Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879


# Numerical Investigation of Hydrostatic Pressure on Free Vibrating Rectangular Cantilever Plates Partially Submerged in Viscous Media

Open Access

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**ARTICLE INFO**

***Article history:***
Received 21 May 2019
Received in revised form 29 June 2019
Accepted 5 July 2019
Available online 15 August 2019

***Keywords:***
Fluid-structure interaction; cantilever plates; natural frequency; mode shape

**ABSTRACT**

In the present study, the properties of the free vibration, represented by natural frequency, and the mode shape of cantilever plates partially submerged in the fluid was studied and analyzed. The viscosity, density, and compressibility of fluid were taken into account in numerical analysis using (ANSYS Package), which is basically based on (Finite Element Technique) in the analysis, by making the interaction between the plate and fluid at the contact surface. The results were compared with the properties of the plate in the case of non-contact with the fluid, which was obtained through the use of the same analytical program using the method of analysis named (Modal Analyses). The natural frequencies and mode shape of cantilever plates were found at different immersion and aspect ratios. The results showed the influence of the immersion and aspect ratios on the dynamic properties of the plates. The simulated data showed that the immersion of the cantilever plates in a fluid leads to a decrease in natural frequency, and this decrease varies with the type of fluid, the ratio of the submerged part and the aspect ratio, and that the fluid density is the most important characteristic of the dynamic properties of the plates.



## 1. Introduction

A thorough understanding of the kinetic interconnection between a flexible structure and fluid is very important and necessary in many engineering applications. An example of such applications is the parts of ships and submarines that are connected or submerged in seawater, water tanks, turbine fins, dams and others. This type of study falls within the style of geometric analysis called (Fluid-Structure Interaction). It is a type of analysis that deals with the study and analysis of the behavior of two or more fluid in the case of contact with each other and study the effect of the behavior of each fluid on the behavior of the other fluid, provided that there is a clear difference in behavior between the contact fluids. In this study, the mechanical kinetic behavior between fluid and plate was examined in case of contact. This type of study takes one of two directions either studying the effect

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of the plate on the fluid or vice versa. Since the research is part of the applied mechanics branch, the effect of fluid on the dynamic behavior of the plate has been studied, specifically affecting the natural frequency and mode shape. There are many former researchers have addressed this type of studies and most of these studies were especially studying reservoirs contain fluid as well as some research on submerged plates in the fluid. Some research was practical and theoretical using numerical analysis method.

Ergin and Uğurlu [1] studied the kinetic properties of a plate immersed in different depths within a fluid using two models of plates at different depths but of the same metal and the (Boundary Element) technique was used in the analysis. The pressure was considered to be the effect of fluid on the dynamic properties of the plate and fluid was considered ideal and non-rotational. In the analysis, the researcher used the packaged software of (ANSYS) for comparison using a different number of elements. The greater the number of elements, the more accurate the results and the closer to the experimental results. When the analysis was done in the case of the submerged plate, the researcher used the so-called (Hydrodynamic Panels) which are distributed along with the plate and the more their number the more accurate. The researcher compared the results with previous research and found that the results are very close.

David [2] has used a new type of element used with the structures for the purpose of achieving the ideal element for non-compressive fluid and using it with the (Finite Element Method). By using this type of fluid element with the standard elements used for solid objects, it has been possible to analyze missiles through a single composite structure containing several distinct elements. The elements involved are cube-shaped and consist of a set of tight edges and compressive strength. These elements are suitable for the fluid inside a rectangular rectangle reservoir.

Danial [3] analyzed large systems of type using a theory of (Fixed Model Interface) and applied it to three cases, the case of direct vibration of the fluid and the fluid state within the body, and the state of conversion and modeling of non-compressive fluid in contact with the surface. Because of the large systems that were addressed, and for the purpose of simplifying the asymmetry in the interconnect matrix, the researcher proposed three techniques to simplify analysis and eliminate impediments. In this research, the researcher considered the nodal vacations as variables in the elements of the solid body while pressure in the nodal was seen as a variable in the fluid elements. The researcher developed the equation of the movement of the solid body formed as usual from the matrix of the stiffness and masses, and the equation of motion of the fluid consisting of a corresponding matrix to the matrix of stiffness based on the inertia of the fluid and matrix corresponding to the matrix of the masses based on the behavior of compressive fluid and the link matrix between the equations is quadratic and asymmetrical.

Anamika [4] studied the effect of fluid density and viscosity on a small cantilever plate immersed in it. Two models of plates were used, the first is a small and simple cantilever made of stainless steel, which is immersed vertically into a fluid that is about one-third the length, the second model is similar to the first model, but the weight of the aluminum is suspended at the end and the cantilever is placed horizontally so that the weight is only immersed in the fluid. Several different types of viscosity and density fluids were used, the difference in natural frequency and the frequency response of the two models and the relationship of this difference with the viscosity and density characteristics of the used fluid. The experimental results were compared with results obtained using ANSYS package and there is a good agreement between the two studies

Lindholm *et al.,* [5] carried out practical research to clarify the frequency response of a plate stabilized by one side in the air and then in case it is partially or totally immersed in the fluid and then compare the practical results with the theoretical results. The approximate segmentation theory of the fluid field was used to determine the effect of fluid on the dynamic properties of the plate. The overall damping of cantilever plate vibration is increased significantly in water as compared with air. The results of this study would indicate that for those problem areas in the field of Naval dynamics wherein elastic plate vibrations are involved, and for which predictions of resonant frequencies and damping factors are derived, knowledge is either inadequate or unavailable.

Kiefling and Feng [6] developed and improved fluid analysis using finite element technology, including the potential energy of compression, while neglecting the change in density. They formulated the kinetic and potential energy in terms of nodal motions. The researchers studied two types of elements used for fluid analysis, the tetrahedron and the hexahedron elements and the characteristics of each type. They applied the method to water inside a cylindrical tank and reached the ripple pattern, the compression pattern of the fluid and the axial vibration pattern of the flexible-walled tank.

Cristoph [7] studied the approximate method of analyzing the interaction between a solid structure and fluid acting in linear and non-compressive behavior. This method depends on the fact that the normal frequency of the body that does not contact with the fluid is higher than the accompanying frequency in the case that the body in contact with non-compressive fluid, also the normal frequency in the case of compressed fluid is lower than that of non-compressive fluid.

Lorraine and Klaus [8] formulated the analog equation using the technique of finite elements method of type (FSI) using a square object with eight nodes element for both fluid and surface so that the nodal shared between fluid and surface are three. On the basis that the effort resulting from speed and pressure represent the nodal variables in the fluid and the nodal motion represent the nodal variables of the surface.

Balendra *et al.,* [9] studied and analyzed the vibratory behavior of cylindrical fluid reservoirs using the theory of shells, which is based on the fact that these reservoirs were fixed on their base. The researcher used the technique of the finite element method of both the fluid and the corresponding reservoir wall, considering the wall of the reservoir in the form of a thin shell.

Khorshidi [10] studied and analyzed the kinetic behavior of a flexible plate separating two fluid-filled reservoirs at different altitudes, as the plate is fixed from the bottom and from the sides only. He studied the effect of the hydrostatic pressure of the fluid on the natural frequency of the plate and the effect of the dimensions of the reservoir on the natural frequency and the effect of the free surface ripple of the fluid on the natural frequency of the plate. The researcher has used the analysis method developed for (Rayleigh-Ritz method). It was found that the low natural frequency is not linear with the depth of the fluid and the dimensions of the reservoir and the plate.

Michal and Ryszard [11] studied and analyzed the kinetic behavior of a plate completely surrounded by fluid and the researcher used technique (boundary element method) in the analysis. The fluid was considered non-viscous and compressive. The researchers have used (boundary integral equation) to describe the movement of the plate and hydrodynamic pressure surrounding the plate.

Casey [12] performed experimental and numerical study on surface-piercing struts constructed of aluminum and PVC to find the influence of partial-immersion and multi-phase flow on the modes of free-vibration. Experiments were conducted with the struts suspended in a water-filled drum and excited with hammer strikes. A shape-sensing methodology was used to experimentally infer mode shapes of the PVC strut. The finite element method (FEM) model used acoustic elements to simulate the fluid domain. The data revealed that the resonant frequencies generally decreased as immersion depth increased as a result of increasing hydrodynamic added mass. The percentage-change in resonant frequencies varied between modes.

Haddara and Cao [13] studied the plates dynamic responses when it exposed to air and in viscous fluid under different conditions. An approximate solution for the motion mathematical expression of a plate and fluid has been presented and provided a factor of analytical added-mass based on the free surface height and the fluid depth. They derived an approximate equation to calculate the modal added mass. The results revealed that there is a good agreement with published data at a depth of 25% from its length.

Liang *et al.,* [14] investigated numerically the free vibration by employing method of Rayleigh Ritz to derive the factor of the added mass of vibrated cantilever plate. Hybrid finite element method depending on the Sanders' thin plate method was used to analyze the case study of solid-fluid interaction for various plates in fluid. The interactive influences are distinctly derived and include inertial influences resulting in a pressure expression whose product with the structural shape function produces a virtual added mass schematic.

All previous research has concluded that contact or immersion of objects within the fluid leads to a clear reduction in the natural frequency of those objects, in addition, the compressive fluid has a greater effect than the non-compressive fluid. The aim of this work is to study and analyze the effect of different fluid properties on three different aspect ratio plates, its thickness and length are fixed and are submerged in different proportions within the fluid.

## 2. Link Equations of Plate and Fluid Behavior

The kinetic behavior of the plate and fluid each has its own governing equations, which has been derived and formulated in numerical form by using the finite element technique. The behavior equation of the plate (solid body) is as follows

$$M\ddot{\tilde{u}} + C\dot{\tilde{u}} + K\tilde{u} = 0 \tag{1}$$

where (K, C, M) representing mass, damping and stiffness matrices for plate elements and ($u$) represents the displacement vector. The equation of fluid behavior is as follows

$$S\ddot{\tilde{p}} + \tilde{C}\dot{\tilde{p}} + H\tilde{p} = 0 \tag{2}$$

Where: (H, $\tilde{C}$, S) represent matrices corresponding to plate matrices but differ in terms of the properties that form and ($P$) represent the pressure vector within the fluid. However, when the plate is in contact with the fluid, the equations change due to the effect of fluid pressure on the behavior of the plate and the effect of the displacement of the plate on fluid behavior, and formed a new matrix within the equations is called the link matrix (Q) between the plate and fluid domains and the two equations are as follows

$$M\ddot{\tilde{u}} + C\dot{\tilde{u}} + K\tilde{u} - Q\tilde{p} = 0 \tag{3}$$

$$S\ddot{\tilde{p}} + \tilde{C}\dot{\tilde{p}} + H\tilde{p} + \rho_o Q^T \ddot{\tilde{u}} = 0 \tag{4}$$

By neglecting the damping of the equations and rewriting them in the form of a matrix, it becomes as follows

$$\begin{bmatrix} M & 0 \\ \rho_o Q^T & S \end{bmatrix} \begin{Bmatrix} \ddot{\tilde{u}} \\ \ddot{\tilde{p}} \end{Bmatrix} + \begin{bmatrix} K & -\mathrm{Q} \\ 0 & \mathrm{H} \end{bmatrix} \begin{Bmatrix} \tilde{u} \\ \tilde{p} \end{Bmatrix} = 0 \quad (5)$$

Because of the lack of symmetry that characterizes this equation, some treatments and modifications are made and converted into a symmetrical form that can be solved. There are several methods used for this purpose and the most widely used **(**Ohayon) [15] method and through the use of this method becomes the equation as follows

$$\left\{ \begin{bmatrix} K & 0 & 0 \\ 0 & 1/\rho_o S & 0 \\ 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} M & 0 & Q \\ 0 & 0 & 1/\rho_o S \\ Q^{\mathrm{T}} & 1/\rho_o S^T & -1/\rho_o H \end{bmatrix} \right\} \begin{Bmatrix} \breve{u} \\ \breve{p} \\ \breve{q} \end{Bmatrix} = 0 \quad (6)$$

These relationships can be applied with the use of package software, especially software based on the technology of finite elements method. By understanding these relationships, it is possible to know the input of these programs in such a way as to lead to the results of analysis with a small error rate and with appropriate accuracy.

## 3. Theoretical Analysis

### *3.1 Elements Type for the Middle Plate and Fluid*

For the purpose of creating a model representing the plate in ANSYS a program, the element was selected from a 3D type (SOLID 45) which is used to create solid objects. This type of element is characterized by having eight nodes and there are three degrees of freedom in each node towards (x, y and z). This type of element is characterized by the elasticity and the ability to crawl and inflation and resistance to stress and large deviation and high capacity of strain, so it is suitable in the case of metals with relatively high flexibility. As for the fluid, an element of 3D type (FLUID 80) was selected. It is an element developed from the solid element (SOLID 45). This element consists of eight nodes in each node, three degrees of freedom similar to the solid element. This element was chosen because of its suitability to create fluid segments if it is present in a reservoir without flow and its relevance to the state of interaction between fluid and solid objects (FSI).

### *3.2 Create a Three-Dimensional Interaction System*

After determining the appropriate element type for each field, the system consisting of the plate and the fluid surrounding it is constructed in whole or in part and a mesh connection is performed between the two domains (plate and fluid) for the purpose of linking the behavior of the two domains with each other so that the effect of one of them moves to the other. This operation was performed using the program (ANSYS- Metaphysics-ver.14.0). The aim of this procedure is to obtain the effect of fluid on the dynamic properties of the plate.

The elements of the plate and fluid within the dividing surface share the same nodes. The fluid domain near the surface of the plate is divided and connected to the net by small and precise elements so that it can capture fluid movement details during plate vibration. This procedure is done after drawing the two domains. The line within the dividing surface is divided into a group of subdivisions that represent the baseline from which to divide the two domains and represent the boundary between two different types of network partitioning per domain. The greater the number of subdivisions, the greater the number of interrelated elements between the two domains and thus the greater the accuracy, so this surface is the joint surface of the plate and fluid. Since the reciprocal effect between the fluid and the plate decreases whenever it is moving away from the dividing surface and the upper and lower fluid surface is divided into extended and increasing divisions and gradually increasing in width for the purpose of achieving this requirement.

The fluid domain is then divided according to these divisions so that the fluid domain near the dividing surface is divided into small elements their dimensions are increased as they move away from that surface. This procedure is very necessary to simulate the behavior of fluid during vibration of the plate in the fluid, as the fluid domain near the plate has a greater impact on its behavior from the far domain, so the smaller the components, the less distortion. Thus, the elements close to the separation surface when moving the plate during the vibration reaches up to the stage of maximum deformation and stop and resist that movement and move the deformation to the element farthest away until the total distortions equal to the amount of displacement needed by the plate. After this process, the plate and fluid domains were obtained in the form of interconnected mesh and the meshed geometry was then exported to FLUENT[16]. The bond between the plate and the fluid was obtained from the separation surface and according to the number of channels selected, as well as the interaction between the fluid and the walls. After this procedure, it is possible to obtain the final form of the system through a reflection of the two related domains and then create an extension of all the spaces with their specifications and according to the dimensions of each domain to obtain a three-dimensional system and then apply boundary conditions as in Figure 1. As for the post-processing, the various simulation data visualization tools of the Computer Fluid Dynamics solver setup had been applied to observe the results [17].

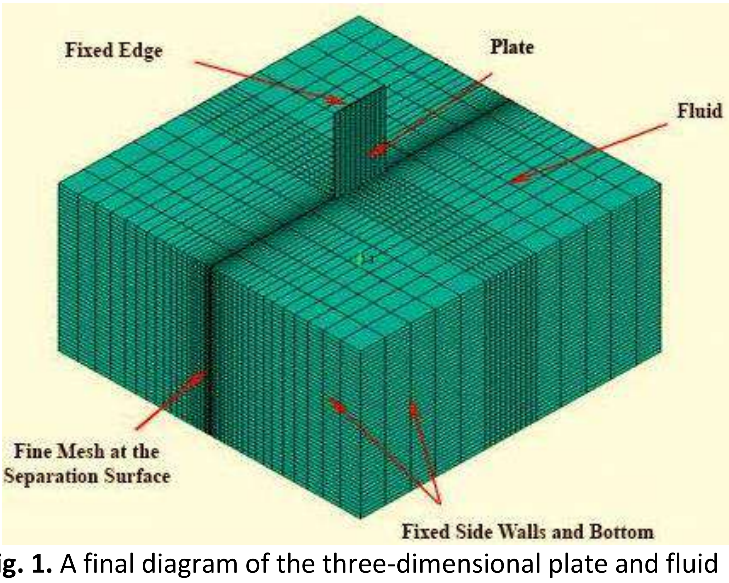



**Fig. 1.** A final diagram of the three-dimensional plate and fluid domains

## 4. Dimensions and Properties Description of the System

In this study, three plates of fixed length (35cm), thickness (3mm) and different width were selected according to three aspect ratios (L/b= 0.25, 0.5, 0.7), starting from non-immersion. Four submerged ratios were selected (S/L= 0, 0.2, 0.4, 0.8). The sheet metal was chosen to be made of (Aluminium). Two fluids were chosen to immerse the plates, namely water and gas oil.

## 5. Results and Discussions

The first three frequencies of the plates when they were not immersed in the fluid and when immersed in the fluid (Figure 2) in three different ratios were found numerically. In order to study the effect of fluid on the natural frequencies of the plates, the relationship between the ratio of immersion and natural frequency was determined (Figure 3-5). It is possible to observe from these figures that the decrease in frequency has occurred at a higher rate at the beginning of the immersion and then the rate is lower and in most cases closer to stability at a certain value, for example, in Figure 3(a), the first frequency of the first plate when was not immersed in the fluid (27.2 Hz) and when it was immersed in water, the greatest drop in the frequency value at the immersion ratio (0.20) became frequency (20.3 Hz), Then, when it is submerged at the immersion ratios (0.40 and 0.80), the frequency becomes (18.4 and 17.1 Hz) respectively, and we observe the same in other cases. The mode shape was not affected by immersion and remained the same type.

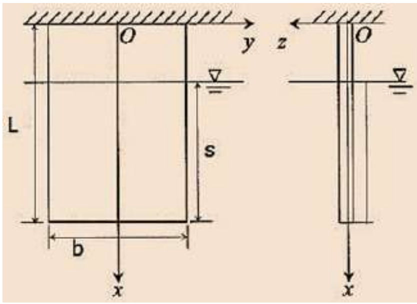


**Fig. 2.** Cantilever plate partially submerged in fluid

It can also be seen that immersion in the water has resulted in a greater reduction than the decrease in immersion within the gas oil, indicating that the amount of decrease in the normal frequency of the plate depends directly on the type of fluid used, that is, depends on the properties of fluid. For example, in Figure 3(a) and 3(b) when the first plate is immersed in water and gas oil at immersion ratio (0.20), the frequency decreases from (27.2) to (20.3 and 22.6) respectively, i.e. the water has a greater effect than the gas oil. This means that the difference in the properties of the fluid leads to a difference in the amount of impact on the frequency of the plate. The fluid has three main properties: density, viscosity and bulk modulus. These properties differ between the fluids and each of these properties has a certain effect on the decrease in frequency.

It can be observed that the frequency drop in the cases of water or gas oil use was similar. It can be seen that the shape of the curves was very similar and this means that the fluid type does not affect the frequency drop pattern, but when this decreasing is taken relatively, it is different. When plotting the ratio between the immersion ratio and the decrease in frequency of the three plates theoretically for the water and gas oil cases, Figure 6, it is possible to observe that the decrease in the first frequency was the highest compared to the second and third frequencies, while the decrease in the third frequency was the lowest. For example, in the first frequency of the three plates when immersed in the gas oil and at the immersion ratio (0.20) (Figure 6(b)), we note that the first frequency drop rate of the first plate was (17.2 %) while the decrease in the second and third frequencies were (14.2 % and 16.6 %) respectively.

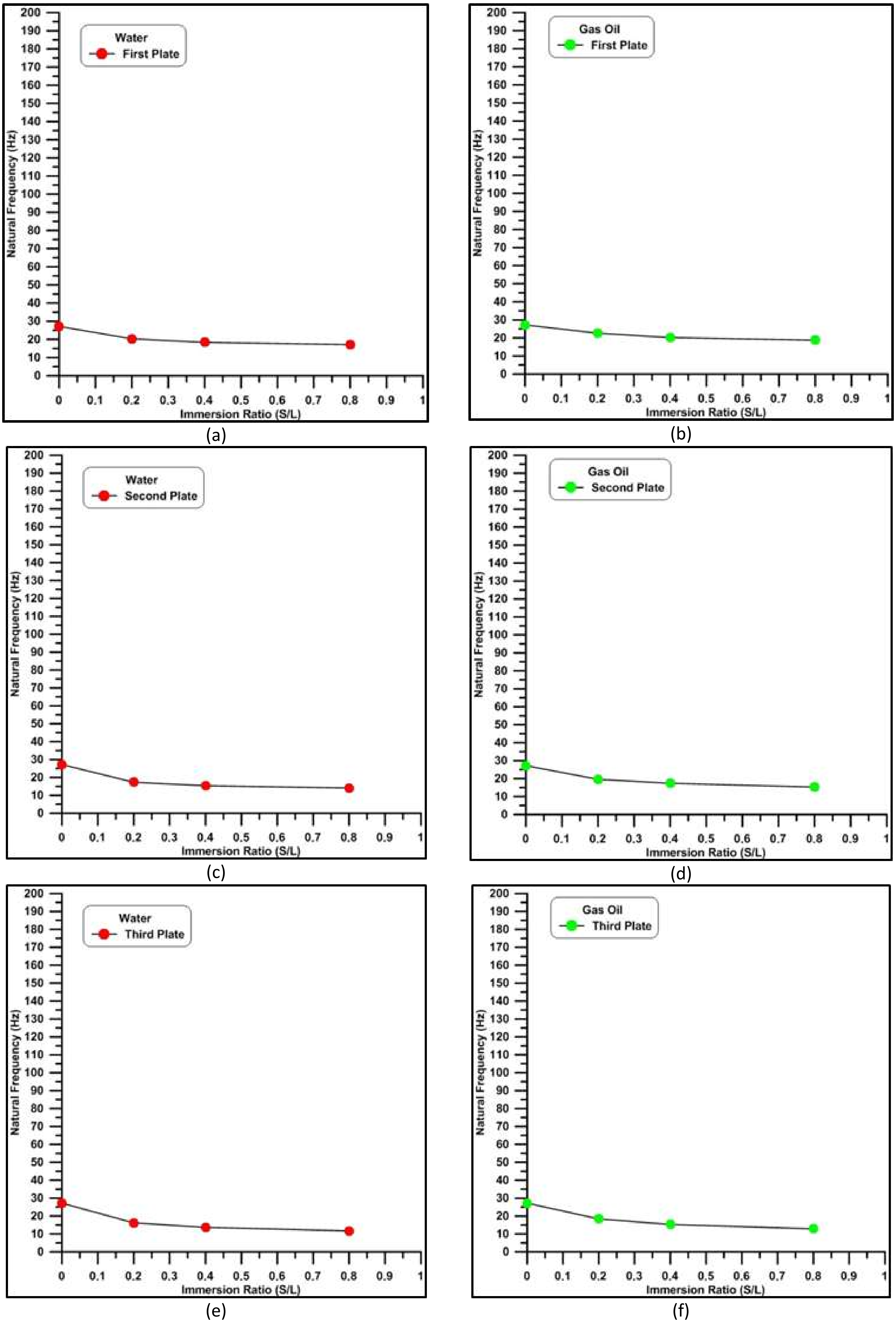



**Fig. 3.** Reduction in first natural frequency for three plates in water and gas oil

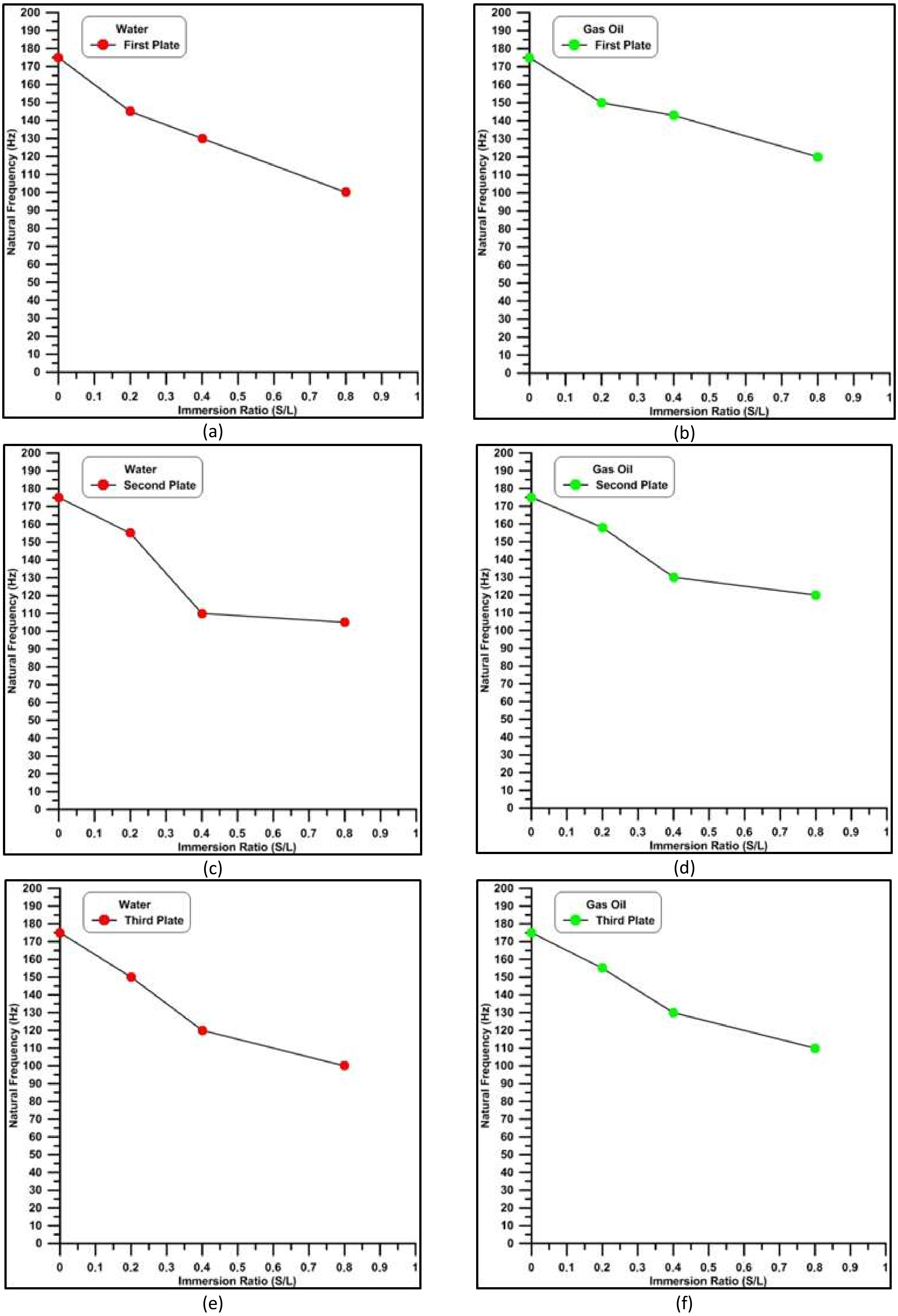
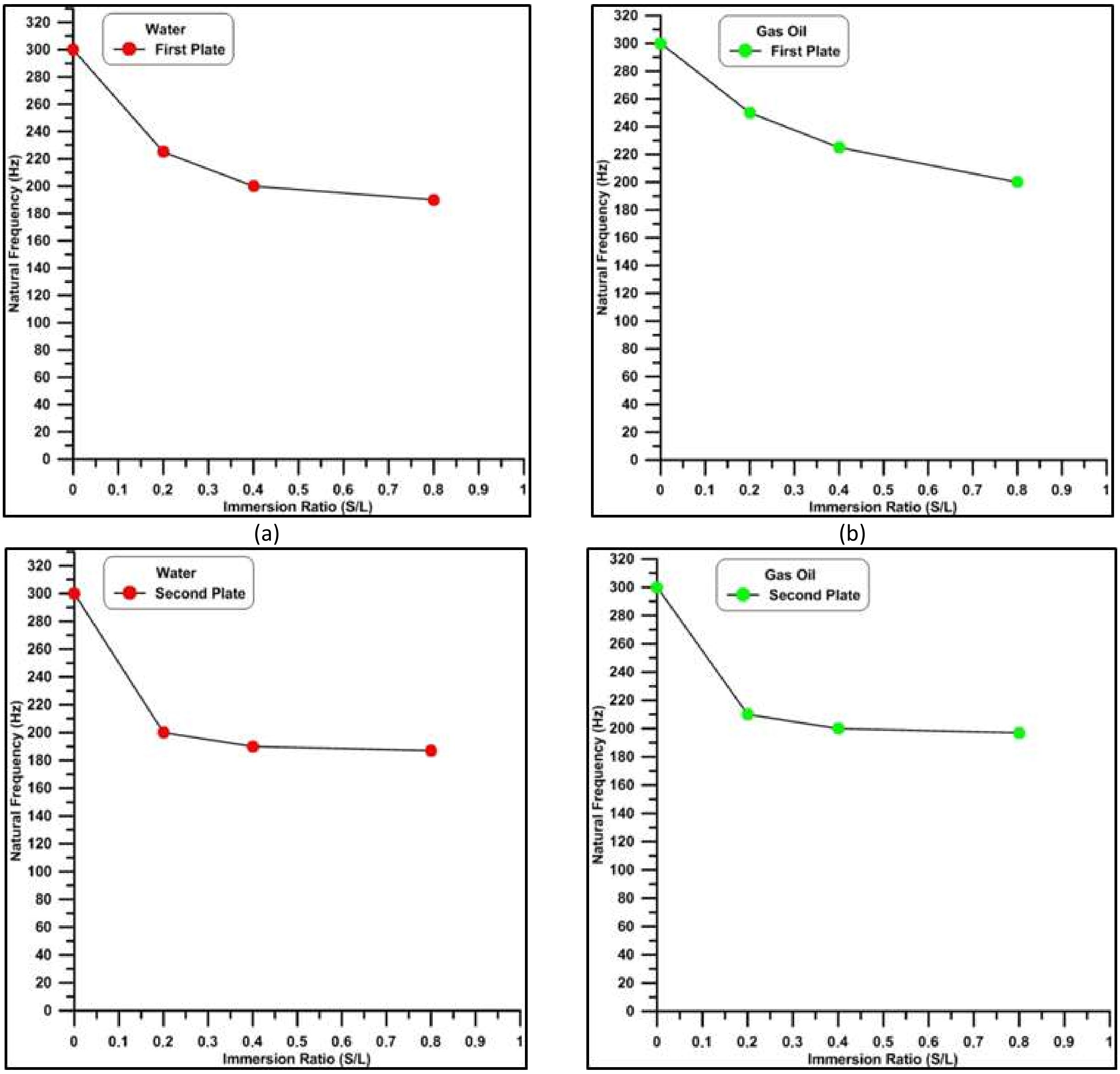
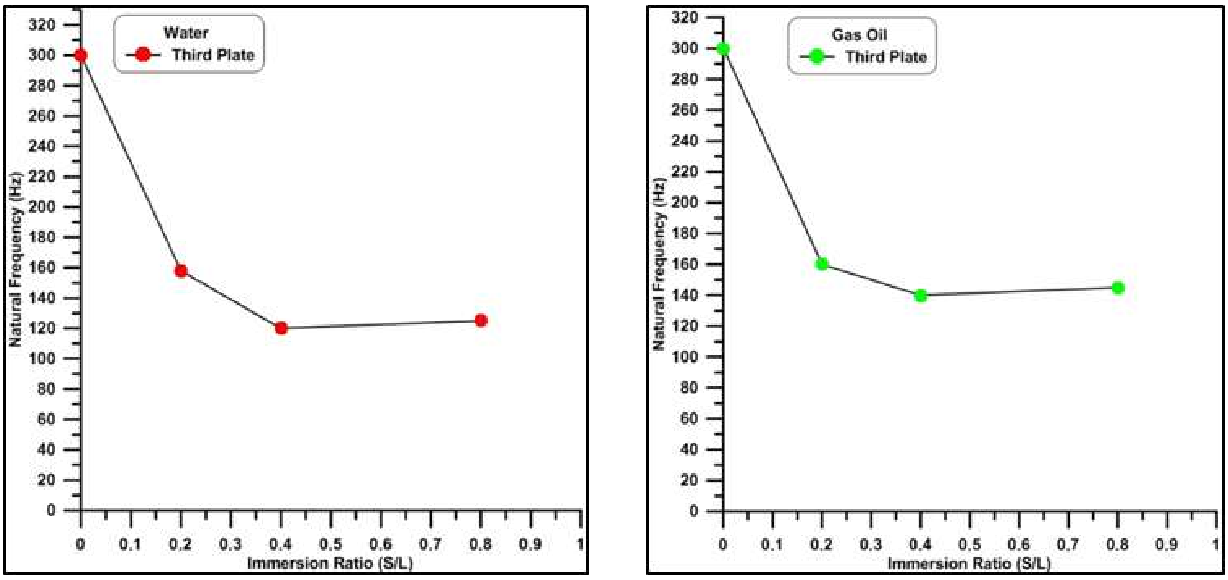



**Fig. 4.** Reduction in second natural frequency for three plates in water and gas oil

From these figures, it is possible to observe that the aspect ratio have a clear effect. The lower the aspect ratio, the lower the frequency. For example, in Figure 5(d), which represents the rate of decrease in the second frequency with the percentage of immersion of the three plates when immersed in gas oil, we note that the first plate was less effective and the rate of decrease in the second frequency of this plate between (14.2 - 31.4 %), while the decrease in the second frequency of the second and third plates ranged from (9.7 - 33.6 %) and (11.4 - 37.1 %), respectively. But we note that this is true in the case of similar mode shapes of the three plates, but when the mode shapes are different, it is different.



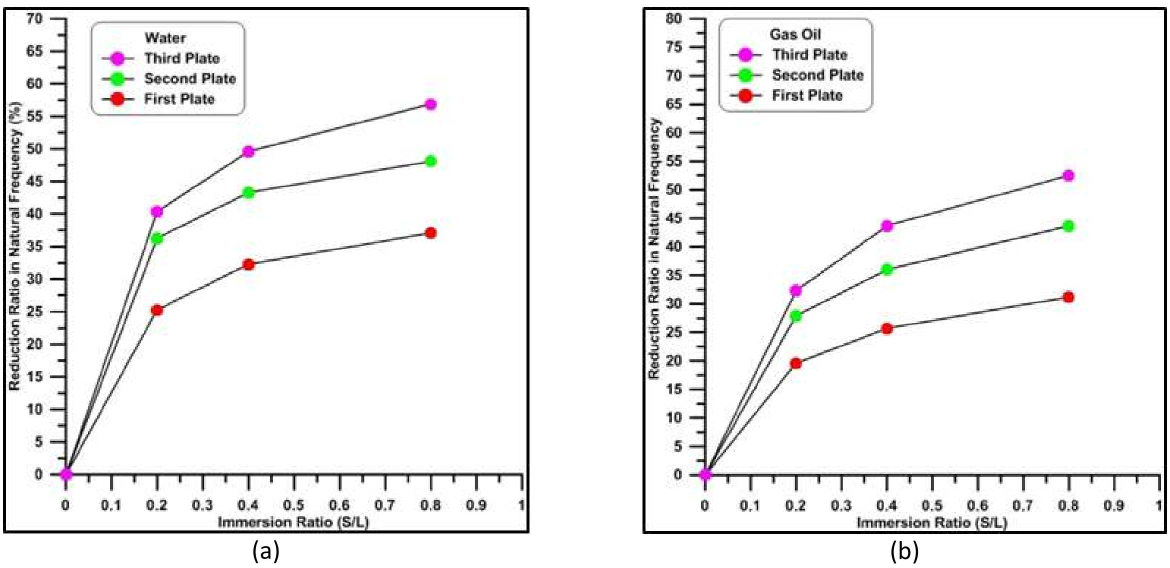

(a) (b) (c) (d)




(e) (f)

**Fig. 5.** Reduction in third natural frequency for three plates in water and gas oil

Figure 6(e) and 6(f) shows that the decrease in frequency of the third plate was the lowest with the knowledge that it has a larger dimension. The decrease in the third frequency of the three plates when immersed in water was ranged between (25 – 36.6 %), (33.3 – 37.6 %) and (47.3 – 58.2 %), respectively. It was found that the mode shape of the third plate was different from the first and second plates, the mode shape of the third plate at the third frequency was twisted, while the mode shape of the first and the second plate was bended, which means that the effect of fluid on the frequency is less in the case of Bending motion pattern. This can be seen in the first and second frequencies of the plates, in the first frequency the mode shape was bended for the three plates, the effect of the aspect ratio is the controller of the decline ratio, in the second frequency, although the mode shape of the third plate is different compared to the mode shape of the first and second , the mode shape of the first and second plate was bended while the mode shape of the third plate was twisted, so the effect of the aspect ratio and the bended mode shape were combined in the first and second plates, for example, when observed in Figure 6(c), the decrease in the second frequency of the three plates when immersed in water is ranged between (17.1 – 42.8 %), (11.4 – 40 %) and (14.2 – 42.8 %), respectively.




(a) (b)

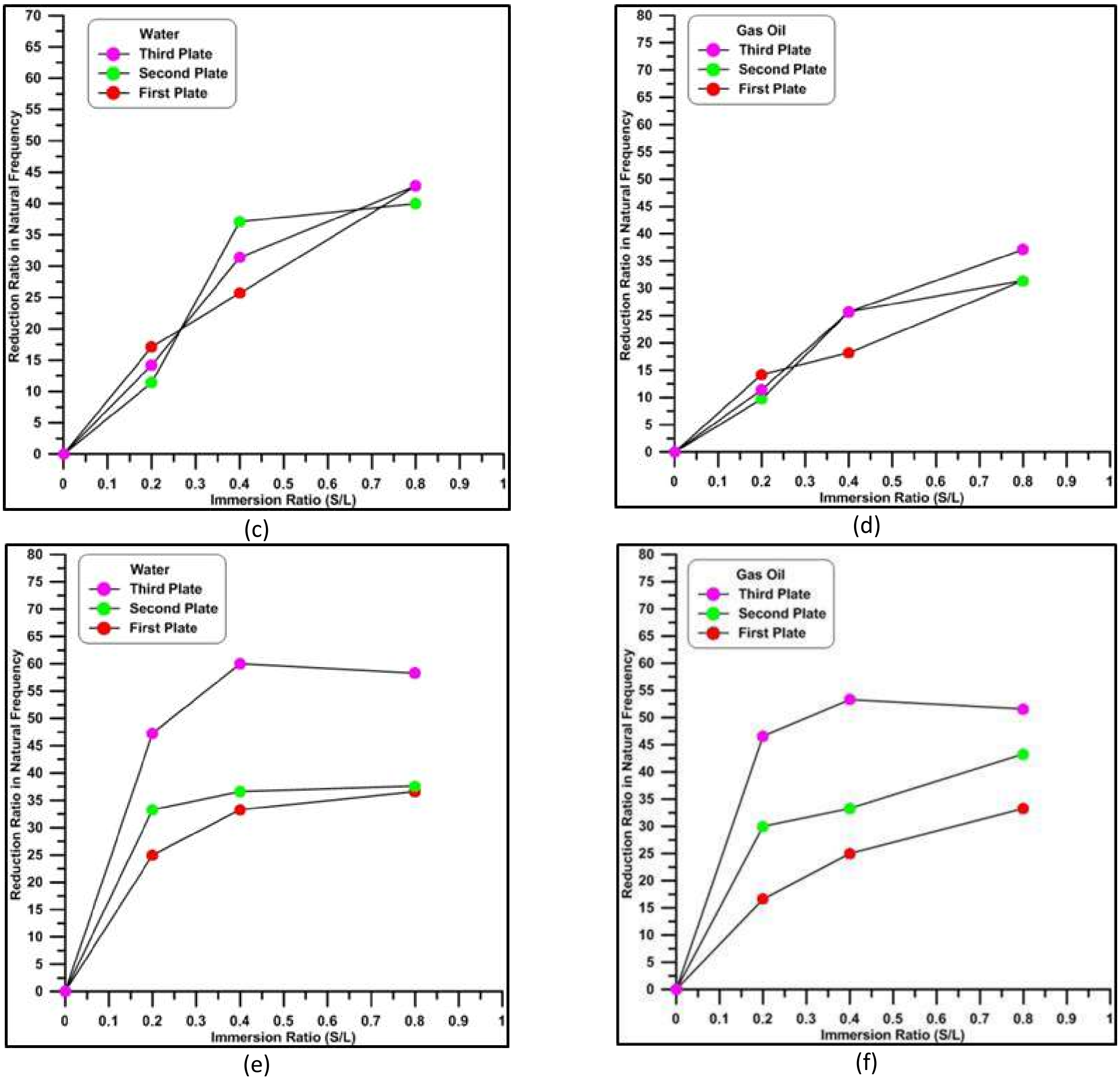



**Fig. 6.** Reduction ratio in three natural frequencies versus immersion ratio for three plates in water and gas oil

From this we conclude that the mode shape has an effect greater than the aspect ratio of the decrease in frequency, in the third plate, although their aspect ratio is the largest, but the rate of decline in frequency was lower. Figure 7-9 illustrates the mode shape of the three plates.

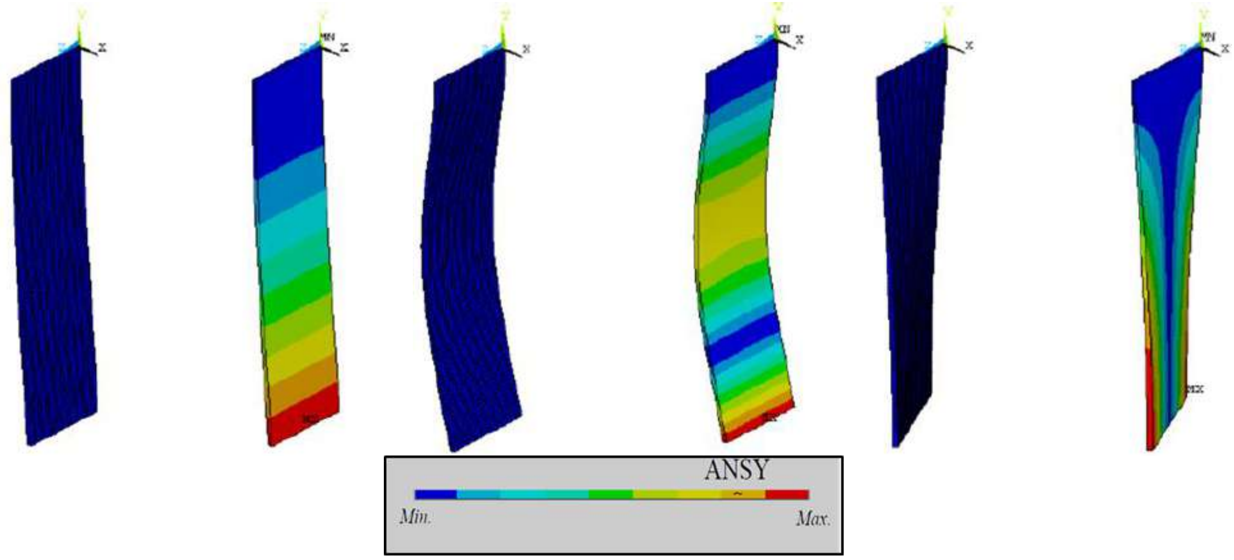



**Fig. 7.** Three mode shapes for first plate

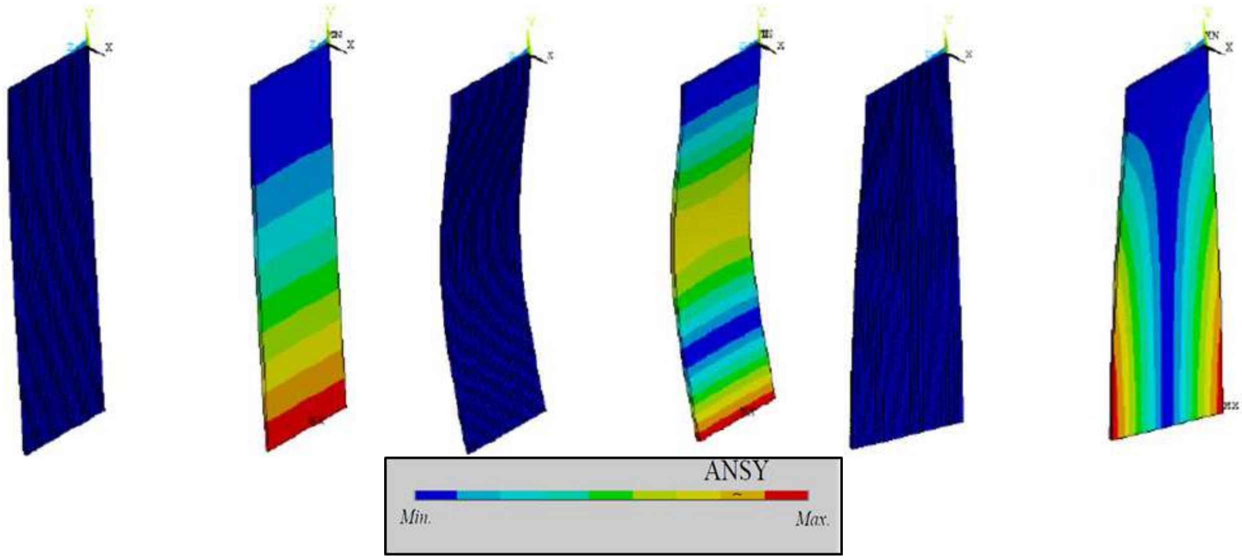



**Fig. 8.** Three mode shapes for second plate

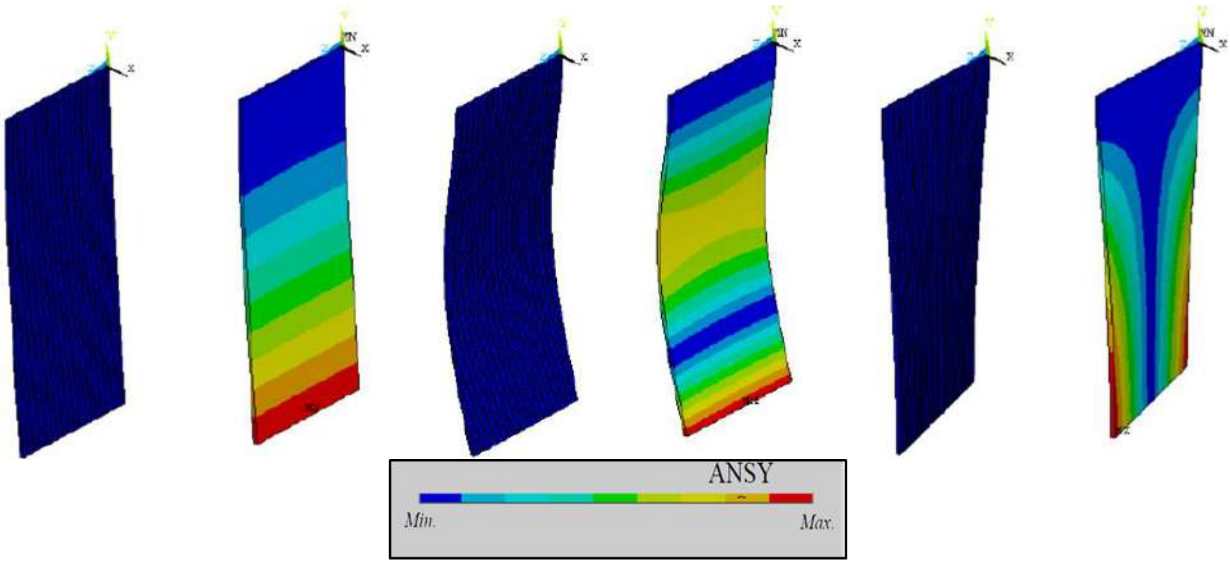



**Fig. 9.** Three mode shapes for third plate

## 6. Conclusions

The effect of partial or total submersion of a cantilever plate on the kinetic properties inside fluids was studied theoretically. The results revealed that the submersion of the plates in the fluid leads to a significant decrease in the natural frequencies of the plates and this decrease increases with increasing the percentage of immersion. It was found that the rate of decrease in frequency is related to the aspect ratio, the higher the aspect ratio, the lower the frequency. It was found that the decrease in frequency is related to the mode shape, the twisted mode shape is less affected than the bended mode shape. It was also found that the decrease in natural frequency is related to the type of fluid (fluid properties).

**Acknowledgement**
The authors wish to thank (Dr. Bassim M. Majel) for their continued support of this research.

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