

Effects of Chemical Reaction and Heat Generation on The Unsteady Free Convection Flow Past an Infinite Vertical Permeable Moving Plate with Variable Temperature

Open
Access

Bommanna Lavanya^{1,*}, Madduleti Nagashashikala²

¹ Department of Mathematics Assistant Professor Senior scale MIT MAHE Manipal Karnataka 576104, India

² Department of Mathematics, Govt. Degree College (Autonomous), Anantapuramu-515001, A.P, India

ARTICLE INFO

ABSTRACT

Article history:

Received 20 August 2019

Received in revised form 1 October 2019

Accepted 4 October 2019

Available online 28 December 2019

We have studied the analytical solutions for the effect of chemical reaction on the unsteady free convection flow past an infinite vertical permeable moving plate with variable temperature has been studied. The plate is assumed to move with a constant velocity in the direction of fluid flow. The highly nonlinear coupled differential equations governing the boundary layer flow, heat and mass transfer are solved using two-term harmonic and non-harmonic functions. The parameters that arise in the perturbation analysis are prandtl number, Schmidt number, Grashof number, modified Grashof number, chemical reaction parameter, skin friction coefficient and sherwood number. The impact of various parameters on velocity, temperature and concentration fields are shown graphically and tables and analyzed in detail.

Keywords:

Heat and Mass transfer; Chemical
Reaction; Heat Generation

Copyright © 2019 PENERBIT AKADEMIA BARU - All rights reserved

1. Introduction

The Magneto-Hydrodynamics (MHD) boundary layers with heat and mass transfer over flat surfaces are found in many engineering and geophysical applications such as geothermal reservoirs, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors. Because of this reason, many researchers tend to apply the MHD flow into their problems. For instance, Amkadni and Azzouzi [1] studied the similarity solution of MHD boundary layer flow over a moving vertical cylinder. Bestman [2] examined the natural convection boundary layer with suction and mass transfer in a porous medium. His results confirmed the hypothesis that suction stabilises the boundary layer and affords the most efficient method in boundary layer control yet known. Lavanya [3] studied Unsteady MHD Convective laminar flow between two Vertical Porous plates with mass transfer.

The effects of chemical reaction and radiation absorption on free convective flow through a porous medium with a variable suction in the presence of uniform magnetic field were studied by

* Corresponding author.

E-mail address: lavanya.b@manipal.edu (Bommanna Lavanya)

Sudheer Babu and Satyanarayana [4]. Ahmed and Alam Sarker [5] presented the problem of a steady two - dimensional natural convective flow of a viscous incompressible and electrically conducting fluid past a vertical impermeable flat plate in the presence of a uniform transverse magnetic field. Lavanya [6] studied MHD rotating flow through a porous medium with heat and mass transfer. Saravana *et al.*, [7] studied the effects of mass transfer on the MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux.

The study of heat generation or absorption effects in moving fluids is important in the view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reactions. Hiremath [8] studied the heat transfer characteristics in the laminar boundary layer of a viscous fluid over a stretching sheet with viscous dissipation or frictional heating and internal heat generation. Nagashashikala *et al.*, [9] studied Effects of dissipation and radiation on heat transfer flow of a convective rotating cuo-water nano-fluid in a vertical channel. Hossain *et al.*, [10] studied problem of the natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption. Alam *et al.*, [11] studied the problem of the free convection heat and mass transfer flow past an inclined semi-infinite heated surface of an electrically conducting and steady viscous incompressible fluid in the presence of magnetic field and heat generation. Lavanya [12] studied Unsteady MHD convective laminar flow between two vertical porous plates with mass transfer. Exact Solution of MHD free convection flow and Mass Transfer near a moving vertical porous plate in the presence of thermal radiation was investigated by Das [13]. Sonth *et al.*, [14] studied Heat and Mass transfer in a viscoelastic fluid over an accelerated surface with heat source/sink and viscous dissipation. Shateyi *et al.*, [15] investigated the effects of thermal Radiation, Hall currents, Soret and Dufour on MHD flow by mixed convection over vertical surface in porous medium. Ahmad *et al.*, [16] studied Unsteady MHD Boundary Layer Flow and Heat Transfer of Ferrofluids Over A Horizontal Flat Plate with Leading Edge Accretion. Siti *et al.*, [17] studied Numerical Solution on MHD stagnation point flow in ferrofluid with newtonian heating, and thermal radiation effect.

It is proposed to study, the flow past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion in the presence of a homogeneous chemical reaction and heat generation. The main reason for the lack of study of this problem is due to difficult mathematical and numerical procedures in dealing with the non-similar boundary layers. The highly non-linear coupled differential equation governing the boundary layer flow, heat and mass transfer are solved using two-term harmonic and non-harmonic functions. Details of the velocities, temperature and concentration field as well as the local skin friction and the local Sherwood number for the various values of the parameters of the problem are presented.

2. Mathematical Analysis

Consider unsteady two-dimensional flow of a laminar, viscous and heat absorbing fluid past an infinite vertical permeable moving plate. The axial coordinates x' is measured vertically upward along the plate, and the y' axis is taken normal to the plate. at time $t' \leq 0$ the plate and fluid are at the same temperature T'_{∞} and concentration C'_{∞} . At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with uniform velocity u_0 , the plate temperature is made raised to C'_w . it is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. but here we assume the level of the species concentration to be very low and hence heat generated during chemical reaction can be neglected. in this reaction the reactive component given off by the surface, occurs only in very dilute form.

Hence, any connective mass transport to or from the surface due to a net viscous dissipation effects in the energy equation are assumed to be negligible. under these assumptions, the boundary layer flow with Boussinesq's approximation is governed by

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta(C - C_\infty) \quad (1)$$

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (2)$$

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2} - kC' \quad (3)$$

where, u is the velocity of the fluid in the x' direction, g is the acceleration due to gravity, β is the volumetric coefficient of expansion with concentration, T' is the temperature of the fluid near the plate, T'_∞ is the temperature of the fluid away from the plate, T'_w is the surface temperature t' time, t is the dimensionless time, ν is the kinematic viscosity, c species concentration, C dimensionless species concentration, C'_∞ species concentration away from the plate C'_w the surface species concentration, d mass diffusion coefficient, k thermal conductivity, k_i chemical reaction parameter, ρ density of fluid, c_p specific heat at constant pressure, u dimensionless velocity, y' co-ordinate axis normal to the plate, y dimensionless coordinate axis normal to the plate. With the following initial and boundary conditions

$$u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty, \quad \text{for all } y', t' \leq 0$$

$$t' > 0, \quad u' = u_0, \quad T' = T'_\infty + (T'_w - T'_\infty), \quad \text{at } C' = C'_w \quad \text{at } y' = 0$$

$$u' = 0, \quad T' \rightarrow T'_\infty, \quad C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \quad (4)$$

where $A = \frac{u_0^2}{\nu}$, u_0 velocity of the plate.

we now introduce the following non dimensional quantities.

$$u' = uu_0 G, \quad t' = \frac{tv}{u_0^2}, \quad y' = \frac{yv}{u_0}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad G = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3},$$

$$G = \frac{g\beta\nu(T'_w - T'_\infty)}{u_0^3}, \quad p_r = \frac{\mu C_p}{k}, \quad S_c = \frac{\nu}{D}, \quad k = \frac{\nu k_1}{u_0^2} \quad (5)$$

where G Grashof number, G_0 modified Grashof number, S_c Schmidt number, μ coefficient of viscosity, θ dimensionless temperature.

in Eq. (1) to (4), which leads to

$$\frac{\partial u}{\partial t} = \theta + \frac{G_0}{G} C + \frac{\partial^2 u}{\partial y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = pr^{-1} \frac{\partial^2 \theta}{\partial y^2} - Q\theta \quad (7)$$

$$\frac{\partial C}{\partial t} = Sc^{-1} \frac{\partial^2 C}{\partial y^2} - kC \quad (8)$$

Eq. (6) to (8) represent a set of partial differential equations that cannot be solved in enclosed form. However, it can reduce to a set of ordinary differential equations in dimensional form that can be solved analytically, this can be done by representing the velocity, temperature and the concentration as

$$u = u_0 + \varepsilon e^{i\omega t} u_1 + \varepsilon^2 e^{2i\omega t} u_2 \quad (9)$$

$$\theta = \theta_0 + \varepsilon e^{i\omega t} \theta_1 + \varepsilon^2 e^{2i\omega t} \theta_2 \quad (10)$$

$$C = C_0 + \varepsilon e^{i\omega t} C_1 + \varepsilon^2 e^{2i\omega t} C_2 \quad (11)$$

Substituting Eq. (9) to (11) into Eq. (6) to (8), equating the harmonic and non-Harmonic terms and neglecting the higher order of ε^3 and simplifying we obtain the following set of differential equations for u , θ and c .

$$u_0'' = -\theta_0 - \frac{G_0}{G} C_0 \quad (12)$$

$$u_1'' - i\omega u_1 = -\theta_1 - \frac{G_0}{G} C_1 \quad (13)$$

$$u_2'' - 2i\omega u_2 = -\theta_2 - \frac{G_0}{G} C_2 \quad (14)$$

$$pr^{-1} \theta_0'' - Q\theta_0 = 0 \quad (15)$$

$$\theta_1'' - (i\omega pr - Q)\theta_1 = 0 \quad (16)$$

$$\theta_2'' - (i\omega pr - Q)\theta_2 = 0 \quad (17)$$

$$C_0'' - ScKC_0 = 0 \quad (18)$$

$$C_1'' - Sc(K + i\omega)C_1 = 0 \quad (19)$$

$$C_2'' - Sc(K + 2i\omega)C_2 = 0 \quad (20)$$

In the above equations, the primes denote differentiation with respect to y . The boundary conditions in Eq. (4) after substitutions in Eq. (9) to (11) are reduced to

$$\begin{aligned} \text{at } y=0 \quad u_0 = \frac{1}{G}, \quad u_1 = 0, \quad u_2 = 0, \quad \theta_0 = 1, \quad \theta_1 = 0, \quad \theta_2 = 0, \quad C_0 = 1, \quad C_1 = 0, \quad C_2 = 0, \\ \text{at } y \rightarrow \infty \quad u_0 = 0, \quad u_1 = 0, \quad u_2 = 0, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0, \quad \theta_2 \rightarrow 0, \quad C_0 \rightarrow 0, \quad C_1 \rightarrow 0, \quad C_2 \rightarrow 0 \end{aligned} \quad (21)$$

Hence from Eq. (18) to (20) under the respective boundary conditions in Eq. (21) and substituting the solutions into Eq. (11) the solution for concentration distribution is given by:

$$C = e^{-\sqrt{Ksc}y} + \varepsilon e^{i\omega t} e^{-\sqrt{Sc(k+i\omega)}y} + \varepsilon^2 e^{2i\omega t} e^{-\sqrt{Sc(k+2i\omega)}y} \quad (22)$$

Also, by solving the differential Eq. (15) to (17), under the boundary conditions in Eq. (21), and substituting the solutions into Eq. (10). we have the temperature distribution is given by

$$\theta = e^{-\sqrt{pr}y} + \varepsilon e^{i\omega t} e^{-\sqrt{i\omega pr - Q}y} + \varepsilon^2 e^{2i\omega t} e^{-\sqrt{2i\omega pr - Q}y} \quad (23)$$

and either from Eq. (12) to (14) under the respective boundary conditions in Eq. (21) and substituting the solutions into Eq. (9). We have the velocity distribution.

$$\begin{aligned} u = -e^{-\sqrt{py}} - \frac{G_0}{G} e^{-\sqrt{Ksc}y} + \varepsilon e^{i\omega t} \left(a_1 G_1 e^{-\sqrt{i\omega}y} - \frac{e^{-\sqrt{i\omega pr}y}}{i\omega(pr-1)} - \frac{G_0 e^{-\sqrt{Sc(k+i\omega)}y}}{G(Sc(k+i\omega)-i\omega)} + \right. \\ \left. \varepsilon^2 e^{2i\omega t} \left(a_1 e^{-\sqrt{2i\omega}y} - \frac{e^{-\sqrt{2i\omega Pr}y}}{2i\omega(pr-Q)} - \frac{G_0 e^{-\sqrt{sc(k+2i\omega)}y}}{G(sc(k+2i\omega)-2i\omega)} \right) \right) \end{aligned} \quad (24)$$

by knowing velocity, temperature and concentration profiles, it is interesting to study about local and average values of skin friction, in non-dimensional quantities, the skin friction.

$$\begin{aligned} \tau = \sqrt{p} - \frac{G_0 \sqrt{KSc}}{G} + \varepsilon e^{i\omega t} \left(-a_1 \sqrt{i\omega} + \frac{\sqrt{i\omega Pr}}{i\omega(Pr-1)} + \frac{G_0 Sc(K+2i\omega)}{GSc(k+i\omega)-i\omega} \right) + \\ \varepsilon^2 e^{2i\omega t} \left(-a_1 \sqrt{2i\omega} + \frac{\sqrt{2i\omega Pr}}{2i\omega(Pr-Q)} + \frac{G_0 Sc(K+2i\omega)}{GSc(k+2i\omega)-2i\omega} \right) \end{aligned} \quad (25)$$

where

$$a_1 = \frac{1}{i\omega(pr-Q)} + \frac{G_0}{G(Sc(K+i\omega)-i\omega)} \quad a_1 = \frac{1}{2i\omega(pr-Q)} + \frac{G_0}{G(Sc(K+2i\omega)-2i\omega)} \quad (26)$$

3. Results and Graphs

In order to get a physical understanding of the problem and for purpose of discussing the results, numerical calculations have been performed for the concentration, velocity, temperature, rate of mass transfer, skin friction and rate of heat transfer. The results are represented graphically in the following figures.

Figure 1 shows the velocity profiles for different values of the permeability parameter, clearly as permeability parameter increases the peak values of the velocity tends to increase. Figure 2 shows the velocity profiles for different values of Grashof number, clearly as radiation parameter increases the peak values of the velocity tends to increase.

For different values of the Schmidt number the velocity profiles are plotted in Figure 3. It is obvious that an increase in the Schmidt number results in decrease in the velocity within the boundary layer. It is observed that the temperature decreases as an increasing the Prandtl number. The reason is that smaller values of Prandtl number are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of Prandtl number. Hence in the case of smaller Prandtl number the thermal boundary layer is thicker and the rate of heat transfer is reduced. Figure 4 has been plotted to depict the variation of temperature profiles for different values of radiation parameter by fixing other physical parameters. From this Graph we observe that temperature decrease with increase in the radiation parameter R . Figure 5 indicates that increase in temperature shows the increase in heat generation. Figure 6 displays the effect of Schmidt number Sc on the concentration profiles respectively. As the Schmidt number increases the concentration decreases. Figure 7 displays the effect of the chemical reaction on concentration profiles. We observe that concentration profiles decrease with increasing chemical reaction parameter. Local skin friction values are plotted in the Table 1 against the Grashof number G . A similar reaction is noted for destructive reaction.

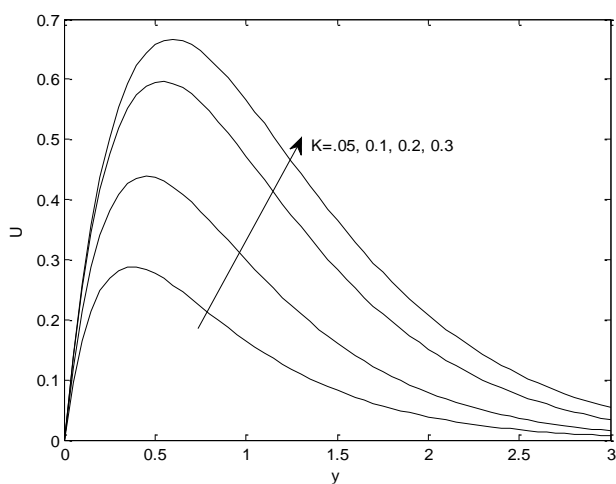


Fig. 1. Velocity profiles for different values of permeability parameter

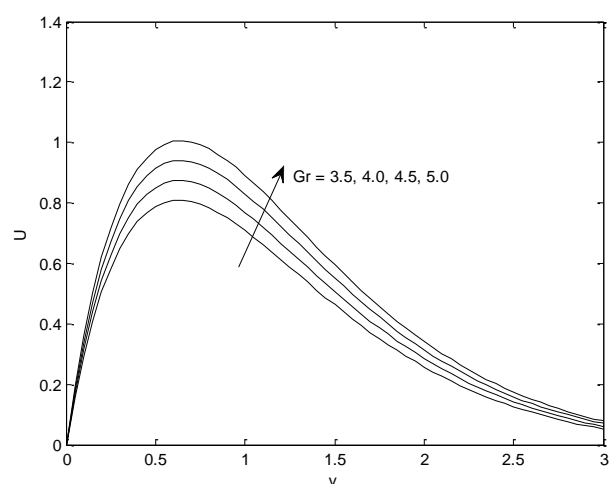


Fig. 2. Velocity profiles for different values of Grashof number

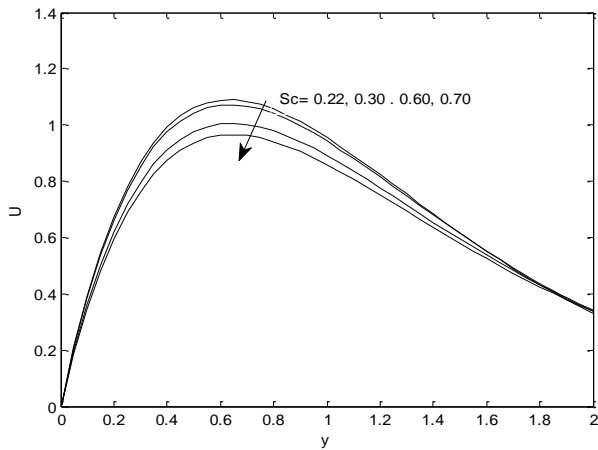


Fig. 3. Velocity profiles for different values of Schmidt number

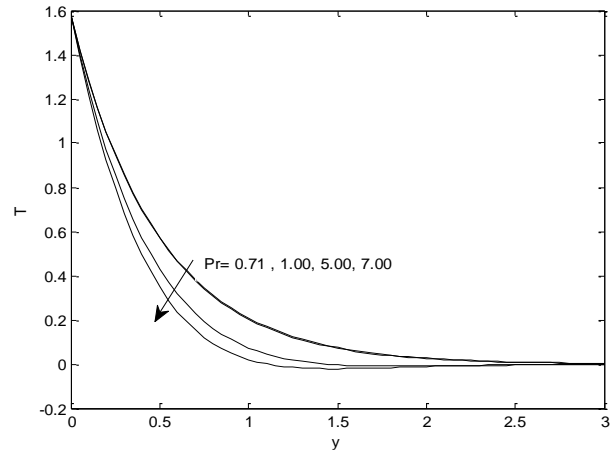


Fig. 4. Velocity profiles for different values of radiation parameter

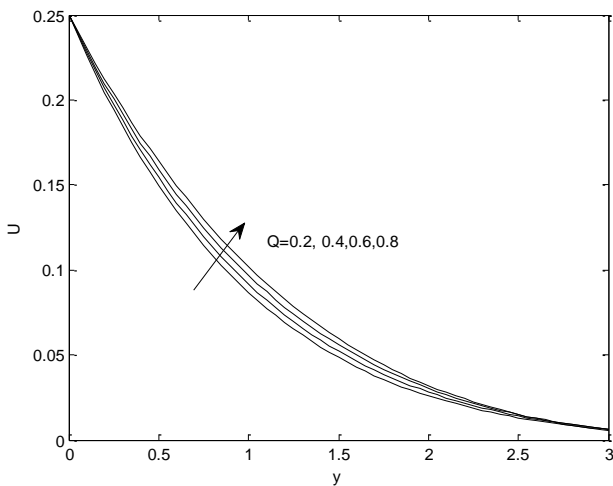


Fig. 5. Temperature profiles for different values of heat generation

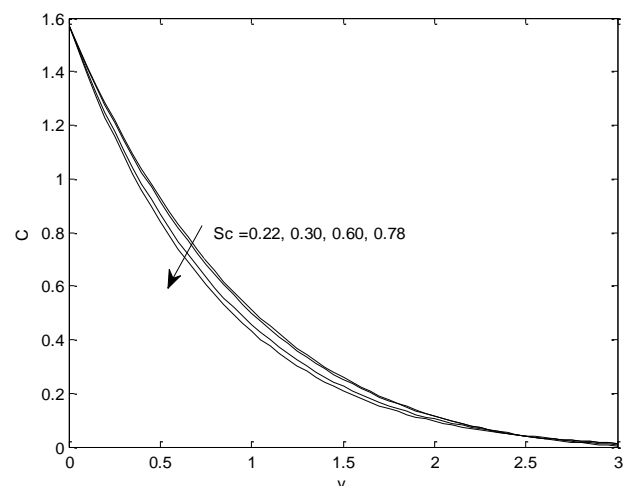


Fig. 6. concentration profiles for different values Schmidt number

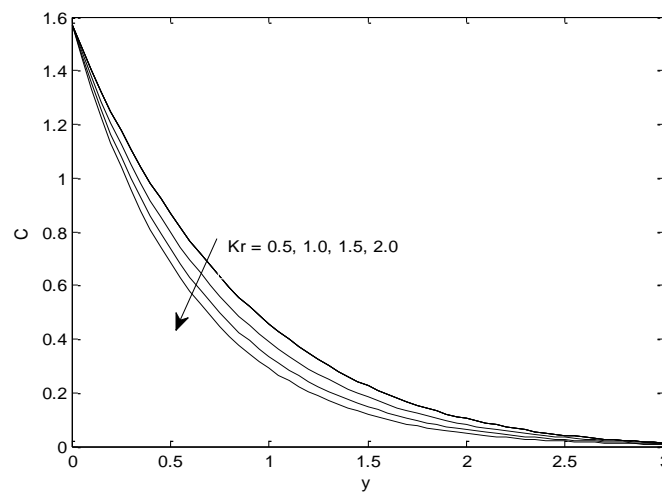


Fig. 7. Concentration profiles for different values of chemical reaction parameter

Table 1
Effects of Grashof number on skin friction

Gr	C_f
1.0	-0.3445
1.2	-0.3446
1.3	-0.3418
1.4	-0.3393

4. Conclusions

A detailed analytical study has been carried out for the flow past an impulsively started infinite vertical plate with variable temperature and mass diffusion. These circumstances are of interest in several manufacturing processes. The dimensionless governing equations are solved by perturbation technique. Numerical evaluations of the closed form results were performed and some graphical results were performed and some graphical results were obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some of physical parameters. The study has been compared with available exact solution the literature and they are found to be in good agreement. It is observed that, the concentration increases during generative reaction and decreases in destructive reaction. The concentration increases with decreasing Schmidt number. The effect of increasing values of k leads to a fall in velocity decreases with increasing values of the Schmidt number. An increase in modified Grashof number leads to an increase in velocity profiles. The skin friction increases with decreasing Schmidt number. In generative reaction the skin friction decreases and in destructive reaction the skin friction increases.

References

- [1] Amkadni, Maryem, and Adnane Azzouzi. "On a similarity solution of MHD boundary layer flow over a moving vertical cylinder." *International Journal of Differential Equations* 2006 (2006): 1-9.
- [2] Bestman, A. R. "Natural convection boundary layer with suction and mass transfer in a porous medium." *International journal of energy research* 14, no. 4 (1990): 389-396.
- [3] Lavanya, B. "Effect of radiation on free convection heat and mass transfer flow through porous medium in a vertical channel with heat absorption/generation and chemical reaction." In *AIP Conference Proceedings*, vol. 1859, no. 1, p. 020023. AIP Publishing, 2017.
- [4] Sudheer Babu, M., and P. V. Satyanarayana. "Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field." *JP Journal of Heat and mass transfer* 3, no. 3 (2009): 219-234.
- [5] Ahmmed, S. F., and M. S. Sarker. "MHD natural convection flow of viscous incompressible fluid from a vertical flat plate." *Journal of Physical Science* 13, (2009): 77-85.
- [6] Lavanya, Bommanna. "MHD Rotating Flow Through a Porous Medium with Heat and Mass Transfer." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 54, no. 2 (2019): 221-231.
- [7] Saravana, R., S. Sreekanth, S. Sreenadh, and R. Hemadri Reddy. "Mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux." *Advances in Applied Science Research* 2, no. 1 (2011): 221-229.
- [8] Hiremath, P. S., and P. M. Patil. "Free convection effects on the oscillatory flow of a couple stress fluid through a porous medium." *Acta Mechanica* 98, no. 1-4 (1993): 143-158.
- [9] Nagasakala, Madduleti, and Bommanna Lavanya. "Effects of Dissipation and Radiation on Heat Transfer Flow of a Convective Rotating Cuo-Water Nano-fluid in a Vertical Channel." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 50, no. 2 (2018): 108-117.
- [10] Molla, Md Mamun, Md Anwar Hossain, and Lun Shin Yao. "Natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption." *International Journal of Thermal Sciences* 43, no. 2 (2004): 157-163.
- [11] Alam, M. S., M. M. Rahman, and M. A. Sattar. "MHD free convective heat and mass transfer flow past an inclined surface with heat generation." *Science & Technology Asia* (2006): 1-8.

-
- [12] Lavanya, B. "Unsteady MHD Convective laminar flow between two Vertical Porous plates with mass transfer." *Journal of Mechanical Engineering Research and Developments* 41, no. 1 (2018): 97-109.
- [13] Das, K., and S. Jana. "Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium." *Bull. Soc. Math. Banja Luka* 17, no. 10 (2010): 15-32.
- [14] Sonth, R. M., Sujit Kumar Khan, M. S. Abel, and K. V. Prasad. "Heat and mass transfer in a visco-elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation." *Heat and Mass Transfer* 38, no. 3 (2002): 213-220.
- [15] Shateyi, Stanford, Sandile Sydney Motsa, and Precious Sibanda. "The effects of thermal radiation, hall currents, solet, and dufour on MHD flow by mixed convection over a vertical surface in porous media." *Mathematical Problems in Engineering* 2010 (2010).
- [16] Ahmad, Sayyid Zainal Abidin Syed, Wan Azmi Wan Hamzah, Mohd Rijal Ilias, Sharidan Shafie, and Gholamhassan Najafi. "Unsteady MHD Boundary Layer Flow and Heat Transfer of Ferrofluids Over A Horizontal Flat Plate with Leading Edge Accretion." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 59, no. 2 (2019): 163-181.
- [17] Siti Hanani Mat Yasin, Muhammad Khairul Anuar Mohamed, Zulhibri Ismail@Mustofa, Basuki Widodo, Mohd Zuki Salleh. "Numerical solution on MHD stagnation point flow in ferrofluid with Newtonian heating and thermal radiation effect." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 57, no. 1 (2019): 12-22.