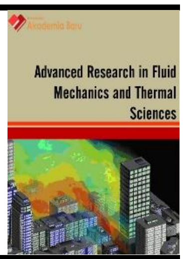
Penerbit Akademia Baru

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage: www.akademiabaru.com/arfmts.html
ISSN: 2289-7879



Influence of Slip Velocity on Micropolar Fluid Through a Porous Channel: Using Keller-Box Method

Open Access

Ashwini Bhat[1], Nagaraj N Katagi[1,*]

[1] Department of Mathematics, Manipal Institute of Technology, Manipal Academy of Higher Education, Udupi Karkala Road, Manipal, Karnataka-576104, India

ARTICLE INFO

Article history:
Received 4 October 2019
Received in revised form 15 December 2019
Accepted 18 December 2019
Available online 4 March 2020

Keywords:
Slip coefficient; micropolar fluid; microrotation; pressure gradient; Keller-box method

ABSTRACT

Micropolar fluid flow through porous channel assuming slip velocity conditions has been analyzed. An efficient numerical scheme based on Keller-box method is used as the main tool for solution approach. The primary objective is to study the influence of non-zero tangential slip velocity on velocity field, micro-rotation and pressure field. The dimensionless similarity equations of momentum, angular momentum are solved numerically using Keller-box scheme. The effect of slip and other pertinent parameters on velocity, micro-rotation field and also pressure are elaborated graphically.

1. Introduction

The examinations on laminar flow through channels and pipes have acquired a significant amount of interest among researchers due to its wide spread applications in the field of biophysical flows. Berman [1] initiated the study on laminar flow of Newtonian fluid through a porous channel for small Reynolds number, obtained a first order perturbation solution. Subsequently several authors White [2], Sellar [3], Yuan [4], Terrill [5], Brady [6], Robinson [7], Cox [8] have extended the above Berman's problem for large value of suction and injection. Later King and Cox[9] investigated time independent steady state Berman problem using asymptotic analysis.

Most of the above studies have included Newtonian fluid in the flow phenomena. The theory of Newtonian fluids cannot accurately describe the coarse structure in the fluid, fiber materials such as colloidal fluids etc. To this end, non-Newtonian fluids have been widely used in industrial applications in the recent past. Micropolar fluid model is one of the prominent non-Newtonian fluid model that has acquired the special status in recent years. Investigation of such liquids speaks to a decent scientific model for classical and biological liquids. The theory of micropolar fluids introduced by Eringen[10] is an exceptional case of the theory of simple microfluids. These fluid elements can sustain stress moments, body couples and are also influenced by the spin inertial. The stress tensor

[*] *Corresponding author.*
E-mail address: nn.katagi@manipal.edu (Nagaraj N Katagi)

is not symmetric for such fluids. Due to the natural structure of elements in micropolar fluids, they represent a good mathematical model for many natural and industrial fluids. The applications of such fluids are in blood flow, lubricants, porous media, turbulent shear flows and flows in capillaries and microchannels. A very good review of this subject and its applications can be found in Lukaazewicz [10], Eringen [11-12], Ariman [13-14].

Kelson *et al.,* [15] explored two dimensional stream of a micropolar liquid in a permeable channel with high mass transfer utilizing the perturbation method and numerical computations. Following researchers, Sastry and Rao [16], Takhar [17], Ashraf [18], and Rashidi [19] have contemplated the micropolar liquid through permeable channel utilizing Semi-numerical and numerical strategies for suction or injection. Later Xin-hui Si *et al.,* [20] have studied the above problem with expanding and contracting walls.

However, the previous literature on micropolar fluids flow between porous boundaries has been limited to no-slip boundary conditions. Beavers and Joseph [21] in their mass efflux experiments proved the existence of slip at the porous boundaries. The historical background to these Beavers - Joseph conditions were also reported by Nield [22]. In our present analysis we considered the slip boundary conditions at the porous wall and obtained the numerical solution for micropolar flow in channel using an implicit Keller-box method. The Keller box method is an implicit scheme with second-order accuracy in both space and time. This scheme differs from other techniques, where the second and higher order derivatives are replaced by first order derivatives through the introduction of additional variables, which results in a system of first order equations. The finite difference approximations to these derivatives results in an algebraic system involving unknown grid points at three levels. The resulting system of equations is a block tri-diagonal system and can be solved with the general block tri-diagonal algorithm. Keller [23] used the above scheme for solving diffusion problems, but it has subsequently been applied to a broad class of problems. A sketch of applications of this box scheme to a variety of boundary layer flow problems is given in [24-26]. This technique is used to solve the present problem and the effects of various pertinent parameters upon the flow field are presented.

Organization of the present paper proceeds as follows. The formulation of problem is given in section (2). In section (3) we give the Keller-box solution of the governing equation which is described in detail. Section (4) is devoted to discussion of various results for all physical parameters.

2. Mathematical Formulation of the Problem

Let us consider an incompressible laminar flow of a micropolar fluid through two infinite parallel porous channel separated by a distance $2h$ (Figure 1). Let the magnitude of uniform velocity of suction at channel walls be V_0. Considering (x, y) coordinate system, with center of channel axis as the origin, with x axis is in a plane parallel to channel walls and y axis is perpendicular it.

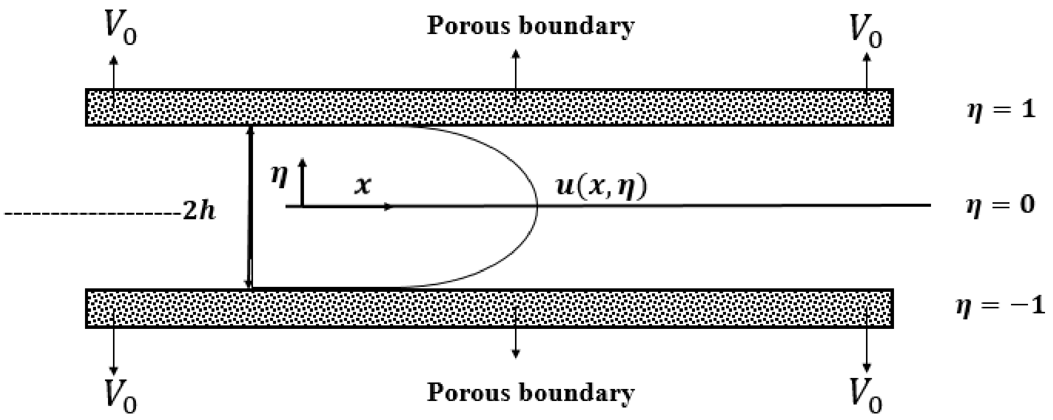


Fig. 1. Geometry of micropolar fluid flow through a porous channel

The velocity and micro-rotation of the flow are,

$$u_x = u(x,y),\ u_y = v(x,y),\ u_z = 0 \tag{1}$$

$$\nu_x = \nu_y = 0,\ \nu_z = \nu(x,y) \tag{2}$$

The equations of continuity and momentum are given as follows

$$\frac{\partial u}{\partial x} + \frac{1}{h}\frac{\partial v}{\partial \eta} = 0 \tag{3}$$

$$\rho(u\frac{\partial u}{\partial x} + \frac{v}{h}\frac{\partial u}{\partial \eta}) = -\frac{\partial p}{\partial x} + \frac{k}{h}\frac{\partial \nu}{\partial \eta} + (\mu + k)(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}) \tag{4}$$

$$\rho(u\frac{\partial v}{\partial x} + \frac{v}{h}\frac{\partial v}{\partial \eta}) = -\frac{\partial p}{h\,\partial \eta} - k\frac{\partial \nu}{\partial x} + (\mu + k)(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 v}{\partial \eta^2}) \tag{5}$$

$$\rho j(u\frac{\partial \nu}{\partial x} + \frac{v}{h}\frac{\partial \nu}{\partial \eta}) = \gamma(\frac{\partial^2 \nu}{\partial x^2} + \frac{1}{h^2}\frac{\partial^2 \nu}{\partial \eta^2}) + k(\frac{\partial v}{\partial x} - \frac{1}{h}\frac{\partial u}{\partial \eta}) - 2k\nu \tag{6}$$

where, $\eta = \frac{y}{h}$, ρ is the density of fluid, p is the pressure, j is the microinertia, μ is the coefficient of viscosity, k and γ are material constants of micropolar fluid.

For velocity slip at the porous wall, we incorporate the slip conditions due to Beavers and Joseph [14], that is slip velocity at porous boundary is proportional to wall shear rate and is given by,

$$u(x,\pm 1) = u_{\text{slip}} = -\phi\frac{\partial u}{\partial \eta} \tag{7}$$

where $\phi = \frac{\sqrt{K}}{\alpha h}$, in which α is a dimensionless constant which depends on porous material and K is the permeability. Eq. (7) reduces to no-slip condition if $\phi = 0$.

The other appropriate boundary conditions for the governing problem are,

$$v(x,\pm 1) = \pm v_0,\ \nu(x,\pm 1) = 0 \tag{8}$$

Assuming a suitable stream function satisfying from Eq. (3) to (5), the similarity solutions can be written as,

$$u = (v_0 - v_0 \frac{x}{h})\frac{df}{d\eta},\ v = v_0 f(\eta)$$

$$\nu = \frac{1}{h}(U_0 - V_0 \frac{x}{h})g(\eta) \tag{9}$$

where, U_0 is the average velocity at the entry of flow. Eq. (4) to (6) can be reduced to,

$$\frac{\partial p}{\partial x} = \frac{\mu}{h^2}(U_0 - V_0 \frac{x}{h})[(1+R)f''' + Rg' + S(f'^2 - ff'')] \tag{10}$$

$$\frac{\partial p}{\partial \eta} = \frac{\mu V_0}{h}((1+R)f'' + Rg - Sff') \tag{11}$$

$$Ag'' - Rf'' - 2Rg = BS(fg' - gf') \tag{12}$$

where, the suction Reynolds number is $S = \frac{\rho V_0 h}{\mu}$, $R = \frac{k}{\mu}, A = \frac{\gamma}{\mu h^2}$, $B = \frac{j}{h^2}$ are the micropolar parameters. The effects of microrotation on velocity and the couple stress on microrotation are given by the parameters R and A respectively. Eliminating pressure from Eq. (10) and (11) we have,

$$\frac{d}{d\eta}[(1+R)f''' + Rg' + S(f'^2 - ff'')] = 0 \tag{13}$$

implies,

$$(1+R)f''' + Rg' + S(f'^2 - ff'') = c(\text{constant}) \tag{14}$$

on differentiation,

$$(1+R)f'''' + Rg'' + S(f'f'' - ff''') = 0 \tag{15}$$

The corresponding boundary conditions in (7) take the form,

$$f'(\pm 1) = -\phi f''(\pm 1),\ f(\pm 1) = \pm 1, g(\pm 1) = 0 \tag{16}$$

where ϕ is the velocity slip coefficient. The dimensionless axial velocity and the microrotation are,

$$\bar{u} = \frac{\rho u h}{\mu} = (\frac{R_e \xi}{4} - SX)f'(\eta) \tag{17}$$

and

$$\bar{v} = \frac{\rho \nu h^2}{\mu} = (\frac{R_e \xi}{4} - SX)g(\eta) \tag{18}$$

where $\xi = \frac{1}{R+2}(\frac{3A}{R+2}(1 - q\ coth\ q) + 2)$, $q = \sqrt{\frac{R(R+2)}{A(1+R)}}$, $R_e = \frac{4\bar{U}_0 \rho h}{\mu}$ denotes Reynolds number at the flow entry of channel, and $X = \frac{x}{h}$.

The non-dimensional stream function describing the flow can be written as,

$$\bar{\psi} = \frac{\psi}{h\bar{V}_0} = (\xi - \frac{4SX}{R_e}) f(\eta) \tag{19}$$

The expression for pressure difference in the channel is given by,

$$C_p = \frac{(p(0,\eta) - p(x,\eta))h^2 \rho}{\mu^2} = \frac{-cX}{4}(R_e \xi - 2SX) \tag{20}$$

Similarly, coefficient of shear stress C_f and couple stress C_m at the walls of the channel are respectively,

$$C_f = (1 + R)(\frac{R_e \xi}{4} - SX) f''(\pm 1) \tag{21}$$

$$C_m = A(\frac{R_e \xi}{4} - SX) g'(\pm 1) \tag{22}$$

Since the flow is symmetric with respect to the center line, boundary conditions in Eq. (16) reduces to,

$$f'' = f = g = 0, \quad \text{at } \eta = 0$$

$$f' = -\phi f'', f = 1, g = 0, \quad \text{at } \eta = 1 \tag{23}$$

The governing Eq. (12) and (15) are not amenable to solve analytically, in a closed form solution. Thus, numerical solution is obtained using Keller-box scheme. In the following section we have elaborated in detail about Keller-box method.

3. Method of Solution

The system of non-linear coupled ordinary equations represented by Eq. (12) and (15) for different values of micropolar parameter R and suction Reynolds number S are developed using Newtons linearization technique. The solution procedure first uses finite difference approximations and solves over a box scheme. This method is also called Keller-box technique. We initialize the procedure by writing in terms of system of first order equations by introducing new variables,

$$u = f',\ v = f'',\ w = f''',\quad h = g' \tag{24}$$

The two coupled higher order differential equations and the boundary conditions may be transformed to six equivalent first order differential equation and boundary conditions respectively are given below.

$$f' = u,\quad u' = v,\quad v' = w,\quad g' = h$$

$$w' = \frac{-R}{(1+R)} h' - \frac{S}{1+R}(uv - fw)$$

$$h' = \frac{BS}{A}(fh - ug) + \frac{R}{A}v + \frac{2R}{A}g \tag{25}$$

And the boundary conditions are,

$$\text{at} \quad \eta = 0, \quad f = 0, \quad v = 0, \quad g = 0$$
$$\text{at} \quad \eta = 1, \quad f = 1, \quad u = -\phi v, \quad g = 0 \tag{26}$$

We initiate discretization process by writing the finite difference approximations of the ordinary differential equations in Eq. (25) as,

$$f_j - f_{j-1} = \frac{\eta_j}{2}(u_j + u_{j-1}) \tag{27}$$

$$u_j - u_{j-1} = \frac{\eta_j}{2}(v_j + v_{j-1}) \tag{28}$$

$$v_j - v_{j-1} = \frac{\eta_j}{2}(w_j + w_{j-1}) \tag{29}$$

$$g_j - g_{j-1} = \frac{\eta_j}{2}(h_j + h_{j-1}) \tag{30}$$

$$w_j - w_{j-1} = \frac{-\eta_j R}{1+R}(h_j - h_{j-1}) - \frac{\eta_j S}{4(1+R)}[(u_j + u_{j-1})(v_j + v_{j-1}) - (f_j + f_{j-1})(w_j + w_{j-1})] \tag{31}$$

$$h_j - h_{j-1} = \frac{\eta_j BS}{4A}[(f_j + f_{j-1})(h_j + h_{j-1}) - (u_j + u_{j-1})(g_j + g_{j-1})] - \frac{R\eta_j}{2A}(v_j + v_{j-1}) + \frac{2\eta_j R}{2A}(g_j + g_{j-1}) \tag{32}$$

We introduce Newton's method by linearizing the system of equations using following iterates,

$$f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)} \tag{33}$$

$$u_j^{(i+1)} = u_j^{(i)} + \delta u_j^{(i)} \tag{34}$$

$$v_j^{(i+1)} = v_j^{(i)} + \delta v_j^{(i)} \tag{35}$$

$$w_j^{(i+1)} = w_j^{(i)} + \delta w_j^{(i)} \tag{36}$$

$$g_j^{(i+1)} = g_j^{(i)} + \delta g_j^{(i)} \tag{37}$$

$$h_j^{(i+1)} = h_j^{(i)} + \delta h_j^{(i)} \tag{38}$$

Substituting the iterates in Eq. (33) to (38) into Eq. (27) to (32) and neglecting the terms that are quadratic in $(\delta f_j^n, \delta u_j^n, \delta v_j^n, \delta w_j^n, \delta g_j^n, \delta h_j^n,)$, and rewriting, we get the following system of algebraic equations.

$$\delta f_j - \delta f_{j-1} - d_j(\delta u_j + \delta u_{j-1}) = (r_1)_j \tag{39}$$

$$\delta u_j - \delta u_{j-1} - d_j(\delta v_j + \delta v_{j-1}) = (r_2)_j \tag{40}$$

$$\delta v_j - \delta v_{j-1} - d_j(\delta w_j + \delta w_{j-1}) = (r_3)_j \tag{41}$$

$$\delta g_j - \delta g_{j-1} - d_j(\delta h_j + \delta h_{j-1}) = (r_4)_j \tag{42}$$

$$(a_1)_j \delta w_j + (a_2)_j \delta w_{j-1} + (a_3)_j(\delta h_j + \delta h_{j-1}) + (a_4)_j(\delta u_j + \delta u_{j-1}) \\ +(a_5)_j(\delta v_j + \delta v_{j-1}) + (a_6)_j(\delta f_j + \delta f_{j-1}) = (r_5)_j \tag{43}$$

$$(b_1)_j \delta h_j + (b_2)_j \delta h_{j-1} + (b_3)_j(\delta f_j + \delta f_{j-1}) + (b_4)_j(\delta g_j + \delta g_{j-1}) \\ +(b_5)_j(\delta v_j + \delta v_{j-1}) + (b_6)_j(\delta u_j + \delta u_{j-1}) = (r_6)_j \tag{44}$$

where,

$$(a_1)_j = 1 - \frac{d_j S}{2(1+R)}(f_j + f_{j-1}) \tag{45}$$

$$(a_2)_j = -2 + (a_1)_j \tag{46}$$

$$(a_3)_j = \frac{R}{1+R} \tag{47}$$

$$(a_4)_j = \frac{-S d_j}{2(1+R)}(v_j + v_{j-1}) \tag{48}$$

$$(a_5)_j = \frac{S d_j}{2(1+R)}(u_j + u_{j-1}) \tag{49}$$

$$(a_6)_j = \frac{-S d_j}{2(1+R)}(w_j + w_{j-1}) \tag{50}$$

$$(b_1)_j = 1 - \frac{d_j\ BS}{2A}(f_j + f_{j-1}) \tag{51}$$

$$(b_2)_j = -2 + (b_1)_j \tag{52}$$

$$(b_3)_j = \frac{-B d_j S}{2A}(h_j + h_{j-1}) \tag{53}$$

$$(b_4)_j = \frac{BS d_j}{2A}(u_j + u_{j-1}) - \frac{2R d_j}{A} \tag{54}$$

$$(b_5)_j = \frac{d_j R}{A} \tag{55}$$

$$(b_6)_j = \frac{d_j BS}{2A}(g_j + g_{j-1}) \tag{56}$$

and

$$(r_1)_j = d_j(u_j + u_{j-1}) + f_{j-1} - f_j \tag{57}$$

$$(r_2)_j = d_j(v_j + v_{j-1}) + u_{j-1} - u_j \tag{58}$$

$$(r_3)_j = d_j(w_j + w_{j-1}) + v_{j-1} - v_j \tag{59}$$

$$(r_4)_j = d_j(h_j + h_{j-1}) + g_{j-1} - g_j \tag{60}$$

$$(r_5)_j = w_{i-1} - w_i - \frac{R}{1+R}(h_i - h_{i-1}) - \frac{Sd_j}{2(1+R)}[(u_i + u_{i-1})(v_i + v_{i-1}) -(f_i + f_{i-1})(w_i + w_{i-1})] \tag{61}$$

$$(r_6)_j = \frac{BSd_j}{2A}[(f_i + f_{i-1})(h_i + h_{i-1}) - (u_i + u_{i-1})(g_i + g_{i-1})] + \frac{d_j R}{A}(v_i + v_{i-1}) + \frac{d_j 2R}{A}(g_i + g_{i-1}) + h_{i-1} - h_i \tag{62}$$

The boundary conditions takes the form,

$$\delta f_0 = 0, \quad \delta v_0 = 0, \quad \delta g_0 = 0, \quad \delta f_J = 0, \quad \delta u_J = 0, \quad \delta g_J = 0$$

The linearized system of Eq. (39) to (44) have a block tri-diagonal structure which can be solved with the help of block elimination method. In vector -matrix form, Eq. (39) to (44) can be written as,

$$A\delta = r \tag{63}$$

where,

$$A = \begin{bmatrix} [A_1] & [C_1] & 0 & \dots & 0 & 0 & 0 \\ [B_1] & [A_2] & [C_2] & \dots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & [B_{J-1}] & [A_{J-1}] & [C_{J-1}] \\ 0 & 0 & 0 & \dots & 0 & [B_J] & [A_J] \end{bmatrix} \tag{64}$$

$$\delta = \begin{bmatrix} [\delta_1] \\ \vdots \\ [\delta_J] \end{bmatrix} \quad r = \begin{bmatrix} [r_1] \\ \vdots \\ [r_J] \end{bmatrix} \tag{65}$$

where in Eq. (64) the elements are defined by,

$$[A_j] = \begin{bmatrix} -d_j & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & -d_j & 0 \\ 0 & -d_j & 0 & 0 & 1 & 0 \\ 0 & 0 & -d_j & 0 & 0 & 1 \\ (a_4)_j & (a_2)_j & -(a_3)_j & (a_6)_j & (a_5)_j & 0 \\ (b_6)_j & 0 & (b_2)_j & (b_3)_j & (b_5)_j & (b_4)_j \end{bmatrix}, \quad 1 \le j \le J-1$$

$$[A_J] = \begin{bmatrix} -d_J & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & -d_J & 0 & 0 \\ 0 & -d_J & 0 & 1 & -d_J & 0 \\ 0 & 0 & -d_J & 0 & 0 & -d_J \\ (a_4)_J & (a_2)_J & -(a_3)_J & (a_5)_J & (a_1)_J & (a_3)_J \\ (b_6)_J & 0 & (b_2)_J & (b_5)_J & 0 & (b_1)_J \end{bmatrix}$$

$$[B_j] = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -d_j & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & (a_6)_j & (a_5)_j & 0 \\ 0 & 0 & 0 & (b_3)_j & (b_5)_j & (b_4)_j \end{bmatrix}, \quad 2 \le j \le J$$

$$[C_j] = \begin{bmatrix} -d_j & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -d_j & 0 & 0 & 0 & 0 \\ 0 & 0 & -d_j & 0 & 0 & 0 \\ (a_4)_j & (a_1)_j & (a_3)_j & 0 & 0 & 0 \\ (b_6)_j & 0 & (b_1)_j & 0 & 0 & 0 \end{bmatrix}, \quad 1 \le j \le J-1$$

$$[\delta_j] = [\delta u_{j-1} \;\; \delta w_{j-1} \;\; \delta h_{j-1} \;\; \delta f_j \;\; \delta v_j \;\; \delta g_j]^T, \quad 1 \le j \le J-1$$

and

$$[\delta_J] = [\delta u_{J-1} \;\; \delta w_{J-1} \;\; \delta h_{J-1} \;\; \delta v_J \;\; \delta w_J \;\; \delta h_J]^T$$

$$[r_j] = [(r_1)_j \;\; (r_2)_j \;\; (r_3)_j \;\; (r_4)_j \;\; (r_5)_j \;\; (r_6)_j]^T, \quad 1 \le j \le J$$

Now we write,

$$A = Lu \tag{66}$$

where,

$$L = \begin{bmatrix} [\alpha_1] & 0 & \dots & 0 & 0 \\ [B_1] & [\alpha_2] & \dots & 0 & 0 \\ \dots & \ddots & \ddots & \dots & \dots \\ 0 & 0 & \cdots & [B_J] & [\alpha_J] \end{bmatrix}; \quad u = \begin{bmatrix} [I] & [\Gamma_1] & 0 & \dots & 0 & 0 \\ 0 & [I] & [\Gamma_2] & \dots & 0 & 0 \\ \dots & \dots & \ddots & \ddots & \dots & \dots \\ 0 & 0 & 0 & \dots & [I] & [\Gamma_{J-1}] \\ 0 & 0 & 0 & \dots & 0 & [I] \end{bmatrix}$$

Here, $[I]$ is unit matrix and $[\alpha_i]$ and $[\Gamma_i]$ are 6×6 matrices whose elements are determined by the following equations.

$$[\alpha_j] = [A_1]$$
$$[A_1][\Gamma_1] = [C_1]$$
$$[\alpha_j] = [A_j] - [B_j][\Gamma_{j-1}], \quad j = 2,3, \dots, J$$
$$[\alpha_j][\Gamma_j] = [C_j], \quad j = 2,3, \dots, J-1$$

Using Eq. (66) and (63) we have,

$$Lu\delta = r \tag{67}$$

Defining $u\delta = w$, we have,

$$Lw = r \tag{68}$$

where,

$$w = [[w_1] \ [w_2] \ [w_3] \ [w_4] \ [w_5] \ [w_6] \ ]^T \tag{69}$$

and the $[w_j]$ are 6×1 column matrices. The elements w can be solved from Eq. (68)

$$[\alpha_1][w_1] = [r_1]$$
$$[\alpha_j][w_j] = [r_j] - [B_j][w_{j-1}]$$

Once the elements of w are found, solution for δ can be obtained using relations

$$[\delta_J] = [w_J]$$
$$[\delta_j] = [w_j] - [\Gamma_j][\delta_{j+1}]$$

Theses solution for δ can be used in Eq. (33) to (38) to find $(i+1)^{th}$ iteration.

4. Results and Discussions

This section examines the influence of velocity slip on microploar fluid through a porous channel. Further, the effect of slip coefficient on velocity, pressure gradient, and microrotation for for different micropolar parameters and suction Reynolds number. The governing equations are rendered dimensionless and are solved by an efficient difference scheme, Keller-box method in conjugation with quasi-linearization process. Figure 2 to 9 explain variation of velocity and micro-rotation for different micropolar parameter and slip coefficient.

From Figure 2 and 3, it is seen that velocity profiles are flattened as Reynolds number S or micro-rotation parameter R increases. However, a general decrease in the magnitude of velocity profiles are noticed with an increase in the slip coefficient.

Figure 4 and 5 shows that micro-rotation profiles $\bar{\nu}$ increases with decrease in S upto $\eta = 0.6$ and then decreases towards the wall by approaching to zero as $\eta \to 1$. Also, we note that micro-rotation profiles decreases in magnitude with an increase in slip coefficient ϕ.

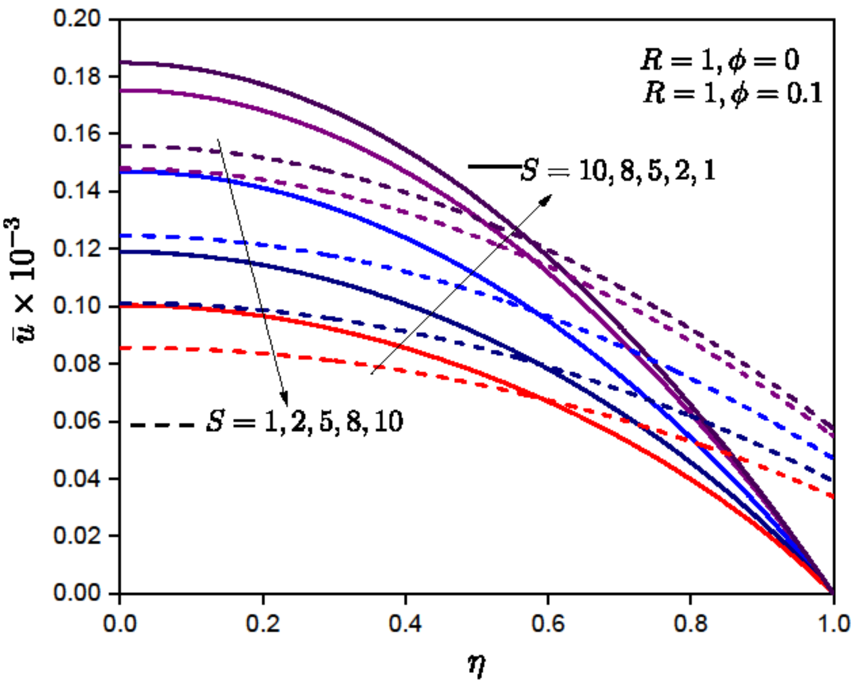


Fig. 2. Velocity profiles for different values of suction with $X = 6$, for $\phi = 0{,}0.1$

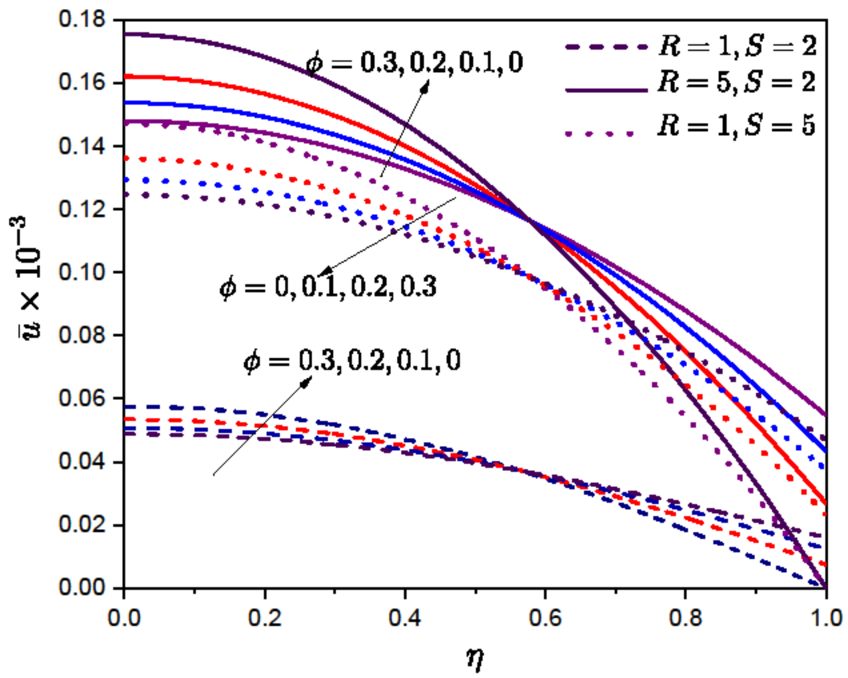


Fig. 3. Velocity profiles for different values of slip coefficient with $X = 7$, for $R = 1{,}5$

Flow lines for various slip coefficient ϕ have been plotted in Figure 6 to 8. It is noticed that an increase in the value of microrotation parameter R results in diminishing the distance of particle which are moving parallel to the walls of channel from the entry of flow. It is also evident that the same nature can be observed in the stream lines as slip coefficient, parameter A and suction Reynolds number S increases. The above behavior of curves shows that micro-rotation and slip coefficient plays a vital role in affecting the fluid flow.

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences
Volume 66, Issue 2 (2020) 49-64

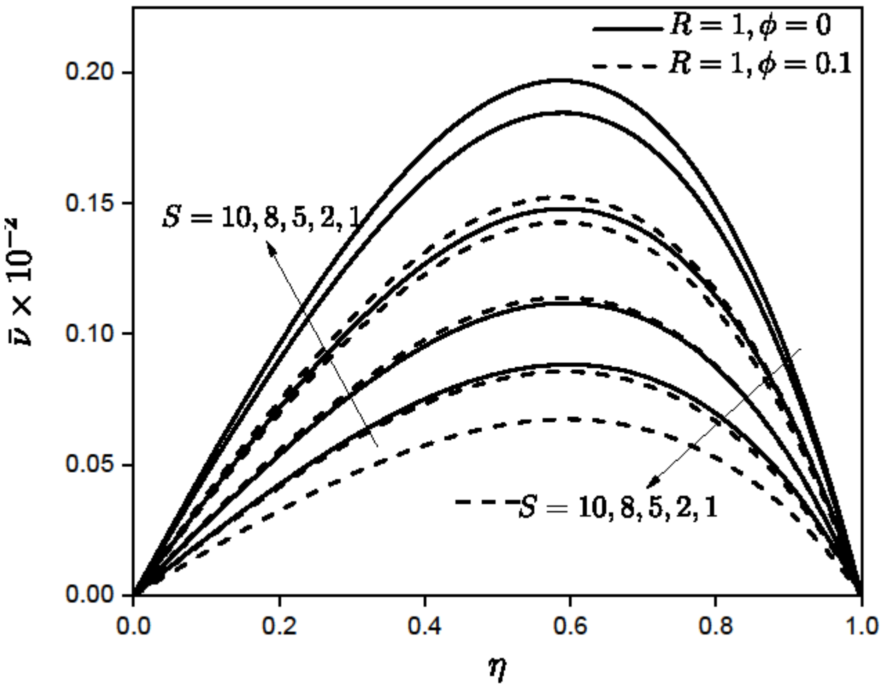


Fig. 4. Micro-rotation profiles for different values of suction with $X = 6$, for $\phi = 0{,}0.1$

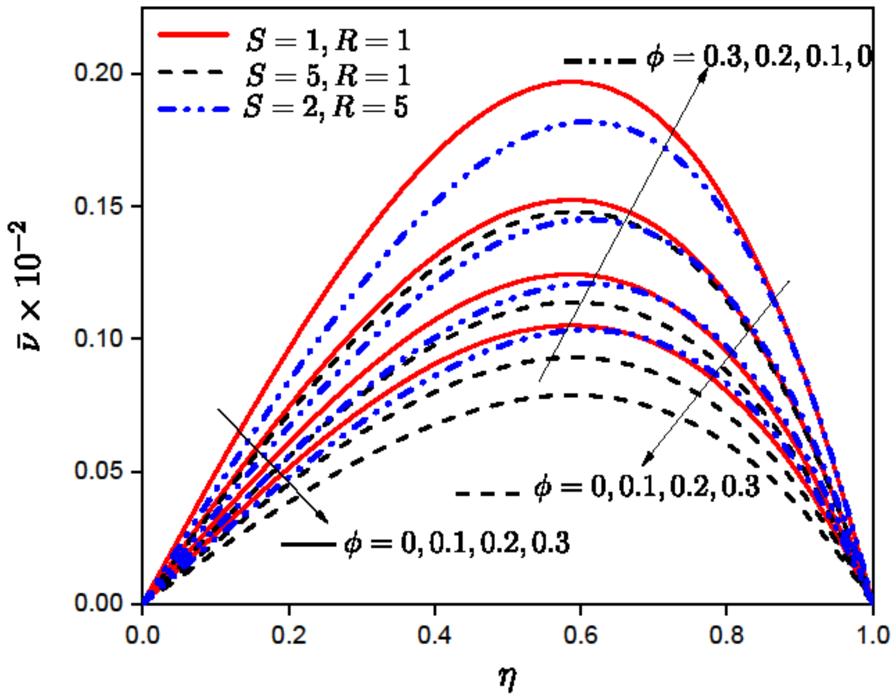


Fig. 5. Micro-rotation profiles for different values of slip coefficient with $X = 7$, for $R = 1{,}5$

Magnitude of the pressure coefficient C_p for different slip velocity and suction Reynolds number S are shown in Figure 9. From the graph, it is seen that an increase in the value of slip coefficient ϕ decreases the value of C_p. Further, it is also noticed that a large increment in ϕ significantly reduces the C_p. This trend is connected with diminution in the shear stress at the porous surface.

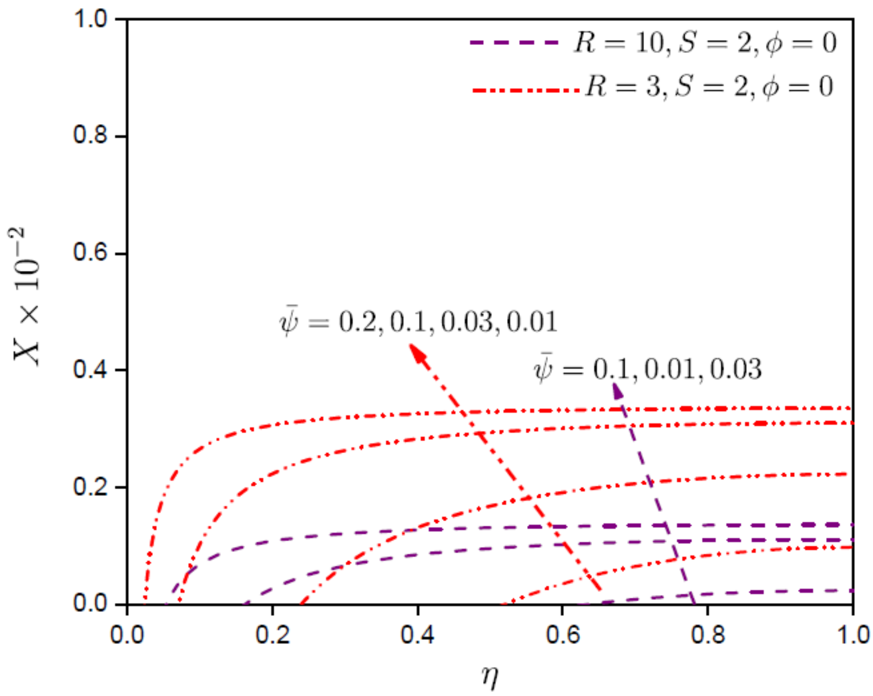


Fig. 6. Flow lines in the upper half of channel when $S = 2$ and $\phi = 0$

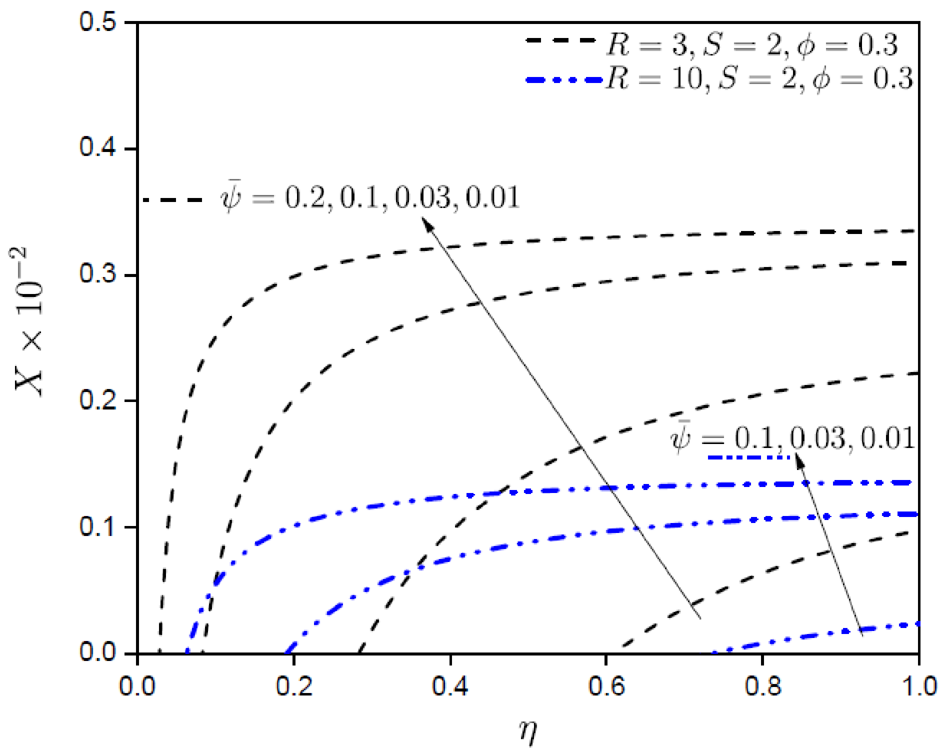


Fig. 7. Flow lines in the upper half of channel when $S = 2$ and $\phi = 0.3$

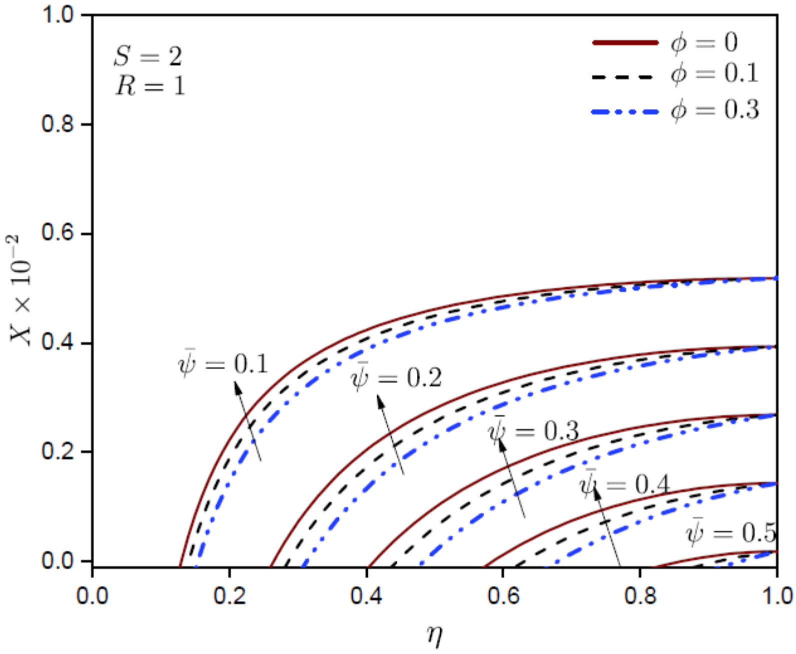


Fig. 8. Effect of slip coefficient on flow lines in the upper half of channel for $S = 2, R = 1$

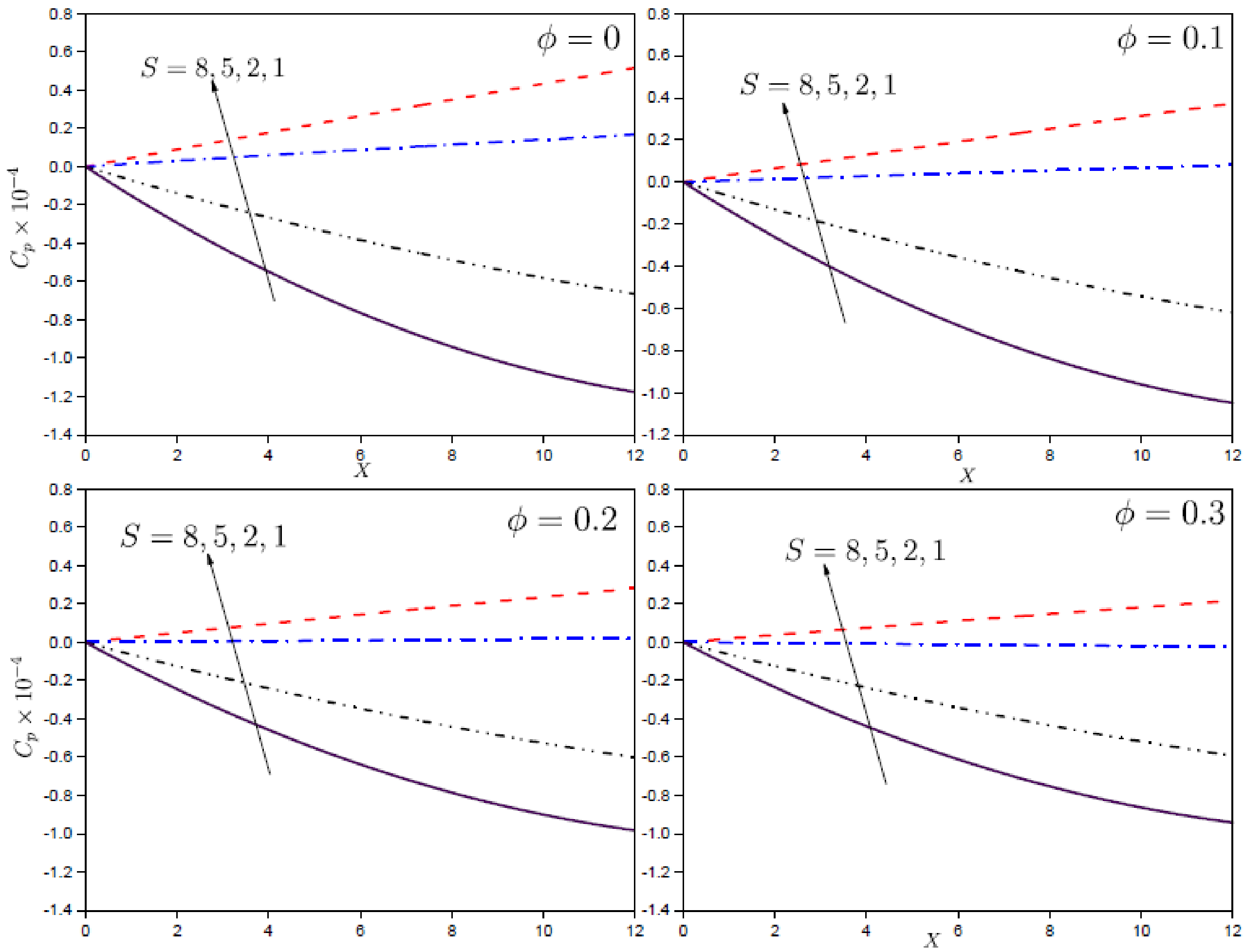


Fig. 9. Effect of velocity slip on pressure coefficient C_p, for different S and ϕ

5. Conclusions

From the above discussion, we conclude that the presence of micro-rotating elements with slip conditions at the porous boundary influences characteristics features of the flow. The problem of channel flow with micropolar fluid [16] is a special case of the present problem when slip coefficient reduces to zero. Also, that when micropolar parameter also reduces to zero, it is Berman [1] channel flow solution. The solution is in good agreement even for Newtonian fluid flow between porous channel with velocity slip [27].

References

[1] Berman, Abraham S. "Laminar flow in channels with porous walls." *Journal of Applied physics* 24, no. 9 (1953): 1232-1235.
[2] White Jr, F. M. "Laminar flow in a uniformly porous tube." (1962): 201-204.
[3] Sellars, John R. "Laminar flow in channels with porous walls at high suction Reynolds numbers." *Journal of Applied Physics* 26, no. 4 (1955): 489-490.
[4] Yuan, S. W. "Further investigation of laminar flow in channels with porous walls." *Journal of Applied Physics* 27, no. 3 (1956): 267-269.
[5] Terrill, R. M. "The flow between two parallel circular disks, one of which is subject to a normal sinusoidal oscillation." (1969): 126-131.
[6] Brady, J. F. "Flow development in a porous channel and tube." *The Physics of fluids* 27, no. 5 (1984): 1061-1067.
[7] Robinson, W. A. "The existence of multiple solutions for the laminar flow in a uniformly porous channel with suction at both walls." *Journal of Engineering Mathematics* 10, no. 1 (1976): 23-40.
[8] Cox, Stephen M. "Analysis of steady flow in a channel with one porous wall, or with accelerating walls." *SIAM Journal on Applied Mathematics* 51, no. 2 (1991): 429-438.
[9] King, J. R., and S. M. Cox. "Asymptotic analysis of the steady-state and time-dependent Berman problem." *Journal of engineering mathematics* 39, no. 1 (2001): 87-130.
[10] Lukaszewicz, Grzegorz. *Micropolar fluids: theory and applications*. Springer Science & Business Media, 1999.
[11] Eringen, A. Cemal. "Theory of micropolar fluids." *Journal of Mathematics and Mechanics* (1966): 1-18.
[12] Eringen, A. Cemal. "Theory of anisotropic micropolar fluids." *International Journal of Engineering Science* 18, no. 1 (1980): 5-17.
[13] Ariman, T. M. A. N. D., M. A. Turk, and N. D. Sylvester. "Microcontinuum fluid mechanics—a review." *International Journal of Engineering Science* 11, no. 8 (1973): 905-930.
[14] Ariman, T. T. N. D., M. A. Turk, and N. D. Sylvester. "Applications of microcontinuum fluid mechanics." *International Journal of Engineering Science* 12, no. 4 (1974): 273-293.
[15] Kelson, Neil A., Andre Desseaux, and Troy W. Farrell. "Micropolar flow in a porous channel with high mass transfer." *ANZIAM Journal* 44 (2002): 479-495.
[16] Sastry, V. U. K., and V. Rama Mohan Rao. "Numerical solution of micropolar fluid flow in a channel with porous walls." *International Journal of Engineering Science* 20, no. 5 (1982): 631-642.
[17] Takhar, Harmindar S., R. Bhargava, R. S. Agrawal, and A. V. S. Balaji. "Finite element solution of micropolar fluid flow and heat transfer between two porous discs." *International journal of engineering science* 38, no. 17 (2000): 1907-1922.
[18] Ashraf, Muhammad, M. Anwar Kamal, and K. S. Syed. "Numerical study of asymmetric laminar flow of micropolar fluids in a porous channel." *Computers & Fluids* 38, no. 10 (2009): 1895-1902.
[19] Rashidi, M. M., SA Mohimanian Pour, and N. Laraqi. "A semi-analytical solution of micropolar flow in a porous channel with mass injection by using differential transform method." *Nonlinear analysis: Modelling and Control* 15, no. 3 (2010): 341-350.
[20] Si, Xin-hui, Lian-cun Zheng, Xin-xin Zhang, and Ying Chao. "Analytic solution to the micropolar-fluid flow through a semi-porous channel with an expanding or contracting wall." *Applied Mathematics and Mechanics* 31, no. 9 (2010): 1073-1080.
[21] Beavers, Gordon S., and Daniel D. Joseph. "Boundary conditions at a naturally permeable wall." *Journal of fluid mechanics* 30, no. 1 (1967): 197-207.
[22] Nield, D. A. "The Beavers–Joseph boundary condition and related matters: a historical and critical note." *Transport in porous media* 78, no. 3 (2009): 537.
[23] Keller, Herbert B. "A new difference scheme for parabolic problems." In *Numerical Solution of Partial Differential Equations–II*, pp. 327-350. Academic Press, 1971.

[24] Keller, Herbert B. "Some computational problems in boundary-layer flows." In *Proceedings of the Fourth International Conference on Numerical Methods in Fluid Dynamics*, pp. 1-21. Springer, Berlin, Heidelberg, 1975.

[25] Keller, Herbert B. "Numerical methods in boundary-layer theory." *Annual Review of Fluid Mechanics* 10, no. 1 (1978): 417-433.

[26] Vajravelu, Kuppalapalle, and Kerehalli V. Prasad. *Keller-box method and its application*. Vol. 8. Walter de Gruyter GmbH & Co KG, 2014.

[27] Ashwini, B., N. N. Katagi, and A. S. Rai. "Analysis of Laminar Flow through a Porous Channel with Velocity Slip." *Malaysian Journal of Mathematical Sciences* 11, no. 3 (2017): 423-439.