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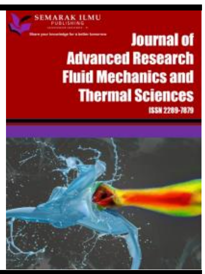

Investigating the Effects of Wu's Slip and Smoluchowski's Slip on Hybrid TiO_2/Ag Nanofluid Performance

Lim Yeou Jiann[1,*], Sharena Mohamad Isa[2], Noraihan Afiqah Rawi[1], Ahmad Qushairi Mohamad[1] Sharidan Shafie[1]

[1] Department Mathematical Sciences, Faculty Sciences, Universiti Teknologi Malaysia, Skudai, 81310 Johor Bahru, Johor, Malaysia
[2] Manufacturing Engineering Technology Section, Universiti Kuala Lumpur Malaysia Italy Design Institute (UniKL MIDI), Kuala Lumpur, 56100, Malaysia

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ABSTRACT

The high thermal conductivity of Titanium dioxide (TiO_2) makes it ideal for heat transfer, particularly in a solar collector. However, nanofluids containing dissolved TiO_2 nanoparticles tend to agglomerate. Thus, nanoparticle silver (Ag) is used to stabilize the nanofluid. This study looks at a hybrid nanoparticle of TiO_2 and Ag, which are put into water to make a hybrid nanofluid, over a stretching plate. The effects of Wu's velocity slip and Smoluchowski's temperature slip are taken into consideration. Xue's thermal conductivity model for the hybrid nanofluid is employed. The governing equations are transformed by applying the similarity transformation technique and solved semi-analytically using the Homotopy Analysis Method. The effect of the multiple slips on the fluid profiles is graphically displayed and discussed. Results illustrate that the temperature profile is diminished by 42% due to Smoluchowski's temperature slip parameter, while the velocity profile is reduced by more than 40% due to the first and second-order Wu's slip parameters. These findings are important for optimizing the performance of the hybrid nanofluids. For example, in the solar collector, where a better understanding of heat transfer fluid characteristics can improve the stability of the hybrid nanofluid and the efficiency of eco-friendly energy storage.

1. Introduction

Solar thermal energy is one of the green alternatives for solving the vanishing of fossil fuels and the global warming problem. It is an effective alternative way in supplying the massive consumption of energy in maintaining the human quality of life criteria. Indeed, the performance of solar collectors has to be improved. The collection of solar energy has suffered various losses due to heat transmission. This deficiency can be overcome if the thermal properties of the heat transfer fluid (HTF) in the solar collector, which conventionally are the water, ethylene glycol (EG), and thermal oil, can be improved. Recently, the fluid containing nanoparticles, called nanofluid, has been the concentration of researchers as the inclusion of these nanoparticles has substantially improved the

* *Corresponding author.*
E-mail address: jiann@utm.my

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thermophysical properties of the heat transfer liquid. The concept of nanofluid has gained recognition after the first proposed by Choi and Eastman [1]. They observed that the thermal conductivity of the HTF with copper nanoparticles is enhanced. Choi *et al.,* [2] detected that the thermal conductivity of the HTF was approximately improved twice by the nanoparticles. The investigation of the nano-HTF has remarkably increased since then. A recent study has demonstrated that the use of hybrid nanoparticles in fluid results in significantly improved heat transfer properties compared to pure nanofluids [3-6]. Maskeen *et al.,* [4] observed that the thermal conducted activity rate has been improved greater as a minor amount of various stabilized nanoparticles are mixed. Besides that, the increment of viscosity in fluid with nanoparticles has been reduced by the hybridization of the nanoparticles [7]. Several studies have extensively reviewed the potential use of hybrid nanofluids in solar energy storage and solar systems [8-12].

Xuan *et al.,* [13] experimentally explore the feature of the plasmonic hybrid nanoparticles Titanium dioxide/ Silver, (TiO_2/Ag) and plasmonic nanofluids (Ag) or plasmonic nanofluids (TiO_2) in solar energy absorption. They found that the nano-TiO_2 fluid has a lower temperature as compared to the hybrid nanofluid. Despite both nanofluid Ag and composite nanoparticles TiO_2/Ag having the same temperature, the hybrid nanofluid offers a cost advantage. TiO_2 is chemically and thermally stable, readily available for purchase, and economically priced [14]. TiO_2 has good thermal conductivity and has frequently been used in thermal absorption. However, as compared with the metallic nanoparticle silver (Ag), TiO_2 has a lower thermal conductivity [15]. Moreover, TiO_2 is easy to agglomerate [14]. To overcome these shortages, Ag and TiO_2 are normally combined to provide a stabilized HTF [15,16]. Jamshed *et al.,* [17] used the hyperbolic tangent fluid model to investigate the properties of the heat-transmitting hybrid nanofluid. The thermal performance of a hybrid nanofluid composed of copper (Cu) and TiO_2 nanoparticles suspended in ethylene glycol (EG) was evaluated. The findings indicate that the hybrid nanofluid exhibits superior thermal performance. Kho *et al.,* [18] investigated the impact of viscous dissipation on hybrid nanofluid Cu/TiO2 for two cases: along stretching and shrinking surface. Then the effect of a rotating sheet with shrinking and stretching on a three-dimensional hybrid nanofluid was discussed by Teh *et al.,* [19]. Reddy *et al.,* [20] acknowledged the convection flow of the hybrid TiO_2/Ag hybrid nanofluids with water (H_2O) as the based fluid inside an annulus cylinder. The effect of the magnetic field and heat generation were analyzed. Kho *et al.,* [21] studied the hybrid TiO_2/Ag nanofluid over a stretchable wedge. The author detected that the thermal conductivity of the hybrid fluid is enhanced when the percentage of the TiO_2 in the hybrid fluid increase. A characteristic of the Ag nanofluid, TiO_2 nanofluid, and TiO_2/Ag hybrid nanofluid was compared by Ahmad [22]. The volume fraction of the hybrid nanoparticle was observed to significantly affect thermal transmission.

The assumption of a smooth surface by most of the researchers in the literature appears impractical, especially when the nano-scaled particles are incorporated into the fluid. To fulfill the realistic, the slip effect at the surface of the solar collector must be considered. It has a significant impact on thermal and momentum distribution in the fluid. Wu [23] has yielded an improved slip model that demonstrates practical utility compared to existing models, including the Maxwell model, the 2nd order slip model, and the 1.5 order slip model. Fang and Aziz [24] had applied Wu's slip boundary condition to study the viscous fluid flow over a stretching sheet. The introduced slip effect was detected to decrease the wall shear stress and then change the movement of the liquid. Fang *et al.,* [25] extended the investigation to consider a shrinking sheet. Dual solutions of the velocity profiles were determined. Similar to the previous discussion, the finding showed that the fluid velocity and the skin friction at the surface are strongly dependent on the slip velocity. The effect of Wu's slip condition on a tangent hyperbolic nanofluid flow was discussed by Khan *et al.,* [26]. The results indicate that the velocity distribution is reduced by the slip and the rheology behavior of the fluid. However, the temperature profile gives an opposite tendency. Shaw *et al.,* [27] investigated the impact of Wu's slip and irregular heat source on a nanofluid flow through a thin needle. Farooq *et al.,* [28] recently elucidated the characteristic of a bioconvection Williamson nanofluid under the impact of Wu's slip and Cattaneo-Christov heat flux over a cylinder. Much more discussion on Wu's slip boundary condition is shown in previous studies [29-35].

Le *et al.,* [36] and Assam *et al.,* [37] presented the concept of temperature jump to improve the accuracy of computational fluid dynamics in simulating the nonequilibrium hypersonic gas flow. Accordingly, the heat flux acting on the surface has a normal direction. Sajid *et al.,* [38] studied the influence of a temperature jump on the flow of Carreau fluid over a stretching plate. Then, Sajid *et al.,* [39] extended the work done by considering the effect of the stretchable surface on the Sutterby nanofluid fluid flow. The nanofluid's heat transfer rate was found to decrease the Smoluchowski's parameter increased. A similar trend was observed by Atif *et al.,* [40]. Atif *et al.,* [40] had studied the effect of the slip velocity and Smoluchowski temperature jump on a tangent hyperbolic nanofluid. The thermal transmission of a Sutterby hybrid nanofluid in solar energy was discussed by Hussain *et al.,* [41]. The authors detected that the temperature of the hybridization nanofluid is improved with a negative variation in Smoluchowski's parameter. Jamshed *et al.,* [42] applied a numerical approach to restudy the fluid problem of Hussain *et al.,* [41]. Different type of hybrid nanofluid was considered by Jamshed *et al.,* [42]. They found that the volume fraction of the hybrid nanoparticle has a positive function with the temperature field but an inverse tendency with the temperature slip effect.

The literature survey indicates that no efforts were made to study the tangent Hyperbolic hybrid nanofluid flows and heat transfer with the influence of Wu's slip and Smoluchowski temperature jump slip. Therefore, following previous studies, the present study aims to theoretically investigate the impact of those slip effects on a hybrid nano-HTF (TiO_2/Ag) in a solar collector [17,20,40]. The developed partial governing equations of the proposed fluid problem are initially transformed into ordinary governing equations by the similarity transformation technique. The Homotopy analysis technique is then practised to determine the results. The most significant parameter that affects heat transmission and fluid flow is evaluated by conducting a parametric study for the embedded parameters. The present study's finding is essential in optimizing the performance of the hybrid nano-HTF in the solar collector.

2. Mathematical Formulation

A two-dimensional incompressible steady boundary layer tangent hyperbolic fluid flow over a stretching plate is discussed in the present research. The fluid is dissolved with a water-based hybrid nanofluid containing silver (Ag) and Titanium dioxide (TiO_2) nanoparticles. Smoluchowski's temperature jump and Wu's slip on the surface of the plate are considered. As illustrated in Figure 1, the horizontal axis (*x*-axis) is oriented parallel to the surface of the plate, while the vertical axis (*y*-axis) is oriented perpendicular to the surface. Magnetic strength B_0 is imposed vertically on the plate. T_∞ is assumed for the fluid that is located far from the plate, and the fluid is static at the initial. The rheology relationship of the hyperbolic tangent fluid is written as [17,26]:

$$\tau^* = \left\{ \left[\mu_\infty + \mu_0 \right] \tanh\left(\Gamma \tilde{\gamma} \right)^\alpha + \mu_\infty \right\} \tilde{\gamma} \tag{1}$$

where $\mu_0$is the zero-shear-rate viscosity, τ^* is the stress tensor, $\mu_\infty$is the infinite-shear-rate viscosity, Γ is the time material constant, and α is the power law index and $\tilde{\gamma}$ is expressed as:

$$\tilde{\gamma} = \sqrt{\frac{1}{2}\sum_i \sum_j \tilde{\gamma}_{ij}\tilde{\gamma}_{ij}} = \sqrt{\frac{1}{2}\Pi} \quad (2)$$

where Π is the second invariant strain tensor. We consider $\mu_\infty = 0$ in the constitutive Eq. (1) since the shear rate viscosity is impossible to infinite. Furthermore, the shear thinning effect is assumed, thus we take $\Gamma\tilde{\gamma} < 1$. Then, by binomial expansion, Eq. (1) becomes:

$$\tau^* = \mu_0\left(1 + \alpha\left[\Gamma\tilde{\gamma} - 1\right]\right)\tilde{\gamma} \quad (3)$$

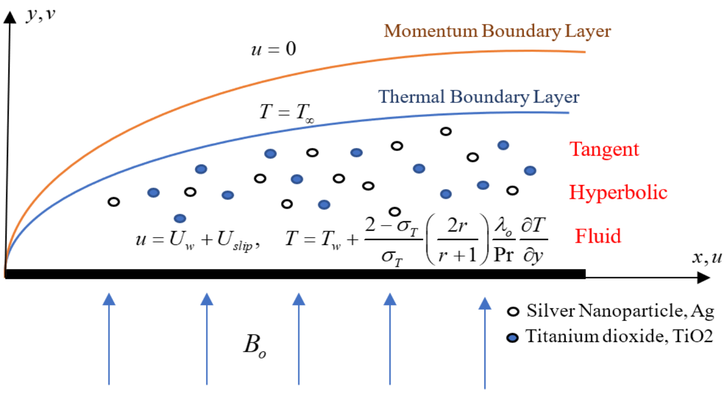


Fig. 1. The fluid flow of the hyperbolic tangent hybrid nanofluid through a stretchable sheet

The boundary layer approximation governing equations of the present considered tangent hyperbolic fluid are represented as:

$$\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0, \quad (4)$$

$$v\frac{\partial u}{\partial y} + u\frac{\partial u}{\partial x} = \left(\frac{\mu}{\rho}\right)_{hnf}\left\{(1-\alpha)\frac{\partial^2 u}{\partial y^2} + \Gamma\alpha\sqrt{2}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right\} - \left(\frac{\sigma}{\rho}\right)_{hnf} B_0^2 u, \quad (5)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = k_{hnf}\frac{\partial^2 T}{\partial y^2} + \left(\frac{\sigma}{\rho c_p}\right)_{hnf} B_0^2 u^2 + \left(\frac{\mu}{\rho c_p}\right)_{hnf}\left\{(1-\alpha)\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\alpha\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^3\right\}. \quad (6)$$

The heat capacitance, effective density, effective dynamics viscosity, and effective electrical conductivity of the hybrid nanofluid are indicated by $\left(\rho c_p\right)_{hnf}$, ρ_{hnf}, μ_{hnf} and σ_{hnf} respectively. Following [23,27,40], the boundary conditions are:

$$\begin{aligned} &u = U_w + U_{slip}, \quad v = 0, \quad T = T_w + T_{slip}, \quad at \quad y = 0, \\ &u \to 0, \quad T \to T_\infty, \quad as \quad y \to \infty. \end{aligned} \quad (7)$$

where

$$U_w = \frac{ax}{l}, \qquad T_{slip} = \frac{2\Lambda\left(2-\varpi_T\right)}{\varpi_T\left(\Lambda+1\right)}\frac{\beta}{\Pr}\frac{\partial T}{\partial y}$$

$$U_{slip} = \beta\left\{\frac{2\left(3-\varpi_u \mathrm{I}^2\right)}{3\varpi_u} - \frac{1-\mathrm{I}^2}{K_n}\right\}\frac{\partial u}{\partial y} + \frac{\beta^2}{4}\left\{\frac{2}{K_n^2}\left(\mathrm{I}^2-1\right)-\mathrm{I}^4\right\}\frac{\partial^2 u}{\partial y^2} = A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2},$$

a is a positive constant, ϖ_u and ϖ_T are the momentum and temperature accommodation coefficient, respectively, *l* is the characteristic length, β is the molecular mean free path, and Λ is the specific heat ratio. *Pr* is the Prandthl number and K_n is the Knudsen number and I is defined as $\min(1/K_n,1)$.

The Eq. (4) - Eq. (7) are reduced into ordinary differential equations by using the following similarity variables;

$$\eta = y\sqrt{\frac{a}{l\nu_f}}, \qquad \Phi = \sqrt{\frac{a\nu_f}{l}}xf\left(\eta\right), \qquad u = \frac{\partial\Phi}{\partial y}, \qquad v = -\frac{\partial\Phi}{\partial x}, \qquad \theta = \frac{T-T_w}{T_w-T_\infty}, \tag{8}$$

where ν_f is the fluid kinemetric viscosity. Eq. (4) and Eq. (7) are converted into

$$ff'' - \left(f'\right)^2 + \left(\frac{\nu_{hnf}}{\nu_f}\right)\left\{\left(1-\alpha\right)f''' + \alpha We f'' f'''\right\} - \left(\frac{\sigma_{hnf}\rho_f}{\sigma_f\rho_{hnf}}\right)M^2 f' = 0 \tag{9}$$

$$\begin{aligned}&\left(\frac{k_{hnf}}{k_f}\right)\theta'' + \left(\frac{\sigma}{\rho c_p}\right)_{hnf}\left(\frac{\rho c_p}{\sigma}\right)_f \Pr EcM^2\left(f'\right)^2 + \\ &\left(\frac{\nu}{c_p}\right)_{hnf}\left(\frac{c_p}{\nu}\right)_f\left\{(1-\alpha)\Pr Ec\left(f''\right)^2 + \frac{We}{2}\alpha\Pr Ec\left(f''\right)^3\right\} + \Pr f\theta' = 0\end{aligned} \tag{10}$$

with boundary conditions

$$\begin{aligned}&f\left(0\right)=0, \quad f'\left(0\right)=1+\delta_1 f''\left(0\right)+\delta_2 f'''\left(0\right), \quad f'\left(\infty\right)=0,\\ &\theta\left(0\right)=1+\delta_3\theta'\left(0\right), \quad \theta\left(\infty\right)=0.\end{aligned} \tag{11}$$

where the derivative with respect to *η* is represented by a prime symbol. $\left(\rho c_p\right)_f$, σ_f , ρ_f and μ_f represent the heat capacitance, electrical conductivity, density, and dynamic viscosity of the fluid, respectively. The embedded parameters are defined as:

$$M^2=\frac{\sigma_f}{\rho_f}\frac{B_0^2}{a}l \ (Magnetic\ Parameter),\quad We=\sqrt{\frac{2}{\nu_f}}\left(\frac{a}{l}\right)^{1/2}U_w\Gamma \ (Weissenberg\ Number),$$
$$\Pr=\frac{\nu_f}{k_f}\ (Prandtl\ number),\quad \mathrm{Ec}=\frac{U_w^2}{c_p\left(T_w-T_\infty\right)}\ (Eckert\ number),$$
$$\delta_1=A\sqrt{\frac{a}{l\nu_f}},\quad \delta_2=B\frac{a}{l\nu_f}\ (1st\ and\ 2nd\ order\ velocity\ slip\ parameter),$$
$$\delta_3=\frac{2-\varpi_T}{\varpi_T}\left(\frac{2\Lambda}{\Lambda+1}\right)\frac{\beta}{\Pr}\sqrt{\frac{a}{l\nu_f}}\ (temperature\ slip\ parameter). \tag{12}$$

For the hybrid nanofluid flow, the thermophysical formula is given as [18,20,43]:

$$\frac{\rho_{hnf}}{\rho_f}=\left(1-\phi_2\right)\left\{(1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right\}+\phi_2\frac{\rho_{s2}}{\rho_f},\quad \frac{\mu_{hnf}}{\mu_f}=\frac{1}{(1-\phi_1)^{2.5}\left(1-\phi_2\right)^{2.5}},$$
$$\frac{\sigma_{hnf}}{\sigma_{bf}}=\frac{\sigma_{s2}+2\sigma_{bf}-2\phi_2(\sigma_{bf}-\sigma_{s2})}{\sigma_{s2}+2\sigma_{bf}+\phi_2(\sigma_{bf}-\sigma_{s2})},\ where\ \frac{\sigma_{bf}}{\sigma_f}=\frac{\sigma_{s1}+2\sigma_f-2\phi_1(\sigma_f-\sigma_{s1})}{\sigma_{s1}+2\sigma_f+\phi_1(\sigma_f-\sigma_{s1})},$$
$$\frac{\left(\rho c_p\right)_{hnf}}{\left(\rho c_p\right)_f}=\left(1-\phi_2\right)\left\{(1-\phi_1)+\phi_1\frac{\left(\rho c_p\right)_{s1}}{\left(\rho c_p\right)_f}\right\}+\phi_2\frac{\left(\rho c_p\right)_{s2}}{\left(\rho c_p\right)_f}. \tag{13}$$

The thermophysical properties of the based fluid and the hybrid nanoparticles are given in Table 1. The Xue model [44, 45] of thermal conductivity is given as

$$\frac{k_{hnf}}{k_{bf}}=\left[1-\phi_2+2\phi_2\left(\frac{k_{s2}}{k_{s2}-k_{bf}}\right)\ln\left(\frac{k_{s2}+k_{bf}}{2k_{bf}}\right)\right]\left[1-\phi_2+2\phi_2\left(\frac{k_{bf}}{k_{s2}-k_{bf}}\right)\ln\left(\frac{k_{s2}+k_{bf}}{2k_{bf}}\right)\right]^{-1},$$
$$\frac{k_{bf}}{k_f}=\left[1-\phi_1+2\phi_1\left(\frac{k_{s1}}{k_{s1}-k_f}\right)\ln\left(\frac{k_{s1}+k_f}{2k_f}\right)\right]\left[1-\phi_1+2\phi_1\left(\frac{k_f}{k_{s1}-k_f}\right)\ln\left(\frac{k_{s1}+k_f}{2k_f}\right)\right]^{-1}, \tag{14}$$

where ϕ_1 is the volume fraction of the Ag nanoparticle and ϕ_2 is for the TiO_2 nanoparticle. $\mathrm{Re}_x=ax^2\left(l\nu_f\right)^{-1}$ is the Reynold number. The physical quantity of interest local skin friction ($\frac{1}{2}C_f Re_x^{1/2}$) and the local Nusselt number ($NuRe_x^{-1/2}$), are denoted by

$$C_f\,\mathrm{Re}_x^{1/2}=\frac{\nu_{hnf}}{\nu_f}\left[\left(1-\alpha\right)f''(0)+\frac{\alpha}{2}Wef''(0)^2\right],\quad Nu\,\mathrm{Re}_x^{-1/2}=-\frac{k_{hnf}}{k_f}\theta'(0). \tag{15}$$

Table 1
The thermophysical properties of the water, H_2O base fluids, and silver Ag and Titanium dioxide, TiO_2, nanoparticles [5,20,21]

Base fluid and Nanoparticles	H_2O	Ag	TiO_2
$\rho\left[kg\cdot m^{-3}\right]$	997.1 $\left[\rho_f\right]$	10500, $\left[\rho_{s1}\right]$	4250, $\left[\rho_{s2}\right]$
$\sigma\left[\Omega^{-1}\cdot m^{-1}\right]$	0.055, $\left[\sigma_f\right]$	6.3 x 10^7, $\left[\sigma_{s1}\right]$	2.6 x 10^6, $\left[\sigma_{s2}\right]$
$k\left[W\cdot m^{-1}\cdot K^{-1}\right]$	0.613, $\left[k_f\right]$	429, $\left[k_{s1}\right]$	8.953, $\left[k_{s2}\right]$
$\left(c_p\right)\left[J\cdot kg^{-1}\cdot K^{-1}\right]$	4179, $\left[\left(c_p\right)_f\right]$	235, $\left[\left(c_p\right)_{s1}\right]$	686.2, $\left[\left(c_p\right)_{s2}\right]$

3. Homotopy Analysis Solutions

The governing Eq. (9) – Eq. (11) are solved by using the Homotopy analysis method (HAM). The solutions are assumed to be functions $f(\eta:p)$ and $\theta(\eta:p)$ that depends on an embedded parameter *p*. The solutions can be expanded at η = 0 by using Taylor's series expansion and produce

$$f(\eta:p)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)p^m,\ \theta(\eta:p)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta)p^m, \tag{16}$$

where

$$f_m(\eta)=\frac{1}{m!}\frac{\partial^m f(\eta:p)}{\partial p^m}\bigg|_{p=0},\qquad \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\theta(\eta:p)}{\partial p^m}\bigg|_{p=0}. \tag{17}$$

The solutions (16) are assumed converge to $f(\eta)$ and $\theta(\eta)$ respectively, at $p=1$ and possess analytic in $p\epsilon[0,1]$. Therefore, we have;

$$\begin{aligned}&\text{at}\quad p=0,\qquad \theta(\eta:p)=\theta_0(\eta),\qquad f(\eta:p)=f_0(\eta),\\&\text{at}\quad p=1,\qquad \theta(\eta:p)=\theta(\eta),\qquad f(\eta:p)=f(\eta).\end{aligned} \tag{18}$$

The solutions are:

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta),\qquad \theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta). \tag{19}$$

The initial guesses are taken as:

$$f_0(\eta)=\frac{1-e^{-\eta}}{1+\delta_1-\delta_2},\qquad \theta_0(\eta)=\frac{e^{-\eta}}{1+\delta_3}, \tag{20}$$

The zeroth-order deformation of the governing equations are

$$(1-p)L_f\left\{f(\eta:p)-f_0(\eta)\right\}=p\hbar_f N_f\left\{f(\eta:p)\right\}, \tag{21}$$

$$(1-p)L_\theta\{\theta(\eta:p)-\theta_0(\eta)\}=p\hbar_\theta N_\theta\{f(\eta:p),\theta(\eta:p)\}. \tag{22}$$

$\hbar_\theta$ and $\hbar_f$ are known as the convergence parameters for the heat equation and momentum equation, respectively. These parameters play a crucial role in defining the convergence rate of solutions to these equations. L_f and L_θ are the linear differential operators and are presented as

$$L_f=\frac{\partial^3 f}{\partial\eta^3}+\frac{\partial^2 f}{\partial\eta^2},\qquad L_\theta=\frac{\partial^2\theta}{\partial\eta^2}+\frac{\partial\theta}{\partial\eta}. \tag{23}$$

N_f and N_θ are nonlinear operators and are defined as

$$N_f\{f(\eta:p)\}=\left(\frac{\nu_{hnf}}{\nu_f}\right)f'''(\eta:p)\{(1-\alpha)+\alpha We f''(\eta:p)\}-\left[f'(\eta:p)\right]^2+f(\eta:p)f''(\eta:p)-M^2\left(\frac{\sigma_{hnf}\rho_f}{\sigma_f\rho_{hnf}}\right)f'(\eta:p), \tag{24}$$

$$N_\theta\{f(\eta:p),\theta(\eta:p)\}=\left(\frac{\nu}{c_p}\right)_{hnf}\left(\frac{c_p}{\nu}\right)_f\left\{(1-\alpha)\Pr Ecf''(\eta:p)^2+\frac{We}{2}\alpha\Pr Ecf''(\eta:p)^3\right\}+\left(\frac{k_{hnf}}{k_f}\right)\theta''(\eta:p)+\left(\frac{\sigma}{\rho c_p}\right)_{hnf}\left(\frac{\rho c_p}{\sigma}\right)_f\Pr EcM^2f'(\eta:p)^2+\Pr f(\eta:p)\theta'(\eta:p), \tag{25}$$

f_m and θ_m are calculated by differentiating *m*-times with respect to *p* of the Eq. (21) and Eq. (22), and dividing them by *m*! respectively. Then, the *m*-th order deformation equations are obtained by taking *p* = 0, and we have

$$L_f\{f_m(\eta)-\chi_m f_{m-1}(\eta)\}=\hbar_f\left\{\begin{array}{l}\left(\frac{\nu_{hnf}}{\nu_f}\right)\left\{(1-\alpha)f'''_{m-1}+\alpha We\left[\sum_{i=0}^{m-1}f''_{m-1-i}f'''_i\right]\right\}+\left[\sum_{i=0}^{m-1}f_{m-1-i}f''_i\right]\\-\left[\sum_{i=0}^{m-1}f'_{m-1-i}f'_i\right]-\left(\frac{\sigma_{hnf}\rho_f}{\sigma_f\rho_{hnf}}\right)M^2f'_{m-1}\end{array}\right\}. \tag{26}$$

$$L_\theta\{\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)\}=\hbar_\theta\left\{\begin{array}{l}\left(\frac{\nu}{c_p}\right)_{hnf}\left(\frac{c_p}{\nu}\right)_f\left\{(1-\alpha)\Pr Ec\sum_{i=0}^{m-1}f''_{m-1-i}f''_i+\sum_{i=0}^{m-1}f''_{m-1-i}\frac{We}{2}\alpha\Pr Ec\sum_{j=0}^{i}f''_{i-j}f''_i\right\}+\\\left(\frac{k_{hnf}}{k_f}\right)\theta''_{m-1}+\Pr\left(\sum_{i=0}^{m-1}f_{m-1-i}\theta'_i+\left(\frac{\sigma}{\rho c_p}\right)_{hnf}\left(\frac{\rho c_p}{\sigma}\right)_f EcM^2\sum_{i=0}^{m-1}f'_{m-1-i}f'_i\right)\end{array}\right\}. \tag{27}$$

where $\chi_m=0\ (k\le 1),\ 1\ (k>1)$. Eq. (26) and Eq. (27) are commonly known as high-order deformation equations. Solving Eq. (28) and Eq. (29) give the solutions of the f_m and θ_m, respectively. The Mathematica package BVPh 2.0 developed by S.J. Liao [46, 47] was employed to calculate the θ_m and f_m for $k \ge 1$.

4. Results

The current study has discussed the impact of Wu's slip and Smoluchowski's temperature slip on a magnetohydrodynamic hyperbolic tangent hybrid nanofluid flow on a stretchable plate. The water is used as the based fluid, and the commonly used nanoparticle silver, Ag, and Titanium dioxide, TiO_2 in the solar collector are dissolved in the fluid to form the tangent hyperbolic hybrid nanofluid. The parameters Pr = 7.38, M = 0.1, We = 0.2, α = 0.2, δ_1 = 0.5, δ_2 = -0.5, δ_3 = 0.1, Ec = 0.2, ϕ_1 = 0.01 and ϕ_2 = 0.01 are utilized as default for the analysis. Besides that, following previous studies, the parameters are taken in range ϕ_1 = ϕ_1 = {0.01, 0.02, 0.03, 0.04}, δ_1 = δ_3 = {0.1, 0.5, 1.0, 1.5, 2.0}, δ_2 = {-0.5, -1.0, -1.5, -2.0}, to investigate fluid flow behavior and heat transfer [27,40,42,48,49].

Table 2 shows the convergences test of the approximation order used in the HAM. As depicted in the table, the solutions of the local skin friction and the local Nusselt number do not show large differences when the order of the approximation is increased from 10 to 50. The error between the solutions for both physical interests is more than 1 x 10^{-4} after order 30. Therefore, the solutions presented in this study were computed using a 30th-order approximation, which is much more efficient for time consumption. The precision of the proposed solutions has been validated by benchmarking them against results from the literature in specific situations. By taking $We = \alpha = \delta_1 = \delta_2 = 0$, the computed local C_f solutions for different magnetic strengths, M is found well agree with the results given in previous studies [49-52]. The comparison is illustrated in Table 3. Furthermore, Table 4 depicts that the results for the local Nusselt number are consensus with the finding in previous studies for Pr = 1,3,5,10 [49,53,54]. This has validated and verified the computed solutions in the present investigation and further enhances the reliability of the analysis and discussion.

Table 2
Convergence test Pr = 7.38, $\alpha = We = Ec = 0.2$, M = 0.1, δ_1 = 0.5, δ_2 = -0.5, δ_3 = 0.1, ϕ_2 = 0.01

m	C_f	Nu	Time Consume (s)
10	-0.483182	-0.694297	5.56138
20	-0.483109	-0.689064	18.7434
30	-0.483065	-0.688776	45.4952
40	-0.483057	-0.688978	86.971
50	-0.483055	-0.689197	151.992

Table 3
Validation for $We = \alpha = \delta_1 = \delta_2 = 0$

M	Akbar *et al.,* [50]	Fathizadeh *et al.,* [52]	Ibrahim [51]	Ganesh *et al.,* [49]	Present
0	1	1	1	-	1
1	-1.41421	-1.41421	-1.41421	-1.41421	-1.41415
5	-2.44948	-2.44948	-2.44958	-2.44949	-2.44949
10	-3.31662	-3.31662	-3.31660	-3.31662	-3.31660

Table 4
Validation for $M = We = Ec = \alpha = \delta_3 = 0$

Pr	Ali [53]	Chen [54]	Ganesh *et al.,* [49]	Present
1	-0.5801	-0.58199	-0.58223	-0.58347
3	-1.1599	-1.16523	-1.16522	-1.16573
5	-	-	-1.56803	-1.56810
10	-2.2960	-2.30796	-2.30798	-2.30951

The effect of the volume fraction of the nanoparticles on the fluid flow and heat transfer is recognized in Figure 2 and Figure 3. Comparing Figure 2(b) and Figure 3(b), the increment of the volume fraction of nanoparticles silver (Ag), ϕ_1 has generated a much alteration to the temperature distribution. This is because nanoparticle Ag has a higher thermal conductivity. Though the change in velocity profile is small, a contrary phenomenon is observed for both nanoparticles. Figure 2(a) illustrates that the velocity of the fluid is increased by the volume fraction ϕ_1 but ϕ_2 reduces the distribution, as seen in Figure 3(a). This variation happens because the inclusion of the nanoparticles in the fluid has physically changed the density of the fluid and increased the collision between the nanoparticles [55].

The influence of Wu's slip on the velocity profile of the fluid is given in Figure 4(a) and Figure 5(a). The first and second-order velocity parameters, δ_1 and δ_2, have reduced the velocity profile. According to Fang and Aziz, [24] and Fang *et al.,* [25], the slip velocity in the first order gives physically meaningful results with a positive value and the slip velocity in the second order only happens when the number is in the negative root. The figures have shown that the impact of the δ_2 is more significant than the first-order velocity slip. δ_1 has reduced the velocity profile by about 43.33% when the number is changed from 0.5 to 2.0. On the other hand, 51.61% of the velocity profile is decreased when the number of δ_2 altered from -0.5 to -2.0. The effect of Wu's velocity slip on the temperature distribution reversely behaves, as demonstrated in Figure 4(b) and Figure 5(b). Either the increment of the first-order slip parameter or the decrement of the second-order slip velocity has increased the resistance to the velocity. Thus, the velocity profiles show a negative function with δ_1 and δ_2, respectively, but a positive function with the temperature distribution. Li *et al.,* [35] had observed alike characteristics.

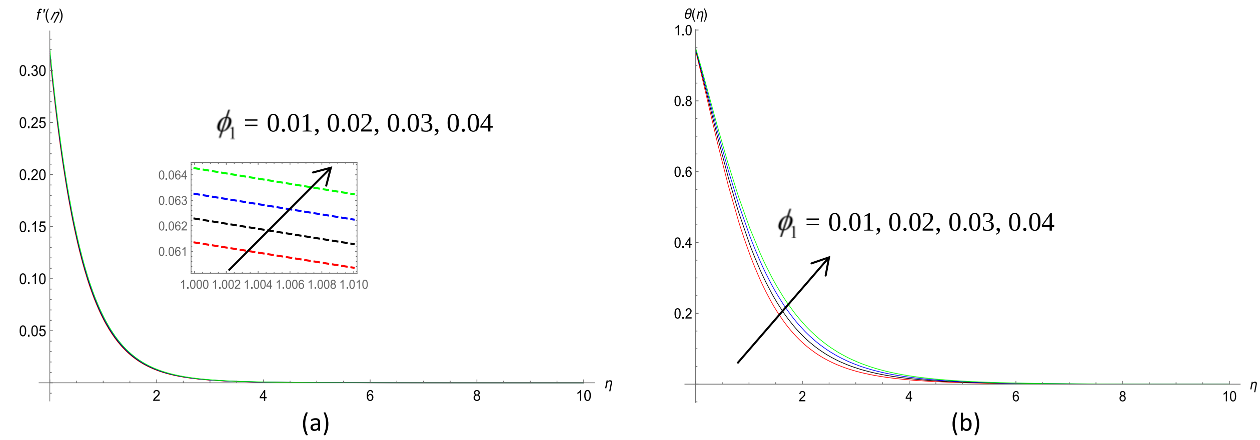
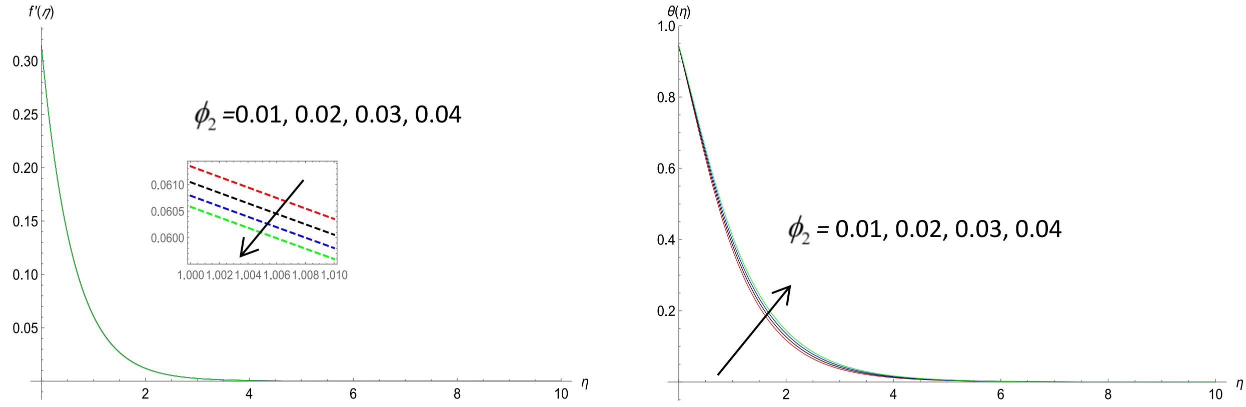
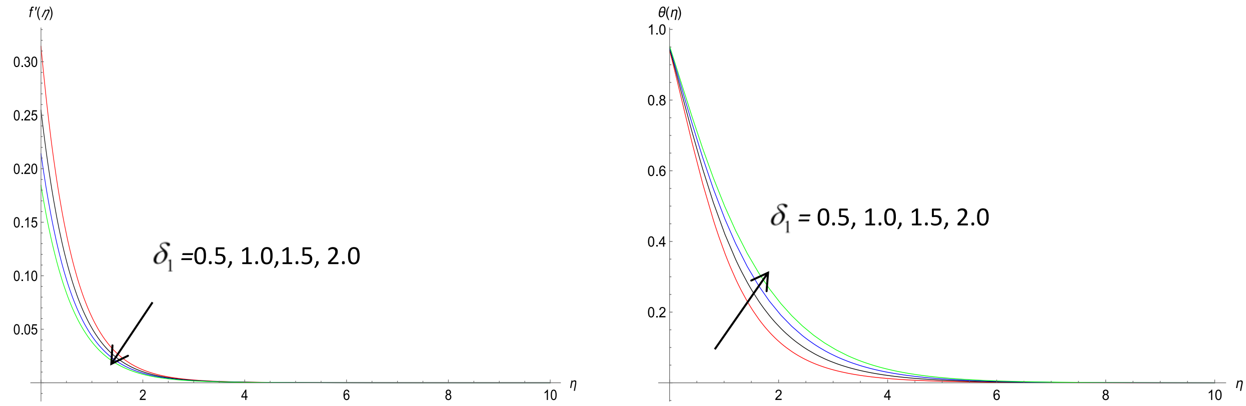


Fig. 2. Profile of different ϕ_1 values with $M=0.1$, $We=0.2$, $\alpha=0.2$, $\delta_2=-0.5$, $\delta_1=0.5$, and $\phi_2=0.01$ (a) Velocity (b) Temperature



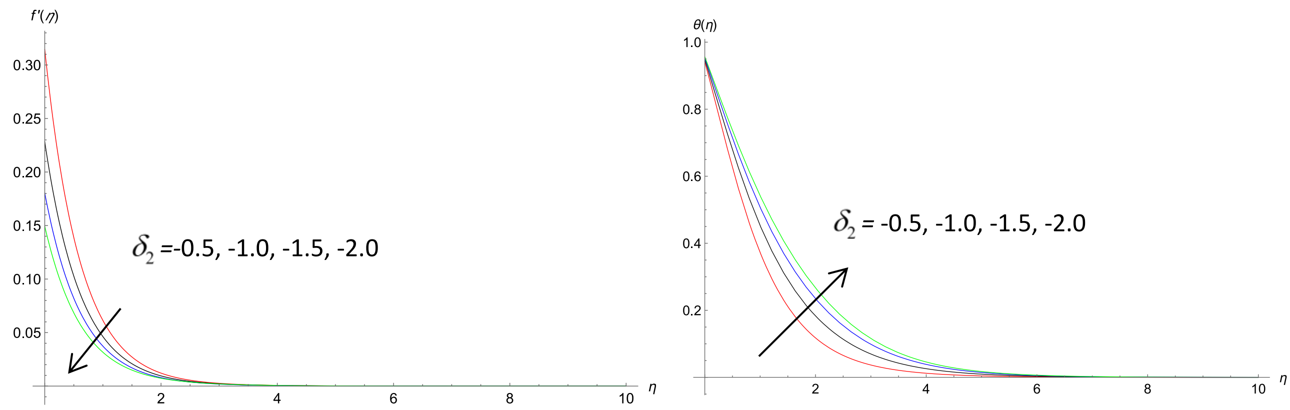
(a) (b)

Fig. 3. Profile for different ϕ_2 values with $M = 0.1$, $We = 0.2$, $\alpha = 0.2$, $\delta_1 = 0.5$, $\delta_2 = -0.5$, and $\phi_1 = 0.01$ (a) Velocity (b) Temperature



(a) (b)

Fig. 4. Distribution for various δ_1 values with $M = 0.1$, $We = 0.2$, $\alpha = 0.2$, $\delta_2 = -0.5$, $\phi_1 = 0.01$, and $\phi_2 = 0.01$ (a) Velocity and (b) Temperature



(a) (b)

Fig. 5. Profile for different δ_2 values with $M = 0.1$, $We = 0.2$, $\alpha = 0.2$, $\delta_1 = 0.5$, $\phi_1 = 0.01$, and $\phi_2 = 0.01$ (a) Velocity (b) Temperature

The effect of the temperature slip parameter is shown in Figure 6. The temperature profile is decreased by 42% when the number of the parameter δ_3 increase from 0.1 to 1.5. A larger value δ_3 reduces the conduction area of the plate surface with the ambient fluid. Thus, the heat transfer from the plate to the fluid is decreased due to the fluid temperature decline [41,42].

Table 5 demonstrates the characteristic of the local skin friction, $C_f \mathrm{Re}_x^{1/2}$ due to the alteration of the embedded parameters. The parameters *M*, *We*, *α* and *δ₁* give a positive function with the local skin friction. Besides that, the decrease of δ_2 from -0.5 to -2.0 has also enhanced the value of $C_f \mathrm{Re}_x^{1/2}$. On the other hand, the value of the local skin friction is reduced by the volume fraction of the nanoparticles ϕ_1 and ϕ_2 respectively. The ϕ_1 and ϕ_2, have enhanced the heat transfer rate as displayed in Table 6. The increase ϕ_1 and ϕ_2 from 0.01 to 0.04 has improved the heat transfer rate by 12.58% and 2.01%, respectively. The nanoparticle silver Ag has much increment because the material has a higher thermal conductivity. The second order Wu's slip parameter δ_2 has diminished the local Nusselt number, $Nu\,\mathrm{Re}_x^{-1/2}$. Similar to the parameter *M*, *We*, *α*, *Ec*, δ_1 and δ_3 decrease the number of the $Nu\,\mathrm{Re}_x^{-1/2}$.

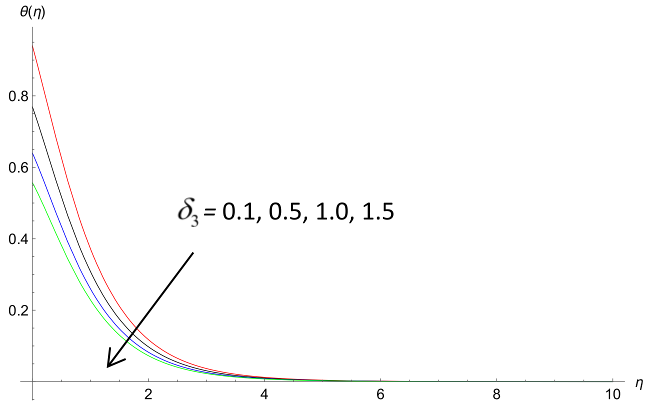


Fig. 6. Temperature profile for different δ_3 values with $M = 0.1$, $We = 0.2$, $\alpha = 0.2$, $\delta_1 = 0.5$, $Ec = 0.2$, $\phi_1 = 0.01$, and $\phi_2 = 0.01$

Table 5
Local Skin Friction for various embedding parameter values

M	*We*	α	δ_1	δ_2	ϕ_1	ϕ_2	*Cf/Re*
0.1	0.2	0.2	0.5	-0.5	0.01	0.01	-0.483065
			1.0				-0.385783
			1.5				-0.320909
			2.0				-0.274632
				-1.0			-0.343395
				-1.5			-0.267990
				-2.0			-0.220133
					0.02		-0.537290
					0.03		-0.594186
					0.04		-0.653895
						0.02	-0.509306
						0.03	-0.536726
						0.04	-0.565388

Table 6
Local Nusselt number for various embedding parameter values

Ec	*We*	α	*M*	δ_1	δ_2	δ_3	ϕ_1	ϕ_2	*Nu/Re*
0.2	0.2	0.2	0.1	0.5	-0.5	0.1	0.01	0.01	0.688776
				1.0					0.635446
				1.5					0.589220
				2.0					0.550761
					-1.0				0.607198
					-1.5				0.546547
					-2.0				0.506790
						0.5			0.534817
						1.0			0.418020
						1.5			0.343094
							0.02		0.718239
							0.03		0.747014
							0.04		0.775416
								0.02	0.693491
								0.03	0.698105
								0.04	0.702640

5. Conclusions

This study analyzed the impact of Wu's slip and Smoluchowski's temperature jump slip on the hybridization tangent hyperbolic nanofluid TiO_2/Ag. A mathematical formulation of the fluid flow over a stretching plate is developed to model the heat transfer fluid (HTF) in a solar collector. The governing partial differential equations have been transformed into nonlinear ordinary differential equations, and we have applied the HAM method to calculate the semi-analytical solutions to the problem. The influence of the embedded essential parameters, such as the first and second-order slip number and Smoluchowski's parameters on the temperature and velocity profile has been explicitly demonstrated and deliberated. Based on the results of this investigation, we can conclude:

i. δ_1 and δ_2, have a negative function with the velocity profile. δ_1 reduces the velocity profile by about 43.33%, while 51.61% of the velocity profile is decreased by δ_2
ii. δ_1 and δ_2, give a positive function with the temperature distribution
iii. δ_3 reduces 42% of the temperature profile
iv. The local skin friction is enhanced by δ_2 and δ_1.
v. The nanoparticles ϕ_1 and ϕ_2 volume fraction, respectively, decline the local skin friction but increase the heat transfer rate.
vi. δ_1, δ_2 and δ_3 diminishes the local Nusselt number.

Overall, these parameters have a significant impact on the behavior of the HTF in the solar collector. The finding in the present study provides valuable insights into the fundamental understanding of fluid dynamics and heat transfer in hybrid nanofluids, which can be used to enhance the efficiency and performance of solar collectors.

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