solutions exist for all value of stretching and shrinking considered in this study, whereas for exponentially shrinking sheet, the solution only occurs up to its critical value, ε_c. Over these critical values, the flow will be isolated from the surface and thus the solutions based upon the boundary layer approximation are not valid. Similar with other studies (Merkin [40], Weidman *et al.,* [41], Harris *et al.,* [42], Ishak [43], Nazar *et al.,* [44], etc) where dual solutions exist, we conclude that the first solutions are stable and physically realizable, while the second solutions are not stable and therefore, not realizable in practice.

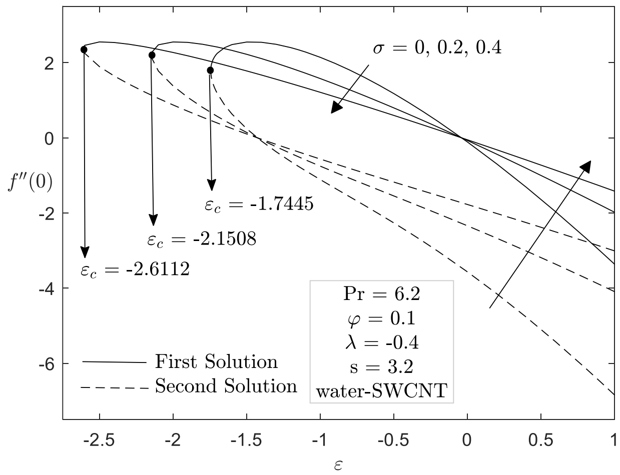


Fig. 2. Variation of skin friction with for different slip parameter

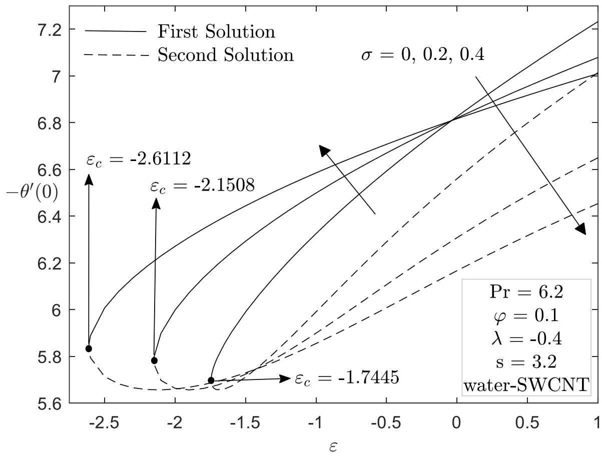


Fig. 3. Variation of heat transfer with for different slip parameter

Figure 4 and 5 show the effects of nanoparticle volume fraction φ on variation of skin friction $f''(0)$ and heat transfer $-\theta'(0)$ in the presence of slip. It is observed that skin friction and heat transfer decrease significantly with increasing values of nanoparticle volume fraction for the case of exponentially shrinking sheet. The reason behind this is that an increase of nanoparticle volume fraction in the base fluid cause the increases of viscosity in the mixture and hence produce a resistance through flow. The study also reveals that for a viscous flow, the steady flow due to an exponentially shrinking sheet is possible when the mass suction parameter $s \geq 1.5564$. However, when the value of nanoparticle increase, the critical value for mass suction also increases for the solution to exist. This is consistent with the physics of the flow where the mass suction is needed in a way to retain the higher amount of vorticity created due to an exponential stretching/shrinking inside the boundary layer flow. Therefore, when the mass suction parameter s satisfies the condition $s \geq 1.5564$, the similarity solutions are happened to exist. However, for $s < 1.5564$ the flow has no similarity solution and dual similarity solutions can be obtained for $s \geq 1.5564$. We can conclude here that nanoparticle volume fraction shortens the range of s for which the solution exists. Figure 4 and 5 also shows that skin friction increase for first solution but decreases for second solution with an increasing value of suction. Heat transfer rate also increase with suction for both solutions.

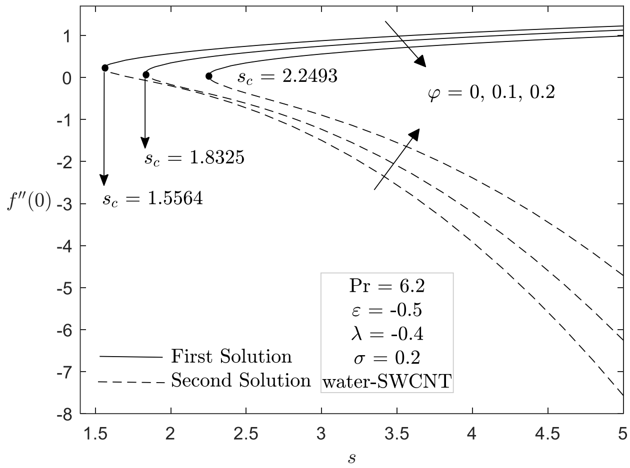


Fig. 4. Variation of skin friction with s for different nanoparticle volume fraction parameter

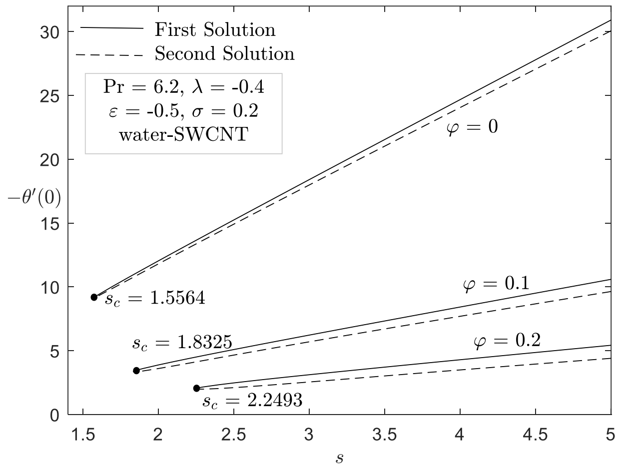


Fig. 5. Variation of heat transfer with s for different nanoparticle volume fraction parameter

Figure 6-11 display the dual solutions obtained on the dimensionless velocity and temperature profiles. From Figure 6 and 7, it can be seen that SWCNT exhibit higher velocity rate and temperature compared to MWCNT because of it has higher density and thermal conductivity (Table 1). Figure 8 and 9, respectively, present the velocity and temperature distributions for some values of mixed convection parameter, i.e. $\lambda < 0$ (opposing flow). The velocity profile reduces with the increasing magnitude of mixed parameter $|\lambda|$ and the opposite trend are observed for temperature profile. Figure 10 and 11 display the effect of suction for velocity and temperature profiles. Momentum boundary layer thickness decreases with increasing value of suction for the first solution, while the opposite behavior is observed in the second solution. The temperature profile also decreases with increasing values of suction for both solutions and consequently the thermal boundary layer thickness will decrease. This is due to the fact that the flow in the first solution of boundary layer is taken up by suction, and thus leads to an acceleration of transverse fluid motion and decrease the frictional force as well as a higher heat transfer rate. Furthermore, the first solutions display a thinner boundary layer thickness compared to the second solutions. Finally, it is worth mentioning that these figures satisfy the boundary conditions (9), therefore, it confirms the validity of numerical results and supports the dual nature of solutions as illustrated in Figure 2-5.

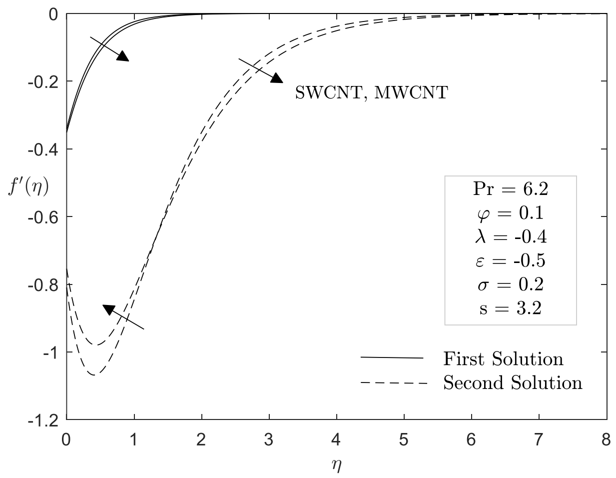


Fig. 6. Velocity profile for different carbon nanotube

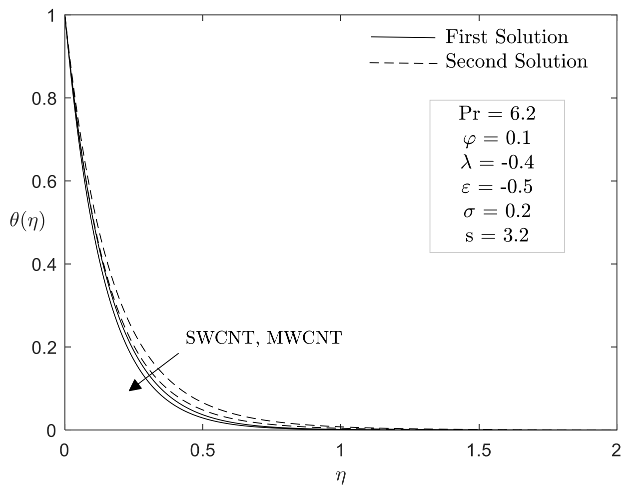


Fig. 7. Temperature profile for different carbon nanotube

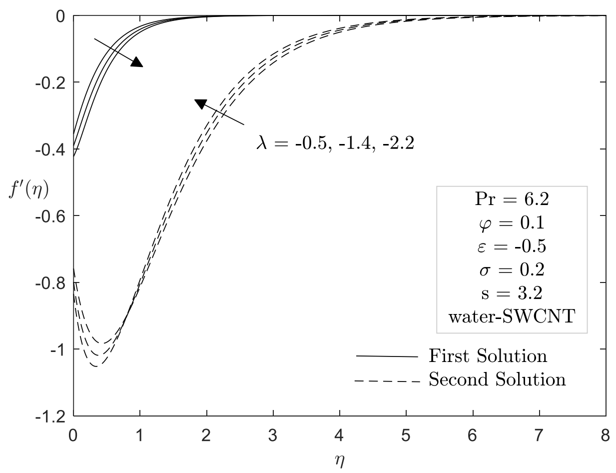


Fig. 8. Velocity profile for different mixed parameter

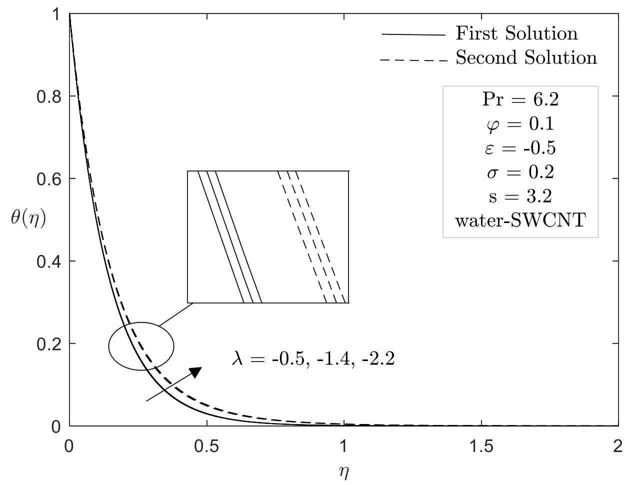


Fig. 9. Temperature profile for different mixed parameter

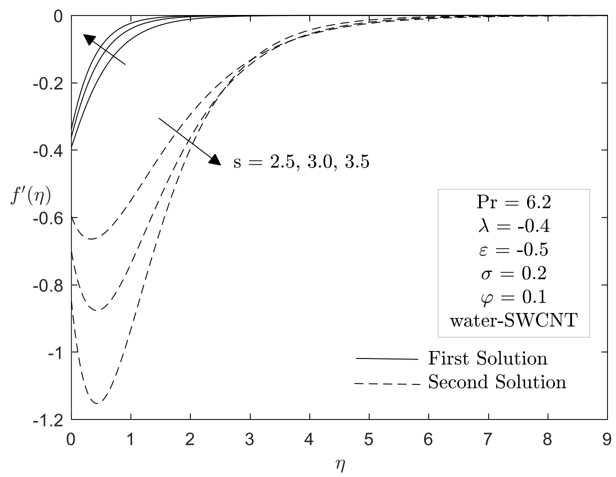


Fig. 10. Velocity profile for different suction parameter

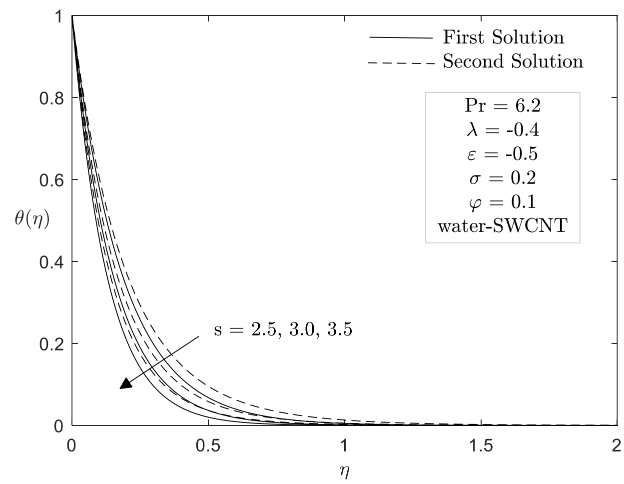


Fig. 11. Temperature profile for different suction parameter

4. Conclusions

A numerical study on mixed convection flow of a carbon nanotube towards an exponentially stretching/shrinking sheet is investigated. Analysis is carried out to investigate these effects on two types of carbon nanotube, namely, single-wall and multi-wall with water as a base fluid. The following are brief summary conclusions drawn from the analysis

i. Dual solutions exist for both stretching and shrinking sheet up to a critical value, ε_c where the existence of solutions depending on the parameter considered, namely, suction parameter $(s>1)$ and mixed parameter $(\lambda<0)$.
ii. Suction increase the skin friction and heat transfer rate at the surface.
iii. The velocity slip parameter increases the heat transfer rate at the surface and decrease the skin friction for the case of shrinking sheet.
iv. Slip parameter acts in widening the range of solution.
v. Single-wall carbon nanotube offer a higher skin friction and heat transfer rate compared to multi-wall carbon nanotube.

Acknowledgement
The authors acknowledged the financial support received in the form of a fundamental research grant scheme (FRGS/1/2018/STG06/UPM/02/4/5540155), (FRGS/01-01-16-1841FR5524948) and MyBrainSc from the Ministry of Higher Education, Malaysia. The authors wish to express their very sincere thanks to the reviewer for the valuable comments and suggestions.

References

[1] Ishak, Anuar, Roslinda Nazar, and Ioan Pop. "Dual solutions in mixed convection boundary-layer flow with suction or injection." *IMA journal of applied mathematics* 72, no. 4 (2007): 451-463.
[2] Ishak, Anuar, J. H. Merkin, Roslinda Nazar, and I. Pop. "Mixed convection boundary layer flow over a permeable vertical surface with prescribed wall heat flux." *Zeitschrift für angewandte Mathematik und Physik* 59, no. 1 (2008): 100-123.
[3] Ali, F. M., R. Nazar, N. M. Arifin, and I. Pop. "MHD mixed convection boundary layer flow toward a stagnation point on a vertical surface with induced magnetic field." *Journal of Heat Transfer* 133, no. 2 (2011): 022502.
[4] Yasin, M. HM, N. M. Arifin, Roslinda Mohd Nazar, F. Ismail, and I. Pop. "Mixed convection boundary layer flow on a vertical surface in a porous medium saturated by a nanofluid with suction or injection." *Journal of Mathematics and Statistics* 9, no. 2 (2013): 119-128.
[5] Rahman, M. M., J. H. Merkin, and I. Pop. "Mixed convection boundary-layer flow past a vertical flat plate with a convective boundary condition." *Acta Mechanica* 226, no. 8 (2015): 2441-2460.
[6] Mustafa, Irfan, Tariq Javed, and Abid Majeed. "Magnetohydrodynamic (MHD) mixed convection stagnation point flow of a nanofluid over a vertical plate with viscous dissipation." *Canadian Journal of Physics* 93, no. 11 (2015): 1365-1374.
[7] Abdullah, A. A., F. S. Ibrahim, A. A. Gawad, and A. Batyyb. "Investigation of unsteady mixed convection flow near the stagnation point of a heated vertical plate embedded in a nanofluid-saturated porous medium by self-similar." *Am. J. Energy Eng* 3 (2015): 42-51.
[8] Sandeep, N., C. Sulochana, and V. Sugunamma. "Radiation and magnetic field effects on unsteady mixed convection flow over a vertical stretching/shrinking surface with suction/injection." *Industrial Eng. Letters* 5, no. 1 (2015): 127-136.
[9] Sivasankaran, S., Niranjan, H. and Bhuvaneswari, M. "Chemical reaction, radiation and slip effects on MHD mixed convection stagnation-point flow in a porous medium with convective boundary condition." *International Journal of Numerical Methods for Heat & Fluid Flow* 27, no. 2 (2017): 454-470.
[10] Jamaludin, A., Nazar, R. and Pop, I. "Three-Dimensional Magnetohydrodynamic Mixed Convection Flow of Nanofluids over a Nonlinearly Permeable Stretching/Shrinking Sheet with Velocity and Thermal Slip." *Applied Sciences* 8, no. 7 (2018): 1128.

[11] Turkyilmazoglu, M. "Analytical solutions to mixed convection MHD fluid flow induced by a nonlinearly deforming permeable surface" *Communications in Nonlinear Science and Numerical Simulation* 63, (2018): 373-379.
[12] Bhattacharyya, K. "Boundary layer flow and heat transfer over an exponentially shrinking sheet." *Chinese Physics Letters* 28, no. 7 (2011): 074701.
[13] Bhattacharyya, K. and Vajravelu, K. "Stagnation-point flow and heat transfer over an exponentially shrinking sheet." *Communications in Nonlinear Science and Numerical Simulation* 17, no. 7 (2012): 2728-2734.
[14] Bachok, N., Ishak, A. and Pop, I. "Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid." *International Journal of Heat and Mass Transfer* 55, no. 25-26 (2012): 8122-8128.
[15] Hafidzuddin, E.H., Nazar, R., Arifin, N.M. and Pop, I. "Boundary layer flow and heat transfer over a permeable exponentially stretching/shrinking sheet with generalized slip velocity." *Journal of Applied Fluid Mechanics* 9, no. 4 (2016): 2025-2036.
[16] Naramgari, S. and Sulochana, C. "Dual solutions of radiative MHD nanofluid flow over an exponentially stretching sheet with heat generation/absorption." *Applied Nanoscience* 6, no. 1 (2016): 131-139.
[17] Alavi, S.Q., Hussanan, A., Kasim, A.R.M., Rosli, N. and Salleh, M.Z. "MHD Stagnation Point flow Towards an Exponentially Stretching Sheet with Prescribed wall Temperature and Heat Flux." *International Journal of Applied and Computational Mathematics* 3, no. 4 (2017): 3511-3523.
[18] Rehman, A. and Sheikh, N. "Boundary Layer Stagnation-Point Flow of Micropolar Fluid over an Exponentially Stretching Sheet." *International Journal of Fluid Mechanics & Thermal Sciences* 3, no. 3 (2017): 25-31.
[19] Jusoh, R., Nazar, R. and Pop, I. "Magnetohydrodynamic rotating flow and heat transfer of ferrofluid due to an exponentially permeable stretching/shrinking sheet." *Journal of Magnetism and Magnetic Materials* 465 (2018): 365-374.
[20] Partha, M.K., Murthy, P.V.S.N. and Rajasekhar, G.P. "Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface." *Heat and Mass transfer* 41, no. 4 (2005): 360-366.
[21] Pal, D. "Mixed convection heat transfer in the boundary layers on an exponentially stretching surface with magnetic field." *Applied Mathematics and Computation* 217, no. 6 (2010): 2356-2369.
[22] Rohni, A.M., Ahmad, S., Ismail, A.I.M. and Pop, I. "Boundary layer flow and heat transfer over an exponentially shrinking vertical sheet with suction." *International Journal of Thermal Sciences* 64 (2013): 264-272.
[23] Patil, P.M., Roy, S. and Momoniat, E. "Thermal diffusion and diffusion-thermo effects on mixed convection from an exponentially impermeable stretching surface." *International Journal of Heat and Mass Transfer* 100 (2016): 482-489.
[24] Patil, P.M., Latha, D.N., Roy, S. and Momoniat, E. "Double diffusive mixed convection flow from a vertical exponentially stretching surface in presence of the viscous dissipation." *International Journal of Heat and Mass Transfer* 112 (2017): 758-766.
[25] Patil, P.M. and Kumbarwadi, N. "Effects of MHD mixed convection with non-uniform heat source/sink and cross-diffusion over exponentially stretching sheet." *International Journal of Numerical Methods for Heat & Fluid Flow* 28, no. 6 (2018): 1238-1255.
[26] Turkyilmazoglu, M. "Free and circular jets cooled by single phase nanofluids." *European Journal of Mechanics-B/Fluids* 76, (2019): 1-6.
[27] Turkyilmazoglu, M. "Latitudinally deforming rotating sphere." *Applied Mathematical Modelling* 71 (2019): 1-11.
[28] Turkyilmazoglu, M. "Dual and triple solutions for MHD slip flow of non-Newtonian fluid over a shrinking surface." *Computers & Fluids* 70 (2012): 53-58.
[29] Zulkifli, S.N., Sarif, N.M. and Salleh, M.Z. "Numerical Solution of Boundary Layer Flow over a Moving Plate in a Nanofluid with Viscous Dissipation: A Revised Model." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 56, no. 2 (2019): 287-295.
[30] Iijima, S. "Helical microtubules of graphitic carbon." *nature* 354, no. 6348 (1991): 56.
[31] Khan, W.A., Khan, Z.H. and Rahi, M. "Fluid flow and heat transfer of carbon nanotubes along a flat plate with Navier slip boundary." *Applied Nanoscience* 4, no. 5 (2014): 633-641.
[32] Haq, R.U., Nadeem, S., Khan, Z.H. and Noor, N.F.M. "Convective heat transfer in MHD slip flow over a stretching surface in the presence of carbon nanotubes." *Physica B: condensed matter* 457 (2015): 40-47.
[33] Hayat, T., Hussain, Z., Alsaedi, A. and Ahmad, B. "Heterogeneous-homogeneous reactions and melting heat transfer effects in flow with carbon nanotubes." *Journal of Molecular Liquids* 220 (2016): 200-207.
[34] Arabpour, A., Karimipour, A. and Toghraie, D. "The study of heat transfer and laminar flow of kerosene/multi-walled carbon nanotubes (MWCNTs) nanofluid in the microchannel heat sink with slip boundary condition." *Journal of Thermal Analysis and Calorimetry* 131, no. 2 (2018): 1553-1566.
[35] Anuar, N.S., Bachok, N. and Pop, I. "A Stability Analysis of Solutions in Boundary Layer Flow and Heat Transfer of Carbon Nanotubes over a Moving Plate with Slip Effect." *Energies* 11, no. 12 (2018): 3243.

[36] Anuar, N.S., Bachok, N., Arifin, N.M. and Rosali, H. "Stagnation Point Flow and Heat Transfer over an Exponentially Stretching/Shrinking Sheet in CNT with Homogeneous–Heterogeneous Reaction: Stability Analysis." *Symmetry* 11, (2019): 522.

[37] Jamil, M.M., Sidik, N.A.C., Umar, U.S., Hamisu, M.T. and Sa'ad, A. "Carbon Nanotube for Solar Energy Applications: A Review." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 56, no.2 (2019): 233-247.

[38] Xue, Q.Z. "Model for thermal conductivity of carbon nanotube-based composites." *Physica B: Condensed Matter* 368, no. 1-4 (2005): 302-307.

[39] Elbashbeshy, E.M.A. "Heat transfer over an exponentially stretching continuous surface with suction." *Archives of Mechanics* 53, no. 6 (2001): 643-651.

[40] Merkin, J.H. "On dual solutions occurring in mixed convection in a porous medium." *Journal of Engineering Mathematics* 20, (1985): 171–179.

[41] Weidman, P.D., Kubitschek, D.G. and Davis, A.M.J. "The effect of transpiration on self-similar boundary layer flow over moving surfaces." *International Journal of Engineering Science* 44, (2006): 730-737.

[42] Harris, S.D., Ingham, D.B. and Pop, I. "Mixed convection boundary-layer flow near the stagnation point on a vertical surface in a porous medium: Brinkman model with slip." *Transport in Porous Media* 77, no. 2 (2009): 267-285.

[43] Ishak, A. "Flow and heat transfer over a shrinking sheet: A stability analysis." *International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering* 8, no. 5 (2014): 905-909.

[44] Nazar, R., Noor, A., Jafar, K. and Pop, I. "Stability analysis of three-dimensional flow and heat transfer over a permeable shrinking surface in a Cu-water nanofluid." *International Journal of Mathematical, Computational, Physical, Electrical and Computer Engineering* 8, no. 5 (2014): 782-788.