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Numerical Modelling of Cavitation in an Elastic Pipe

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ABSTRACT

This study is devoted to a theoretical and numerical modeling of transient vaporous cavitation in a horizontal pipeline. The model approach is, essentially, based on that of the column separation model (CSM). The basic system of partial differential equations to solve is a hyperbolic type and adapts perfectly to the method of characteristics. This code, allows us, taking into account the unsteady part of the friction term, to determine at any point of the pipe, and at each instant, the average piezometric head, the average discharge and the change in volume of the vapor cavity. This study illustrates the coupled effect of the presence of air pockets resulting in cavitation and the unsteady term of friction, on the amplitude of the pressure wave. The calculation results are in good agreement with those reported in the literature.

1. Introduction

Vaporous cavitation occurs when the pressure drops to the saturated vapor pressure. This type of cavitation is rapidly changing, as it only takes place during the duration of the reduced pressure. In this phenomenon small and largely empty cavities are generated in a fluid, which expand to large size and then rapidly collapse. The cavitation usually appears when a liquid is subjected to rapid changes of pressure that cause the formation of cavities in the liquid where the pressure is relatively low. When subjected to higher pressure, the voids implode and can generate an intense shock wave and choke. Cavitation occurs normally in pumps, propellers, impellers and piping systems [1-7]. In the literature, there is a group of mathematical models based on this type of cavitation, the so-called discrete vapor cavity models (DVCM). There are two types of vaporous cavitation, localized vaporous cavitation (high vacuum) and distributed vaporous cavitation (low vacuum). In some regions where evaporation is produced by pressure drop, the continuous medium is ruptured by creating columns separated from the fluid, this is the Column Separation Model (CSM). Several studies have been made using this model, notably the work of Anton Bergant and al [8,9], Shu [10], Paquette [11], Adamkowski *et al.,* [12]. Cavitation can be avoided by careful design of hydraulic system such as avoiding high fluid velocities, low pressures and high temperatures. In this article we will study the

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vaporous cavitation in pipe with the discrete vapor cavity model (DVCM) with taking into account the unsteady shear stress that reflects the unsteady friction term introduced by Brunone *et al.,* [13], Zielke [14]. We then examine the evolution of the pressure head at the valve and at the midpoint of the pipeline, and the change in the volume of air bubbles resulting from cavitation.

2. Assumptions and Basic Equations

The flow is assumed unidirectional compressible barotropic and constant entropy. Assume also that the water hammer speed is very large compared to the average flow velocity, and it is not affected by its biphasic nature. The column separation model in systems (tank-horizontal-line valve) are described by the hyperbolic system formed by Eq. (1) and Eq. (2) derived from the conservation of mass and momentum averaged over a cross section of the pipe [1-6]

$$\frac{\partial\ P}{\partial\ t}+\rho\ C^2\frac{\partial\ V}{\partial\ x}=0 \tag{1}$$

$$\frac{1}{\rho}\frac{\partial\ P}{\partial\ x}+\frac{\partial\ V}{\partial\ t}=-\left(f_q+f_u\right)\ \frac{V|V|}{2D} \tag{2}$$

$$C=\sqrt{\frac{\kappa/\rho}{1+(1-\mu^2)\kappa D/eE}} \qquad Water\ hammer\ speed$$

With $f = f_q + f_u$, where f_q is the part related to the quasi—stationary flow, f_u is the unstationary part. The first term is the solution of the Coolbrok equation [1,2].

$$\frac{1}{\sqrt{f_q}}=-2\,log_{10}\left(\frac{2{,}51}{Re\sqrt{f_q}}+\frac{\left(\frac{\varepsilon}{D}\right)}{3{,}71}\right) \tag{3}$$

3. Numerical Solution

By introducing the water hammer velocity [1,2] $C=\left(\frac{\partial\rho}{\partial P}+\frac{\rho}{S}\frac{\partial S}{\partial P}\right)^{-\frac{1}{2}}$ the system of Eq. (1) and Eq. (2), Transformed along the characteristic curves of slopes $\left.\frac{dx}{dt}\right|_{C^\pm}=\pm\ C$ we obtain at each calculation node at each instant the algebraic system

$$P_{i,t}-P_{i-1,t-\Delta t}+\rho\ C\ \left((V_u)_{i,t}-(V_d)_{i-1,t-\Delta t}\right)=\rho\ C\ \Delta\ t\ T_f \tag{4}$$

$$P_{i,t}-P_{i+1,t-\Delta t}-\rho\ C\left((V_d)_{i,t}-(V_u)_{i+1,t-\Delta t}\right)=-\rho\ C\ \Delta\ t\ T_f \tag{5}$$

The continuity equation for the volume of the discrete vapor cavity is described by

$$\left(\forall_g\right)_{i,t}=\left(\forall_g\right)_{i,t-2\Delta t}+\left(\psi\left((V_d)_{i,t}-(V_u)_{i,t}\right)+(1-\psi)\ \left((V_d)_{i,t-2\Delta t}-(V_u)_{i,t-2\Delta t}\right)\right)\ S*(2\Delta\ t) \tag{6}$$

The unsteady term of friction is given by the convolution product [6]

$$f_u(t) = \frac{32\ \nu}{D\ V\ |\ V\ |} \int_0^t \frac{\partial\ V}{\partial\ \tau} W_0(t-\tau)\ d\ \tau \tag{7}$$

$$W_{app}(\tau) = \sum_{l=1}^{N} m_l e^{-n_l\ \tau} \qquad \tau = \frac{4\nu}{D^2} t \tag{8}$$

where coefficients m_l and n_l relative to the weighting function W are determined for laminar flow by Zielke [14], Vardy and Brown [15], Vitkovsky, Zhou and others [16-22] for turbulent flow. From the function f_u a function y_l is defined by

$$f_u(t) = \frac{32\nu}{DV|V|} \sum_{l=1}^{N} y_l(t) \qquad y_l(t) = \int_0^t \frac{\partial V}{\partial t^*} m_l e^{-n_l \frac{4\nu}{D^2}\left(t-t^*\right)} dt^*$$

The explicit expression of the y_l function is

$$y_l(t+2\Delta t) = e^{-n_l \frac{4\nu}{D^2}\Delta t}\{e^{-n_l K \Delta t} y_l(t) + m_l[V((t+2\Delta t) - V(t)]\} \tag{9}$$

4. Application and Results

In this application, we consider a turbulent flow, for two steady-state flow values $V_0 = 0.3\ m/s$ and $V_0 = 1.4\ m/s$. The flow is in the horizontal copper pipe anchored to the upstream to a tank filled with water and of height H_0, ending at the downstream to a valve that closes abruptly. The parameters of the fluid and the pipe are summarized in the following Table 1.

Table 1
Installation data

Tank height H_0 (m)	26
Internal pipe diameter (mm)	22.1
Length of pipe (m)	37.23
Thickness of the pipe (mm)	1.63
Dynamic viscosity of water at 20 ° (Pa. s)	1.11×10^{-3}
Poisson's ratio	0.34
Bulk modulus of water (GPa)	2.2
Young's modulus of copper (GPa)	120
Density of water at 20°C (Kg/m^3)	1000
Vapour pressure of water at 20 ° C (bar)	0.023

In this study, we analysis the evolution over time, of the pressure, the volume of the discrete vapour cavity and friction stress. Figure 1 shows the valve, for permanent flow regimes of velocity $V_0 = 0.3\ m/s$ and $V_0 = 1.4\ m/s$, the change in friction stress with taking into account or not the unsteady part, over time. The Figure 2 shows the same conditions, the pressure variation versus time at the valve and at the pipe midpoint with taking into account or not unsteady term. The Figure 3 shows the same conditions, the behaviour of the volume of the discrete vapour cavity formed at the valve, with taking into account or not the part of the unsteady friction term. The Figure 4 shows the evolution in time of pressure, taking into account or not the term unsteady for permanent regime velocity $V_0 = 1.4 m/s$. Figure 5 corresponds to the evolution of air pockets formed over time at the valve and at the pipe midpoint with taking into account or not the unsteady friction term.

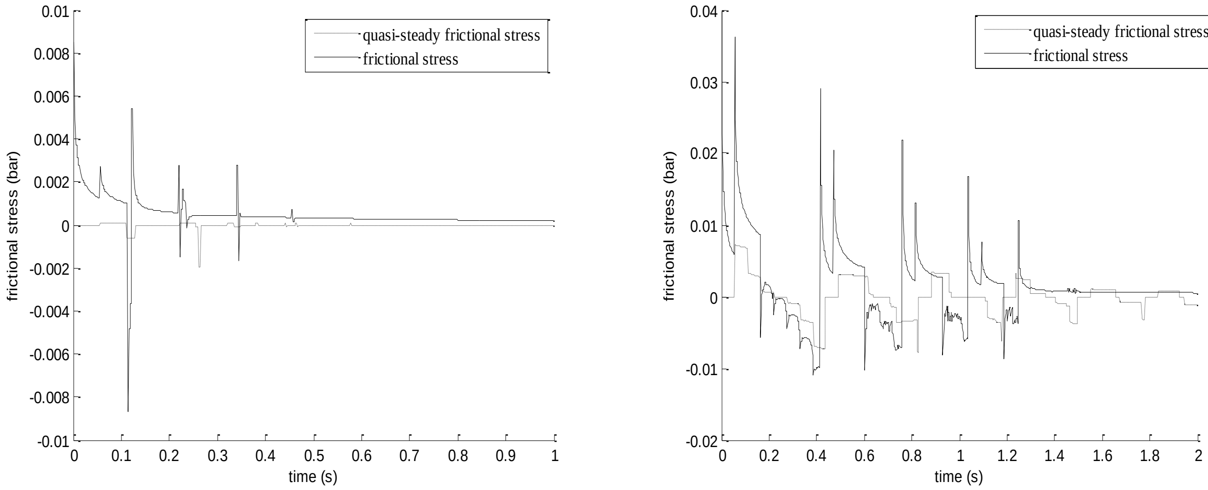


Fig. 1. Variation of the shear stress for V_0 = 0.3 m/s (left) and for V_0 = 1.4 m/s (right)

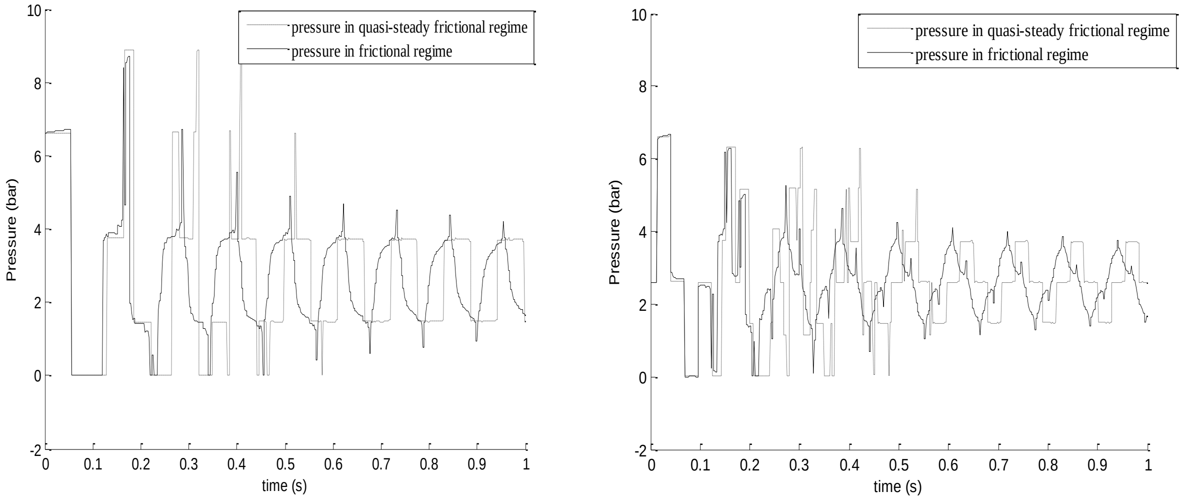


Fig. 2. pressure at the valve (left) and at the pipe midpoint (right) for V_0 = 0.3 m/s

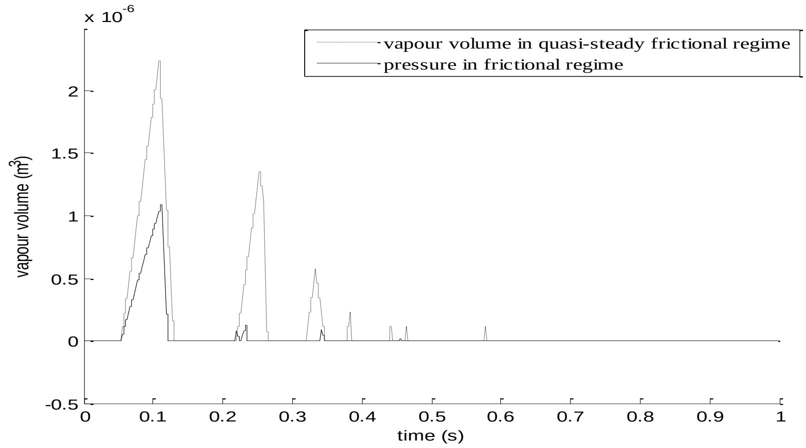
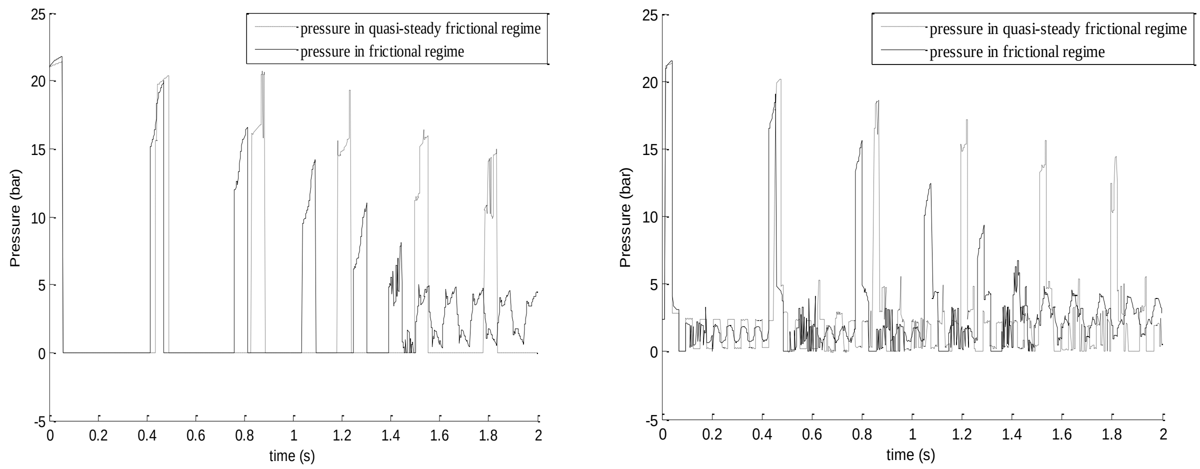
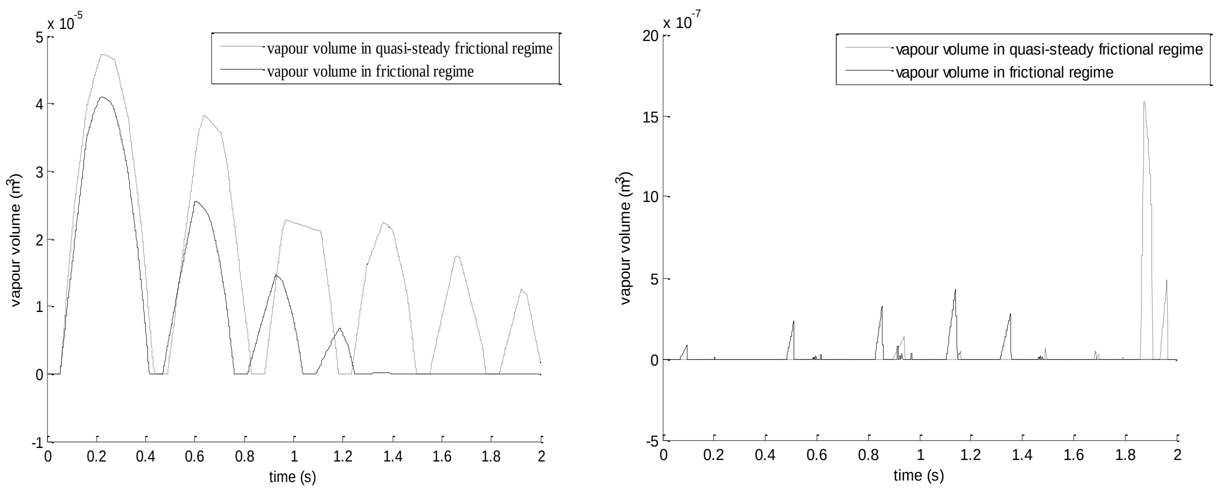


Fig. 3. evolution of the vapour pocket at the valve for V_0 = 0.3 m/s



(a) (b)

Fig. 4. Pressure at (a) the valve and at (b) the pipe midpoint in the model (DVCM) for V0 = 1.4 m/s



(a) (b)

Fig. 5. Evolution of the vapour pocket at (a) the valve and at (b) the pipe midpoint for V_0 = 1.4 m/s

This study clearly shows the influenced effect of unsteady term shear stress. Also, we see that for high speeds, the maximum pressure is reached after a period of 2L / a closure of the valve, while for lower speeds, the maximum pressure is observed after the first collapse of the first pocket air. The values of the maximum pressure and maximum volume of vapour for both speeds are grouped in the following tables

V_0 = 0.3 m/s	$\forall_{max}$ $(10^{-6}\mathrm{m}^3)$	$\Delta t_{\forall_{max}}$	1ère collapse (s)	$(P_{valve})_{max}$ (bar)	$t(P_{Valve})_{max}$ (s)
N = 20	1.0856	65.1	0.1234	8.7168	0.1774

V_0 = 1.4 m/s	$\forall_{max}$ $(10^{-6}m^3)$	$\Delta t_{\forall_{max}}$	1ère collapse (s)	$(P_{valve})_{max}$ (bar)	$t(P_{Valve})_{max}$ (s)
N = 20	40.908	359	0.4172	21.804	0.0554=2L/ a

5. Conclusion

The aim of the current study is to modelize a transient vaporous cavitation in an elastic pipe using a discrete vapour cavitation model with taking into account the unsteady friction term of flow. This shows that the cavitation is a phenomenon that results from turbulent flow. It also, highlights the combined effects of the vapour bubbles resulting from cavitation and shear stress on reducing the pressure wave in the flow. The unsteady friction losses contribute to a significant reduction of pressure. Compared to the classical waterhammer, the cavitation generates de pressure pulses due to the collapse of vapour cavities formed. This code clearly shows the relative effect of unsteady term shear stress.

In view of a practical, the important of this study is the knowledge of the maximum pressure likely to occur and it is logical to think that this computer code is a relevant understanding of the tool calculus in transient cavitation flow in pipes. This code can facilely be used alongside the existing codes to simulate the cavitation of transient flow in complex networks: water supply, hydraulics, heating, etc.

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