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Blood Conveying Ferroparticle Flow on a Stagnation Point Over a Stretching Sheet: Non-Newtonian Williamson Hybrid Ferrofluid

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ABSTRACT

The present research investigated the characteristics of convective boundary layer flow and heat transfer of the blood carrying ferroparticle modelled known as the non-Newtonian Williamson hybrid ferrofluid. The fluid flow and the heat transfer of a stagnation point over a stretching surface are considered. The similarity transformation approach is used to reduce the partial differential equation system to an ordinary differential equation. The transformed equations are then solved numerically by Runge-Kutta-Felberg (RKF45) method in Maple software. The flow characteristic and the heat transfer of the non-Newtonian Williamson hybrid nanofluid are tested from various pertinent fluid parameters. The temperature distribution, velocity profiles, as well as variation of the Nusselt number and the skin friction coefficient are analysed and discussed. The study reveals that the non-Newtonian Williamson Hybrid ferrofluid potentially provided better performance in heat transfer capability compared to ferrofluid with the same volume of nanoparticle volume fraction.

1. Introduction

Ferrofluid is one of the magnetically controllable fluids composed of nanoscale ferromagnetic particles in a carrier fluid. It has a long history of usage which was first used to cope with zero gravity in space. The capability of this fluid to transfer heat has been studied by many researchers in the past decade such as Blennerhassett *et al.,* [1] and Barclay [2]. This fluid is designed to be directed in a specific direction using magnetic field strength. This made ferrofluid suitable to be manipulated as the transport medium fluid and as the heat transfer agent to maintain the operating temperature devices. The latest study which investigates heat, thermal radiation, and slip flow of ferrofluid towards various geometry like stagnation, the stretching surface including a flat surface with heat flux and Newtonian heating boundary conditions were by Mohamed *et al.,* [3], Yasin *et al.,* [4], Tlili

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et al., [5], Jamaludin *et al.,* [6], and Hosseinzadeh *et al.,* [7]. Research on the behaviour of ferro-copper oxide/ water hybrid nanofluid was done by Waqas *et al.,* [8] and found that dispersion of both particles improved the heat transfer of water. Zeeshan *et al.,* [9] stated that ferrofluid is used as the heat-controlling agent in the electric motor. Rashad [10] noted that ferrofluid is used in medical treatment for its role in cancer therapy, bleeding, stopping agents, magnetic resonance imaging, and other diagnostic tests.

In performing the medical treatment, the medication and proteins transportation must be delivered to the only targeted tissue. It is prevention to the missed delivery to the healthy tissue which can cause the treatment implication. Specifically, blood is an electrically conductive fluid The electro-magnetic force acting on blood results in a strand of blood movement inside the body [11]. The obstruction of blood flow combined with the efficiency of the dosage targeting is the challenge in these treatment procedures.

Considering the human blood flow, the ordinary Navier-Stokes's equations did not fulfil the characteristics to represent the fluid flow since human blood is a non-Newtonian fluid that acts like a pseudoplastic. To cater for this pseudoplastic characteristic, Navier-Stokes's equations needs to modify. Williamson [12] is the first who modelled the pseudoplastic fluid. Known as the Williamson model, Cramer and Marchello [13] concluded that this model fits the experimental data of polymer solutions and particle suspensions better than other models. The Williamson model was then extended to a thin oblique layer flow, the fluid pulsatile flow in rock fracture, and the peristaltic flow with radially varying MHD, respectively [14-16]. The latest studies of heat transfer of Williamson fluid include the works by Song *et al.,* [17], Asjad *et al.,* [18], and Almaneea [19]. Learning all the above studies, the blood-based Williamson hybrid ferrofluid flow has not been researched.

In several engineering and industrial applications, it is significant to study the stagnation point flow of non-Newtonian fluid. With the knowledge, it can be contributed to designing transpiration cooling and thermal fluid recovery [20]. Stagnation point flows exists on all solid bodies moving in a fluid and located near the stagnation region. Researchers often study this problem with stretching and shrinking surface [20,21]. Haq *et al.,* [22] had study the thermal radiation and slip effects on MHD stagnation point flow of nano-fluid over stretching sheet using RKF45 method and shooting technique to solve coupled ordinary differential equations. Sandeep *et al.,* [23] solved the stagnation-point flow of a Jeffery nano-fluid over a stretching surface with an induced magnetic field and chemical reaction.

To sum up, this research investigates the upgraded fluid called non-Newtonian Williamson hybrid ferrofluid. A small amount of metal nanoparticles, copper (*Cu*) is blended with ferrofluid (Fe_3O_4) together in a non-Newtonian Williamson fluid represented by blood is believed to provide better thermal properties compared to its conventional ferrofluid.

2. Mathematical Formulation

Figure 1 illustrates a steady two-dimensional blood flow on a stagnation point over a stretching sheet with ambient temperature, T_∞ Assuming that u and v are the velocity components along the $x-$ and $y-$axes, respectively.

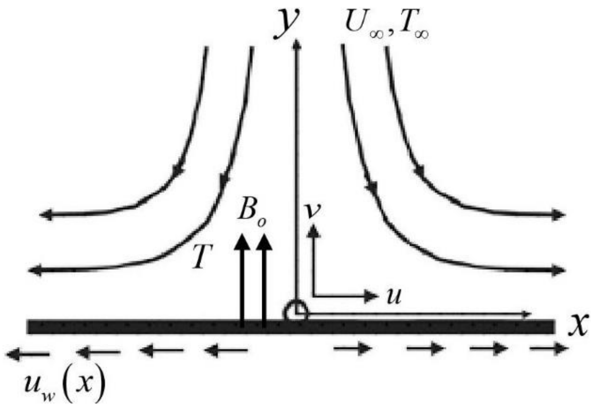

Fig. 1. Physical Model and the coordinate system

Next, the stretching velocity $u_w(x) = ax$ and the free stream velocity $U_\infty = bx$ are assumed in a linear form where a and b are positive constants [24]. The Navier-Stoke equations are governed as follows [3,25]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U_\infty \frac{dU_\infty}{dx} + \nu_{hnf}\frac{\partial^2 u}{\partial y^2} + \sqrt{2}\ \nu_{hnf}\ \Gamma \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2(x)}{\rho_{hnf}}(u - U_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{hnf}}{\left(\rho C_p\right)_{hnf}}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subjected to boundary conditions

$$\begin{aligned} &u = u_w,\ v = 0,\ T = T_w \text{ at } y = 0,\\ &u = U_\infty,\ T \to T_\infty \text{ as y} \to \infty. \end{aligned} \tag{4}$$

The hybrid ferrofluid kinematic viscosity, dynamic viscosity, and density are denoted as $\nu_{hnf},$ μ_{hnf} and ρ_{hnf} respectively. T is the temperature inside the boundary layer, $(\rho C_p)_{hnf}$ is the heat capacity of hybrid ferrofluid and k_{hnf} is the thermal conductivity of Williamson hybrid ferrofluid. Next, B_0 is the uniform magnetic field strength and σ is the electric conductivity. Other properties related to base fluid and the nanoparticles are denoted with subscript $_{bf}$ and $_{s1,s2}$ respectively [26]

$$\begin{aligned} &\nu_{hnf} = \frac{\mu_{hnf}}{\rho_{hnf}}, \quad \rho_{hnf} = (1-\phi_2)\left[(1-\phi_1)\rho_f + \phi_1\rho_{s1}\right] + \phi_2\rho_{s2}, \quad \mu_{hnf} = \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}},\\ &\left(\rho C_p\right)_{hnf} = (1-\phi_2)\left[(1-\phi_1)\left(\rho C_p\right)_f + \phi_1\left(\rho C_p\right)_{s1}\right] + \phi_2\left(\rho C_p\right)_{s2},\\ &\frac{k_{hnf}}{k_{bf}} = \frac{k_{s2} + 2k_{bf} - 2\phi_2(k_{bf} - k_{s2})}{k_{s2} + 2k_{bf} + \phi_2(k_{bf} - k_{s2})}, \quad \frac{k_{bf}}{k_f} = \frac{k_{s1} + 2k_f - 2\phi_1(k_f - k_{s1})}{k_{s1} + 2k_f + \phi_1(k_f - k_{s1})}, \end{aligned} \tag{5}$$

where ϕ_1, ϕ_2 are the nanoparticles volume fractions for Fe_3O_4 and Cu respectively. Noticed that Eq. (1) to Eq. (3) are non-linear partial differential equations that consist of many dependent and independent variables. It is also in dimensional forms which is difficult to solve directly. Therefore, the similarity transformation approach is applied

$$\eta = \left(\frac{b}{\nu_f}\right)^{1/2} y, \quad \psi = \left(b\nu_f\right)^{1/2} xf(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{6}$$

Eq. (6) shows the similarity variables where η and θ is a non-dimensional variable while ψ is the stream function. The similarity variables (6) satisfy the Eq. (1) by definition

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{7}$$

Next, the Eq. (6) to Eq. (7) are substituted into governing Eq. (2) to Eq. (3), thus gives the following transformed ordinary differential equations

$$\frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left[(1-\phi_2)+\left[(1-\phi_1)+\phi_1(\rho_{s1}/\rho_f)\right]+\phi_2(\rho_{s2}/\rho_f)\right]}(f''' + \lambda f'' f''') + ff'' - f'^2 + 1 - M(f'-1) = 0 \tag{8}$$

$$\frac{k_{hnf}/k_f}{(1-\phi_2)\left[(1-\phi_1)+\phi_1(\rho C_p)_{s1}/(\rho C_p)_f\right]+\phi_2(\rho C_p)_{s2}/(\rho C_p)_f}\theta'' + \Pr f\theta' = 0. \tag{9}$$

The boundary conditions become

$$\begin{aligned} &f(0) = 0, \; f'(0) = \varepsilon, \; \theta(0) = 1, \\ &f'(\eta) \to 1, \; \theta(\eta) \to 0, \; as \; y \to \infty \end{aligned} \tag{10}$$

By definition, $\Pr = \frac{\nu_f\left(\rho C_p\right)_f}{k_f}$ is a Prandtl number that is set to 21 corresponds to blood, $\varepsilon = \frac{a}{b}, (\varepsilon > 0)$ is a stretching parameter, $\lambda = x\Gamma\sqrt{\frac{2b^3}{\nu_f}}$ is the non-Newtonian Williamson fluid parameter and $M = \frac{\sigma B_o^2(x)}{b\rho_{nf}}$ the magnetic parameter [27-29]. The physical quantities interested are the skin friction coefficient C_f and local Nusselt number Nu_x which are given by

$$C_f = \frac{\tau_w}{\rho_f U_\infty^2}, \quad Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \tag{11}$$

with the surface shear stress τ_w and the surface heat flux q_w given by

$$\tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}+\frac{\Gamma}{\sqrt{2}}\left(\frac{\partial u}{\partial y}\right)^2\right)_{\bar{y}=0}, \quad q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}.$$

Using variables in Eq. (6) and Eq. (12) give

$$C_f \operatorname{Re}_x^{1/2} = \frac{1}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}\left(f''+\frac{\lambda}{2}f''^2\right) \text{ and } Nu_x \operatorname{Re}_x^{-1/2} = -\frac{k_{hnf}}{k_f}\theta'(0) \tag{12}$$

where $\operatorname{Re}_x = \frac{U_\infty x}{\nu_f}$ is the Reynold's number.

3. Results

Using the Runge-Kutta-Felberg (RKF45) method in Maple software, the non-linear ordinary differential Eq. (8) and Eq. (9) with boundary conditions (10) were solved numerically. The effect of physical parameters which are the stretching parameter ε, magnetic parameter M, and non-Newtonian Williamson fluid parameter λ on the velocity profiles, the temperature profiles, the reduced skin friction coefficient $C_f \operatorname{Re}_x^{1/2}$, and the reduced Nusselt Number $Nu_x \operatorname{Re}_x^{-1/2}$ were analysed. The thermophysical properties of blood, magnetite (*Fe_3O_4*), and copper (*Cu*) are shown in Table 1. Table 2 shows the comparison of the present result with previously reported numerical results to validate the numerical method accuracy. It is found that the reduced skin friction $C_f \operatorname{Re}_x^{1/2}$ is in good agreement with the previously published.

Table 1
Thermophysical properties of blood, magnetite, and copper nanoparticles

Physical Properties	Blood	Magnetite (Fe_3O_4), ϕ_1	Copper (Cu), ϕ_2
ρ (kg/m^3)	1053	5180	8933
C_p (J/kg·K)	3594	670	385
k(W/m·K)	0.492	9.7	400

Table 2
Comparison values of $C_f \operatorname{Re}_x^{1/2}$ with previously published results when $\varepsilon = \phi_2 = \lambda = 0$ and $\Pr = 6.2$

ϕ_1	Yasin *et al.,* [30]				Present paper			
	M=1	M=2	M=5	M=10	M=1	M=2	M=5	M=10
0.01	1.638754	1.936626	2.636552	3.505884	1.638704	1.936603	2.636534	3.505861
0.1	2.154755	2.546461	3.466805	4.609891	2.154733	2.546441	3.466780	4.609858
0.2	2.841629	3.358193	4.571913	6.079379	2.841595	3.358167	4.571881	6.079338

Figure 2 and Figure 3 show the distributions of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ for various values of *M*. Four types of fluid are used which are blood ($\phi_1 = \phi_2 = 0$), 0.1 vol. of *Fe3O4*/blood ferrofluid ($\phi_1 = 0.1, \phi_2 = 0$), 0.16 vol. of *Fe3O4-Cu*/blood hybrid ferrofluid ($\phi_1 = 0.1, \phi_2 = 0.06$), and 0.16 vol. of *Fe3O4*/blood ferrofluid ($\phi_1 = 0.16, \phi_2 = 0$). It can be seen that for both figures, the increase of *M* and the volume of nanoparticles results to the increase of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$. *Fe3O4-Cu*/blood hybrid ferrofluid produces the highest values of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ compared to other types of fluid while blood-based fluid produces the lowest $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ value. The high values of $Nu_x \mathrm{Re}_x^{-1/2}$ in 0.16 vol. of *Fe3O4-Cu*/blood hybrid ferrofluid contributed by the high thermal conductivity of copper. Mathematically, it is concluded that the values of $Nu_x \mathrm{Re}_x^{-1/2}$ may be increased by adjusting the portion of nanoparticle volume fraction of copper larger than the magnetite.

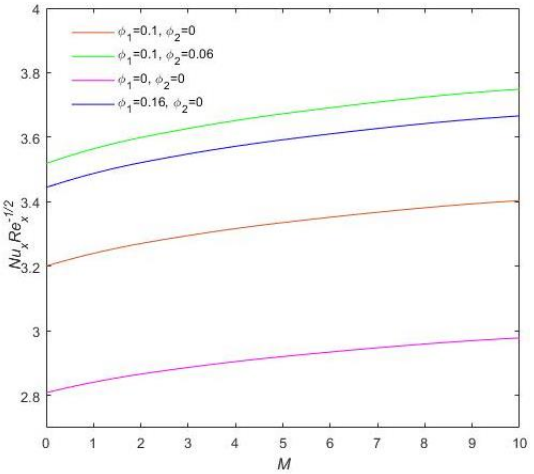

Fig. 2. Distribution of $Nu_x \mathrm{Re}_x^{-1/2}$ for various values of M when $\mathrm{Pr} = 21,\ \lambda = 1,\ \varepsilon = 0.5$

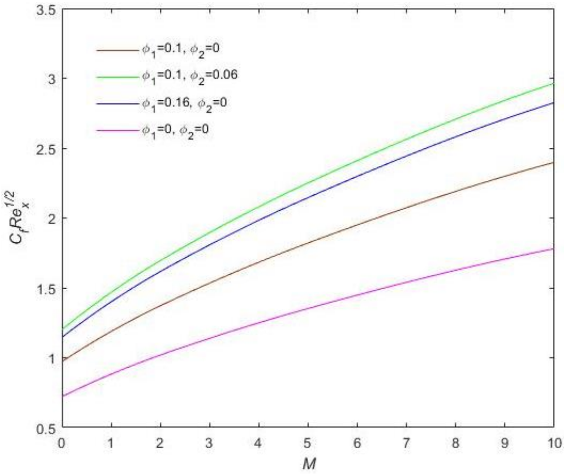

Fig. 3. Distribution of $C_f \mathrm{Re}_x^{1/2}$ for various values M when $\mathrm{Pr} = 21,\ \lambda = 1,\ \varepsilon = 0.5$

The magnetic parameter M effects on a value of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ are tabulated in Table 3. It is found that both quantities increase as M increases. The ferroparticles in fluid are attracted to a magnetic field at the plate surface thus raising the values of $C_f \mathrm{Re}_x^{1/2}$. Next, it is found that the velocity boundary layer thickness is reduced when M is increased due to a drag force known as Lorentz force which retard the fluid flow [31]. The velocity profiles for various values of M are illustrated in Figure 4. Physically, the reduction in velocity boundary layer thickness refers to the increase in velocity gradient thus supporting the increase in $C_f \mathrm{Re}_x^{1/2}$ discussed in Table 3.

Table 3
The values of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ for various values of M when $\mathrm{Pr} = 21, \lambda = 1, \varepsilon = 0.5$

M	$Nu_x \mathrm{Re}_x^{-1/2}$	$C_f \mathrm{Re}_x^{1/2}$
0	3.518923	1.200826
1	3.564319	1.467737
2	3.599269	1.695528
5	3.672800	2.251115

10	3.748940	2.965030

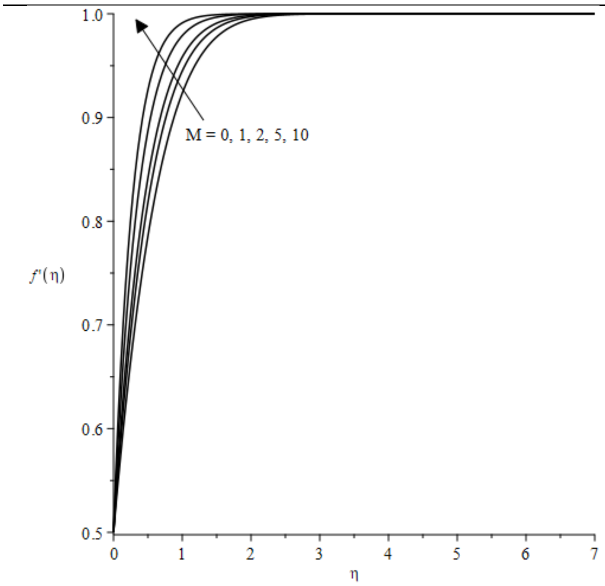

Fig. 4. Velocity profiles for various values of M when $\Pr = 21,\ \lambda = 1,\ \varepsilon = 0.5$

The values of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ for various values of the Williamson fluid parameter, λ are shown in Table 4. It is found that λ gives a small effect on $Nu_x \mathrm{Re}_x^{-1/2}$ as agreed by Hashim *et al.,* [25]. Meanwhile, the values of $C_f \mathrm{Re}_x^{1/2}$ increases with λ, as the fluid becomes more resistant to flow [32]. The non-Newtonian Williamson fluid parameter represents the ratio of relaxation to retardation time [33]. Theoretically, the rise of λ reduced the retardation time which gave a reduction on the velocity profile while increasing the boundary layer thickness. Analysing the value changes on λ in Table 4, it can be concluded that temperature and velocity profile only show small changes in a boundary layer thicknesses.

Table 4
The values of $Nu_x \mathrm{Re}_x^{-1/2}$ and $C_f \mathrm{Re}_x^{1/2}$ for various values of λ when $\Pr = 21,\ \mathrm{M} = 0.5,\ \varepsilon = 0.5$

λ	$Nu_x \mathrm{Re}_x^{-1/2}$	$C_f \mathrm{Re}_x^{1/2}$
0.1	3.543318676	1.340459
0.2	3.537956325	1.358710
0.3	3.533027028	1.376116
0.4	3.528465171	1.392782

Next, Figure 5 and Figure 6 show the temperature profiles and the velocity profiles for various values of stretching parameter, ε. From Figure 5, it is shown that the thermal boundary layer flow is decreasing when ε increases. Increasing in ε reflect to a domination of-the stretching velocity ax towards the free stream velocity bx that causes the thinning of temperature boundary layer thickness which is agreeable with Mohamed *et al.,* [34]. Lastly, in Figure 7, it is concluded that when $\varepsilon < 1$, the free stream velocity is larger than the stretching velocity of the surface causes the flow to have a boundary layer structure. Meanwhile, when $\varepsilon > 1$ the boundary layer flow becomes inverted, and the velocity boundary layer thickness decreases with ε [35].

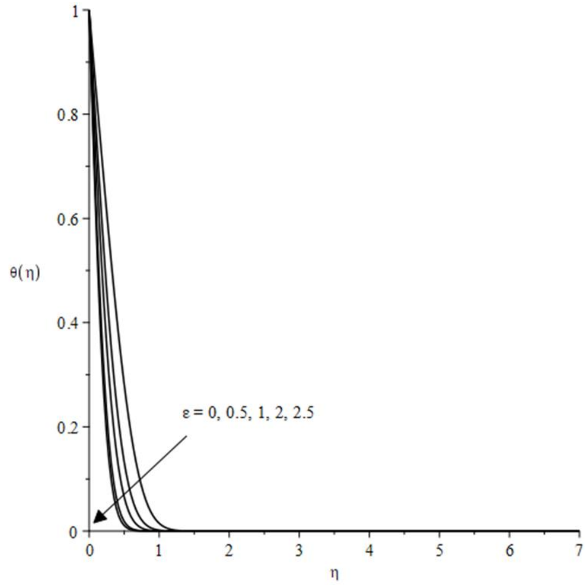

Fig. 5. Temperature profile for various values of ε when $\Pr = 21$, M=0.5, $\lambda = 0.1$

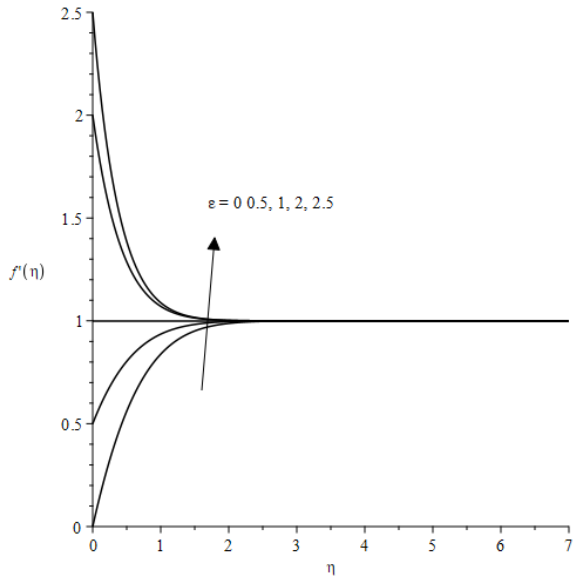

Fig. 6. Velocity profile for various values of ε when $\Pr = 21$, M=0.5, $\lambda = 0.1$

4. Conclusions

Convective boundary layer flow and the heat transfer of non-Newtonian Williamson hybrid ferrofluid on a stagnation point towards a stretching sheet were numerically studied. It was shown how the magnetic parameter M, the non-Newtonian fluid parameter λ, the stretching parameter ε and the nanoparticles volume fractions ϕ_1, ϕ_2 for Fe_3O_4 and *Cu* affect the temperature profiles, velocity profiles as well as the Nusselt number $Nu_x \mathrm{Re}_x^{-1/2}$, and the skin friction $C_f \mathrm{Re}_x^{1/2}$. The summary that can be concluded in this research are as follows

i. The non-Newtonian Williamson Hybrid ferrofluid potentially provided better performance in heat transfer capability compared to ferrofluid with the same volume of nanoparticle volume fraction.
ii. The Lorentz force which retarded the fluid flow has increased the skin friction coefficient as the magnetic parameter increases.
iii. The values of a Nusselt number increase as the magnetic and the stretching parameter increases.
iv. The thermal boundary layer thickness decreases when the stretching parameter increases.
v. The velocity boundary layer thickness decreases when the magnetic parameter increases.

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