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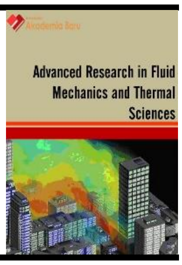

Slip Effect on Mixed Convection Flow Past a Thin Needle in Nanofluid Using Buongiorno's Model

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ABSTRACT

This work examines the effect of the buoyancy force or mixed convection on the steady two dimensional nanofluid flow over a vertical thin needle with the presence of velocity slip on the surface. The nanofluid model used for the present study incorporates the influence of the Brownian motion and thermophoresis. The governing equations in the form of partial differential equations are transformed into nonlinear ordinary differential equations, before being solved numerically using bvp4c package in MATLAB software. Special priority has been given to the physical parameters of interest, including mixed convection, velocity ratio, Brownian motion, thermophoresis, slip and needle thickness. The impacts of those parameters are described in detail through graphs for the velocity, temperature and concentration profiles, skin friction coefficient, local Nusselt number and also local Sherwood number. The outcome of the study shows that the existence of the dual solutions is noticed when the needle and the buoyancy force against the direction of the fluid motion. It is reveals from the study that the presence of the velocity slip increases the magnitude of the skin friction coefficient, heat and mass transfer in the system.

Keywords:
Dual solutions; mixed convection; nanofluid; thin needle; velocity slip

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1. Introduction

In recent years, as energy costs have increased rapidly, there must be a great need for new types of heating or cooling fluid that will enhance thermal efficiency, hence, reduces the overall energy usage. Choi [1] found that an innovative way to improve heat transfer performance is by using nanoscale particles in base fluid. Nanofluid is a new class of fluids created by dispersing nanometer-sized particles such as metal, oxide, carbides and carbon nanotubes in a base fluid (e.g., ethylene glycol, water, propylene glycol). By suspending ultra-fine nanoparticles in a base fluid, the thermal conductivity of the conventional heat transfer fluid can be significantly increased. Nanofluid is said to be a new generation of coolant for various industrial and automotive applications because of their excellent heat transfer performance. Furthermore, nanofluid offer various advantages in industrial

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and engineering applications, for instance, macro and microscale heat exchangers, nuclear reactors, fuel cell, biomedical and transportations [2-4].

The consideration of the boundary layer flow over various surfaces in nanofluid has gained a vast number of researchers in fluid mechanics. Interestingly, nanofluid consist of two models; first is a model proposed by Buongiorno [5], and second is a model proposed by Tiwari and Das [6]. In this work, we only focus on the model proposed by Buongiorno [5]. In Buongiorno's model, an increment in the thermal conductivity occurs due to the presence of two main effects, namely the Brownian diffusion and thermophoretic diffusion of nanoparticles. Several researches have been done to explore the potential of nanofluid in heat transfer performance in the boundary layer flow. For instance, Khan and Pop [7] studied the boundary layer flow of a nanofluid past a stretching sheet. The steady boundary layer flow of a nanofluid past a moving semi-infinite flat plate in a uniform free stream were discussed by Bachok *et al.,* [8]. Besides, Reddy *et al.,* [9] investigated the soret effect on mixed convection flow in a nanofluid over a vertical flat plate under convective boundary conditions. More works regarding the boundary layer flow in a nanofluid are reported in the existing literature [10-16]. Very recently, Mustafa [17] performed an analytical solution for free and circular jets cooled by single phase nanofluids. Hussanan *et al.,* [18] investigated the analysis of heat transfer of sodium carboxymethyl cellulose based nanofluid with titania nanoparticles. Zulkifli *et al.,* [19] studied the problem of the boundary layer flow past a moving plate in nanofluid with the presence of viscous dissipation by considering the revised boundary condition.

The study of the boundary layer flow and heat transfer over a thin needle has been an active area of research nowadays. This is due to the movement of the needle that disturbs the free stream direction, where this phenomenon is the main interest in measuring the velocity profiles and temperature distributions of the flow field in experimental research of flow and heat transfer analysis. Thin needle is categorized as a body of revolution where its diameter is of the same order as the velocity, temperature or concentration boundary layer that it develops. Normally, the measuring tools such as a hot wire anemometer or shielded thermocouple consist of a very thin needle or wire. Hence, the study of the flow past such slender needle-shaped surfaces is of considerable practical interest. Some examples of their applications in industries are transportation, coating of wires, lubrication, geothermal power generation and blood flow problem in biomedical.

The investigation of the flow over a thin needle has been first discussed by Lee [20]. Then after, Narain and Uberoi [21] furthered the Lee [20] work by studying the force convection flow on the thin needle in viscous fluid. It should be noted that there have been published several papers [22-24] on boundary layer analysis over a thin needle in viscous fluid. In addition to viscous fluid, there is one type of fluid that often used by researchers recently. This fluid is known as a nanofluid, where it has higher thermal conductivity compared to the viscous fluid. Grosan and Pop [25] considered the force convection flow past a thin needle in nanofluid with variable wall temperature by solving the problem using boundary value problem solver bvp4c in MATLAB software. Following that, several investigations on nanofluid flow over a thin needle available in recent literatures [26-28]. Very recently, Salleh *et al.,* [29] analyzed the nanofluid flow past a moving thin needle by taken into account the effects of chemical reaction and heat source. In their study, they also performed the stability analysis in order to determine the stability of the solutions obtained.

Inspired by the aforementioned studies, the intention here is to analyze the nanofluid flow due to a moving thin needle using Buongiorno's mathematical model. Using suitable transformations, similarity equations are formulated which are solved numerically via bvp4c package in MATLAB software. The graphical illustrations are presented in order to interpret the function of the physical parameters on the fluid flow and heat transfer characteristics. In the present content, we are also interested in considering surface-fluid interaction where the slip flow regime occurs. However, no-slip assumption is not consistent with all physical behaviors in several flow situations and it must be substituted by a partial slip boundary condition [30]. The study of the slip effect on the permeable surface in viscous fluid is introduced by Beavers and Joseph [31]. The comprehensive literature on the slip effect with various physical situations can be found in recent years [32-35]. Also, it is good to note that no works has been found in the existing papers for studying the slip effect on the boundary layer over a thin needle.

2. Methodology

We consider a steady two dimensional laminar flow of a nanofluid towards a moving vertical thin needle with uniform ambient temperature T_∞ and nanoparticle volume fraction C_∞. Figure 1 illustrates the flow model and physical coordinate system. It is assumed that the uniform temperature and the uniform nanoparticle volume fraction of the needle surface are T_w and C_w, respectively, where $T_w > T_\infty$ for a heated surface (assisting flow) and $T_w < T_\infty$ for a cooled surface (opposing flow) and $C_w > C_\infty$. Here x is the coordinate measured along the needle leading edge in the vertical direction, r is the coordinate measured in the direction normal to the needle surface and $R(x) = \left(\nu cx/U\right)^{1/2}$ represents the needle radius. Since the needle is assumed thin, the influence of its transverse curvature is beneficial, however, the pressure gradient along the needle can be ignored [20]. The needle moves with a uniform velocity U_w in the same or opposite direction with the fluid flow of a uniform velocity U_∞.The sketch of the physical configuration and coordinate system are presented in Figure 1.

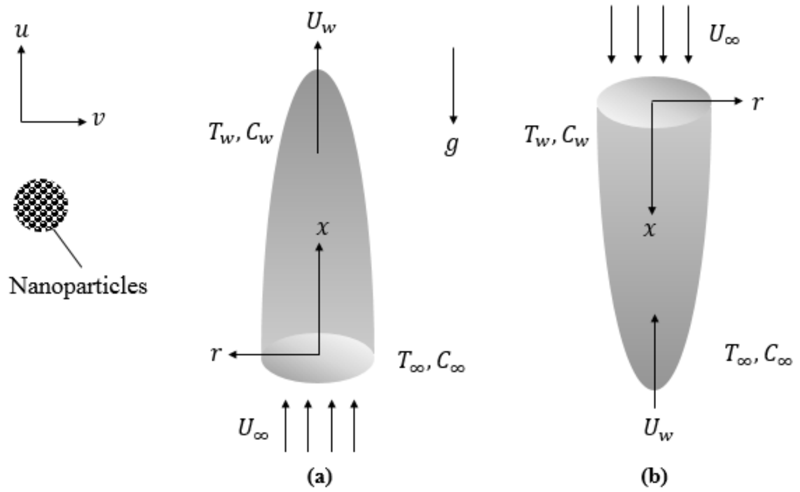

Fig. 1. Flow model and physical coordinate system: (a) assisting flow and (b) opposing flow

In-line with these assumptions, the governing equations describing the conservation of mass, momentum, energy and solid volume fraction can be written as follows.

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{\rho_f}\left[(1-C_\infty)\rho_{f_\infty}\beta(T-T_\infty) - \left(\rho_p - \rho_{f_\infty}\right)(C-C_\infty)\right]g, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\sigma\left[D_B\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right], \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}=\frac{D_B}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{4}$$

with the relevant boundary conditions

$$\begin{aligned} &u=U_w+L\left(\frac{\partial u}{\partial r}\right),\quad v=0,\quad T=T_w,\quad C=C_w \ \text{ at } \ r=R(x),\\ &u\to U_\infty,\quad T\to T_\infty,\quad C=C_\infty \ \text{ as } \ r\to\infty \end{aligned} \tag{5}$$

where u and v are the velocity components along x and r axis, respectively, T and C are nanofluid temperature and concentration in the boundary layer, respectively, D_B and D_T represent the Brownian diffusion and thermophoretic diffusion coefficient, ν is the kinematic viscosity, ρ_f and ρ_p are densities of the base fluid and of nanoparticles, respectively, ρ_{f_∞} is the density of the ambient fluid, β is the volumetric expansion coefficient, g is the gravitational acceleration, α is the thermal diffusivity and $\sigma=(\rho \mathrm{d})_p/(\rho \mathrm{d})_f$ is the ratio of effective heat capacity of the nanoparticle to heat capacity of the base fluid.

We seek the similarity solution of Eq. (1)-(4) of the following form

$$\psi=\nu x f(\eta),\quad \eta=\frac{Ur^2}{\nu x},\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \tag{6}$$

Here, η is the similarity variable, $\theta(\eta)$ and $\phi(\eta)$ are the dimensionless temperature and concentration, respectively, and $\psi(x,r)$ is the dimensionless stream function which is defined as

$$u=\frac{1}{r}\frac{\partial\psi}{\partial r},\quad v=-\frac{1}{r}\frac{\partial\psi}{\partial x}. \tag{7}$$

Using Eq. (7), Eq. (1) is identically satisfied. Effectively, we obtain the following three coupled similarity equations.

$$2(\eta f'')'+ff''+\frac{1}{4}\lambda\theta-\frac{1}{4}Nr\phi=0, \tag{8}$$

$$\frac{2}{\Pr}(\eta\theta')'+f\theta'+2\eta Nb\chi'\theta'+2\eta Nt\theta'^2=0, \tag{9}$$

$$2\eta\chi''+2\chi'+2\eta\frac{Nt}{Nb}\theta''+2\frac{Nt}{Nb}\theta'+Lef\chi'=0, \tag{10}$$

and the transformed boundary conditions are

$$f(c)=\frac{\varepsilon}{2}c+2\delta cf''(c),\quad f'(c)=\frac{\varepsilon}{2}+2\delta f''(c),\quad \theta(c)=1,\quad \phi(c)=1,$$
$$f'(\eta)=\frac{1}{2}(1-\varepsilon),\quad \theta(\eta)\to 0,\quad \phi(\eta)=0 \quad \text{as} \quad \eta\to\infty, \tag{11}$$

where prime denotes differentiation with respect to η and assuming $\eta=c$ to represent the needle size or thickness.

The outcome parameters are defined as

$$\Pr=\frac{\nu}{\alpha},\quad Le=\frac{\nu}{D_B},\quad Nb=\frac{\sigma D_B(C_w-C_\infty)}{\nu},\quad Nt=\frac{\sigma D_T(T_w-T_\infty)}{\nu T_\infty},\quad \varepsilon=\frac{U_w}{U},$$
$$Nr=\frac{(\rho_p-\rho_{f_\infty})(C_w-C_\infty)gx}{U^2\rho_f},\quad \lambda=\frac{Gr_x}{\mathrm{Re}_x^2}, \tag{12}$$

where $Gr_x=(1-C_\infty)(T_w-T_\infty)g\rho_{f_\infty}\beta x^3/\nu^2\rho_f$ is the local Grashof number and $\mathrm{Re}_x=Ux/\nu$ is the local Reynolds number. Here, $\Pr$ is the Prandtl number, Le is the Lewis number, Nb is the Brownian motion parameter, Nt is the thermophoresis parameter, Nr is the buoyancy ratio parameter, ε is the velocity ratio parameter between the free stream and the needle surface with the composite velocity of $U=U_w+U_\infty$, and λ is constant mixed convection (or buoyancy) parameter. Note that, $\lambda>0$ corresponds to the assisting flow and $\lambda<0$ corresponds to the opposing flow.

The skin friction coefficients C_f, local Nusselt number Nu_x and local Sherwood number Sh_x are given by

$$C_f=\frac{\mu}{\rho U^2}\left(\frac{\partial u}{\partial r}\right)_{r=c}=4\mathrm{Re}_x^{-1/2}c^{1/2}f''(c), \tag{13}$$

$$Nu_x=-\frac{x}{(T_w-T_\infty)}\left(\frac{\partial T}{\partial r}\right)_{r=c}=-2\mathrm{Re}_x^{1/2}c^{1/2}\theta'(c), \tag{14}$$

$$Sh_x=-\frac{x}{(C_w-C_\infty)}\left(\frac{\partial C}{\partial r}\right)_{r=c}=-2\mathrm{Re}_x^{1/2}c^{1/2}\phi'(c), \tag{15}$$

3. Results

In this section, the numerical computations are plotted and discussed in detail with the help of graphical illustrations. The dimensionless velocity, temperature, concentration, heat and mass transfer rates as well as the skin friction coefficient for some values of slip parameter δ, needle thickness c, mixed convection parameter λ, velocity ratio parameter ε, Brownian motion parameter Nb and thermophoresis parameter Nt are presented in Figure 1-12. This problem is solved numerically via bvp4c package through MATLAB software. Bvp4c package is a technique used to compute the boundary value problem for ordinary differential equations. This kind of package applies the finite difference methods, where the outcome can be gained by using an initial guess supplied at the initial mesh point and change step size to obtain the specified certainty. Nevertheless, to use this package, we need to reduce these boundary value problems to a system of first order ordinary differential equations. In order to validate our numerical technique, a comparison value of shear stress $f''(c)$ has been made with the previous studies by Ahmad *et al.,* [28] and Salleh *et al.,* [29]. A good agreement is noticed in these studies as can be seen in Table 1.

Table 1
Comparison values of shear stress when $\delta = \lambda = Nr = \varepsilon = Le = 0$ for various values of needle thickness when $\Pr = 1$

c	Ahmad *et al.,* [28]	Salleh *et al.,* [29]	Present Result
0.01	8.4924360	8.4924453	8.4924453
0.1	1.2888171	1.2888300	1.2888259
0.15	-	0.9383388	0.9383386
0.2	-	0.7515725	0.7515724

The effect of the slip parameter δ on the dimensionless velocity distribution $f'(\eta)$, dimensionless temperature distribution $\theta(\eta)$ and the dimensionless concentration distribution $\phi(\eta)$ are shown in Figure 2-4. It is noticed from these figures that the distributions of the velocity, temperature and concentration increase for the increasing value of slip parameter. These situations are influenced by significant reduction in the boundary layer thicknesses as δ increases at the points, where these distributions reach the far field boundary condition. In addition, the presence of slip also increases the reduced skin friction coefficient, local Nusselt number (or heat transfer rate) as well as the local Sherwood number (or mass transfer rate) as can be seen in Figure 5-7. It is worth mentioning that the occurrence of the slip on the needle surface enlarges the region of the dual similarity solutions exist. We have to note that the dual solutions are likely to exist when the flow is opposing or when the mixed convection parameter takes the negative values, $\lambda < 0$. However, the dual solutions only exist in a certain range of $\lambda_c < \lambda \le 0$. According to the previously published work by some researchers, they hypothesized that the upper branch solution is stable and physically relevant, while the lower branch solution is unstable and not physically relevant. More detail on how to determine the stability of the solution can be found in the work of Weidman *et al.,* [36], Rosca and Pop [37] and Salleh *et al.,* [38].

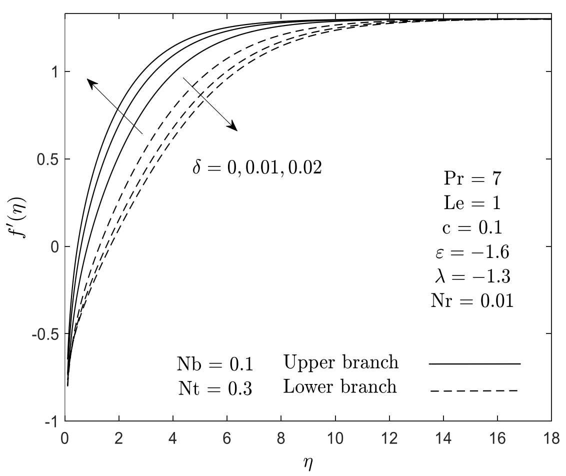

Fig. 2. Velocity distributions for several values of slip parameter δ

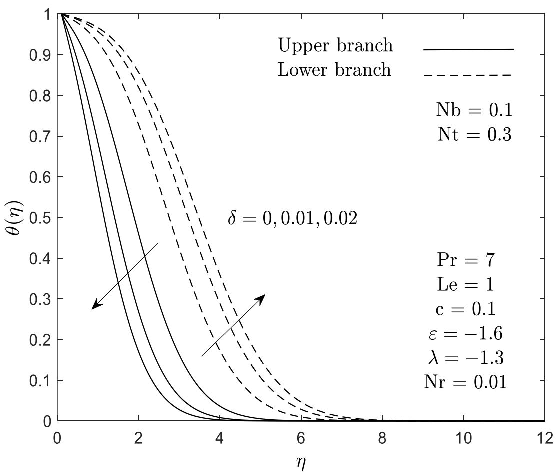

Fig. 3. Temperature distributions for several values of slip parameter δ

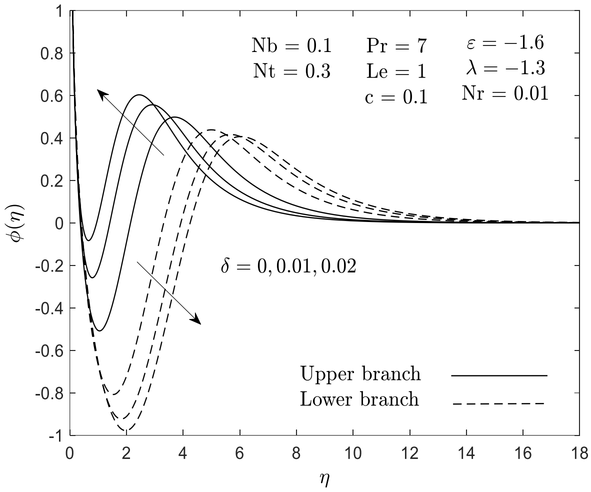

Fig. 4. Concentration distributions for several values of slip parameter δ

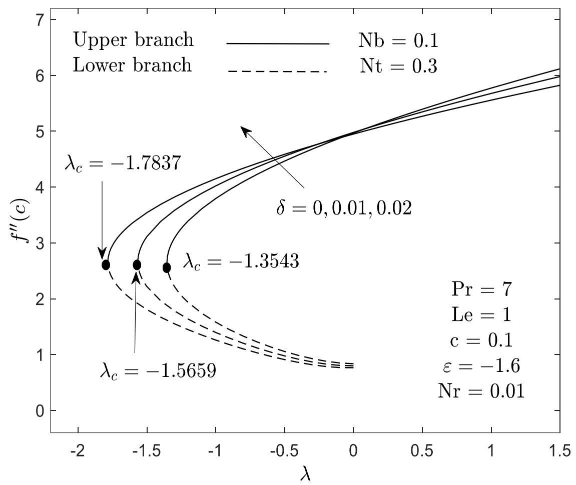

Fig. 5. Reduced skin friction coefficient for several values of slip parameter δ

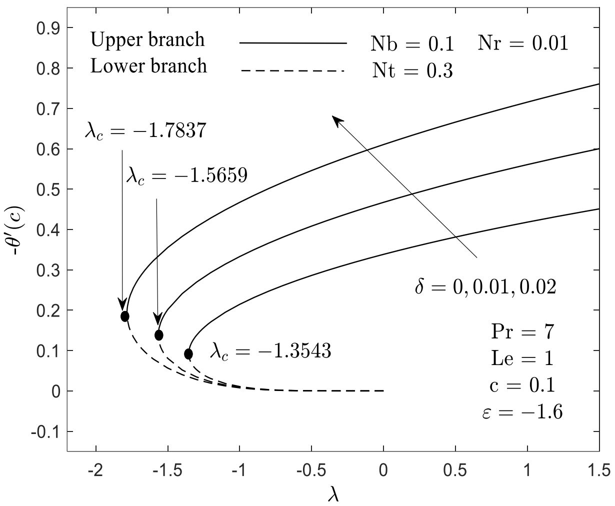

Fig. 6. Reduced local Nusselt number for several values of slip parameter δ

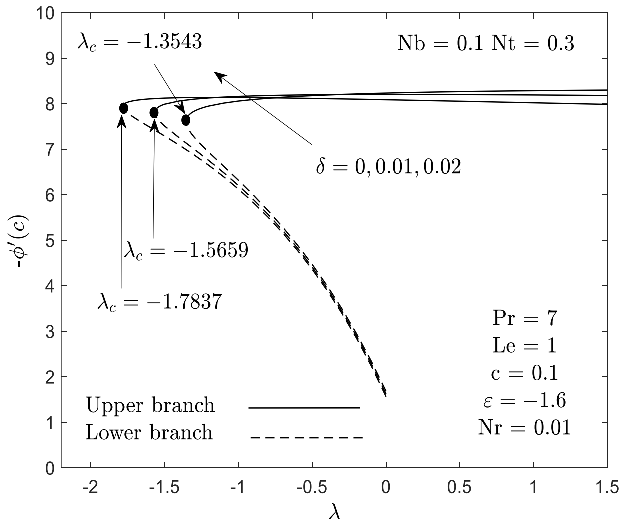

Fig. 7. Reduced local Sherwood number for several values of slip parameter δ

To analyze the influence of needle thickness c within the fluid phases of nanofluid, we plot Figure 8-13 which illustrates the variation of velocity, temperature and concentration distributions, skin friction coefficient, heat transfer rate and mass transfer rate. As expected in Figure 8-10, the momentum, thermal and concentration boundary layer thicknesses for the slender surface of the needle $(c = 0.1)$ is always smaller compared to that of a thick surface $(c = 0.2)$. It is found that the velocity rate, temperature gradient and concentration distributions increase with the reduction in these boundary layer thicknesses. Interestingly, the decrement of the boundary layers also helps to enhance the skin friction coefficient, heat and mass transfer rate occur within the needle and the fluid flow as can be observed in Figure 11-13. From the physical point of view, the thinner the surface of the needle, the less time is required to transfer the heat and mass from the needle to the fluid. As a consequence, increases the rate of heat and mass transfer in the system. Besides that, the dual solutions exist are more pronounced when the needle moves against the direction of the fluid flow or when the velocity ratio parameter is negative $(\varepsilon < 0)$. It should be noted that, the range of the dual solutions exist are in between $\varepsilon_c < \varepsilon \le 0.4$, in which for $\varepsilon < \varepsilon_c$ no similarity solutions obtained for Eq. (8)-(10).

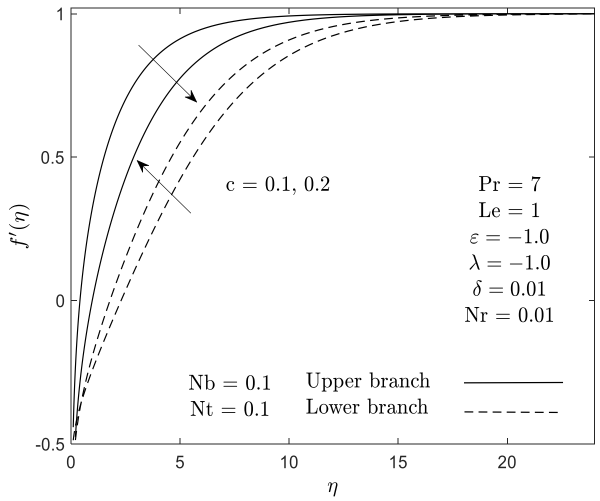

Fig. 8. Velocity distributions for several values of needle thickness c

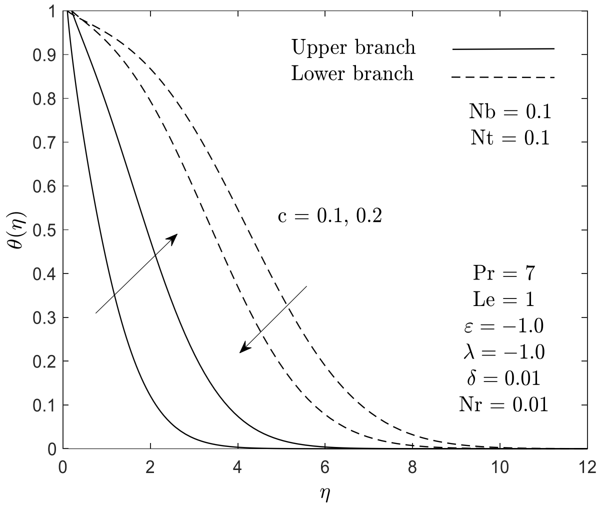

Fig. 9. Temperature distributions for several values of needle thickness c

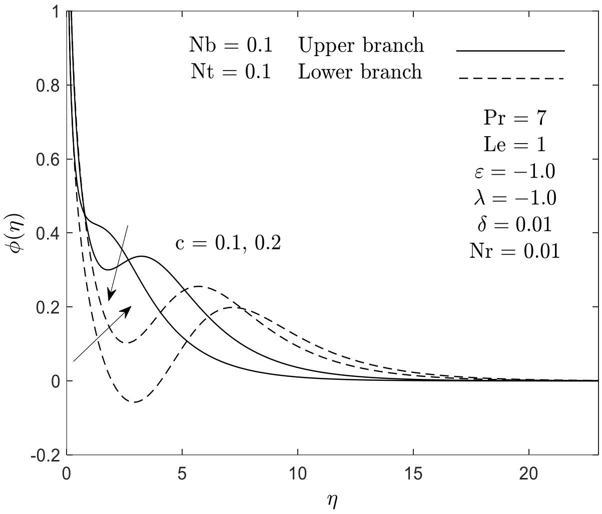

Fig. 10. Concentration distributions for several values of needle thickness c

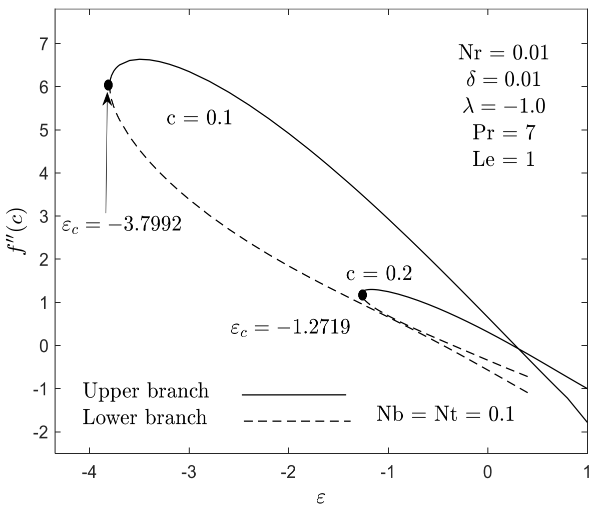

Fig. 11. Reduced skin friction coefficient for several values of needle thickness c

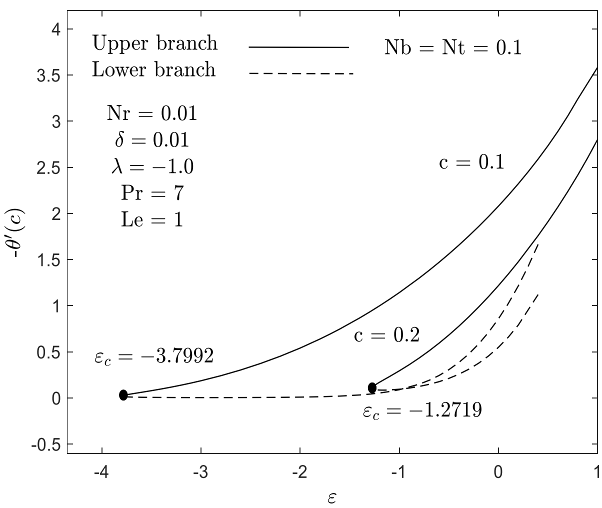

Fig. 12. Reduced local Nusselt number for several values of needle thickness c

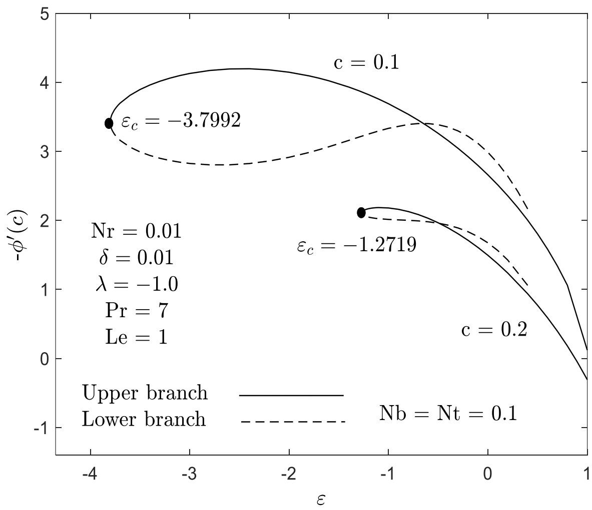

Fig. 13. Reduced local Sherwood number for several values of needle thickness c

Figure 14-15 elucidate the effects of the Brownian motion parameter Nb and thermophoresis parameter Nt on the heat and mass transfer rate in term of local Reynold number. As can be observed in Figure 14-15, the rate of heat and mass transfer decrease as the Brownian motion parameter increase from $Nb = 0.1$ to $Nb = 0.5$. Physically, the higher rate of the Brownian motion slows down the rate of heat and mass transfer occurs in the system. The random movement of the nanoparticles and base fluid particles suspended in the fluid due to the non-stop collision makes such situation happen. Moreover, the presence of the thermophoretic effect in the flow enhances the formation of the thermal boundary layer thickness, thus, decreases the temperature gradient, and consequently decrease the rate of heat transfer on the needle surface. This behavior is illustrated in Figure 14. It is also worth mentioning that as the thermophoretic effect rise up, the nanoparticles will penetrate deeper in the fluid, hence, reduces the concentration boundary layer thickness. This leads to an increment in the concentration gradient as well as the rate of mass transfer as shown in Figure 15.

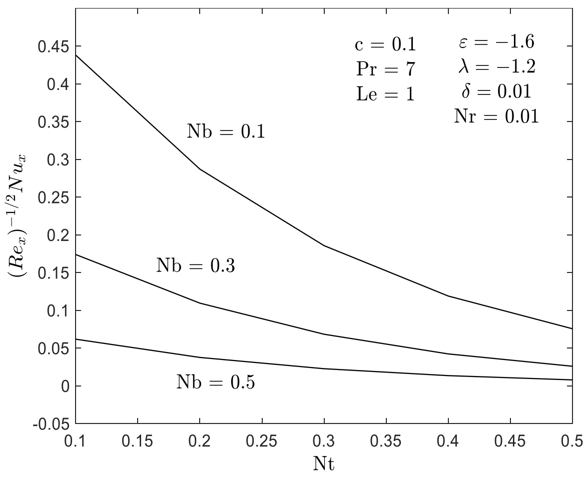

Fig. 14. Local Nusselt number for several values of Brownian motion parameter Nb and thermophoresis parameter Nt

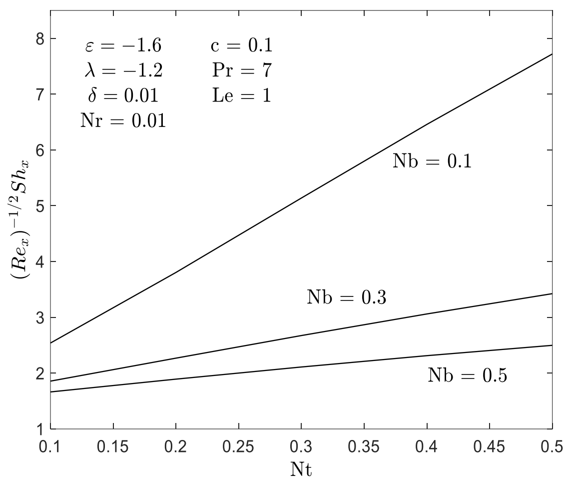

Fig. 15. Local Sherwood number for several values of Brownian motion parameter Nb and thermophoresis parameter Nt

4. Conclusions

In this research, the influence of buoyancy forces on steady laminar nanofluid flow passing through a moving thin needle in the occurrence of slip effect is numerically studied. This problem is solved by using the bv4pc package in MATLAB software. The governing PDEs have been transformed into a set of ODEs by employing pertinent similarity transformations and solving the equations together with boundary conditions by bvp4c package in MATLAB software. A parametric analysis is executed to investigate the impact of several emerging parameters on the fluid flow characteristic. The followings are conclusion that can be drawn from the study:

i. The region of the dual solutions exist increases as the slip parameter increase and when the needle thickness decrease.
ii. The existence of the dual similarity solutions is noticed when the needle moves against the direction of the free stream flow and when the needle is being cooled (opposing flow).
iii. The higher values of the Brownian motion parameter reduce the rate of heat and mass transfer between the surface of the needle and the fluid flow.
iv. An increment in the slip parameter enhances the skin friction coefficient, heat and mass transfer rate at the needle surface.

v. The slender surface of the needle allows heat and mass transfer to take place quickly compared to a thick surface.

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