Unsteady MHD Squeezing Flow of Casson Fluid Over Horizontal Channel in Presence of Chemical Reaction

Nur Azlina Mat Noor1, Mohd Ariff Admon1,*, Sharidan Shafie1

1 Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, 81310 Johor Bahru, Johor, Malaysia

ARTICLE INFO

ABSTRACT

The fluid flow with chemical reaction is one of well-known research areas in the field of computational fluid dynamic. It has been acknowledged by many researchers due to its industrial applications in the modelling of flow on a nuclear reactor. Motivated by the implementation of the flow in the industry problems, the aim of this study is to explore the impacts of chemical reaction on magnetohydrodynamic (MHD) squeeze flow of Casson fluid over a permeable medium in the slip condition with viscous dissipation. The flow is induced due to compress of two plates. Transformation of the partial differential equations (PDEs) to ordinary differential equations (ODEs) is performed by imposing similarity variables. The numerical procedure of Keller-box is used to solve the dimensionless equations. Comparison of the numerical results with former studies to validate the current solutions is conducted. It is shown to be in proper agreement. The results show that the velocity and wall shear stress enhance as both plates moving nearer. Moreover, increase in Hartmann and Casson parameters reducing the velocity, temperature and concentration. The temperature and the rate of thermal transfer boosts with the existence of viscous dissipation. Furthermore, the mass transfer rate is discovered boosts in destructive chemical reaction and adverse outcome is noted in constructive chemical reaction.

Keywords: Casson fluid; Chemical reaction; Squeezing flow; Viscous dissipation; Slip boundary

1. Introduction

The research of squeeze flow between two plates has obtained great interest from scientists because of its engineering applications involving injection molding, moving pistons, hydraulic lifts and lubrication system. The applications implement the concept of squeeze flow in which the flow is induced by the motion of both plates toward and away from one another. The primary research on behaviour of a lubricant within approaches horizontal surfaces was made by Stefan [1]. The basic model of squeeze flow using lubrication theory is discovered mathematically. Reynolds [2] and Archibald [3] continued Stefan’s study by considering elliptic and rectangle geometries. Later, several researchers have studied the thermodynamic behavior of squeezing flow in various flow configurations [4-8].

*Corresponding author.
E-mail address: ariffadmon@utm.my

https://doi.org/10.37934/arfmts.92.2.4960
Casson fluid is a non-Newtonian fluid based on the interaction of solid and liquid behavior. It is categorized as shear thinning flow with high shear viscosity and yield stress [9]. It acts similar as Newtonian fluid at the high values of wall shear stress [10]. Casson [11] introduced the Casson model for the production of silicon and inks suspension. The model is also preferable in the reviewing the properties of blood flow [12]. The primary study on time-dependent squeeze flow of Casson fluid on the two horizontal surfaces was conducted by Sampath et al., [13] and Khan et al., [14] using different methods to achieve the analytical solution.

The analysis on the impacts of magnetic field together with permeable medium is explored in the natural and industrial cases such as geothermal energy recovery, plasma studies, oil extraction and nuclear reactors. Permeable medium is a solid which consists of fluid-filled voids. The squeezing flow of Casson fluid across a permeable medium with the influence of magnetic field via Homotopy perturbation method (HPM) was reported by Khan et al., [15]. The similar work on magnetohydrodynamic flow of nanofluid in a porous medium was analysed by Sobamowo et al., [16]. It was noticed that fluid velocity decelerates as magnetic parameter increases, while it accelerates when permeability parameter increases.

The presence of viscous dissipation is important in the fluid flowing with high speed and viscosity. Viscous dissipation occurs when there is heat generation caused by the frictional force between fluid particles in a viscous fluid [17]. The impacts of viscous dissipation in the squeezing flow of Casson fluid was discovered by Mohyud-Din et al., [18] and Khan et al., [19]. The presence of Eckert number boosts the temperature profile and rate of heat transfer. Mustafa et al., [20] reviewed the combination impacts of joule dissipation and chemical reaction in the squeeze flow of viscous fluid. Further, Naduvinamani and Shankar [21] explored the squeezing flow of Casson fluid in the influences of MHD, joule dissipation, chemical reaction and joule heating. Naduvinamani and Shankar [22] extended their work by taking into account thermal radiation and heat source/sink impacts. Later, the analysis of Soret and Dufour was carried out by Naduvinamani and Shankar [23] along with the previous effects. Noor et al., [24] reported the effects of chemical reaction and viscous dissipation on MHD squeezing flow of Jeffrey nanofluid in horizontal channel with slip condition.

All the above studies show the fluid flow with no velocity slip at the surface. However, the velocity slip at the surface is necessary in the physical problems especially for non-Newtonian flow [25]. For example, the addition of velocity slip is important for flow in the micro devices [26-28]. The fluids with slip condition at wall are exhibited in the internal cavities and polishing of artificial heart valves. Qayyum et al., [29] investigated the presence of velocity slip on squeezing flow of Casson fluid in permeable medium with magnetic field. Khan et al., [30] reported the axisymmetric and two-dimensional flow of nanofluid in squeezed two surfaces under the impacts of velocity slip and joule dissipation. Moreover, Singh et al., [31] explored Khan et al., [30] works by including magnetic field impact on two dimensional nanofluid flow.

Based on the above cited papers, the influences of chemical reaction on squeezing flow of Casson fluid across permeable medium with viscous dissipation has not yet been explored. It is also noticed that less attention is paid involving the slip boundary condition. Therefore, the study concerns on time-dependent MHD squeezing slip flow of Casson fluid across porous medium with chemical reaction and joule dissipation. The numerical solutions are obtained by imposing the method of Keller-box. Moreover, the precision of the current outputs is validated with the reported results in the journals. The influences of significance parameters on concentration, temperature and velocity are investigated.

The present work is mostly implemented in the modelling of flow in a nuclear reactor. The presence of chemical reaction in the mathematical model is important to investigate the flow with nuclear reaction in the nuclear reactor. It is discovered that the lack of control of nuclear reaction
may lead to the widespread contamination of air and water. Hence, the nuclear reaction flow is instantaneously stopped when the nuclear power plant accidents happen [32]. Moreover, the significance of MHD and porous medium is analyzed in the fluid flow. Song et al., [33] reported that MHD possesses the highest efficiency among various of power conversion system. Power conversion efficiency is essential to NEP system. Nuclear electric propulsion (NEP) is simply electric propulsion in which the electricity is derived from a nuclear reactor. Saadati et al., [34] reported that safety margin of nuclear reactor using porous media approach. One of the main concerns in nuclear power plant operation is to increase the heat removal from the reactor. Hence, the implementation of porous medium boosts the heat removal in the nuclear reactor.

2. Mathematical Formulation

The time dependent MHD flow of Casson fluid through porous medium with chemical reaction, joule dissipation and velocity slip are explored. The squeezing of two surfaces generates the flow in the channel. The distance of two surfaces is 
\[ y = \pm h(t) = \pm l(1 - \alpha t)^{1/2} \]
The two surfaces are moving further as \( \alpha < 0 \) and the surfaces are moving closer as \( \alpha > 0 \) till \( t = 1/\alpha \) with velocity \( \nu_w(t) = \frac{\partial h(t)}{\partial t} \).

The lower plate is exerted with the magnetic field \( B(t) \) vertically [35]. Figure 1 portrays the geometrical model of Casson fluid flow.

![Geometrical model of Casson fluid flow](image)

The governing equations of Casson fluid are adapted from Mohyud-Din et al., [18] and Naduvinamani and Shankar [22] and reduced to the following equations using boundary layer approximation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_f \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(t)}{\rho_f} u - \nu_f \left( 1 + \frac{1}{\beta} \right) \frac{\varphi}{k_1(t)} \frac{\partial T}{\partial t} \]  
\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_f \frac{\partial^2 T}{\partial y^2} + \nu_f \left( 1 + \frac{1}{\beta} \right) \left[ 4 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 \right], \]
\[
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D_m \frac{\partial^2 c}{\partial y^2} - k_c(t) c, \tag{4}
\]

Here, the thermal diffusion of Casson fluid is \( \alpha_f = \frac{k}{(\rho c)_f} \), the rate of chemical reaction is \( k_c(t) = k_2(1 - \alpha t)^{-1} \) and the permeability of porous medium is \( k_1(t) = k_0(1 - \alpha t) \).

The correlated boundary conditions (BCs) are

\[
u = N_1 v_f \left(1 + \frac{1}{\beta_f} \frac{\partial u}{\partial y}\right), \quad \rho = \rho_f, \quad A = A_f, \quad \theta = \frac{T}{T_w}, \quad \phi = \frac{c}{c_w} \tag{7}
\]

Substitute Eq. (7) into Eq. (2), Eq. (3) and Eq. (4) yields to obtain the subsequent non dimensional equations

\[
\left(1 + \frac{1}{\beta_f}\right) f^\prime \prime \prime - S(\eta f^\prime \prime + 3f'' + f'f'' - 2f''') - Ha^2 f'' - \left(1 + \frac{1}{\beta_f}\right) \frac{1}{Da} f'' = 0, \tag{8}
\]

\[
\frac{1}{Pr} \theta'' + S(f \theta' - \eta \theta') + Ec \left[1 + \frac{1}{\beta_f}\right] (f''')^2 + 4 \delta^2 (f')^2 = 0, \tag{9}
\]

\[
\frac{1}{Sc} \phi'' + S(f \phi' - \eta \phi') - R \phi = 0, \tag{10}
\]

with the non-dimensional BCs

\[
f(\eta) = 0, f''(\eta) = 0, \quad \theta'(\eta) = 0, \quad \phi'(\eta) = 0, \quad \text{at} \quad \eta = 0, \tag{11}
\]

\[
f(\eta) = 1, f'(\eta) = \gamma \left(1 + \frac{1}{\beta_f}\right) f''(\eta), \quad \theta(\eta) = 1, \quad \phi(\eta) = 1, \quad \text{at} \quad \eta = 1. \tag{12}
\]

The pertinent terms in the non-dimensional equations are described by

\[
S = \frac{\alpha l^2}{2 \nu_f}, \quad Ha = l B_0 \sqrt{\frac{\sigma}{\rho_f \nu_f}}, \quad Da = \frac{k_0}{\varphi_l}, \quad \gamma = \frac{N_0 v_f}{l}, \quad \delta = \frac{l}{\chi} (1 - \alpha t)^{3/2},
\]

\[
Ec = \frac{\alpha^2 l^2}{4 c_f T_w (1 - \alpha t)^2}, \quad Pr = \frac{v_f}{\alpha_f}, \quad Sc = \frac{v_f}{D_m}, \quad R = \frac{k_2 l^2}{v_f}.
\]

Physically, the motion of surfaces is illustrated by squeeze parameter with \( S > 0 \) signifies the plates approaching closer and \( S < 0 \) signifies the plates separating further. The velocity profile is
managed by Darcy and Hartmann numbers. Moreover, the temperature field is determined by Eckert and Prandtl numbers. The effects of chemical reaction are discovered in the fluid concentration. Also, Schmidt number indicates the simultaneous flow with momentum and mass transfer.

3. Results and Discussion

The governing Eq. (8) to Eq. (10) in corresponded BCs of Eq. (11) and Eq. (12) are resolved via Keller-box procedure. This method has been implemented by many researchers, such as Zaib et al., [38] and Rosali et al., [39] because it is an unconditionally stable and succeed in obtaining the accurate results. It is also recommended to be used in solving the nonlinear parabolic problems. The four steps involved to solve the governing equations are as follows

(i) The ordinary differential equations are reduced to a system of first order equations.
(ii) The first order system is discretized to obtain the equations in the finite difference form by using central difference scheme.
(iii) The nonlinear equations are linearized using Newton’s method and then written in matrix-vector form.
(iv) Finally, the linear system can be solved via block tri-diagonal elimination technique.

MATLAB software is used to obtain the graphical and numerical outputs. Proper values of the step size, $\Delta \eta = 0.01$ and thickness of boundary layer, $\eta_\infty = 1$ are compulsory to get the accurate outputs. The variation in the current and former results of concentration, temperature and velocity is described as convergence criterion. Calculation is ended when the numerical outputs converging to $10^{-5}$ [40].

The calculation is done by varying the parameters of $S$, $\beta$, $Ha$, $Da$, $\gamma$, $Ec$, $Pr$, $R$ and $Sc$ to discover the behaviour of concentration, temperature and velocity profiles. The previous results are represented as limiting case for justification of the present results. Table 1 displays the results of $-f''(1)$, $-\theta'(1)$ and $-\phi'(1)$ in Shankar and Naduvinanami [41] journal is compared with the present results and it is noticed in proper agreement.

<table>
<thead>
<tr>
<th>$S$</th>
<th>Present results</th>
<th>Shankar and Naduvinanami [41]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f''(1)$</td>
<td>$\theta'(1)$</td>
</tr>
<tr>
<td>-1.0</td>
<td>2.170255</td>
<td>3.319904</td>
</tr>
<tr>
<td>-0.5</td>
<td>2.617512</td>
<td>3.129556</td>
</tr>
<tr>
<td>0.01</td>
<td>3.007208</td>
<td>3.047166</td>
</tr>
<tr>
<td>0.5</td>
<td>3.336504</td>
<td>3.026389</td>
</tr>
<tr>
<td>2.0</td>
<td>4.167412</td>
<td>3.118564</td>
</tr>
</tbody>
</table>

The effect of $S$ on axial velocity for contraction and expansion cases is presented in Figure 2. The surfaces approaching closer is indicated by $S > 0$ and the surfaces separating further is indicated by $S < 0$. It is shown declines for $\eta < 0.5$ and it rises for $\eta \geq 0.5$ when $S > 0$. In contrast, the velocity increases for $\eta < 0.5$ and it reduces for $\eta \geq 0.5$ when $S < 0$. The regions $0 \leq \eta \leq 0.45$ and $0.45 < \eta \leq 1$ refer to the region away and nearby the upper surface, respectively. The raise in velocity caused by the flow across the narrow channel rapidly when the surfaces are squeezed. On the contrary, the velocity declines because the fluid confronts high resistancy in a wide channel due to the plates are separated. The cross-flow is shown at the centre of boundary layer. Squeeze number
shows no impact on the velocity graph at $\eta_c = 0.45$, critical point. The variation of $\beta$ on axial velocity is illustrated in Figure 3. For $\eta \leq 0.5$, the velocity decreasing and for $\eta > 0.5$, it accelerating with $\beta$ rises. The deceleration of fluid velocity is due to the enhancement of flow viscosity strengthen the intermolecular forces of fluid particles. Hence, the decrease in kinetic energy led to the flow temperature declines. The influence of $Ha$ on axial velocity is discovered in Figure 4. It is noted that for $\eta \leq 0.5$, the velocity reduces and for $\eta > 0.5$, it elevates with enhancing $Ha$. The magnetic field in the electrical conducted fluid generates Lorentz force. This force offers more resistancy in the fluid, which resulting the flow in the channel decelerates. The effect of $Da$ on axial velocity is depicted in Figure 5. For $\eta \leq 0.5$, the velocity accelerating and for $\eta > 0.5$, it declines with raise in $Da$. The increase in $Da$ boosts the permeability of porous medium, which lead to the enhancement of fluid flow. The influences of $\gamma$ on axial velocity exhibited in Figure 6. The slip velocity at the upper boundary is indicated by $\gamma \neq 0$. It is discovered that the velocity of flow nearby the plate is distinct from the velocity of plate if slip arises. The velocity increases for $\eta \leq 0.65$ and it reduces for $\eta > 0.65$ when $\gamma$ elevates. The behaviour indicates that the increment of $\gamma$ indicates that the fluid slip across the upper boundary increases. Hence, it enhances the resistance within the boundary and fluid particles, that cause decelerating the flow near the vicinity of upper boundary.
Fig. 4. Impact of $Ha$ on $f' (\eta)$

Fig. 5. Impact of $Da$ on $f' (\eta)$

Fig. 6. Impact of $\gamma$ on $f' (\eta)$
The influences of \( Ec \) on temperature graph is portrayed in Figure 7. The temperature is shown boosts due to the enhancement in \( Ec \). The heat induction caused by the friction of fluid particles in the high viscosity flow is described as joule dissipation effect. High values of \( Ec \) cause an increment in the kinetic energy. Thus, it raises the friction amongst fluid particles, which resulting in the boost of the temperature. Figure 8 illustrates the influences of \( R \) on concentration region. It decreasing in destructive chemical reaction \((R > 0)\) and it rises in constructive chemical reaction \((R < 0)\). It is discovered that the strength of chemical reaction changes the diffusion rate. The increment of mass transfer rate in the case \( R > 0 \) has led to reduction in the concentration field.

![Fig. 7. Impact of \( Ec \) on \( \theta(\eta) \)](image)

![Fig. 8. Impact of \( R \) on \( \phi(\eta) \)](image)

4. Physical Quantities of Interest

Physically, the dimensionless parameters in the fluid are skin friction, Nusselt and Sherwood terms. The friction force at the surface boundary is described by skin friction. Moreover, Nusselt and Sherwood terms represent the rate of thermal and mass transfer in the fluid and surfaces. The terms of \( Cf_x, Nu_x \) and \( Sh_x \) are denoted by [42]
\( C_{f_x} = \frac{\tau_w}{\rho_f v_w^2}, \quad N_u_x = \frac{q_w}{a_f \tau_w}, \quad S_h_x = \frac{q_s}{D_m c_w}, \)

where \( \tau_w, q_w \) and \( q_s \) are the skin friction, thermal and mass fluxes at the surface. The terms of \( \tau_w, q_w \) and \( q_s \) are indicated by

\[
\tau_w = \mu_B \left( 1 + \frac{1}{\beta} \right) \left[ \frac{\partial u}{\partial y} \right]_{y=h(t)}, \quad q_w = -\alpha_f \left( \frac{\partial T}{\partial y} \right)_{y=h(t)}, \quad q_s = -D_m \left( \frac{\partial C}{\partial y} \right)_{y=h(t)}.
\]

The non-dimensional of \( C_{f_x}, N_u_x \) and \( S_h_x \) are

\[
\frac{l^2}{x^2} (1 - at) Re_x C_{f_x} = \left( 1 + \frac{1}{\beta} \right) f''(1), \quad \sqrt{(1 - at) N_u_x} = -\theta'(1), \quad \sqrt{(1 - at) S_h_x} = -\phi'(1),
\]

Graphical results of the dimensionless parameters are shown in Figure 9 and Figure 10. The influence of \( S, Ha \) and \( \beta \) on skin friction is plotted in Figure 9. It is noted that the wall shear stress caused by the raise in \( S \) and \( Ha \), while it decreases with increasing in \( \beta \). The Lorentz force led to the enhancement in the friction within the fluid and solid boundary, which results in the wall shear stress rises. On the contrary, the decline in wall shear stress owing to the high viscosity of Casson fluid. It has enhanced the intermolecular forces in fluid particles, which causes the kinetic energy decreases. The mass transfer rate elevates with increase in \( Sc \) for \( R > 0 \), whereas it declines for \( R < 0 \).
5. Conclusions

The current investigation explores the time-dependent MHD slip flow of Casson fluid in squeezed two surfaces with chemical reaction and joule dissipation. Keller-box technique is applied to solve the governing equations numerically. The numerical results are compared with the reported journal outputs and noted in close agreement. The impacts of pertinent parameters, which are $S$, $\beta$, $Ha$, $Da$, $\gamma$, $Ec$, $Pr$, $R$ and $Sc$ on concentration, temperature and velocity are studied. The important outcomes in the analysis are deduced as:

i. The velocity decelerates when the surfaces separate further ($S < 0$) and it accelerates when the surfaces approach closer ($S > 0$).
ii. The wall shear stress boosts as $S$ and $Ha$ rises, whereas it decreases when $\beta$ increasing.
iii. The concentration, temperature and velocity decline due to the enhancement of $\beta$ and $Ha$.
iv. The velocity near to the upper boundary decelerating when $Da$ and $\gamma$ increases.
v. The heat transfer rate and flow temperature boost for increasing $Ec$ values.
vi. The concentration enhancing in the constructive chemical reaction ($R < 0$) and it declines in the destructive chemical reaction ($R > 0$).
vii. The rate of mass transfer increases when $R > 0$ and it decreases when $R < 0$.

Acknowledgement

This research was funded by Ministry of Education (MOE) and Research Management Centre of Universiti Teknologi Malaysia (UTM), FRGS/1/2019/STG06/UTM/02/22, 5F004, 5F278, 07G70, 07G72, 07G76, 07G77 and 08G33.

References


