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Numerical Analysis on the Flow Depth Behaviour in Dividing Open-Channel for Trapezoidal and Rectangular Cross-Sectional Channel

Puteri Nadia Shafiqah Harmizi¹, Siti Aimi Asyarah Zakaria², Iskandar Shah Mohd Zawawi^{1,*}, Mohd Ridza Mohd Haniffah²

¹ School of Mathematical Sciences, College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia

² Department of Water and Environment Engineering, School of Civil Engineering, Faculty of Engineering, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

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ABSTRACT

Open-channel with dividing flow refers to any side water withdrawals from streams, rivers or main channel. It has become an important issue, especially when measuring the flow rate and depth of water as part of environmental management schemes and many practical projects such as irrigation and drainage systems, water and wastewater treatment plants, and other water resources projects. Although the behavior of water flow has been studied by previous researchers, most studies have only been carried out in simulation and lab experiments for a straight prismatic main channel. The mathematical research has not yet been fully understood, particularly in relation to the behavior of water flow. Therefore, in this study, the modified general equations of dividing open-channel flow for rectangular and trapezoidal in the form of non-linear polynomial equations are proposed to determine the upstream flow depth in rectangular and trapezoidal cross-sectional channels. The computations of water flow depth influenced by Froude number and discharge ratio are performed with the Newton-Raphson procedure in Maple software. The numerical solution shows good agreement with the experimental data as reported in the literature. When the discharge ratio is low, the flow depth in trapezoidal and rectangular channels exhibits similarity. However, as the discharge ratio increases, the flow depth in the rectangular channel surpasses that in the trapezoidal channel. Furthermore, as the Froude number increases, the rectangular channel experiences a greater flow depth in comparison to the trapezoidal channel.

1. Introduction

An open-channel flow is a type of watercourse that permits some of the flow to pass through a right-angled junction, also known as a T-junction, which is frequently found in water engineering applications. This type of flow is mainly characterized by inflow and outflow discharges, upstream and downstream water depth, and a lateral outflow. Open-channel flow becomes increasingly

* Corresponding author.

E-mail address: iskandarshah@uitm.edu.my

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complex and challenging due to the vast array of parameters and intricate flow characteristics involved. The hydraulic behavior of open-channel flow depends on the system environments such as the shape of the cross-sectional channel and the type of the flow, which can be classified into the critical flow, subcritical flow, and supercritical flow [1]. Other characteristics of open-channel junctions are dividing and combining types of flow [2-14]. In dividing types of junctions, the main channel is divided into two branches, while for the combining type, the two branches channel is combined into one channel.

One-dimensional (1D) theoretical models represent a straightforward and useful approach to analysing open-channel combining flows. These models typically involve the application of the mass continuity equation, along with a simplified momentum or energy conservation equation, specifically at the junction area [15]. In a study conducted by Shabayek *et al.*, [16], their model pioneered the development of a general non-linear model based on the theory of momentum with more physical effects such as boundary friction forces and non-right-angle junctions. The model of two control volumes using the approach shown by Shabayek *et al.*, [16] has been verified analytically for dividing steady flow by Rashwan [5]. Furthermore, Ghostine *et al.*, [17] presented a comparison with the 1D dynamic model proposed by Shabayek *et al.*, [16] with the 2D model, while Luo *et al.*, [18] made a comparison with the 3D model. In earlier studies, most of the equations of right-angled and short branch channels with equal width have been developed to determine the upstream-to-downstream depth ratio at the junction [19]. However, researchers have attempted to derive a model of the straight prismatic channel with different branching angles of the junction [17,18,20].

The most common channel sections of open channel flow that have been studied by the previous research are trapezoidal and rectangular. For instance, Mohammed [21] proposed a numerical solution of two ordinary differential equations for discharge and flow depth in a rectangular cross-sectional channel using Euler's method. Results referred to the possibility of correlating water surface profile, water depth, and flow discharge numerically as well as the possibility of increasing flow discharge passing through the branch channel. Schindfessel *et al.*, [22] elaborated on the influence of the cross-sectional shape (semi-circular, rectangular, and trapezoidal) of the separation zone in a junction using large-eddy simulation models to simulate the complex turbulent flow. The diversion flow ratio under different frequencies was calculated using the Mike11HD hydrodynamic model [23]. The governing equation for hydrodynamic calculations is the Saint-Venant equation, which describes the open channel's one-dimensional unsteady flow, including the continuity equation and the momentum equation.

Research on dividing open-channel flow later advanced with the exploration of mathematical and numerical modelling approaches. A mathematical model based on the continuity and momentum principle was proposed by Pandey and Mishra [24] to investigate the relationship between depth ratio and complex flow features of combining and dividing flow in rectangular and trapezoidal cross-sectional channels. The results proved that the flow depth ratio in rectangular and trapezoidal increases when the discharge ratio increases. Shahari *et al.*, [25] have detailed the derivation of the dividing flow equation proposed by Pandey and Mishra [24] using techniques of algebraic manipulation and ratio principle to investigate the relationship between flow rate ratio and dividing angles. The study's results revealed an appropriate dividing angle to avoid overflow, in which the flow rate after the junction is below critical flow.

In a recent study, Zawawi *et al.*, [26] investigated the relationship between flow depth, Froude number, and flow rate ratio by solving the non-linear polynomial equation of combining flow for trapezoidal and V-shaped channels. As the Froude number and flow rate ratio increase, the value of flow depth in the trapezoidal channel is higher than in the V-shaped channel. However, most existing general equations of open-channel flow with various parameters are complicated to be solved.

Furthermore, the aim of this research is to derive the simplified version of general equations of dividing flow for trapezoidal and rectangular cross-sectional shapes in the form of polynomial equations. By solving the resulting equation using the Newton-Raphson procedure, the relationship between discharge ratio, Froude number, flow depth ratio, and shape of the cross-sectional is analyzed and discussed.

2. Methodology

2.1 Description of Channel

The characteristics of the open channel dividing flow and its cross-sectional properties have to be considered for the equation of the mathematical model. The schematic layout of the dividing channel is illustrated in Figure 1. The notation of 0 is main channel, while 1 and 2 are channel 1 and channel 2 respectively. Q is the flow rate in each channel, b is the bottom width of the channel, θ_1 and θ_2 are angles of the branch channel.

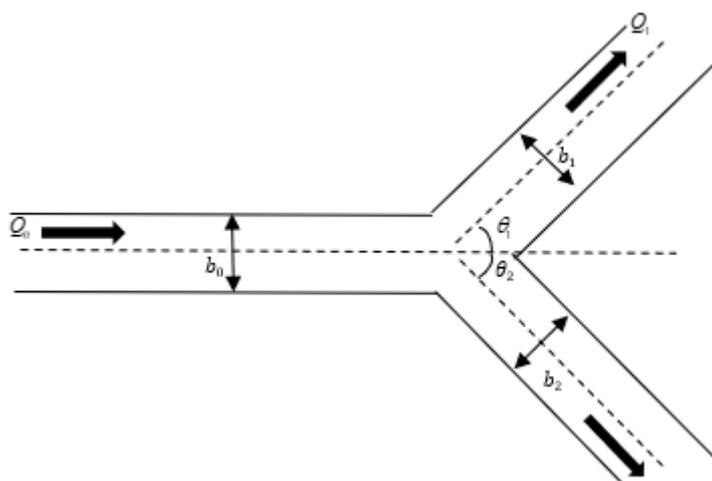


Fig. 1. The open channel dividing flow

The dividing flow from the main channel to two branch channels may be determined with the aid of momentum principle and mass continuity with the following assumptions:

- i. All channels are straight in which the cross-sectional shape and size, also the bottom slope is constant (prismatic channel).
- ii. The main channel is connected to two branch channels.
- iii. The flow takes place from the main channel to the channel 1 and channel 2.
- iv. The flow is parallel to channel walls and the velocity is uniformly distributed immediately above and below the junction.
- v. The wall friction is neglected as compared to other forces.
- vi. The depth of the flow in the main channel is equal to channel 1 and channel 2.

2.2 Formulation of the Dividing Flow Equation

The determination of flow behaviour can be calculated based on the continuity of flow equation for steady one-dimensional flow:

$$Q_0 = Q_1 + Q_2 \tag{1}$$

where $Q_0 = A_0V_0$, $Q_1 = A_1V_1$, and $Q_2 = A_2V_2$ are discharge in the main channel, channel after junction, and branch channel respectively. For each channel, these equations state that the discharge, Q is equivalent to the product of the channel velocity, V and area of the flow, A . While it is possible to estimate or survey the channel shape, it may not be as practical to manually, or physically measure the flow velocity. When measurements of the channel or flow velocity are unavailable, Manning's equation can be used to determine the velocity. Manning's equation, $V = \frac{1}{n} R_h^{2/3} S^{1/2}$, where $n = 0.035$, is Manning's roughness coefficient for the major river, V is the mean velocity of flow in meter (m) per second, R_h is hydraulic radius (area of the flow section per wetted perimeter, W_p of flow in channel), S is the slope of the channel in meter (vertical) per foot (horizontal) [27].

As the dividing of open channel flow model is an application of fluid dynamics, the momentum principle used is slightly different from classical physics. Momentum is usually a product of mass and velocity. In the open channel flow model, the momentum is generally known as the product of mass flow rate and velocity. The general equation of the momentum principle is given as follows:

$$-\rho QV_1 + \rho QV_2 = \bar{P}_1 A_1 - \bar{P}_2 A_2 \tag{2}$$

The terms of where ρ is the fluid density and \bar{P} is the average pressure. By applying the Eq. (2) and Eq. (1) in the flow direction of the main channel, the following equation is obtained:

$$P_0 - P_2 \cos \theta_2 - P_1 \cos \theta_1 - \Delta P = \frac{\gamma}{g} (Q_1 V_1 \cos \theta_1 + Q_2 V_2 \cos \theta_2 - Q_0 V_0) + U_1 + U_2 \tag{3}$$

where U_1 and U_2 are the momentum transfer from the main channel to branch channel and can be written as:

$$U_1 = \rho Q_1 V_0 C \sin \theta_1, U_2 = \rho Q_2 V_0 C \sin \theta_2$$

The geometric characteristics of trapezoidal and rectangular channels to derive the models are presented in Table 1.

Table 1
 The geometric properties of typical trapezoidal and rectangular cross-sectional channels

Geometric properties	Trapezoidal	Rectangular
Cross-sectional area, A	$by + zy^2$	by
Wetted perimeter, W_p	$b + 2y\sqrt{1 + z^2}$	$b + 2y$
Hydraulic radius, R_h	$\frac{by + zy^2}{b + 2y\sqrt{1 + z^2}}$	$\frac{by}{b + 2y}$
Top width, T	$b + 2zy$	b

2.2.1 The polynomial equation for trapezoidal channel

The shape of trapezoidal cross-sectional is illustrated in Figure 2, where the side slope of the channel, z , top width, T , bottom width, b and depth of flow, y are the geometric details of the cross-sectional channel.

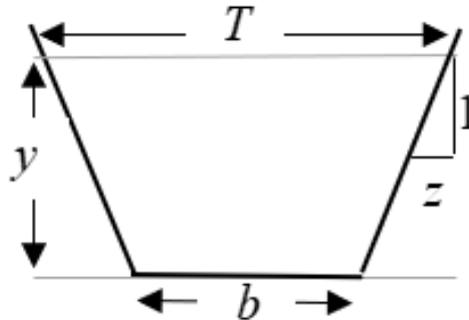


Fig. 2. Diagram of trapezoidal cross-sectional channel

The general equation for the trapezoidal channel is defined as follows [25]:

$$(1 + 2k_0) \left[\frac{1}{2} (1 - y_r^2 - Br_2 y_r^2 \cos \theta_2) + \frac{k_0}{3} (1 - y_r^3 - y_r^3 \cos \theta_2) \right] = F_0^2 (1 + k_0)^2 \left[C (q_r \sin \theta_2 + (1 - q_r) \sin \theta_1) + \frac{1 + k_0}{y_r} \left(\frac{(1 - q_r)^2}{Br_1 + k_0 y_r} \cos \theta_1 + \frac{q_r^2}{Br_2 + k_0 y_r} \cos \theta_2 \right) - 1 \right]. \quad (4)$$

where $k_0 = \frac{zy_0}{b_0}$, $Br_1 = \frac{b_1}{b_0}$, $Br_2 = \frac{b_2}{b_1}$. Expand Eq. (4), yields:

$$-\frac{F_0^2 (1 + k_0)^3 (1 - q_r)^2}{(Br_1 y_r + k_0 y_r^2)} \cos \theta_1 - \frac{F_0^2 (1 + k_0)^3 q_r^2}{(Br_2 y_r + k_0 y_r^2)} \cos \theta_2 = \frac{1}{2} (1 + 2k_0) - \frac{y_r^2}{2} (1 + 2k_0) - \frac{Br_2 y_r^2}{2} (1 + 2k_0) \cos \theta_2 + \frac{k_0}{3} (1 + 2k_0) - \frac{k_0}{3} (1 + 2k_0) y_r^3 - \frac{k_0}{3} (1 + 2k_0) y_r^3 \cos \theta_2 + F_0^2 (1 + k_0)^2 - F_0^2 C (1 + k_0)^2 (1 - q_r) \sin \theta_1 - F_0^2 C (1 + k_0)^2 q_r \sin \theta_2 \quad (5)$$

Eq. (5) cannot be solved for any values of angles θ_1 and θ_2 unless one of the angles is at 90° . Hence, the bifurcation angle in the interval $0^\circ \leq \theta_1 \leq 90^\circ$ and $\theta_2 = 90^\circ$ is considered, in which one of the trigonometry in the equations is eliminated. Then, both sides of the resulting equation are multiplied with $Br_1 y_r + k_0 y_r^2$, yields:

$$\begin{aligned}
 & -F_o^2(1+k_o)^3(1-q_r)^2 \cos \theta_1 = \\
 & \left[\frac{1}{2}(1+2k_o) - \frac{y_r^2}{2}(1+2k_o) + \frac{k_o}{3}(1+2k_o) - \frac{k_o}{3}(1+2k_o)y_r^3 + F_o^2(1+k_o)^2 - F_o^2C(1+k_o)^2(1-q_r) \sin \theta_1 \right] \\
 & \left[-F_o^2C(1+k_o)^2q_r \right. \\
 & \left. (Br_1y_r + k_oy_r^2) \right] \tag{6}
 \end{aligned}$$

Expand Eq. (6) and rearrange in the following form:

$$\begin{aligned}
 & -\frac{k_o^2}{3}(1+2k_o)y_r^5 + \left[-\frac{Br_1k_o}{3}(1+2k_o) - \frac{k_o}{2}(1+2k_o) \right] y_r^4 - \frac{Br_1}{2}(1+2k_o)y_r^3 \\
 & + \left[\frac{k_o}{2}(1+2k_o) + \frac{k_o^2}{3}(1+2k_o) + F_o^2k_o(1+k_o)^2 - F_o^2Ck_o(1+k_o)^2(1-q_r) \sin \theta_1 - \right. \\
 & \left. F_o^2Ck_o(1+k_o)^2q_r \right] y_r^2 \\
 & + \left[\frac{Br_1}{2}(1+2k_o) + \frac{Br_1k_o}{3}(1+2k_o) + Br_1F_o^2C(1+k_o)^2q_r \right] y_r + F_o^2(1+k_o)^3(1-q_r)^2 \cos \theta_1 = 0 \tag{7}
 \end{aligned}$$

The similar techniques as described in the previous study are applied to simplify the general equation of dividing flow in a trapezoidal cross-sectional channel [26]. Therefore, the simplified of Eq. (7) is given by:

$$r(y_r) = ay_r^5 + by_r^4 + dy_r^2 + fy_r + g \tag{8}$$

where:

$$r(y_r) = 0,$$

$$a(k_o) = -\frac{k_o^2}{3}(1+2k_o),$$

$$b(k_o, Br_1) = -\frac{Br_1k_o}{3}(1+2k_o) - \frac{k_o}{2}(1+2k_o),$$

$$d(k_o, Br_1) = -\frac{Br_1}{2}(1+2k_o),$$

$$\begin{aligned}
 e(k_o, C, F_o, \theta_1, q_r) = & \frac{k_o}{2}(1+2k_o) + \frac{k_o^2}{3}(1+2k_o) + F_o^2k_o(1+k_o)^2 - F_o^2Ck_o(1+k_o)^2(1-q_r) \sin \theta_1 \\
 & - F_o^2Ck_o(1+k_o)^2q_r,
 \end{aligned}$$

$$f(k_o, C, F_o, Br_1, q_r) = \frac{Br_1}{2}(1+2k_o) + \frac{Br_1k_o}{3}(1+2k_o) + Br_1F_o^2C(1+k_o)^2q_r,$$

$$g(k_o, F_o, \theta_1, q_r) = F_o^2(1+k_o)^3(1-q_r)^2 \cos \theta_1.$$

2.2.2 The polynomial equations for a rectangular channel

Figure 3 shows the typical rectangular cross-sectional channel. The geometric details of the rectangular channels consist of top width, T , bottom width, b and depth of flow, y .

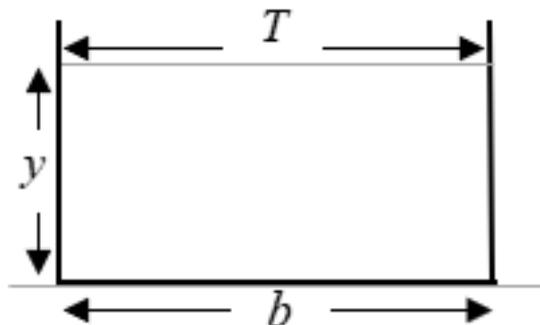


Fig. 3. Diagram of rectangular cross-sectional channel

The general equations for a rectangular cross-sectional channel can be retrieved from Eq. (4) by letting $k_0 = 0$. Hence, the following equation is obtained:

$$\frac{1}{2}(1 - y_r^2 - Br_2 y_r^2 \cos \theta_2) = F_0^2 \left[C(q_r \sin \theta_2 + (1 - q_r) \sin \theta_1) + \frac{1}{y_r} \left(\frac{(1 - q_r)^2}{Br_1} \cos \theta_1 + \frac{q_r^2}{Br_2} \cos \theta_2 \right) - 1 \right]. \quad (9)$$

The same steps from the previous section are taken to derive the polynomial equation for the rectangular cross-sectional channel. Expand Eq. (9), yields:

$$\frac{-F_0^2(1 - q_r)^2}{Br_1 y_r} \cos \theta_1 - \frac{-F_0^2 q_r^2}{Br_2 y_r} \cos \theta_2 = -\frac{1}{2} + \frac{y_r^2}{2} + \frac{Br_2 y_r^2}{2} \cos \theta_2 - F_0^2 + F_0^2 C(1 - q_r) \sin \theta_1 + F_0^2 C q_r \sin \theta_2 \quad (10)$$

Then, a similar branching angle as trapezoidal cross-sectional is assumed. By eliminating one of the trigonometry in Eq. (10) and multiplying with $Br_1 y_r$, the polynomial equation for rectangular cross-sectional channel is obtained:

$$\frac{Br_1}{2} y_r^3 + \left[-\frac{Br_1}{2} - Br_1 F_0^2 + Br_1 F_0^2 C(1 - q_r) \sin \theta_1 + Br_1 F_0^2 C q_r \right] y_r + F_0^2 (1 - q_r)^2 \cos \theta_1 = 0 \quad (11)$$

Therefore, the simplified Eq. (11) is:

$$r(y_r) = j \cdot y_r^3 + k \cdot y_r + l \quad (12)$$

where;

$$r(y_r) = 0,$$

$$j(Br_1) = \frac{Br_1}{2},$$

$$k(Br_1, F_0, \theta_1, C, q_r) = -\frac{Br_1}{2} - Br_1 F_0^2 + Br_1 F_0^2 C(1 - q_r) \sin \theta_1 + Br_1 F_0^2 C q_r,$$

$$l(F_0, q_r, \theta_1) = F_0^2 (1 - q_r)^2 \cos \theta_1.$$

3. Results and Analysis

The geometrical properties of the channel junction are given in Table 2, while Table 3 shows the experimental and measured hydraulic details of the trapezoidal and rectangular channels [24]. From Table 2 we can see that the value of the bottom width, b for the trapezoidal and rectangular channels, has been kept different for all channels. Further, the angle of the branch for branch channel 1 and branch channel 2 has been kept at 30° and 90° respectively to the main channel. However, there are four parameters involved in this research which are k_0 , F_0 , q_r , and measured depth (y_0 and y_1).

Table 2
 The geometrical properties of the channel junction

Type of channel	Channel	Bottom width (m), b	Dividing angle from the direction of flow in main channel
Rectangular	0	0.450	NA
	1	0.350	30°
	2	0.250	90°
Trapezoidal	0	0.272	NA
	1	0.172	30°
	2	0.072	90°

Table 3
 Experimental and measured characteristics of the channels

Type of channel		k_0	F_0	q_r	Measured Depth	
					y_0	y_1
Rectangular	R1	0.000	0.238	0.380	0.1097	0.1107
	R2	0.000	0.401	0.600	0.1146	0.1156
	R3	0.000	0.410	0.810	0.1151	0.1163
Trapezoidal	R1	0.226	0.303	0.435	0.1065	0.1097
	R2	0.188	0.188	0.322	0.0885	0.0907
	R3	0.200	0.404	0.635	0.0942	0.0995

3.1 Experiment Validation

Table 2 and Table 3 are used to calculate the predicted flow characteristics for the rectangular and trapezoidal cross-sectional channels by using the proposed equation. However, three sets of experimental data (R1, R2, and R3) taken from the previous study are used to validate the proposed model for the trapezoidal and rectangular channels [24]. In Table 4, the results from the proposed model give a good agreement between experiments and predicted, y_1 with the estimated error of 2.61%, 2.71%, 2.95% for trapezoidal and 0.91%, 1.73%, 2.41% for rectangular. The proposed equation shows that the predicted depth, y_1 matches very well with the experimental data.

Table 4
 Comparison flow depth in experiment and mathematical model

Type of channel		Measured Depth (Experiment), y_1	Depth ratio, y_r	Predicted depth, y_1	Error (%)
Rectangular	R1	0.1107	0.9999	0.1097	0.91
	R2	0.1156	1.0259	0.1176	1.73
	R3	0.1163	1.0348	0.1191	2.41
Trapezoidal	R1	0.1097	1.0032	0.1068	2.61
	R2	0.0907	0.9971	0.0882	2.71
	R3	0.0995	1.0251	0.0966	2.95

3.2 Parametric Investigation: Comparative Flow Characteristics

Comparative prediction of flow depth is studied for trapezoidal and rectangular cross-sectional channels with similar flow conditions. The parametric investigation is based on two parameters which are F_0 and q_r . The R3 data in Table 2 and Table 3 are used as fixed variables to analyse the effect of parameters on y_r . The property values of other cases are obtained by varying each parameter, whilst keeping the other value constant. The effect of F_0 and q_r on y_r has been analysed and shown in Table 5 and Table 6.

Table 5
 The effect of q_r on y_r

q_r	y_r	
	Trapezoidal	Rectangular
0.2	0.96	0.96
0.4	1.00	1.00
0.6	1.02	1.03
0.8	1.03	1.03

Table 6
 The effect of F_0 on y_r

F_0	y_r	
	Trapezoidal	Rectangular
0.2	1.00	1.01
0.4	1.02	1.03
0.6	1.06	1.08
0.8	1.11	1.14

For simplicity, Figure 4 and Figure 5 show the dependence of depth ratio on various parameters, which are Froude number, F_0 and discharge ratio, q_r respectively. Figure 4 shows that the depth ratio increases when the discharge ratio increases for both cross-sectional channels. The trapezoidal and rectangular channels have a similar highest value of y_r when $q_r = 0.8$. In Figure 5, the highest value of y_r is 1.14 for the rectangular channel and 1.11 for the trapezoidal channel with the increase of F_0 . In conclusion, these results give the similar pattern to that in study by Pandey and Mishra [24] in which the main factors to influence the water depth in the branch channel system are the discharge ratio and Froude number.

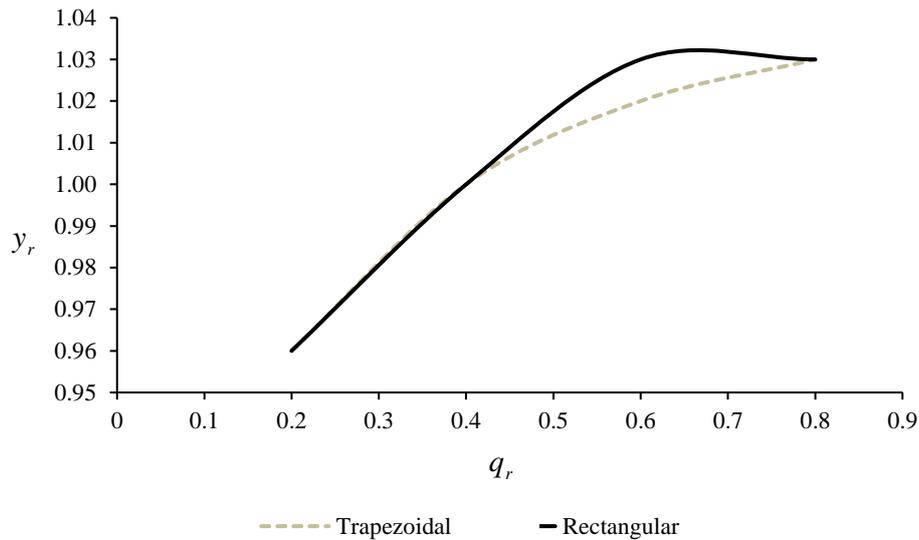


Fig. 4. The graph of effect of discharge ratio on depth ratio for trapezoidal and rectangular

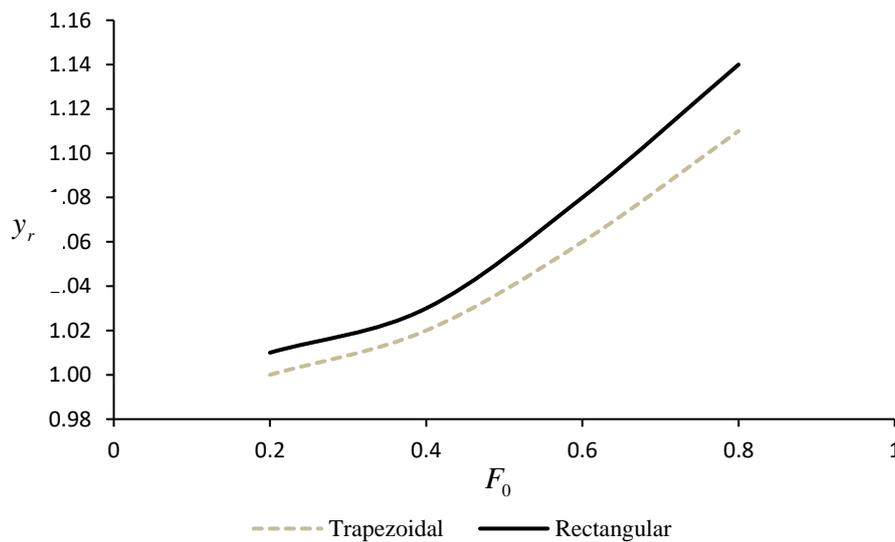


Fig. 5. The graph of effect of Froude number on depth ratio for trapezoidal and rectangular

4. Conclusions

A simple and practical equation has been studied through a mathematical study to help engineers of water resources discover the most appropriate shape of cross-sectional channel and branching angle in dividing flow. The simplified version of the general equation for dividing flow has been developed in the form of polynomial equations by considering the branching angles in channel after junction and branch channel are $\theta_1 = 30^\circ$ and $\theta_2 = 90^\circ$ respectively. Two parameters, which are Froude number and discharge ratio affect the behaviour of the water flow with trapezoidal and rectangular cross-sectional channels. At low discharge ratios, both channels exhibit similar depth ratios. However, as the discharge ratio increases, the flow depth in the rectangular channel surpasses that of the trapezoidal channel. Moreover, as the Froude number increases, the rectangular channel experiences a consistently higher rate of flow depth increase compared to the trapezoidal channel.

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