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Motion of a Thin Film of a Fourth Grade Nanofluid with Heat Transfer Down a Vertical Cylinder: Homotopy Perturbation Method Application

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ABSTRACT

The effects of uniform magnetic field, heat generation and chemical reaction on the flow of non-Newtonian nanofluid down a vertical cylinder is investigated. The fluid is incompressible and electrically conducting and obeys fourth grade model. The viscous and Ohmic dissipation are considered. The problem is modulated mathematically by a system of non-linear partial differential equations which describe the continuity, momentum, and nanoparticles concentration as well as Lorentz equation. This system is transformed into a non-dimensional system of ordinary differential equations and then is solved analytically by using homotopy perturbation method. The obtained solutions are functions of the physical parameters of the problem. The effects of these parameters on the obtained solutions are discussed numerically and illustrated graphically through some figures. It is obvious that these physical parameters play an important rule to control and modify the velocity, temperature and nanoparticles concentration of non-Newtonian nanofluid.

Keywords:
MHD flow; heat transfer; Non-Newtonian nanofluid; homotopy perturbation method

1. Introduction

Recently the nanofluids which have greater thermal conductivity coefficient than ordinary fluid, have been more considered by the researchers. Nanofluids are colloid suspension of base fluid and nanoparticles (1-100nm). These small size particles can improve the coefficient of heat transfer as compared with base fluid. The nanoparticles are made up of metals, such that titanium, copper, gold, iron or their oxides and carbides or carbon nanotubes. Nano fluid dynamics have grabbed the nanoparticles concentration of different researchers due to its wide range of applications in biology, engineering and biomedical as well drug delivery systems, neuro-electronic interfaces and photo dynamic therapy.

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Most of researches deal with viscous Newtonian fluids. However, this is not enough in such a way that nanoparticles device is applied to examine the bio-fluids. The use of nanofluids as coolant is chosen because its size and better placement. Engines could be developed at higher temperatures permitting for more power, due to the higher competence. Such as in medical stream: great amount of heat (2000-3000W/cm^2) created by high density X-rays which should be suitable by nanofluid technology, vehicles cooling: nanofluids are used as lubricant, gear oil, coolant and engine oil. There are many studies which represent the nanofluid applications [1-5]. Muhammad *et al.,* [6] studied the disordered convective flow of Ag–ethylene glycol nanofluid and the analysis it done for a cavity having center as heated. Ahmad *et al.,* [7] discussed boundary layer flow through a curved stretching sheet embedded in porous medium. Effects of non-linear radiation in two–dimensional flow of nanomaterial is discussed by Hayat *et al.,* [8]. Hayat *et al.,* [9] studied the effect of nonlinear mixed convection of rate-type non-Newtonian materials in stretched flows.

The non–Newtonian fluid includes in different procedures such as movement of chyme, blood flow (hemodynamics), propagation of liquid metals, alloys and slurries. heat and mass transfers in porous media with chemical reaction and magnetic field have an important in many scientific fields such as thermal in insulation, exaction of crude oil, chemical catalytic, drying, evaporation and energy transfer in a cooling tower. Eldabe *et al.,* [10] discussed the magneto-hydrodynamic non–Newtonian nanofluid flow over a stretching sheet through a non–Darcy porous medium with radiation and chemical reaction. Boundary layer stagnancy point flow of third-grade fluid across a stretching surface with changing thickness is studied by Hayat *et al.,* [11]. Muhammad *et al.,* [12] discussed the impact of magnetic dipole in a viscous Ferro fluid by linear stretched surface. Effect of heat transfer of Eyring-Powell on MHD boundary layer chemical reacting flow past a stretching sheet is discussed by Eldabe *et al.,* [13]. A hybrid isothermal model in Ferro hydrodynamic boundary layer flow for the homogeneous-heterogeneous reactions is established by Muhammad [14]. Waqas *et al.,* [15] investigated magnetic nanoparticles and their flow characteristics and the implementation prospects is active research area. Hayat *et al.,* [16] discussed the effects of hall currents and slip conditions on peristaltic transport of Cu–water nanofluid in rotating medium. The effect of vertical through flow on a bio-thermal convection in a water-based nanofluid by using more realistic boundary condition is investigated by Saini and Sharma [17]. Waqas [18] discussed effects of double stratification and chemical reaction on the flow induced by a nonlinear stretching surface with changing thickness. Turbulent mixed convection and two-dimensional laminar flows over facing using nanofluids in a heated rectangular channel having a repulsion mounted on its wall are numerically simulated by Mohammed *et al.,* [19]. Eldabe *et al.,* [20] studied the MHD peristaltic flow of Jeffry nanofluid with heat transfer through a porous medium in a vertical tube. The non-linear problem of thin film flow of fourth grade fluid down a vertical cylinder is studied by Siddiqui *et al.,* [21]. There are many researches deal with various kind of the constitutive equations of different types of relation between stress and rate of strain of non-Newtonian fluid [22-26]. The forced convection of nanofluid for heat transfer in grooved and helically ribbed horizontal micro tube solved numerically by using ANSYSFLUENT is studied by Mugilan *et al.,* [27].

In spite of the fourth grade model is suitable for the human blood flow, a little of researchers are used. Then, the main aim of this study is to investigate the flow of fourth grade nanofluid with heat transfer down a vertical cylinder because it has important scientific applications in different fields. The heat and transfer and chemical reaction as well as viscous and Ohmic dissipation with chemical reaction are considered, also, the viscous and Ohmic dissipation with chemical reaction are considered. Analytical solutions for the momentum, energy and nanoparticles concentration equations are obtained by using HPM. The effects of the various parameters of the problem were discussed analytically and graphically in order to control the motion of the fluid by changing these parameters.

2. Formulation of the Problem

Let us consider the steady flow of non-Newtonian nanofluid with heat and mass transfer through porous medium down a vertical cylinder. The fluid obeys fourth grade model. The viscous and Ohmic dissipation are considered as well as the heat generation and chemical reaction. The cylindrical coordinates (r,θ, z) are chosen, see Figure 1.

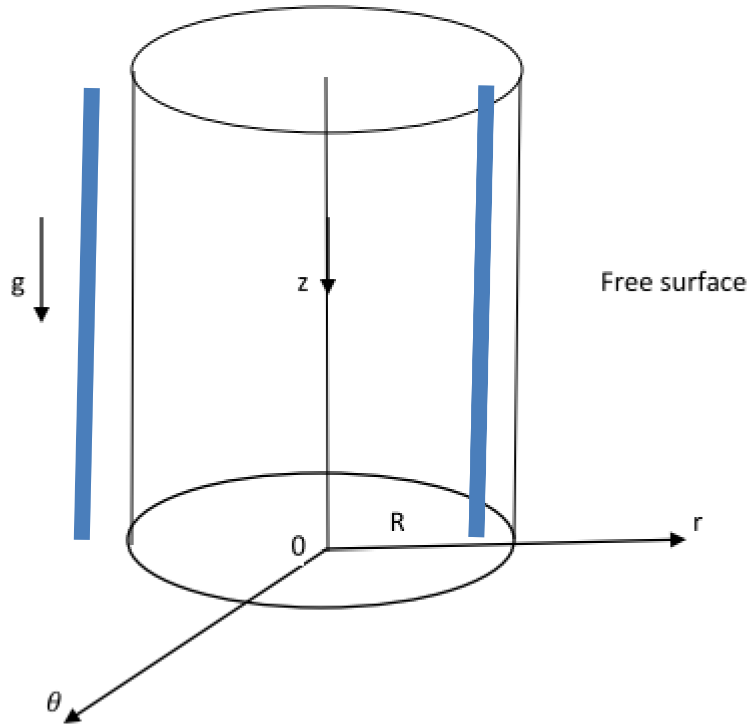

Fig. 1. Schematic of the problem

The constitutive equation for τ in fourth grade model is given by

$$\tau = S_1 + S_2 + S_3 + S_4 \tag{1}$$

where

$$\left.\begin{aligned} &S_1 = \mu A_1 \quad S_2 = \alpha_1 A_2 + \alpha_2 A_1^2, \\ &S_3 = \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) + \beta_3 (trA_2) A_1 \\ &S_4 = \gamma_1 A_4 + \gamma_2 (A_3 A_1 + A_1 A_3) + \gamma_3 A_2^2 + \gamma_4 (A_2 A_1^2 + A_1^2 A_2) \\ &+\gamma_5 (trA_2) A_2 + \gamma_6 (trA_2) A_1^2 + \gamma_7 (trA_3) + \gamma_8 (trA_2 A_1) A_1 \end{aligned}\right\} \tag{2}$$

where μ is the viscosity coefficient of the fluid, α_1, α_2 , $\beta_1 - \beta_3$ and $\gamma_1 - \gamma_8$ are the material constants, $A_0 - A_4$ are defined as

$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1}\nabla V + (\nabla V)^T A_{n-1}$, $n \geq 1$, $A_0 = I$, the identity tensor and $\frac{D}{Dt}$ is the material derivative.

The governing equations of steady motion of an electrically conducting incompressible Nano non-Newtonian fluid with heat and mass transfer are

The equation of continuity

$$\nabla \cdot \underline{V} = 0 \tag{3}$$

The equation of momentum

$$\rho_f(\underline{V} \cdot \nabla)\underline{V} = -\nabla P + \nabla \cdot \tau - \frac{\mu \underline{V}}{k} + \underline{J} \times \underline{B} \tag{4}$$

The equation of heat

$$(\rho C)_f \underline{V} \cdot \nabla T = K_m \nabla^2 T + (\rho C)_p \{D_\beta \nabla C \cdot \nabla T + \frac{D_T}{T_c} |\nabla T|^2\}$$
$$+ \frac{\underline{J} \cdot \underline{J}}{\sigma} + \underline{\tau} \cdot \nabla \underline{V} + Q(T - T_0) \tag{5}$$

The equation of nanoparticles concentration

$$\underline{V} \cdot \nabla C = D_p \nabla^2 C + \frac{D_T}{T_c} \nabla^2 T \pm K(C - C_0) \tag{6}$$

where $\underline{V}$is the fluid velocity, $\rho_f$is the fluid density, $P$is the fluid pressure,τ is the stress tensor defined before, $K$is the permeability of the medium, $\underline{B}$ is the strength of the applied magnetic field, $\underline{J}$is the current density, σ is magnetic conductivity, $T$is temperature field, $K_m$is the thermal conductivity of the medium, $D_\beta$is the Brownian motion coefficient, $D_T$is the thermophoretic diffusion coefficient, $T_c$is the mean temperature of the fluid, C is the fluid nanoparticles concentration, $(\rho C)_p$is the heat capacity of nanoparticles, $(\rho C)_f$ is the heat capacity of the fluid, Q is the coefficient of heat generation andT_0 and C_0 are the temperature and nanoparticles concentration at the free surface. Since the fluid flow is in the form of a thin, uniform axisymmetric film of thickness δ, in contact with stationary air, we shall consider the fluid velocity in the form

$$\underline{V} = (0,0,u(r)),$$

where the continuity equation is satisfied identically and the applied magnetic field of strength $\underline{B}$, is $\underline{B} = (0, B_0, 0)$. It is assumed that no applied polarization voltage exist, this mean that the electric field ignored, also the induced magnetic field assumed to be negligible.

The appropriate boundary conditions of governing the problem are given by

$$\left.\begin{array}{l} w = 0, T = T_1 C = C_1, \text{at} r = R, \\ \frac{dw}{dr} = 0, \frac{d\theta}{dr} = 0, \frac{d\varphi}{dr} = 0, \text{at} r = R + \delta_0. \end{array}\right\} \tag{7}$$

where $T_1$and C_1 are the temperature and nanoparticles concentration at the wall of the cylinder and δ_0 is the thickness of the thin fluid film.

Consider the following non-dimensional variables

$$w^* = \frac{w}{U}, r^* = \frac{r}{R}, T = T_0 + \theta(T_1 - T_0), C = C_0 + \varphi(C_1 - C_0)$$

After using the above assumptions and the dimensionless quantities, the governing equations of the problem (4)-(6) with conditions (7) can be rewriting as follows

$$w'' + \frac{1}{r}w' - \beta w + 2\varepsilon(3w'^2 w'' + \frac{1}{r}w'^3) + \alpha = 0, \tag{8}$$

$$\theta'' + \frac{1}{r}\theta' + q\theta + Nb\theta'\varphi' + Nt\theta'^2 + sw^2 + \omega w'^2 + \eta w'^4 = 0, \tag{9}$$

$$\varphi'' + \frac{1}{r}\varphi' - \lambda\varphi + N_A(\theta'' + \frac{1}{r}\theta') = 0 \tag{10}$$

Subjected the boundary conditions

$$w = 0, \theta = 1, \varphi = 1, \text{at } r = 1,$$

$$w' = 0, \theta' = 0, \varphi' = 0, \text{at} r = 1 + \delta \tag{11}$$

where dash denotes the ordinary differentiation with respect to $r, \lambda = \frac{kR^2}{D\beta}$ is the chemical reaction parameter, $N_A = \frac{D_T(T_1 - T_0)}{T_c D_\beta(C_1 - C_0)}$ is the modified diffusivity ratio, $q = \frac{QR^2}{K_m}$ is the heat generation parameter, $Nb = \frac{(\rho C)_p D_\beta(C_1 - C_0)}{K_m}$is Brownian motion parameter, $Nt = \frac{(\rho C)_p D_T}{T_c K_m}$is the thermophoresis parameter, $s = \frac{\sigma B_0^2 U^2 R^2}{K_m(T_1 - T_0)}$ is Ohm dissipation parameter,$\omega = \frac{\mu U^2}{K_m(T_1 - T_0)}$is the first viscous dissipation parameter and$\eta = \frac{2U^4(\beta_2 + \beta_3)}{R^2 K_m(T_1 - T_0)}$is the second viscous dissipation parameter, $\beta = \frac{1}{k} + M$, $k = \frac{k}{R^2}$ is the permeability parameter and $M = \frac{\sigma B_0^2 R^2}{\mu}$ is the magnetic field parameter, $\varepsilon = \frac{(\beta_2 + \beta_3)U^2}{\mu R^2}$ is the non-Newtonian parameter, $\alpha = \frac{\rho g R^2}{U\mu}$ is the gravity parameter and $\delta = \frac{\delta_0}{R}$ is the thickness of the film.

3. Method of Solution

The homotopy perturbation method (HPM), is a spectrum extension method used in a solution of both non-linear partial and ordinary differential equations. The method appoints a homotopy modify to generate an approximate series solution of differential equations. In view of the HPM [28], Eq. (8)-(10) satisfy the following relations

$$H(P, w) = L_1(w) - L_1(w_{10}) + PL_1(w_{10}) + P(-\beta w + 2\varepsilon(3w'^2 w'' + \frac{1}{r}w'^3)), \tag{12}$$

$$H(P, \theta) = L_2(\theta) - L_2(\theta_{10}) + PL_2(\theta_{10}) + P(n\theta'\varphi' \mp m\theta + q\theta'^2 + sw^2 + s\omega w'^2 + \eta w'^4), \tag{13}$$

$$H(p,\varphi) = L_2(\varphi) - L_2(\varphi_{10}) + PL_2(\varphi_{10}) + P(N_A(\theta'' + \frac{1}{r}\theta') - \lambda\varphi), \quad (14)$$

with $L_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \alpha$, $L_2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}$ as the linear operator, the initial approximations $w_{10}, \theta_{10}, \varphi_{10}$ can be defined

$$w_{10} = \frac{1}{4}(a_1 - a_1 r^2 + 2a_1 b_2\, ln(r)) \quad (15)$$

$$\theta_0 = \varphi_0 = 1 - \frac{ln(r)}{ln(b)} \quad (16)$$

The proposition is that the solution of Eq. (12)-(14) can be developed a power of series in P

$$w(r,P) = w_0 + Pw_1 + P^2 w_2 + \ldots.., \quad (17)$$

$$\theta(r,P) = \theta_0 + P\theta_1 + P^2\theta_2 + \ldots\ldots.., \quad (18)$$

$$\varphi(r,P) = \varphi_0 + P\varphi_1 + P^2\varphi_2 + \ldots\ldots\ldots \quad (19)$$

The solution of velocity, temperature and nanoparticle phenomenon (for P=1) are formed as

$$w(r) = a_0 + \frac{a_1}{r^4} + \frac{a_2}{r^2} + a_3 r^2 + a_4 r^4 + a_5 r^6 + a_6\, ln(r) + a_7 r^2\, ln(r) + a_8 r^4\, ln(r) + a_9\, ln(r)^2, \quad (20)$$

$$\theta(r) = a_{10} + \frac{a_{11}}{r^4} + \frac{a_{12}}{r^2} + a_{13} r^2 + a_{14} r^4 + a_{15} r^6 + a_{16} r^8 + a_{17}\, ln(r) + a_{18} r^2\, ln(r) + a_{19} r^4\, ln(r) + a_{20} r^6\, ln(r) + a_{21} r^2\, ln(r)^2 + a_{22} r^4\, ln(r)^2 + a_{23}\, ln(r)^2 + a_{24}\, ln(r)^3, \quad (21)$$

$$\varphi(r) = a_{25} + \frac{a_{26}}{r^2} + a_{27} r^2 + a_{28} r^4 + a_{29} r^6 + a_{30}\, ln(r) + a_{31} r^2\, ln(r) + a_{32} r^4\, ln(r) + a_{33}\, ln(r)^2 + a_{34} r^2\, ln(r)^2. \quad (22)$$

The constants $a_0 - a_{32}$ are excluded here. However, they are available upon request from the author.

Now, the coefficient of skin friction τ_ω, the coefficient of heat transfer *Nu* and Sherwood number *Sh* at the thin film of tube, are defined, respectively by

$$\tau_\omega = \left[(1+2\varepsilon)\frac{\partial w}{\partial r}\right]_{r=b}, \qquad Nu = \left.\frac{\partial\theta}{\partial r}\right|_{r=b}, \; Sh = \left.\frac{\partial\phi}{\partial r}\right|_{r=b}. \quad (23)$$

The expressions for τ_ω, *Nu* and *Sh* have been calculated from Eq. (20)-(22) into Eq. (23), and they have been evaluated and tabulated numerically for different values of the problem parameters. The obtained results will be discussed in the next section.

4. Results and Discussion

It is clear that we have to choose the thickness of the film less than one. Moreover, the approximation which have been used (boundary-layer approximation) restricted us to choose the following values
g = 9.8 m/sec^2 and δ = 0.4 cm.

To discuss the effect of various parameters involved in the problem on the solution of the considered problem, a numerical results are calculated using Mathematica package Ver. 10, for the axial velocity w, the temperature θ, the nanoparticles concentration φ, the skin friction coefficient τ_ω, the heat transfer coefficient Nu, and the mass transfer coefficient Sh.

Figure 2 and 3 illustrate the change of the axial velocity w versus the radial coordinate r for several values of Darcy number Da and the non-Newtonian parameter ϵ, respectively. It is seen, from Figure 2 and 3, that the axial velocity decreases with the increase of Da, whereas it increases as ϵ increases, respectively. Moreover, it is also noted that for each value of both Da and ϵ, w is always positive, and all obtained curves don't intersect at the end of thin-layer. It is noticed that there is a good agreement between the obtained result in Figure 2 and those obtained by Eldabe and Abouzeid [29]. The effects of the other parameters are found to be similar to them; these figures are excluded here to avoid any kind of repetition.

The variations of the temperature distribution $\theta$with the dimensionless radial coordinate r for various values of heat source parameter q and the dissipation parameter η, respectively, are displayed in Figure 4 and 5. The graphical results of Figure 4 and 5 indicate that the temperature distribution θincreases with increasing in the parameter q, while it decreases by increasing the parameter η, respectively. It is also noted from Figure 4, that all the obtained curves are coincide at the boundary of tube, namely in the interval $r \in [1.0, 1.02]$, while in Figure 5, all the obtained curves are coincide at the boundary of thin film, namely in the interval $r \in [1.28, 1.40]$, then θ increases with r till a definite value $r=r_0$ (represents the maximum value of θ) and it decreases afterwards. This maximum value of θ increases by increasing q, while it decreases by increasing η.

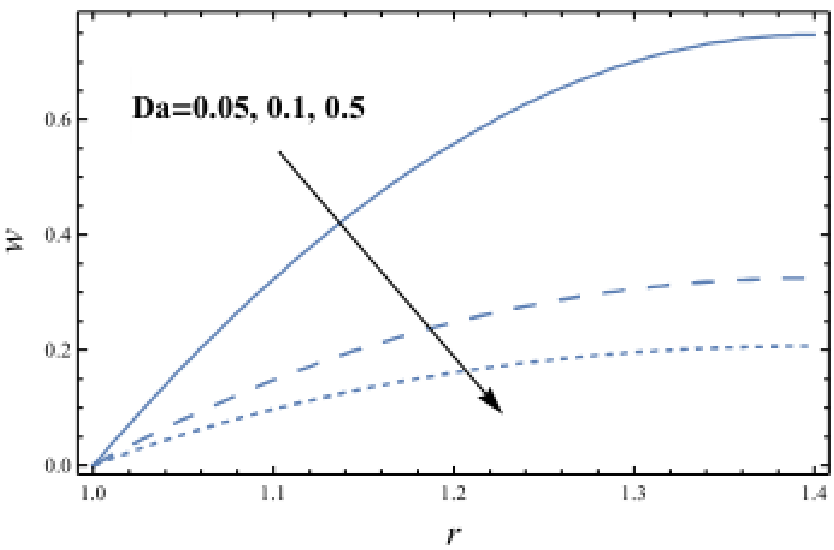

Fig. 2. The velocity is plotted against r for different value of $Da$when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; \eta = 1; N_a = 1.3; \lambda = 1; q = 10; nt = 4.5; s = 10; nb = 3.5;\ m1 = 5$

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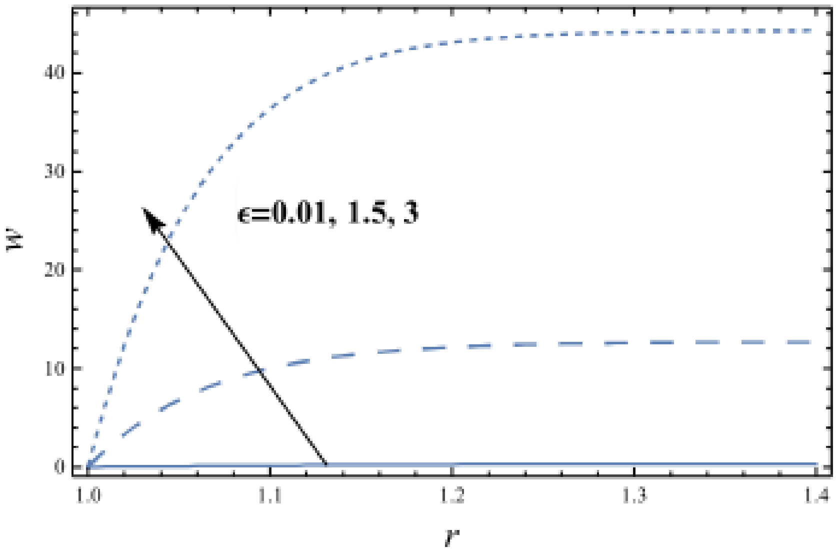

Fig. 3. The velocity is plotted against r for different value of ε when $r_1 = 1; b = 1.4; \alpha = 3; \omega = 1; \eta = 1; N_a = 1.3; \lambda = 1; q = 10; nt = 4.5; s = 10; nb = 3.5; da = 0.05;\ m1 = 5$

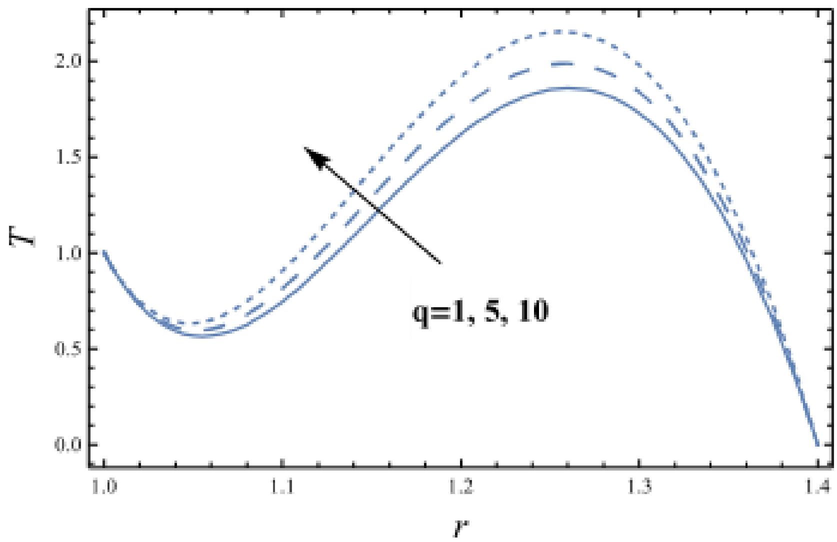

Fig. 4. The temperature is plotted against r for different value of q when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; N_a = 1.3; \lambda = 1; q = 10; nt = 4.5; s = 10; nb = 3.5; da = 0.05; m1 = 5$

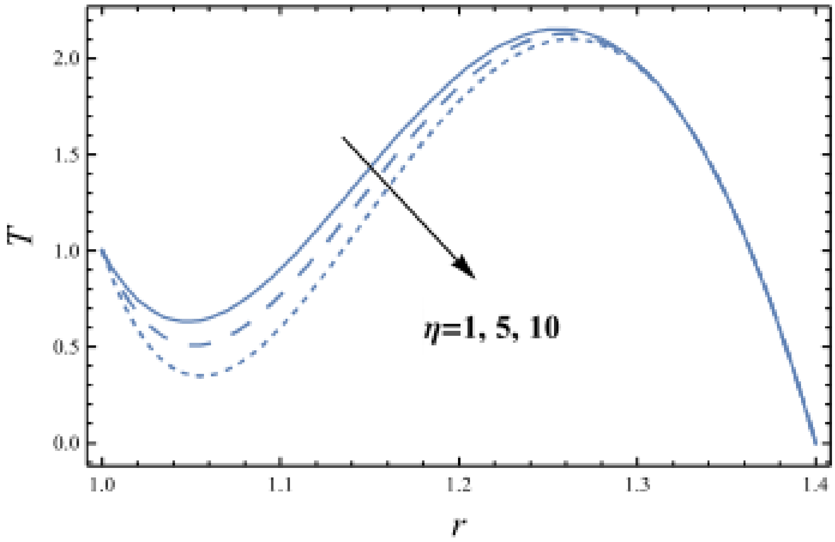

Fig. 5. The temperature is plotted against r for different value of $\eta$when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; N_a = 1.3; \lambda = 1; q = 10; nt = 4.5; s = 10; nb = 3.5; da = 0.05; m1 = 5$

Figure 6 and 7 obtain the influence of the thermophoresis parameter *Nt* and chemical reaction parameter λ on the temperature distribution, respectively. It is observed that near the boundary of the thin film, the temperature increases by increasing *Nt*, whereas it decreases by increasing values of λ, but near the boundary of the tube, namely in the interval [1, 1.04], an opposite behavior occurs, i.e. *T* increases by increasing λ, whereas it decreases by increasing values of *Nt*. Moreover, it is clear that the temperature is always positive and increases by increasing *r* till a maximum value of *r,* after which it decreases. The result in Figure 6 is in agreement with those obtained by Mohamed and Abouzeid [30]. Other parameters affect on the temperature, but they are found to be similar to them. So, figures are excluded to avoid any kind of repetition.

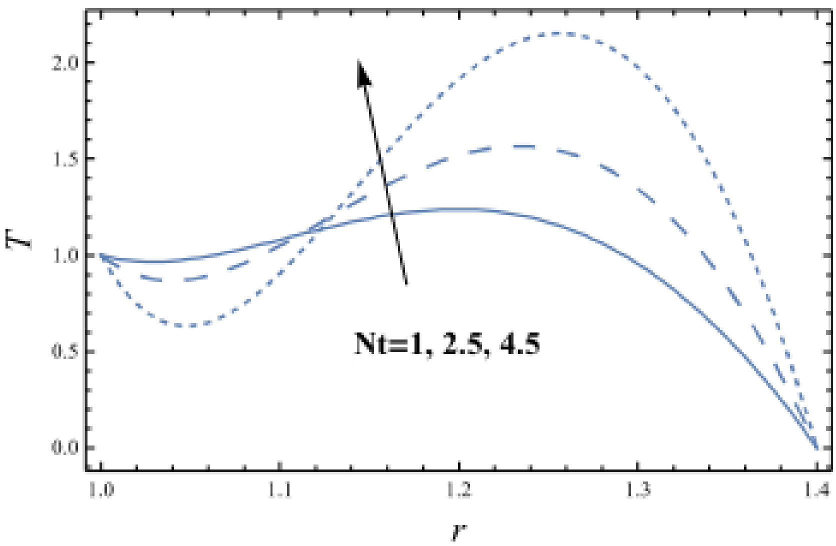

Fig. 6. The temperature is plotted against r for different value of $N_t$when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; \eta = 1; N_a = 1.3; \lambda = 1; q = 10; s = 10; nb = 3.5; da = 0.05; m1 = 5$

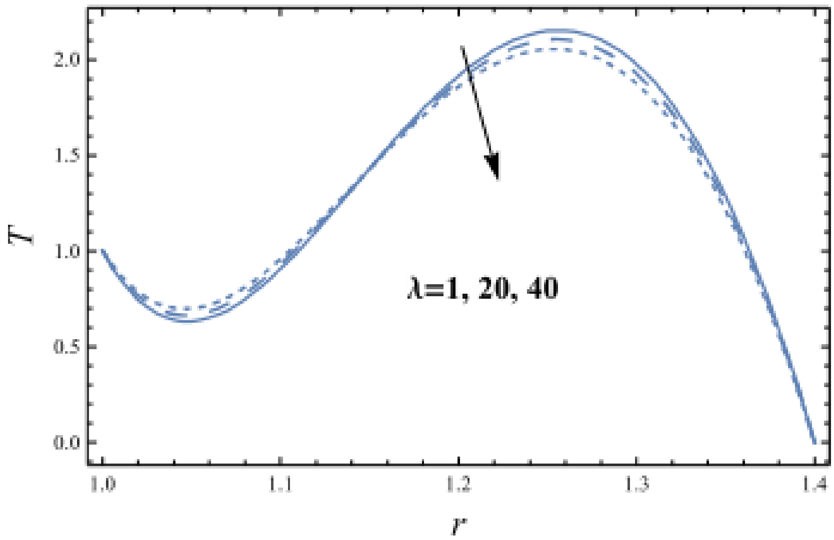

Fig. 7. The temperature is plotted against r for different value of λ when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; \eta = 1; N_a = 1.3; q = 10; nt = 4.5; s = 10; nb = 3.5; da = 0.05; m1 = 5$

Figure 8 and 9 represent the behaviors of the nanoparticles concentration distribution φ with the dimensionless radial coordinate r for different values of Brownian motion parameter Nb and Ohmic dissipation parameter s, respectively. It is indicated from Figure 8 and 9, that the nanoparticles concentration distribution increases with the increase of Nb, whereas it decreases as s increases, respectively. It is also noted that the difference of the nanoparticles concentration distribution for different values of Nb and s becomes greater with increasing the radial coordinate r and reaches minimum value, after which it increases. The result in Figure 8 is in a good agreement with those obtained by Eldabe and Abouzeid [31]. The effects of other parameters (figures are removed) are found to be exactly similar to the effect of s on Nu given in Figure 9, with the only difference that the obtained curves are very close to each other than those obtained in Figure 9.

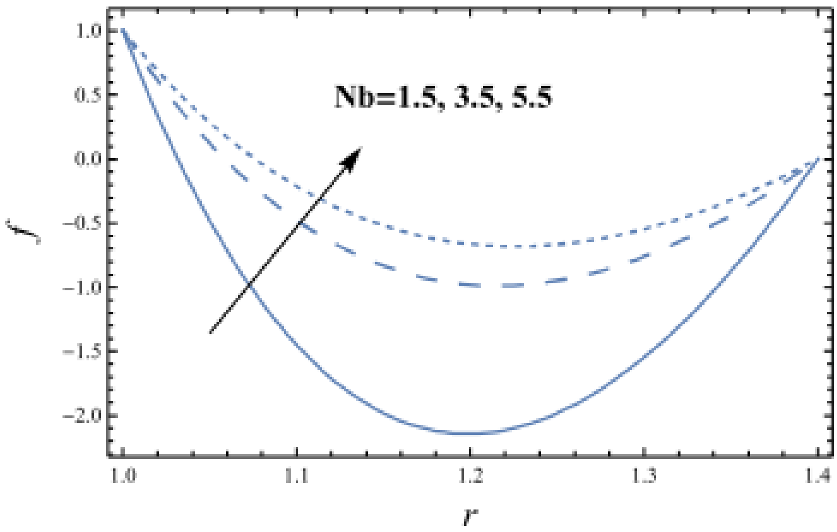

Fig. 8. The nanoparticles is plotted against r for different value of N_b when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; \eta = 1; N_a = 1.3; \lambda = 1; q = 10; nt = 4.5; s = 10; da = 0.05; m1 = 5$

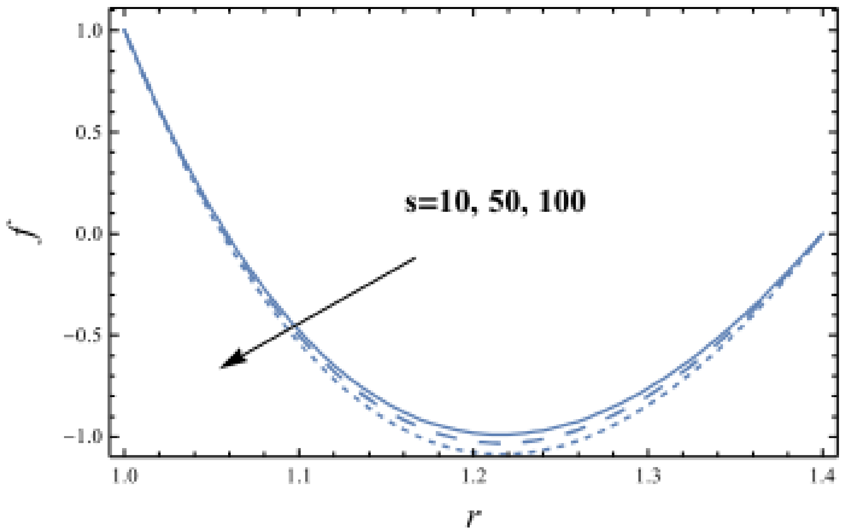

Fig. 9. The nanoparticles is plotted against r for different value of $s$when $r_1 = 1; b = 1.4; \alpha = 3; \varepsilon = 0.01; \omega = 1; \eta = 1; N_a = 1.3; \lambda = 1; q = 10; nt = 4.5; nb = 3.5; da = 0.05; m1 = 5$

5. Conclusion

In this article, MHD boundary-layer flow of an incompressible fourth-grade nanofluid through a porous medium down a vertical cylinder with heat transfer has been studied. The viscous and Ohmic dissipation are considered. The present analysis can serve as a model which may help in understanding the mechanics of physiological flows [32, 33]. The governing partial differential equations are transformed into a set of nonlinear ordinary differential equations using similarity transformations. A homotopy perturbation technique is performed to get analytical solutions for that system of equations. The expressions for the axial velocity, temperature, nanoparticles concentration distributions have been discussed graphically. The following observations have been found.

i. The axial velocity *w* increases with the increase each of g, $\epsilon$and M, whereas it decreases as Da increases.
ii. The axial velocity *w* for different values of all parameters is always positive and becomes greater with increasing the radial coordinate *r*, but all curves don't intersect at the end of thin-layer.
iii. The temperature increases or (decreases) with the increase each of λ, Nt and Nb.
iv. The temperature θ for different values of all parameters becomes greater with increasing the radial coordinate r and reaches maximum value (at a finite value of r: r = r_0) after which it decreases. But it seems to have a minimum value near the cylindrical wall.
v. The nanoparticles concentration behaviors opposite compared to temperature behavior except that it hasn't the inverse value near the cylindrical wall.

References

[1] Shawky, Hameda M., Nabil TM Eldabe, Kawther A. Kamel, and Esmat A. Abd-Aziz. "MHD flow with heat and mass transfer of Williamson nanofluid over stretching sheet through porous medium." *Microsystem Technologies* 25, no. 4 (2019): 1155-1169.
[2] Hayat, T., Rija Iqbal, A. Tanveer, and A. Alsaedi. "Mixed convective peristaltic transport of Carreau-Yasuda nanofluid in a tapered asymmetric channel." *Journal of Molecular Liquids* 223 (2016): 1100-1113.
[3] Abou-Zeid, Mohamed Y. "Homotopy perturbation method for couple stresses effect on MHD peristaltic flow of a non-Newtonian nanofluid." *Microsystem Technologies* 24, no. 12 (2018): 4839-4846.

[4] Abou-Zeid, Mohamed. "Effects of thermal-diffusion and viscous dissipation on peristaltic flow of micropolar non-Newtonian nanofluid: application of homotopy perturbation method." *Results in Physics* 6 (2016): 481-495.
[5] Sayed, Hamed M., Emad H. Aly, and K. Vajravelu. "Influence of slip and convective boundary conditions on peristaltic transport of non-Newtonian nanofluids in an inclined asymmetric channel." *Alexandria Engineering Journal* 55, no. 3 (2016): 2209-2220.
[6] Muhammad, Noor, S. Nadeem, and Alibek Issakhov. "Finite volume method for mixed convection flow of Ag–ethylene glycol nanofluid flow in a cavity having thin central heater." *Physica A: Statistical Mechanics and its Applications* 537 (2020): 122738.
[7] Ahmad, Shafiq, S. Nadeem, and Noor Muhammad. "Boundary layer flow over a curved surface imbedded in porous medium." *Communications in Theoretical Physics* 71, no. 3 (2019): 344.
[8] Hayat, Tasawar, Sajid Qayyum, Muhammad Waqas, and Ahmed Alsaedi. "Thermally radiative stagnation point flow of Maxwell nanofluid due to unsteady convectively heated stretched surface." *Journal of molecular liquids* 224 (2016): 801-810.
[9] Hayat, T., S. Naz, M. Waqas, and A. Alsaedi. "Effectiveness of Darcy-Forchheimer and nonlinear mixed convection aspects in stratified Maxwell nanomaterial flow induced by convectively heated surface." *Applied Mathematics and Mechanics* 39, no. 10 (2018): 1373-1384.
[10] El-Dabe, Nabil, Abeer A. Shaaban, Mohamed Y. Abou-Zeid, and Hemat A. Ali. "Magnetohydrodynamic non-Newtonian nanofluid flow over a stretching sheet through a non-Darcy porous medium with radiation and chemical reaction." *Journal of Computational and Theoretical Nanoscience* 12, no. 12 (2015): 5363-5371.
[11] Hayat, Tasawar, Muhammad Ijaz Khan, Muhammad Waqas, and Ahmed Alsaedi. "Magnetohydrodynamic stagnation point flow of third-grade liquid toward variable sheet thickness." *Neural Computing and Applications* 30, no. 8 (2018): 2417-2423.
[12] Muhammad, Noor, Sohail Nadeem, and M. T. Mustafa. "Impact of magnetic dipole on a thermally stratified ferrofluid past a stretchable surface." *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering* 233, no. 2 (2019): 177-183.
[13] Eldabe, Nabil TM, Ahmed Y. Ghaly, Mona AA Mohamed, and Mohamed SH Mahmoud. "MHD boundary layer chemical reacting flow with heat transfer of Eyring–Powell nanofluid past a stretching sheet." *Microsystem Technologies* 24, no. 12 (2018): 4945-4953.
[14] Muhammad, Noor, S. Nadeem, and M. T. Mustafa. "Hybrid isothermal model for the ferrohydrodynamic chemically reactive species." *Communications in Theoretical Physics* 71, no. 4 (2019): 384.
[15] Waqas, Muhammad, Shagufta Jabeen, Tasawar Hayat, M. Ijaz Khan, and Ahmed Alsaedi. "Modeling and analysis for magnetic dipole impact in nonlinear thermally radiating Carreau nanofluid flow subject to heat generation." *Journal of Magnetism and Magnetic Materials* 485 (2019): 197-204.
[16] Hayat, T., M. Rafiq, and A. Alsaedi. "Investigation of Hall current and slip conditions on peristaltic transport of Cu-water nanofluid in a rotating medium." *International Journal of Thermal Sciences* 112 (2017): 129-141.
[17] Saini, S., and Y. D. Sharma. "A bio-thermal convection in water-based nanofluid containing gyrotactic microorganisms: effect of vertical throughflow." *Journal of applied fluid Mechanics* 11, no. 4 (2018): 895-903.
[18] Hayat, T., M. Zubair, M. Waqas, A. Alsaedi, and M. Ayub. "On doubly stratified chemically reactive flow of Powell–Eyring liquid subject to non-Fourier heat flux theory." *Results in Physics* 7 (2017): 99-106.
[19] Mohammed, H., O. A. Alawi, and NA Che Sidik. "Mixed convective nanofluids flow in a channel having forward-facing step with baffle." *Journal of Advanced Research in Applied Mechanics* 24, no. 1 (2016): 1-21.
[20] El-dabe, Nabil TM, Mohamed Y. Abou-zeid, and Yasmeen M. Younis. "Magnetohydrodynamic peristaltic flow of Jeffry nanofluid with heat transfer through a porous medium in a vertical tube." *Appl. Math. Inform. Sci* 11, no. 4 (2017): 1097-1103.
[21] Siddiqui, Abdul M., R. Mahmood, and Q. K. Ghori. "Homotopy perturbation method for thin film flow of a fourth grade fluid down a vertical cylinder." *Physics Letters A* 352, no. 4-5 (2006): 404-410.
[22] Bhatti, Muhammad Mubashir, Ahmed Zeeshan, and Rahmat Ellahi. "Endoscope analysis on peristaltic blood flow of Sisko fluid with Titanium magneto-nanoparticles." *Computers in biology and medicine* 78 (2016): 29-41.
[23] Hayat, T., Masood Khan, M. Ayub, and Abdul M. Siddiqui. "The unsteady Couette flow of a second grade fluid in a layer of porous medium." *Archives of Mechanics* 57, no. 5 (2005): 405-416.
[24] El-Dabe, Nabil TM, A. Refaie Ali, A. A. El-shekhipy, and G. A. Shalaby. "Non-Linear Heat and Mass Transfer of Second Grade Fluid Flow with Hall Currents and Thermophoresis Effects." *Appl. Math* 11, no. 1 (2017): 267-280.
[25] Hayat, T., Maryam Shafique, A. Tanveer, and A. Alsaedi. "Hall and ion slip effects on peristaltic flow of Jeffrey nanofluid with Joule heating." *Journal of Magnetism and Magnetic Materials* 407 (2016): 51-59.
[26] Abou-Zeid, Mohamed. "Homotopy perturbation method to MHD non-Newtonian nanofluid flow through a porous medium in eccentric annuli with peristalsis." *Thermal Science* 21, no. 5 (2017): 2069-2080.

[27] Mugilan, Thillai, Nor Azwadi Che Sidik, and Wan Mohd Arif Aziz Japar. "The use of smart material of nanofluid for heat transfer enhancement in microtube with helically spiral rib and groove." *Journal of Advanced Research in Materials Science* 32, no.1 (2017): 1-12.

[28] Abou-zeid, Mohamed Y., and Mona AA Mohamed. "Homotopy perturbation method for creeping flow of non-Newtonian power-law nanofluid in a nonuniform inclined channel with peristalsis." *Zeitschrift für Naturforschung A* 72, no. 10 (2017): 899-907.

[29] Eldabe, NT. and Abou-zeid, M.Y. "Numerical treatment and global error estimation of natural convective effects on gliding motion of bacteria on a power-law nanoslime through a non-Darcy porous medium." *Arabian Journal for Science and Engineering* 39, (2014): 5045–5062.

[30] Mohamed, Mona AA, and Mohamed Y. Abou-zeid. "MHD peristaltic flow of micropolar Casson nanofluid through a porous medium between two co-axial tubes." *Journal of Porous Media* 22, no. 9 (2019): 1079-1093.

[31] Eldabe, Nabil, and Mohamed Abou-Zeid. "Radially varying magnetic field effect on peristaltic motion with heat and mass transfer of a non-Newtonian fluid between two co-axial tubes." *Thermal Science* 22, no. 6 Part A (2018): 2449-2458.

[32] Abou-zeid, Mohamed, Abeer A. Shaaban, and Muneer Y. Alnour. "Numerical treatment and global error estimation of natural convective effects on gliding motion of bacteria on a power-law nanoslime through a non-Darcy porous medium." *Journal of Porous Media* 18, no. 11 (2015): 1091–1106.

[33] Abou-zeid, Mohamed. "Magnetohydrodynamic boundary layer heat transfer to a stretching sheet including viscous dissipation and internal heat generation in a porous medium." *Journal of porous Media* 14, no. 11 (2011): 1007-1018.