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Unsteady MHD Casson Fluid Through a Porous Medium over an Inclined Plate in the Present of Chemical Reaction

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ABSTRACT

The unsteady of MHD Casson fluid flow through an infinite inclined plate in the presence of chemical reaction is investigated. In this research, Casson fluid model is chosen to be used in this study to characterize the non-Newtonian fluid behaviour. We are employed the Laplace transform technique with an appropriate boundary condition to convert the governing partial differential equations into ordinary differential equations. All the transformed equations are then solved numerically by using Mathematica. Finally, we were obtained the exact solution of momentum, energy and concentration of the cases. The results of flow features for different values of the governing parameters, unsteadiness parameter, Casson parameter are analysed in graphs and has been discussed in the result section. The results presented that fluid velocity rises with the increment of magnetic parameter due to the present of Lorentz force. Lorentz force helps in reducing heat for electronic system and radiators. Furthermore, the increasing of inclination angle and chemical reaction make the velocity increases. The result for Casson fluid explained that this fluid helps to lower the resistance of the yield stress so that the velocity become higher.

1. Introduction

Non-Newtonian fluids has captured attention due to its applications in various industries such as ground water hydrology, petroleum reservoirs, solid matrix heat, chemical catalytic reactors, transpiration cooling, design of solid matrix heat and geothermal energy production. The difference between Newtonian fluid and non-Newtonian fluid is nonlinear relation between stress and strain rate which make non-Newtonian fluids are more complicated than Newtonian fluids. Casson fluid is one of the non-Newtonian fluids that popular among researchers where this fluid is a shear thinning fluid which assumed to have an infinite rate of shear and a yield stress below which no flow occurs and a zero viscosity an infinite rate of shear. Mukhopadhyay *et al.*, [1] were motivated to investigated

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the unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface having a prescribe surface temperature using Casson fluid model to characterize the non-Newtonian fluid behaviors. They are using shooting method to solve the governing equations. They found out that the effect of increasing values of Casson parameter is to suppress the velocity field but the temperature is enhanced with the increasing of Casson parameter. Shehzad *et al.*, [2] investigated the effect of mass transfer in the magnetohydrodynamic flow of a Casson fluid over a porous stretching sheet in the presence of a chemical reaction and suction. They observed that the Casson parameter and Hartmann number have similar effects on the velocity in a qualitative sense and the concentration profile decreases rapidly in comparison to the fluid velocity when they increased the values of the suction parameter. Later on, Ullah *et al.*, [3] explored the effects of first order chemical reaction and thermal radiation on mixed convection flow of Casson fluid in the presence of magnetic field and the flow is generated due to unsteady nonlinearly stretching sheet placed inside a porous medium. The results show the fluid velocity rises with increase in radiation parameter in the case of assisting flow and is opposite in the case of opposing fluid while radiation parameter has no effect on fluid velocity in the forced convection. Aneja *et al.*, [4] were attracted to analyse numerically the phenomenon of natural convection in a square porous cavity containing Casson fluid. They obtained that an increase in Casson fluid parameter and Darcy number, heat transfer and flow circulation increase. Raju *et al.*, [5] discussed the effects of induced magnetic field and homogeneous-heterogeneous reactions on stagnation flow of Casson fluid.

Magnetohydrodynamics or hydro-magnetics (MHD) is the field of dynamic in the presence of magnetic characteristics and impact of electrically conducting liquids which has significant applications in engineering and biomedical sciences. The examples of MHD are liquid metals, plasma, electrolytes and salt water. Recently, hydromagnetic flows and heat transfer in a porous medium have been considered extensively due to their occurrence in several engineering processes such as filtration of liquid metals, compact heat exchangers, cooling of nuclear reactors, metallurgy, casting and fusion control. Raju *et al.*, [6] studied, the steady magnetohydrodynamic (MHD) forced convective flow of a viscous fluid of finite depth in a saturated porous medium over a fixed horizontal channel with thermally insulated and impermeable bottom wall in the presence of viscous dissipation and joule heating. The mixed boundary layer flow and heat of an MHD viscous fluid due to a nonlinearly deforming body subjected to a uniform magnetic field with heat generation or absorption has been analyzed by Turkyilmazoglu [7]. He concluded that the main physical implication of the results is that both momentum and temperature layers are thinned with strong magnetic fields. Furthermore, upper branch solutions are more cooled leading to higher heat transfer rates compared to the lower branches. Another studied by Muhammad *et al.*, [8] considered electrical conducting viscous fluid flow over a curved surface with second order slip. They obtained that the velocity is decreasing function of first slip parameter while both Bejan number and entropy generation is upsurged versus heterogeneous reaction parameter. Khan *et al.*, [9] were discussed about the mathematical modelling of physical problem focusing on two dimensional magnetohydrodynamics Prandtl fluid flow over a stretching sheet with stratification and heat generation. It is clear from literature review that thermal stratification cools down the fluid temperature while heat generation rises it. Bhatti *et al.*, [10] examined three-dimensional unsteady MHD boundary layer flow of viscous nanofluid having gyrotactic microorganism through a stretching porous cylinder with thermal radiation and chemical reaction. They witnessed that the velocity of the fluid diminishes for large values of magnetic parameter and porosity parameter while radiation effects show an increment in temperature profile.

The study of heat and mass transfer under the effects of chemical reaction has significant importance in various applications in science and engineering such as chemical engineering process,

evaporation at the surface of water body and transfer of energy in a wet cooling tower. Arshad *et al.*, [11] were analyzed the flow of viscous nanofluid with heat and mass transmission above an unsteady infinite porous surface with chemical reaction. The results shows when Prandtl number increase, Sherwood and Nusselt number also increases while Brownian motion reduces the Sherwood number. Moreover, viscoelastic parameter increases the skin friction and decreases the Nusselt number. Also, Afikuzzaman *et al.*, [12] constructed the study about the MHD free convection and heat transfer fluid flow through a semi-infinite vertical porous plate with the effect of chemical reaction. They observed that an increases in Grashof number is to increase the velocity distributions but by increasing the magnetic parameter which reduces the velocity profiles whereas increasing the heat generation parameter which increase the temperature profile. In the other hand, Reddy *et al.*, [13] investigated the boundary layer flow of nanofluids and heat transfer over a nonlinear stretching sheet with magnetic field, chemical reaction in presence of suction/injection. They conclude that the nanoparticle concentration increases with the increasing values of magnetic field while it experienced a faster decreasing in concentration at all point with the increasing values of chemical reaction parameter. Sehra *et al.*, [14] examined the heat mass transfer and MHD flow over a vertical plate with chemical reaction, arbitrary shear stress and exponential heating.

The goal of the recent research is to analyzed about the unsteady Casson fluid through a porous medium over inclined plate in the present of chemical reaction electrically conducting heat fluid near an infinite inclined plate. The main objective of this research is to study about the behaviour of Casson fluid is because it is one of the popular fluids that researcher attracted to study about recently due to its wide applications in industry. To our knowledge, no study has been carried out yet to investigate about this. The exact solutions of momentum, energy and concentration equations, under Boussinesq approximation were obtained by Laplace transform technique. The variations in fluid velocity, fluid temperature and concentration were shown graphically with numerical result.

2. Mathematical Formulation

An unsteady flow of Casson fluid in the presence of chemical reaction electrically conducting heat fluid near an infinite inclined plate is considered. Magnetic field B is applied to the plate and a first-order chemical reaction effect is assumed to occur in this study. In the unsteady boundary layer equations governing the Casson fluid, radiation heat transfer and concentration fields can be written in dimensional form as shown below:

$$\frac{\partial u^*}{\partial t^*} = \nu \left(1 + \frac{1}{\gamma} \right) \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho} u^* - \frac{\nu}{K^*} u^* + g \beta_T \cos \varphi (T^* - T_\infty^*) + g \beta_C \cos \varphi (C^* - C_\infty^*) \quad (1)$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C^*}{\partial t^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} - K_r (C^* - C_\infty^*) \quad (3)$$

where u^* , T^* , C^* are velocity, temperature and concentration respectively. ν is kinematic viscosity, γ is Casson fluid, σ is the electrical expansion, ρ is the fluid density, K^* is the permeability of the porous medium, g is the gavity and its assume to be zero, β_T and β_C are the thermal expansion and

concentration expansion, k is the thermal conductivity, c_p is the specific heat, q_r is the radiative heat flux, D is the mass diffusion and K_r is the chemical reaction parameter.

The approximate boundary conditions are defined as

$$\begin{aligned} u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \text{ for } y^* \geq 0, \text{ and } t^* \leq 0 \\ u^* = 0, T^* = T_w^*, C^* = C_w^* \text{ for } y^* = 0 \text{ and } t^* > 0 \\ u^* \rightarrow 0, T^* \rightarrow T_\infty^*, C^* \rightarrow C_\infty^* \text{ for } y^* \rightarrow \infty \text{ and } t^* \geq 0 \end{aligned} \quad (4)$$

In the above boundary conditions, at time $t^* \leq 0$, the plate and the fluid initially at rest with the same temperature T_∞^* and C_∞^* . When $t^* > 0$, both temperature and concentration is raised to constant temperature and concentration, T_w^* and C_w^* . Temperature and concentration are approaching to zero when $t^* \geq 0$.

The dimensionless variables are introducing as follows:

$$y = \frac{y^*}{L}, t = \frac{t^* (vg)^{1/3}}{L}, u = \frac{u^*}{(vg)^{1/3}}, T = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*} \quad (5)$$

The temperature of the plates T_∞^* and T_w^* are supposed to produce radiative heat flux term and simplified using Rosseland approximation as shown below

$$\frac{\partial q_r}{\partial y} = -\frac{4\sigma^*}{3K^*} \frac{\partial T^{*4}}{\partial y^*} \quad (6)$$

where σ^* represent Stefan-Boltzmann constant and K^* represent mean absorption coefficient. The temperature differences within the flow are assume to be adequately small such that T^{*4} and it can be expressed as a linear function of the temperature. Taylor series is used to expand T^{*4} to T_∞^* and neglecting higher-order terms, hence

$$T^{*4} \cong 4T_\infty^* T - 3T_\infty^4 \quad (7)$$

Rosseland approximation become,

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^*}{3K^*} \frac{\partial^2 T^{*4}}{\partial y^{*2}} \quad (8)$$

Eq. (1), Eq. (2) and Eq. (3) are then reduced to dimensionless form by using Eq. (5) and they become:

$$\frac{\partial u}{\partial t} = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - M.u - \frac{u}{K} + G_r T + G_c C \quad (9)$$

$$\frac{\partial T}{\partial t} = \frac{(1+R)}{Pe} \frac{\partial^2 t}{\partial y^2} \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Pe_c} \frac{\partial^2 C}{\partial y^2} - KrC \quad (11)$$

where

$$M = \frac{\sigma B_0^2}{\mu} \left(\frac{v}{\sqrt{g}} \right)^{4/3}, \quad G_r = \cos \phi \beta_T (T_w^* - T_\infty^*), \quad G_c = \cos \phi \beta_c (C_w^* - C_\infty^*)$$

$$R = \frac{16\alpha\sigma^* T_\infty^* L^2}{k}, \quad Pe = RePr, \quad Pe_c = ReSc$$

From the above equation, M is magnetic parameter known as Hartmann number, K is the porosity parameter, G_r is the thermal Grashof number, G_c is the mass Grashof number, R is radiation parameter Pe known as Peclet's number of mass transfer, Pe_c is known as Peclet's number of concentrations, Re is Reynold number, Pr is Prandtl number and Sc is Schmidt number. The characteristic length can be defined as:

$$L = \frac{v^{2/3}}{g^{1/3}} \quad (12)$$

The initial and boundary conditions reduced to dimensionless form as follows:

$$\begin{aligned} u=0, T=0, C=0 \text{ for } y \geq 0 \text{ and } t \leq 0 \\ u=0, T=1, C=1 \text{ for } y=0 \text{ and } t > 0 \\ u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ for } y \rightarrow \infty \text{ and } t \geq 0 \end{aligned} \quad (13)$$

2.1 Solution of the Problem

The energy Eq. (2) and Eq. (3) is uncoupled from the momentum Eq. (1). Therefore, temperature variable $T(y,t)$ and concentration variable $C(y,t)$ can be solve where the solution $u(y,t)$ can be achieved. In order to solve the equations, Laplace transform is applied to Eq. (11), Eq. (12) and Eq. (13) with respect to t with the present of Eq. (16) and solving the result from different equations, we obtained:

Energy Equation

$$\bar{T} = \frac{1}{s} e^{-y\sqrt{\frac{s(Pe)}{1+R}}} \quad (14)$$

$$\bar{C} = \frac{1}{s} e^{-y\sqrt{Pe_c(Kr+s)}} \quad (15)$$

Momentum Equation

$$\begin{aligned} \bar{u} = & a_1 \left[\frac{1}{a_2(s-a_2)} - \frac{1}{a_2s} \right] e^{-y\sqrt{Prs+R}} + a_3 \left[\frac{1}{a_4(s-a_4)} - \frac{1}{a_4s} \right] e^{-y\sqrt{(Kr+s)Sc}} - a_1 \left[\frac{1}{a_2(s-a_2)} - \frac{1}{a_2s} \right] e^{-y\sqrt{\frac{\lambda+s}{n}}} \\ & - a_3 \left[\frac{1}{a_4(s-a_4)} - \frac{1}{a_4s} \right] e^{-y\sqrt{n}} \end{aligned} \quad (16)$$

where

$$a_1 = \frac{-Gr \cos \phi}{\left(\frac{Pe}{1+R} - 1\right)}, \quad a_2 = \frac{\lambda(1+R)}{Pe-1}, \quad a_3 = \frac{-Gc \cos \phi}{(1-ReSc)}, \quad n = 1 + \frac{1}{\gamma}$$

Hence, we obtained the exact solution for velocity, temperature and concentration from Eq. (14), Eq. (15), and Eq. (16) by using inverse Laplace transform. These solutions are:

$$T = \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{\theta}{t}} \right) \quad (17)$$

$$C = \frac{1}{2} \left[e^{y\sqrt{ReScKr}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{ReSc}{t}} \right) + \sqrt{Krt} + e^{-y\sqrt{ReScKr}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{ReSc}{t} - \sqrt{Krt}} \right) \right] \quad (18)$$

$$U = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 \quad (19)$$

where

$$u_1(y,t) = \frac{a_1 e^{a_2 t}}{2a_2} \left[e^{y\sqrt{\theta a_2}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{\theta}{t}} + \sqrt{a_2 t} \right) + e^{-y\sqrt{\theta a_2}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{\theta}{t}} - \sqrt{a_2 t} \right) \right]$$

$$u_2(y,t) = \frac{a_1}{a_2} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{Pr}{t}} \right)$$

$$u_3(y,t) = \frac{a_3 e^{a_4 t}}{2a_4} \left[e^{y\sqrt{Pe_m(Kr+a_4)}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{Pe_m}{t} + \sqrt{(Kr+a_4)t}} \right) + e^{-y\sqrt{Pe_m(Kr+a_4)}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{Pe_m}{t} - \sqrt{(Kr+a_4)t}} \right) \right]$$

$$u_4(y,t) = \frac{a_3}{2a_4} \left[e^{y\sqrt{Pe_m Kr}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{Pe_m}{t} + \sqrt{Krt}} \right) + e^{-y\sqrt{Pe_m Kr}} \operatorname{erfc} \left(\frac{y}{2} \sqrt{\frac{Pe_m}{t} - \sqrt{Krt}} \right) \right]$$

$$u_5(y,t) = \frac{a_1 e^{a_2 t}}{2a_2} \left[e^{y\sqrt{\lambda+a_4}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda+a_2)t} \right) + e^{-y\sqrt{\lambda+a_4}} \operatorname{erfc} \left(-\frac{y}{2\sqrt{t}} - \sqrt{(\lambda+a_2)t} \right) \right]$$

$$u_6(y,t) = \frac{a_1}{2a_2} \left[e^{y\sqrt{\lambda}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t} \right) + e^{-y\sqrt{\lambda}} \operatorname{erfc} \left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t} \right) \right]$$

$$u_7(y,t) = \frac{a_3 e^{a_4 t}}{2a_4} \left[e^{y\sqrt{\lambda+a_4}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{(\lambda+a_4)t}\right) + e^{-y\sqrt{\lambda+a_4}} \operatorname{erfc}\left(-\frac{y}{2\sqrt{t}} - \sqrt{(\lambda+a_4)t}\right) \right]$$

$$u_8(y,t) = \frac{a_3}{2a_4} \left[e^{y\sqrt{\lambda}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} + \sqrt{\lambda t}\right) + e^{-y\sqrt{\lambda}} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} - \sqrt{\lambda t}\right) \right]$$

$\operatorname{erfc}(x)$ being the complimentary error function defined by

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x), \quad \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-\eta^2} d\eta$$

3. Results and Discussion

The results from the previous section are presented to understanding clearly the physical insight and graphical interpretation. While keeping $\gamma = 0.5$, $R = 0.3$, $m = 1.0$, $Gr = 2.0$, $Gm = 2.0$, $M = 0.5$ and $\phi = \pi/4$. All these values considered constant unless they are cited [15].

Figure 1 shows the velocity profile with magnetic parameter. The graph showed the velocity profiles increase when the magnetic parameter increases. However, the outlines are very closely to describe the slow growth of velocity. This is happened due to the application of the transverse magnetic field that give a resistive type of force called Lorentz force. Therefore, it enhances the thickness of boundary layers of momentum and thermal region which helps to cool the electronic systems and radiators and it slow down the growth of velocity.

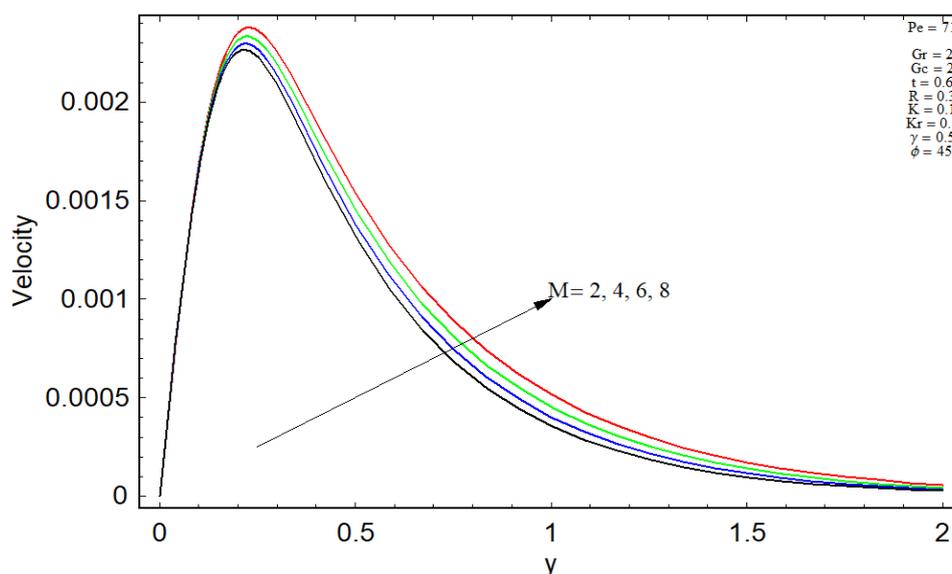


Fig. 1. The influence of magnetic parameter, M on the velocity profile

Figure 2 illustrate the graph of the fluid velocity with the variation values of chemical reaction. As we can see from the graph, the velocity is rising with the increasing of chemical reaction. When the chemical reaction increase, the boundary layer become thin due to the effect of hydrodynamic and concentration along the way.

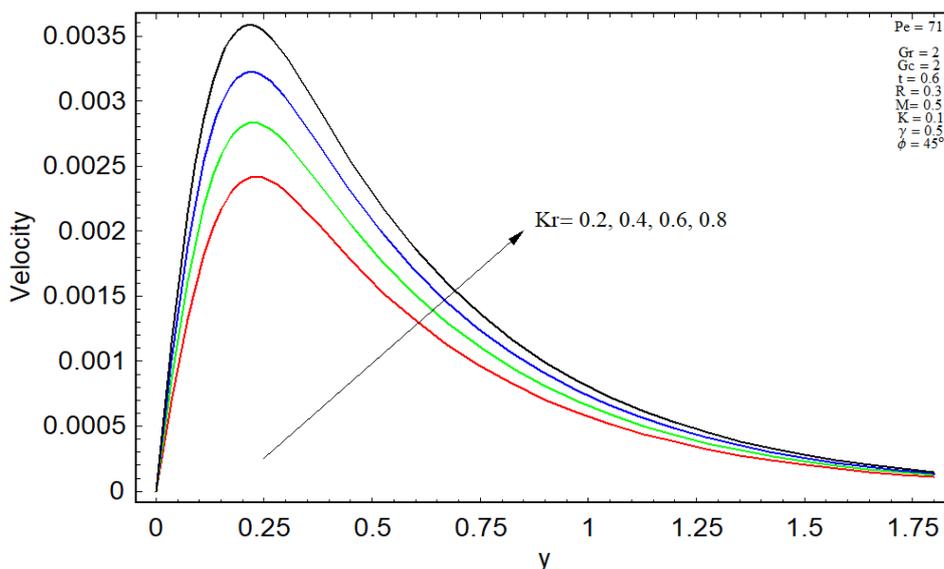


Fig. 2. The influence of chemical reaction, Kr on the velocity profile

Figure 3 shows the velocity increase with the increasing of inclination angle. It happened due to the increasing of inclination angle accelerates the fluid motion along the plate. When the plate is inclined from the vertical, the buoyancy force is affected as well due to the thermal and mass diffusion reduced as $\cos \phi$ decreased.

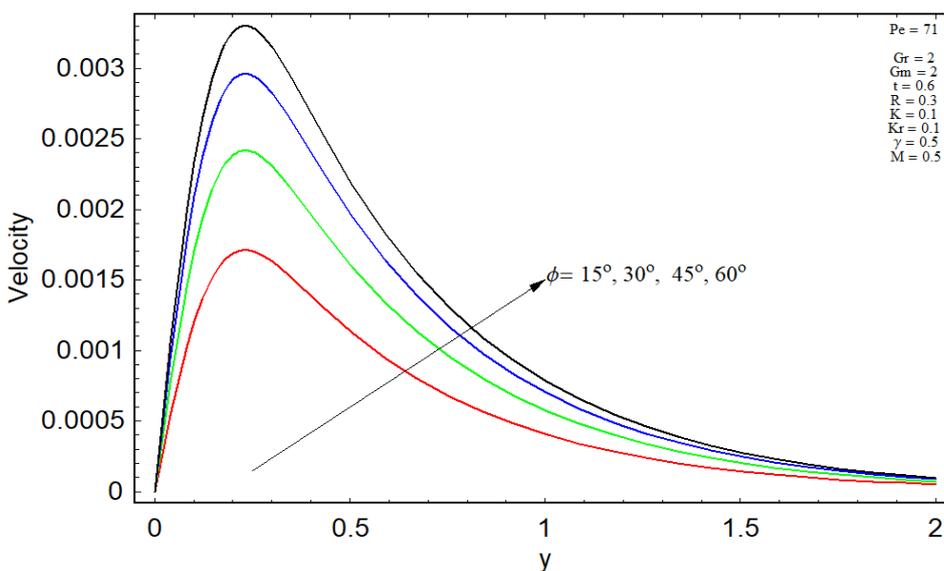


Fig. 3. The influence of inclination angle, ϕ on the velocity profile

Figure 4 displays the velocity profile against Casson fluid. The plot explained when the Casson parameter increase, the velocity also increases. This activity developed a low resistance to the yield stresses, which decreases the flow field. Therefore, velocity profile rise.

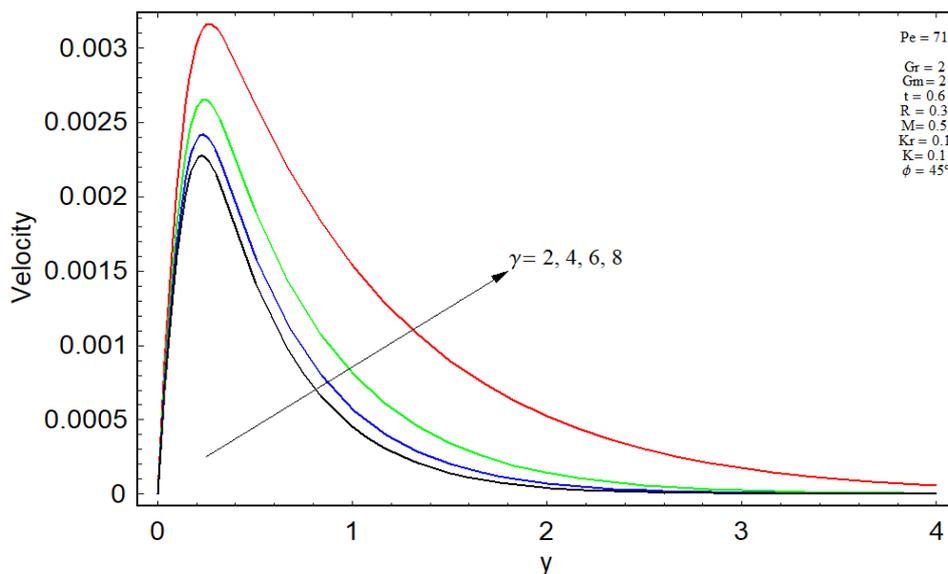


Fig. 4. The influence of Casson fluid, γ on the velocity profile

Figure 5 illustrate the nature of velocity field against the changed values of porous medium.

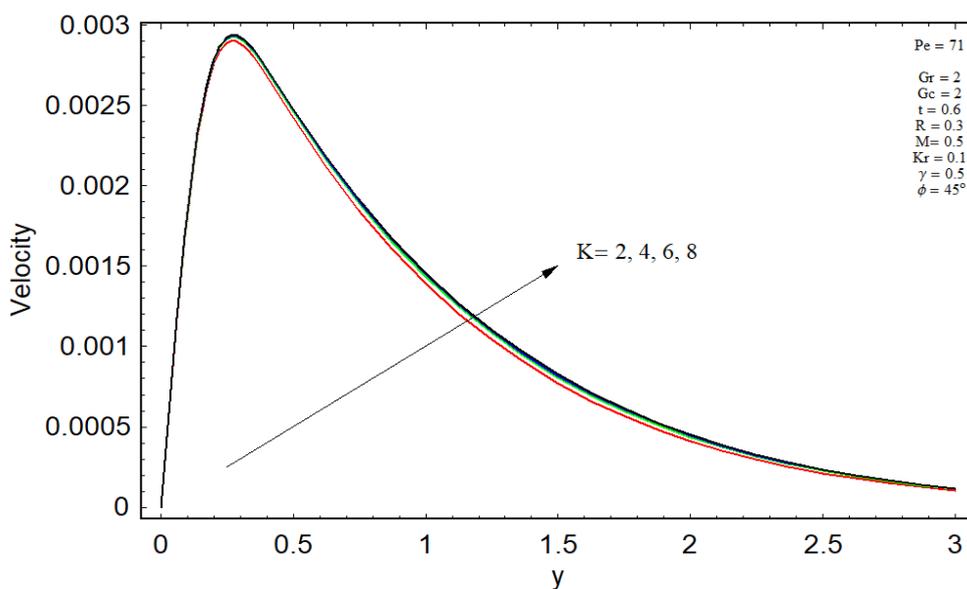


Fig. 5. The influence of porous medium, K on the velocity profile

The rising value of porous medium developed the resistance to flow; hence the flow of velocity is increase. Literally, the frictional force that oppose the flow is decreasing and makes the velocity become higher.

Figure 6 describes the thermal fields rises with an increment in radiation parameter. This is happened due to the increasing of conduction when radiation parameter increases, hence, the fluid temperature inclined at every point away from the surface. Attributes of chemical reaction on concentration field curves are interpreted in Figure 7. As we can see from the graph, the concentration was declined with the larger values of chemical reaction parameter. This situation described when chemical reaction parameter is higher, the chemical species rate is damage, which dissolves fluid specie efficiently. Therefore, fluid concentration reduces.

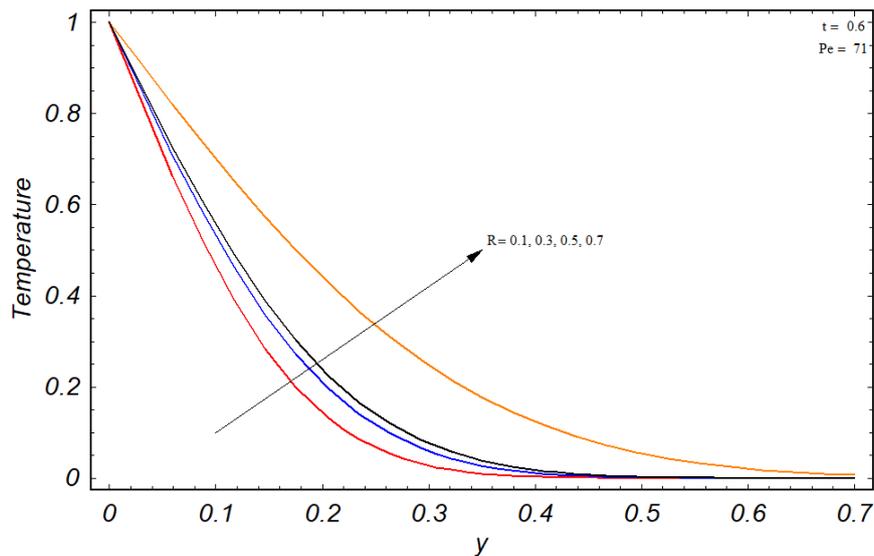


Fig. 6. The influence radiation, R on the temperature profile

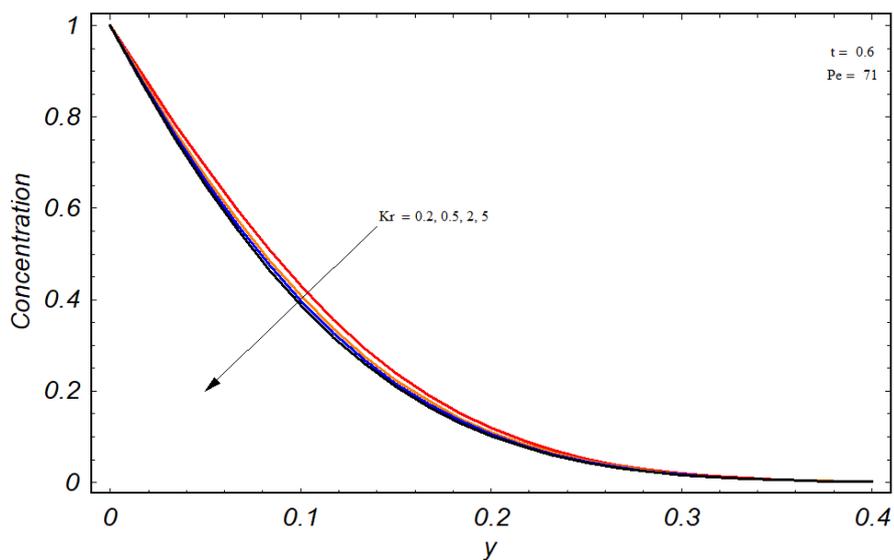


Fig. 7. The influence chemical reaction, Kr on the concentration profile

4. Conclusions

In this paper, the exact solution of unsteady Casson fluid in the present of chemical reaction electrically conducting heat fluid near an infinite inclined plate was presented. From the results, we can see the behaviour of Casson fluid, chemical reaction and porous medium throughout the research. The result of velocity, temperature as well as concentration expressions were gained from Laplace transform method. The outcomes of momentum, temperature and concentration fields are represented graphically. The most decisive findings of the research are summarized below.

- i. Fluid velocity rises with the increment of magnetic parameter. However, the outlines are very closely because of the slow growth of velocity.
- ii. The velocity is increase with the increment of chemical reaction.
- iii. The rising of inclination angle accelerates the fluid motion along the plate.
- iv. The increasing of Casson parameter make the velocity also increases.

- v. The rising value of porous medium developed the resistance to flow, hence the flow of velocity is increase.
- vi. Temperature fields rises with an increment in radiation parameter.
- vii. The concentration was declined with the larger values of chemical reaction parameter.

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