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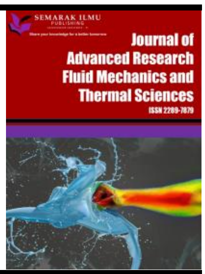

Combined Effects of Chemical Reaction and Radiation on MHD Squeezing Flow of Casson Fluid

M. Harikrushna[1,*], G. Prabhakara Rao[2], A. Mallikarjuna Reddy[1]

[1] Department of Mathematics, Sri Krishnadevaraya University, Ananthapuramu-515003, Andhra Pradsh, India
[2] Department of Mathematics, Government College (A), Ananthapuramu-515001, Andhra Pradsh, India

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ABSTRACT

The impacts of chemical reaction on the magnetohydrodynamic (MHD) squeezing Casson fluid flow through a porous medium under the slip condition with viscous dissipation the presence of radiation parameter have been discussed. The flow is produced when two plates are compressed together in close proximity to one another. Using similarity variables may successfully convert partial differential equations (PDEs) to ordinary differential equations (ODEs). The shooting technique was used to perform the numerical analysis, which entailed solving the competent governing equations with dominating parameters for a thin liquid layer. This was done to determine the results of the study. It is essential to evaluate the numerical results in light of previously conducted research to validate the current answers. According to the results, an increase in the distance between the two plates leads to a rise in the velocity and the wall shear stress. The gas's velocity, temperature, and concentration all drop due to an increase in the Hartmann and Casson parameters. Both the temperature and the rate of heat transfer rise in direct proportion to the amount of viscous dissipation. In addition, it has been shown that the rate of mass transfer rises during destructive chemical interactions, but during constructive chemical reactions, the contrary occurs, which results in adverse effects.

1. Introduction

Convective flow is the term describing the movement of fluid through a membrane with pores that is influenced by the force that is being applied. The term “heat transfer” refers to the movement of thermal energy caused by temperature gradients or differences. The scientific study of heat transfer not only describes how heat energy can be transmitted, but also makes predictions about the pace at which the transfer of heat energy will occur under given particular predetermined conditions. The second law of thermodynamics states that heat always passes from things with higher temperatures to those with less temperature. The design of aircraft systems and components, rockets and weapons of mass destruction, as well as in the fields of aerospace and space technology, civil engineering, chemical engineering, cooling generators are just a few examples of numerous

* *Corresponding author.*
E-mail address: harikrushna.m@gmail.com

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applications of heat transfer. Within mixes comprised of two or more components whose concentrations vary from place to place; it appears that there is a continuous natural tendency for various kinds to be transferred from regions of greater concentration to regions of lesser concentration. Mass transfer is the term used to describe the process of transferring mass due to a variation in species concentration within a mixture. Convective mass transport and diffusion mass transport are the two methods by which mass are transferred. The concept of mass transfer in physics, chemistry and biology includes a wide range of processes, including condensation, solvent extraction on the surface of a plate, as well as in the rocket motors, transpiration cooling of jet engines etc. The biological application of mass transfer is present in many processes, including the food processing, oxygenation of blood, respiration, drug assimilation and a number of other processes. The science of MHD examines the magnetic characteristics and activity of electrically conducting fluids.

The investigation and comprehensive reviews are most significant in the sub regions of the hydrological science, together with water supported administration that impacts onto environment as well as societies. Those comprised, to the physically, chemically, bio-geochemically, stochastically in addition to schemes characteristics of surfaces in addition to groundwater hydrology**,** hydro-meteorology**,** hydro-geology as well as hydro-geophysics. Significant themes are including the imminent as well as methodology of various restraints they are, climatology, water resources system, eco-hydrology, geo-morphology, soil sciences, instrumentations in addition to remote sensing, data along with information science, civil with environmental engineering. Multi as well as inter-disciplinary analyses of hydrological predicaments are pacifically broadening the accepting of hydrologic sciences through integration by the social, economics, and/or behavior sciences.

The modern developments in the technologies required the ground-breaking revolutions into the domains for the heat transports. In spite of the difficulty of non-Newtonians fluids, sensible mathematician along with engineering researcher are connected into the non-Newtonian fluids energetic since, the flow in addition to heat transportations characteristics of those fluids is necessary to abundant in addition to a variety of schemes into bio-technology, pharmaceutically and chemically engineering regions, etc. Non-Newtonian models often feature the non-linearly connection among the stresses in addition to the applied strain rates. The conservation theory may not describe the mechanically properties of the non-Newtonian liquids, such as thinner as well as shearing thicknesses, average stresses difference, in addition to visco-elastics responses. As a result, it is necessary to develop an original and effective prediction. Casson models have gained enormous recognition as a result of this, which can be attributed to the many constitutive type equations suggested to represent the development and heat transfer process. The explorers recognized the Cassons fluids models as the non-Newtonian fluids because of those wide ranging application into biomedical as well as industrials engineering, energies construction, geo-physically fluids dynamics, in addition to the dynamics. The Cassons fluids models are an essential rheologically modeling among these several models that are not based on Newton's laws, and it was primarily created with Casson's help. As the non-Newtonians fluids, Cassons fluids are an example of the shearing-thin liquid used. It exhibited yielding stress. When a lower amounts of shearing stresses than the yielding stress is applied, the fluid will behave approximating solid; more specifically, there will be no flow, but this behavior will be eliminated if the shearing forces applied are more than a yielding stress. In the shear thicken fluids modeling, it was supposed to have had the infinite stickiness near disappears rates of shear, yielding stress underneath which no fluid flowing taken places, as well as vanishes viscosity near an unrestrained paces of shear strains. Additionally, this was presumed to have had yielding stresses below these no flowing occurs. The amounts of repeated models of Casson fluids included things like tomato sauce, jelly, honey, humans' blood, soups, and biologically fluids, among other things. Many researchers are interested in studying the impacts of combined accumulation as well as temperature transportation by the chemically reactions onto the MHD flow of the gelatinous, non-compressible, in addition to electrical performing fluids since of those extensively range of engineering as well as industrially application. Some examples of these applications include MHD generator, plasmas study, nuclear reactor, as well as geo-thermal energy extraction. The influence of the first ordered chemically reactions was investigated by Das *et al.,* [1] on the flow that went through an impulsive initiated unbounded perpendicular plate while maintaining the invariable accumulation transport in addition to temperature efflux. Anjalidevi and Kandasamy [2] considered the accumulation as well as temperature transportations onto continuous laminar flowing over a half-infinitely flat plate into the representing of a phenomenon with chemical reacting. Andersson [3] investigated a magnetic domain influence onto the movement of the visco-elastic fluids as it moves through the stretched sheet. Nandkeolyar *et al.,* [4] studied the unstable MHD temperature as well as accumulations transports flowing of the temperature radiating in additions to chemical reacting fluids as it moved through the smooth permeable plate by the ramping walls temperatures. The theoretically examination of convection accumulations and temperature transportations of unstable MHD movement via the perpendicular semi-unlimited absorbent plate was published by Reddy *et al.,* [5]. The porous plate had varying viscosities as well as thermally conductivities. In past years, the non-Newtonians liquids had seen widespread use into the chemically, pharmaceutically, as well as cosmetics sectors. For example, non-Newtonians liquids had been utilized into manufacturing various chemical, oils, gaseous, paints, syrups, juices [6].

Newtonians fluids contain glutinous stress directly comparative into the locally strains rates at every site, but their uses are restricted because of this relationship. They cannot express the different facts observed for the liquids used in industrially as well as the other technical appliances; they are blood, soaps, definite oil, paint, as well as various emulsions. This was good knowledge that, the mechanism of the non-Newtonian liquids gave a formidable dispute into the works of engineer, physicist, in addition to mathematician. The Naviers–Stokes equation cannot adequately explain the characteristics of such liquids, and there were not the singles constitutive equations that can represent every fluids' characteristics. Non-Newtonians liquids' constitutive equations are used to characterize the rheological characteristics of the juices.

A variety of non-Newtonian fluid representations, they are, visco-plastics, Bingham plastics, Brinkmans types, power laws fluids, Oldroyd-B fluids, as well as Walters-B fluids modeling, have been suggested as a response to the complicated behavior of fluids [7-12]. The Casson fluids modeling were the well-known non-Newtonian modeling that Cassons first presented to predict the flowing performance of pigment-oils suspensions [13]. Cassons is also a name of the person who first introduced the model. Fredrickson [14] explored the Casson fluid's continuous flow in a tube during the early experiments on the substance. By using the Casson fluid model, Boyd *et al.,* [15] could explain blood flow's steady and oscillatory flow patterns. Mernone *et al.,* [16] portrayed the peristaltic flowing of the Cassons fluids into the conduit with just 2 dimensions. Mustaffa *et al.,* [17] used a homotopy analysis model to investigate a Casson fluid's unsteady boundary layer flow in additions temperature transports across the stirring smooth plate with the analogous liberated stream. The impacts of thermally radiating onto the Cassons liquid movement in additions to temperature transports across unevenly stretched surfaces were addressed by Mukhopadhay [18]. The assorted convective stagnations points movement of the Cassons fluids under convection frontier stipulations was investigated by Hayat *et al.,* [19]. Bhattacharyya [20] researched Casson fluid's boundary layer stagnation point flows in additions temperature transports toward the decreasing and stretched sheet. Pramanik [21] investigated the Cassons fluids flows in additions temperature transports across exponential permeable stretched surfaces into the representing of the thermally radiating.

Mukhopadhyay and Mondal [22] explored the forced convective flows of the Cassons fluids across the symmetrically permeable wedge while considering the surfaces temperatures efflux. The flows in additions temperature transports features of the Cassons fluids movement are an essential study field because of their importance for the optimum dispensation of chocolates, coffees, as well as additional types of foods stuffs [23]. Akbar and Khan [24] researched the metachronal thrashing of cilia when the cilia were in the presence of Cassons fluids in additions to the magnetic fields. Also, Akbar [25] explored the impact of the magnetic fields onto the peristaltic flowing of the Cassons fluids into an asymmetrically conduit as the applications into crude oils processing. Noreen and Butt [26] looked at the physiological process of how Casson fluid moves via a plumb duct. Nadeem *et al.,* [27] presented the results of the theoretically investigations onto the oblique stagnations points flowing of the Casson nanofluids towards the stretched surfaces by the temperature transports. In the identical year, Nadeem *et al.,* [28] explored the MHDs 3-dimensionalized frontier layered movement of the Casson nanofluids over the linear stretched sheet by the convection frontier conditions.

The amount of heat transferred gone from the surfaces is related to the temperatures of the surface in an immediate area in many circumstances. This kind of impact is referred to as the Newtonian heating consequence. Merkin [29] looked at conjugate convection flowing with Newtonian heating in his research. Researchers were drawn to explore the Newtonian heating stipulation in those issues as a result of the variety of applications that it has. An accurate study of the flow of accumulation as well as temperature via the perpendicular plate heated using the Newtonians method was reported by Hussanan *et al.,* [30]. The unstable frontier layered flows as well as temperatures transports of the Cassons fluids were examined by Hussanan *et al.,* [31]. This was done underneath the Newtonian heating frontier conditions for the non-Newtonian fluids. Das *et al.,* [32] spoke about the Newtonian heating impact onto an unstable hydro-magnetic Cassons fluids flowing that went across the smooth plate by the temperatures in additions to accumulation transportation. Ghosh and Mukhopadhyay [33] researched a Casson nanofluid's MHD slip movement in additions temperatures and accumulation transportation across an exponential extending permeable sheet. Khan *et al.,* [34] explored the possibility of virtually investigating Casson fluid via homogeneous and heterogeneous processes. By employing the Laplace transformations approach, Khalid *et al.,* [35] describe time-dependent MHD Casson fluid-freed flows that take place across a fluctuating perpendicular plate that is immersed by a porous medium. These findings shed light on a lowered speed and increased skin frictions onto the plate surfaces brought about with an increase in the magnetic fields' parameters.

Rauf *et al.,* [36] investigated the frontier layered movement of the Cassons fluid onto the sheet extending by the two traditions: depictions to an across magnetic field and thermally radiation with coupled convection circumstances using the Range Kutta Fehlbergs technique. Both of these approaches were used to study the frontier layered flow of the Cassons liquids. This is shown by the fact that the systems experienced a thermal expansion due to the more significant radiating. Zaib *et et al.,* [37] studied Casson liquid with the repercussions of viscous dissipations traveling onto an exponentially diminishing sheet. The numerical systems are assessed computationally using the shooting approach, and twofold solutions were achieved of the temperature and velocity distributions of the firing area. This research demonstrated a decrease in the temperature distributions of the system reasoned by an enlarge into the Prandtl quantity. However, the system's temperature rose directly from the enhanced gelatinous dissipation. In their study, Raju *et al.,* [38] contrasted the Casson fluid and the Newtonian fluids as they passed the exponential extending surfaces while experiencing the consequences of heat radiating, gelatinous dissipations, as well as magnetic fields. The growth of heat organisms for higher viscous dissipations repercussions and temperatures transportations rates is optimal for Cassons fluid as opposed to Newtonian fluid. Research that is somewhat equivalent to this one has been presented by Sulochana *et al.,* [39], wherein a 3-dimensional Casson nanofluids are contrasted into a Newtonian liquid utilizing the Range Kutta shooting methodology.

The MHD repercussions are added into the experiment, and fluid flow is seen to be occurring behind the stretched sheet simultaneously. The accumulations rate, and the temperatures transports pace, enhanced for highest stretched relations parameters. Cassons liquid was bigger into the Newtonian liquid in temperature and mass transportation discharges in this context. Kumaran and Sandeep [40] researched the comparison for MHD Cassons fluids to Williamsons liquids as they flowed onto the crest of a greater paraboloids of revolution, considering the thermophoresis in additions to Brownian motions consequences. The MHD Cassons fluids flows across a wedge, and the impact of the binary chemically reactions and commencing energy have been examined by Zaib *et al.,* [41]. The modeling is enhanced and assessed using modified Arrhenius functions to represent the establishment energy. The MHD coupled convection Cassons fluid flows via the effects of the two stratification and temperatures immersion have been explored in Rehman *et al.,* [42] research paper. The Casson fluids are the shear thinning fluids based on the following 3 assumptions: at zero shear rate, the viscosity is unlimited; no fluids flow survives lower than the yielding stress; and this, at an unrestricted shearing, the viscosity is ignorable. As explained by Reddy *et al.,* [43]. In their paper, Hamid *et al.,* [44] described the stretched sheet encased in the MHD Cassons liquids and modulated with linearly performed thermally radiating. The effectiveness of dual solutions is analyzed, showing that an increasing radiating function may provide more significant and lower temperature profiles. Hamid *et al.,* [45] utilized the Galerkin finite elements methodology to study the rates of heat transit for a liberated convection flow of the Cassons liquids passing through the fairly heated trapezoidal chamber.

Khan *et al.,* [46] discussed MHD flow and heat transfer of double stratified micropolar fluid over a vertical permeable shrinking/stretching sheet with chemical reaction and heat source. Teh and Ashgar [47] explored the three dimensional MHD hybrid nanofluid flow with rotating stretching/shrinking sheet and Joule heating. Bakar *et al.,* [48] explored the stability analysis on mixed convection nanofluid flow in a permeable porous medium with radiation and internal heat generation. Hashim *et al.,* [49] discussed the natural convection in trapezoidal cavity containing hybrid nanofluid with important properties. Shahrim *et al.,* [50] explored the exact solution of fractional convective Casson fluid through an accelerated plate. Ahmad *et al.,* [51] are investigated the two new modified variational iteration algorithms for the numerical solution of coupled Burgers' equations. Ahmad *et al.,* [52] explored the analytic approximate solutions for some nonlinear parabolic dynamical wave equations. Inc *et al.,* [53] explored analysing time-fractional exotic options via efficient local meshless method. Attar *et al.,* [54] explored the analytical solution of fractional differential equations by Akbari–Ganji's method. Zangooee *et al.,* [55] discussed hydrothermal analysis of hybrid nanofluid flow on a vertical plate by considering slip condition, Hosseinzadeh *et al.,* [56] discussed the thermal analysis of moving porous fin wetted by hybrid nanofluid with trapezoidal, concave parabolic and convex cross sections. Gamaoun *et al.,* [57] explored the effects of thermal radiation and variable density of nanofluid heat transfer along a stretching sheet by using Keller Box approach under magnetic field. Rasool *et al.,* [58] explored the hydrothermal and mass aspects of MHD non-Darcian convective flows of radiating thixotropic nanofluids nearby a horizontal stretchable surface. Krishna *et al.,* [59] explored the radiative MHD flow of Casson hybrid nanofluid over an infinite exponentially accelerated vertical porous surface. Krishna and Chamkha [60] investigated the diffusion-thermo, radiation-absorption and Hall and ion slip effects on MHD free convective rotating flow of nano-fluids past a semi-infinite permeable moving plate with constant heat source. Krishna *et al.,* [61] discussed the Soret and Joule effects of MHD mixed convective flow of an incompressible and electrically conducting viscous fluid past an infinite vertical porous plate taking Hall effects into account.

According to the papers detailed earlier, it has not yet been established whether or not chemical reactions affect the squeezing flow of Casson fluid across a porous medium with viscous dissipation. In addition to this, it has been seen that the sliding boundary condition is not given as much consideration as it should be. Consequently, the study is concentrated on time-dependent MHD squeezing slip flow of Casson fluid through porous media with chemical reaction and joule dissipation in the presence of radiation parameters. The numerical solutions may be obtained with the use of the shooting method. In addition, the correctness of the current outputs has been checked by comparing them to the results that have been published in peer-reviewed papers. In this study, we investigate the impact that key parameters have on the fluid's concentration, temperature, and velocity.

2. Formulation and Solution of the Problem

To create the governing equations for Casson fluid flow when it is compressed between two parallel plates, the rheological equation shown below is employed.

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{P_y}{\sqrt{2\pi}}\right) e_{ij}, \pi > \pi_{c,} \\ \left(\mu_B + \dfrac{P_y}{\sqrt{2\pi_C}}\right) e_{ij}, \pi < \pi_{c,} \end{cases} \tag{1}$$

In Eq. (1), $\pi = e_{ij}, e_{ij}$, where e_{ij} are the $(i, j)th$ component of the deformation rate, π is the product of the deformation rate with itself, π_C is a critical value of this product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid and P_y the yield stress of the fluid.

$$P_y = \frac{\mu_B \sqrt{2\pi}}{\beta} \tag{2}$$

Denote the yield stress of fluid. Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called Rheopectic, in the case of Casson fluid (Non-Newtonian) flow where $\pi > \pi_c$

$$\mu = \mu_B + \frac{p_y}{\sqrt{2\pi}} \tag{3}$$

When Eq. (2) is substituted into Eq. (3), the kinematic viscosity may be expressed as

$$\vartheta = \frac{\mu}{\rho} = \frac{\mu_B}{\rho}\left(1 + \frac{1}{\beta}\right) \tag{4}$$

where $\beta = \dfrac{\mu_B\sqrt{2\pi_c}}{p_y}$ denotes the Casson fluid parameter. The nature of Non-Newtonian fluid vanishes and it behaves as Newtonian fluid when $\beta \to \infty$.

The time dependent MHD flow of Casson fluid through porous medium with chemical reaction, joule dissipation and velocity slip are explored. The squeezing of two surfaces generates the flow in the channel. The distance of two surfaces is $y = \pm h(t) = \pm(1 - \alpha t)^{1/2}$. The two surfaces are moving further as $\alpha < 0$ and the surfaces are moving closer as $\alpha > 0$ till $t = 1/\alpha$ with velocity $v_w(t) = \partial h(t)/\partial t$. The lower plate is exerted with the magnetic field (t) vertically [62]. Figure 1 portrays the geometrical model of Casson fluid flow. Also, the homogeneous first order chemical reaction effect is accounted in the concentration equation. Further, in the present analysis, symmetric flow is assumed. With these assumptions, the continuity, momentum, energy and concentration equations governing the present physical problem with necessary conditions are as follows [63,64].

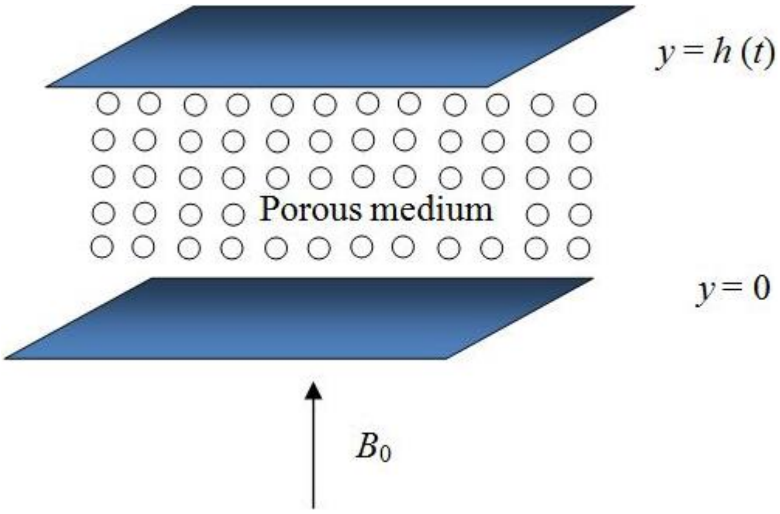

Fig. 1. Physical model of problem

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} v\frac{\partial u}{\partial y} = \nu_f\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(t)}{\rho_f}u - \nu_f\left(1+\frac{1}{\beta}\right)\frac{\varphi}{k_1(t)}u \tag{6}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} v\frac{\partial v}{\partial y} = \nu_f\left(1+\frac{1}{\beta}\right)\frac{\partial^2 v}{\partial y^2} - \frac{\sigma B^2(t)}{\rho_f}v - \nu_f\left(1+\frac{1}{\beta}\right)\frac{\varphi}{k_1(t)}v \tag{7}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} v\frac{\partial T}{\partial y} = \alpha_f\frac{\partial^2 T}{\partial y^2} + \frac{\nu_f}{c_f}\left(1+\frac{1}{\beta}\right)\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right] - \frac{1}{\rho_f c_f}\frac{\partial q_r}{\partial y} + \frac{1}{\rho_f c_f}\frac{Q(t)}{T} \tag{8}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} - k_c(t)C \tag{9}$$

where, $\alpha_f = \dfrac{k}{(\rho c)_f}$ is the thermal diffusion of Casson fluid, $k_c(t) = k_2\left(1-\alpha t\right)^{-1}$ is the rate of chemical reaction and $k_1(t) = k_0\left(1-\alpha t\right)$ is the permeability of porous medium.

The correlated boundary conditions are

$$u = N_1 v_f\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y},\ v = vw = \frac{\partial h(t)}{\partial t} \quad at \quad y = h(t) \tag{10}$$

$$\frac{\partial u}{\partial y} = 0, \quad v = 0, \quad \frac{\partial T}{\partial y} = 0, \quad \frac{\partial C}{\partial y} = 0, \qquad \text{at } y = 0 \tag{11}$$

where, $N_1(t) = N_0\left(1-\alpha t\right)^{1/2}$ represent momentum slip.

For an optically thick fluid, in addition to emission there is also self absorption and usually the absorption co-efficient is wavelength dependent and large so we can adopt the Rosseland approximation for the radiative heat flux vector qr. Thus, qr is given by

$$q_r = \frac{-4\sigma_1}{3k_1}\frac{\partial^2 T^4}{\partial z^2} \tag{12}$$

where k_1 is Rosseland mean absorption co-efficient and σ_1 is Stefan–Boltzmann constant.

We assume that the temperature differences within the flow are sufficiently small so that T^4 can be expressed as a linear function. By using Taylor's series, we expand T^4 about the free stream temperature T_∞ and neglecting higher order terms. This results in the following approximation:

$$T^4 \approx 4T_\infty^3 T^* - 3T_\infty^4 \tag{13}$$

The energy equation is derived using Eq. (12) and Eq. (13) as follows

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} v\frac{\partial T}{\partial y} = \alpha_f\frac{\partial^2 T}{\partial y^2} + \alpha_f\left(\frac{16\sigma T_\infty^3}{3k_f k_1^*}\right)\frac{\partial^2 T}{\partial y^2} + \frac{v_f}{c_f}\left(1+\frac{1}{\beta}\right)\left[4\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]\frac{1}{\rho_f c_f}\frac{Q(t)}{T} \tag{14}$$

The similarity transformation used in a study by Noor *et al.,* [62] is imposed to transform the PDEs to dimensionless ODEs

$$\eta = \frac{y}{l\sqrt{(1-\alpha t)}},\ u = \frac{ax}{2(1-\alpha t)} f'(\eta),\ v = \frac{al}{2\sqrt{(1-\alpha t)}} f(\eta),\ \theta = \frac{T}{T_w},\ \phi = \frac{C}{C_w} \tag{15}$$

Substitute Eq. (11) into Eq. (2), Eq. (3), Eq. (5) and Eq. (10) yields to obtain the subsequent non dimensional equations

$$\left(1+\frac{1}{\beta}\right)f^{iv}-S\left(\eta f'''+3f''+ff''-ff'''\right)-Ha^2f''-\left(1+\frac{1}{\beta}\right)\frac{1}{Da}f''=0 \tag{16}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}R_d\right)\theta''+S\left(f\theta'-\eta\theta'+Q\theta\right)+E_c\left[\left(1+\frac{1}{\beta}\right)\left(f''\right)^2+4\sigma^2\left(f'\right)^2\right]=0 \tag{17}$$

$$\frac{1}{Sc}\varphi''+S\left(f\varphi'-\eta\varphi'\right)-R\varphi=0 \tag{18}$$

The correlated Dimensionless boundary conditions (BCs) are

$$f(\eta)=0,\ f''(\eta)=0,\ \theta'(\eta)=0,\ \varphi'(\eta)=0 \qquad at \quad \eta=0 \tag{19}$$

$$f(\eta)=1,\ f'(\eta)=\gamma\left(1+\frac{1}{\beta}\right)f''(\eta),\ \theta(\eta)=1,\ \varphi(\eta)=1 \quad at \quad \eta=1 \tag{20}$$

The significant parameters in the non-dimensional equations are defines as

$$S=\frac{\alpha l^2}{2v_f},\ Ha=lB_0\sqrt{\frac{\alpha}{\rho_f v_f}},\ Da=\frac{k_0}{\phi l^2},\ \gamma=\frac{N_0 v_f}{l},\ \delta=\frac{1}{x}(1-\alpha t)^{1/2},\ \Pr=\frac{v_f}{\alpha_f},\ Sc=\frac{v_f}{D_m}$$

$$R=\frac{k_2 l^2}{v_f},\ Ec=\frac{\alpha^2x^2}{4c_fT_w\left(1-\alpha t\right)^2},\ R_d=\frac{4\sigma T_\infty^3}{k_f k_1^*},\ \gamma=\frac{2Q_0}{\alpha\left(\rho c\right)_f}$$

Physically, the movement of channel is portrayed by squeezing number with S>0 shows the plates approaches closer and S<0 shows the plates separates further. Darcy and Hartmann numbers are important parameter for velocity profile. Furthermore, thermal radiation, Eckert number and heat generation/absorption parameters are used for regulation of temperature profile. The effect of chemical reaction is exhibited in the nanoparticle's concentration profile. The flow in the simultaneous momentum and mass diffusion is described by Schmidt number.

The physical quantities of interest which govern the flow are the local skin friction coefficient C_{fx}, local Nusselt number Nu_x and local Sherwood number Sh_x, which are defined as

$$Cf_x=\frac{\tau_w}{\rho f u_w^2},\ Nu_x=\frac{xq_w}{\alpha_f\left(T_w-T_\infty\right)},Sh_x=\frac{xq_s}{D_B\left(C_s-C_\infty\right)} \tag{21}$$

where τw, qw and qs are the wall skin friction, wall heat flux and wall mass flux respectively given by

$$\tau_w=\mu_B\left(1+\frac{1}{\beta}\right)\left[\frac{\partial u}{\partial y}\right]_{y=h(t)},\ q_w=-\alpha_f\left[\frac{\partial T}{\partial y}\right]_{y=h(t)},\ q_s=D_m\left[\frac{\partial C}{\partial y}\right]_{y=h(t)} \tag{22}$$

The non-dimensional forms of the skin friction coefficient, Nusselt number and Sherwood number in terms of similarity variable are

$$\frac{l^2}{x^2}(1-at)\mathrm{Re}_x\, Cf_x = \left(1+\frac{1}{\beta}\right) f''(1), \sqrt{(1-at)}\, Nu_x = -\theta'(1), \sqrt{(1-at)}\, Sh_x = -\varphi'(1), \tag{23}$$

3. Results and Discussion

The ordinary differential Eq. (16) to (18) with conditions (19) and (20), governing the radiative squeezing flow of unsteady magneto-hydrodynamic Casson fluid between two parallel plates are coupled and nonlinear in nature. Hence, to solve these complex flow equations, numerically stable Runge-Kutta fourth order integration algorithm with standard shooting technique is used [65]. To confirm the accuracy of the present numerical scheme and correctness of the obtained numerical solutions, the results are compared with those of Noor *et al.,* [63] and Naduvinamani and Shankar [64]. This comparison is shown in Table 1. From Table 1 it is observed that, the present numerical results are exactly matching with the semi-analytical results of Noor *et al.,* [62,63]. Also, it is noticed from Table 1 that, the absolute values of wall shear stress are enhanced for the increasing values of squeezing number, whereas, the Nusselt and Sherwood numbers decrease. Also, it is obvious that, the negative values of the Nusselt number indicate the flow of heat from the surface of parallel plates to the fluid between the plates.

Table 1
Comparison of $-f\,''(1)$, $-\theta\,'(1)$ and $-\phi\,'(1)$ for different values of S when $\beta \to \infty$, $Da \to \infty$ and $Ha = Rd = \gamma = Nb = Nt = 0$

S	Noor *et al.,* [63]			Naduvinamani and Ushashanka [64]			Present Values		
	$-f\,''(1)$	$-\theta\,'(1)$	$-\phi\,'(1)$	$-f\,''(1)$	$-\theta\,'(1)$	$-\phi\,'(1)$	$-f\,''(1)$	$-\theta\,'(1)$	$-\phi\,'(1)$
-1.0	2.170266	3.319955	0.804558	2.170091	3.319899	0.804559	2.170851	3.317499	0.808545
-0.5	2.617511	3.129545	0.781404	2.617404	3.129491	0.781402	2.617547	3.124588	0.787585
0.01	3.007209	3.047168	0.761229	3.007134	3.047092	0.761225	3.006774	3.047758	0.767582
0.5	3.336505	3.026389	0.744229	3.336449	3.026352	0.744224	3.334578	3.024785	0.744785
2.0	4.167445	3.118545	0.701819	4.167389	3.118551	0.701813	4.164755	3.118568	0.701714

To describe the physical insight of the present problem in depth, the thermodynamic flow behaviour of skin-friction coefficient, Nusselt and Sherwood numbers along with velocity, temperature and concentration profiles in the flow region for different set physical parameters namely, squeezing parameter (S), Casson fluid parameter (β), Hartmann number (Ha), radiation parameter (R), heat generation or absorption parameter (Q), Eckert number (Ec), Prandtl number (Pr), chemical reaction parameter (Kr) and Schmidt number (Sc) are investigated. For better understanding of the numerical results, the computer generated numerical data are presented in the form of graphs and tables.

The thermodynamic behaviour of velocity, temperature and concentration profiles for different values of squeezing number (S) is depicted in Figure 2 to Figure 7 with fixed values of β=0.8, Ha=R=Q=Ec=Kr=0.1, Pr=0.1 and Sc=0.7. The movement of the parallel plates moving away from one another (S>0) is illustrated by Figure 2, Figure 3 and Figure 4. Similarly, the movement of the parallel plates coming close to one another (S<0) is described by Figure 5, Figure 6 and Figure 7 in the flow region. For positive values of squeezing number, radial velocity field (F) decreases from η=0 to η=1 and radial velocity profile increases from η=0 to η=1 for negative values of S. These changes in radial velocity fields are clearly shown in Figure 2 and Figure 5, respectively. This increment in velocity field is due to the reason that, when parallel plates move apart from one another, fluid is sucked into the channel consequently which increases the velocity field. In other case, when plates move close to one another, liquid inside the channel is emitted out which gives the liquid dropping inside the channel and hence velocity of the fluid decreases. However, squeezing number is a function of velocity field in the flow region.

Figure 3 and Figure 6 show the thermodynamic differences seen in the temperature profile (θ) for both S > 0 and S0 in the flow zone. These variations were observed in the temperature profile. Figure 2 illustrates how the temperature field is finally suppressed for the cases in which the squeezing number is allowed to increase in a positive direction. This drop in the temperature field is because a higher value of the squeezing number reduces the squeezing force exerted on the flow, which in turn has a negative impact on the temperature field. On the other hand, looking at Figure 6, one can see an increase in the magnitude of the temperature field for S = 0. Since the magnifying value of the squeezing number is closely related to the decaying of the kinematic viscosity, the distance between parallel plates, and the speed at which plates move, it is evident that the temperature field is relatively high when plates move close to one another. This is the case because of the close relationship between these three factors. On the other hand, the temperature profile exhibits the behavior of a function that steadily increases from zero to one. This much is abundantly apparent. Nevertheless, the impact of S on the concentration profile in the flow zone is seen in Figure 4 and Figure 7. An increased concentration field may be seen for S values greater than zero, as seen in Figure 4. Furthermore, looking at Figure 7, one can see that the concentration field goes down when S is made smaller than 0. In addition, Figure 7 reveals that the influence of the compressing number decreases with greater values. This is something that can be seen. Therefore, the concentration profile is a function of the number of times the substance was squeezed.

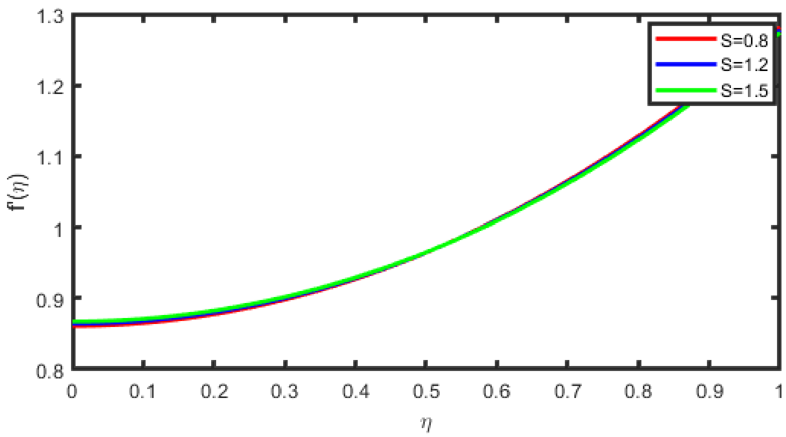

Fig. 2. The velocity profiles for f(η) against S > 0

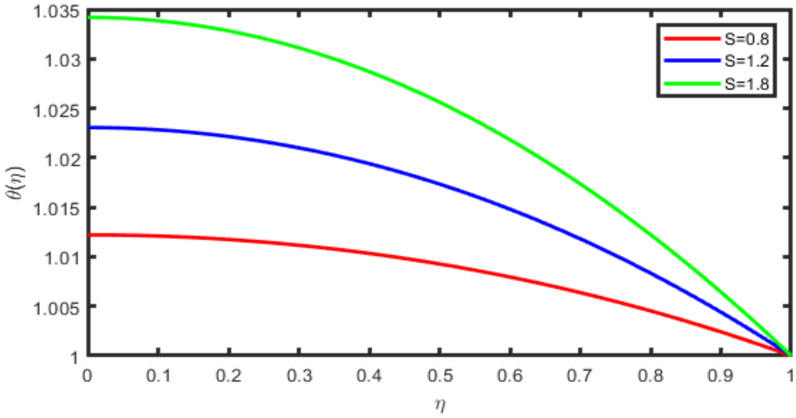

Fig. 3. The temperature profiles for θ(η) against S>0

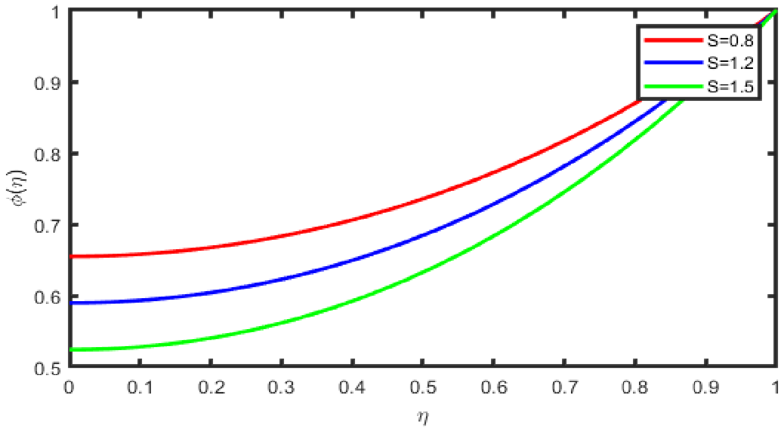

Fig. 4. The concentration profiles for ϕ(η) against S>0

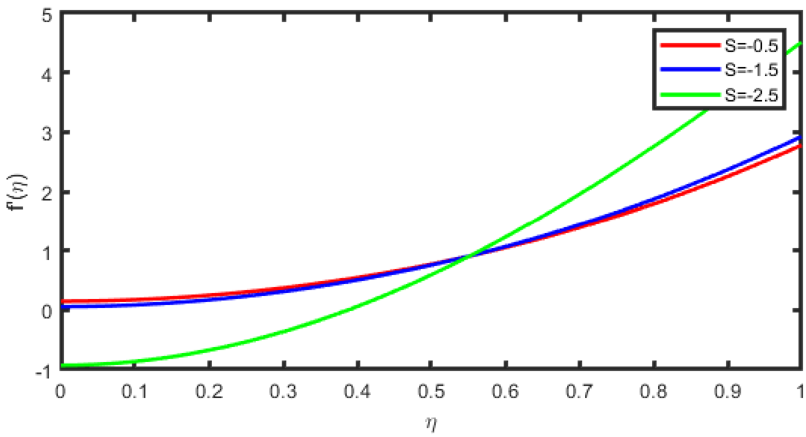

Fig. 5. The velocity profiles for f(η) against S<0

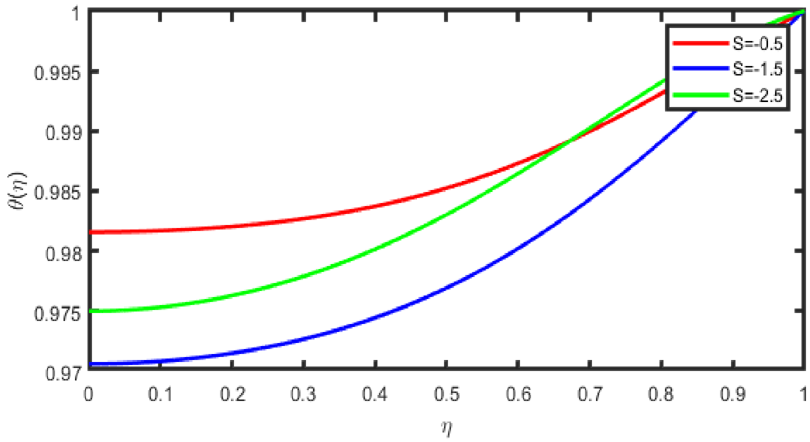

Fig. 6. The temperature profiles for θ(η) against S<0

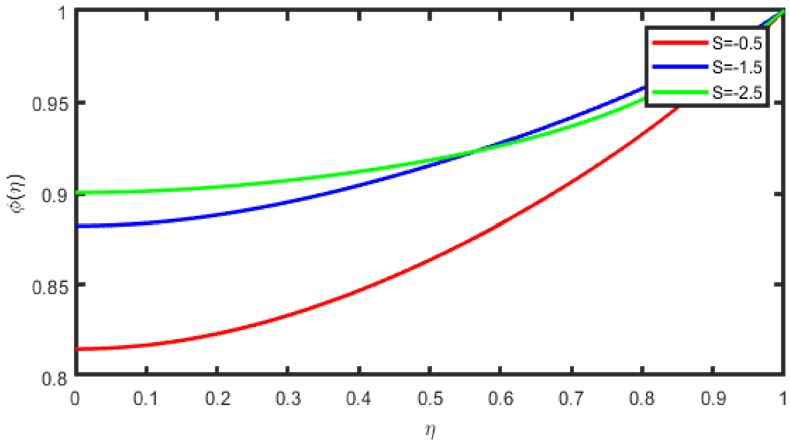

Fig. 7. The concentration profiles for ϕ(η) against S<0

The impact of Casson fluid parameter on velocity, temperature and concentration profiles is described in Figure 8 to Figure 10 with fixed S=Ha=0.5, R=Q=Kr=0.1, Sc=0.7, Pr=0.7 and Ec=0.8.

It can be seen in Figure 8 how the value affects the axial velocity profile. Figure 8 reveals that a rise in the value results in a greater axial velocity field below 0.5, but the opposite is seen in the channel section above 0.5. This can be seen by comparing the two regions. In addition, due to these shifts in the axial velocity field at the channel's edges, there is a tendency toward cross-flow in the channel's center area.

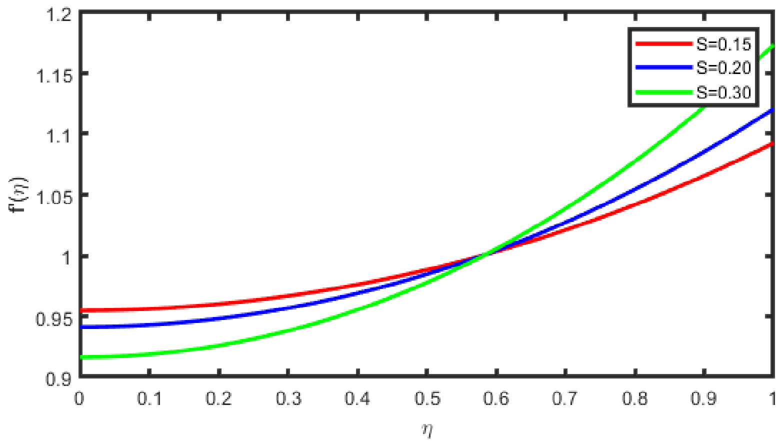

Fig. 8. The velocity profiles for f(η) against β

In addition, Figure 9 illustrates how the value describes how the value influences the temperature profile. Figure 9 indicates quite clearly that there is a negative correlation between an increase in the Casson fluid parameter and a drop in the temperature field in the flow zone. In addition, the thickness of the thermal boundary layer decreases when the Casson fluid parameter is increased to higher and higher values. However, the reduction in thickness of the thermal boundary layer is caused by an increase in the elasticity stress variable. A thinner thermal boundary layer is comparable to lower thermal diffusivity values, which demonstrates a higher temperature differential in the region of parallel plates. This is the case because similar plates have a reduced ability to dissipate heat. In addition to this, the temperature field is a function of that lower, and this decline occurs from =0 to =1.0. The temperature field may be normalized by raising the values. This is possible because it appears directly in the thermal equation (see Eq. (8)). In addition to this, it can be shown from Figure 10 that each of the curves in concentration profiles overlap with one another, which suggests that does not have a significant impact on concentration profiles. This is because it does not explicitly appear in the mass diffusion equation (refer to Eq. (14)). The influence of Hartmann number on flow behaviour is described in Figure 11 to Figure 13 with fixed values of Pr=0.7, Sc=0.7 and β=S=R=Q=Kr=Ec=0.1.

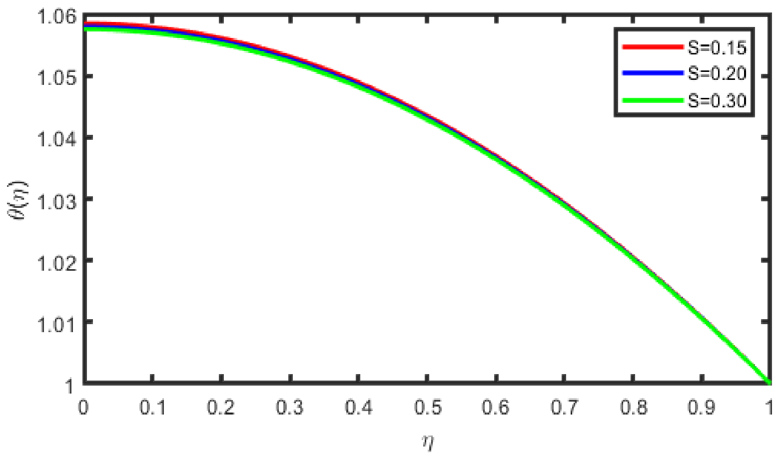

Fig. 9. The temperature profiles for θ(η) against β

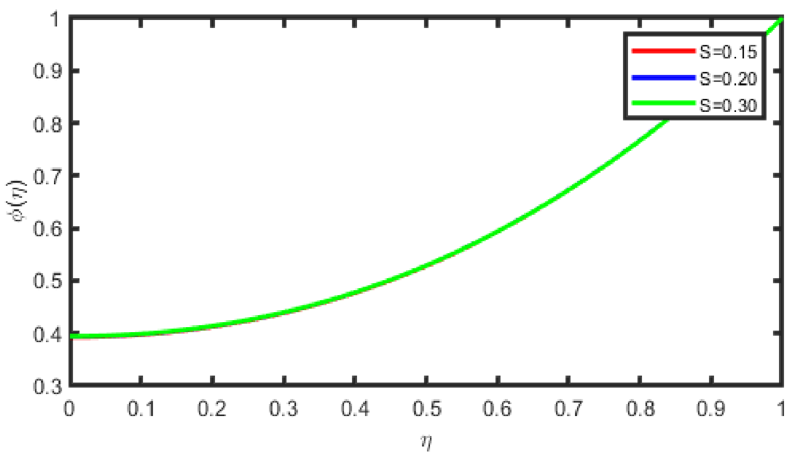

Fig. 10. The concentration profiles for ϕ(η) against β

An increase in the Hartmann number results in a reduction in the standard component of the velocity profile in the flow zone, as can be seen in Figure 11. This is a self-explanatory illustration of the phenomenon. It is reasonable to anticipate that a negligible increase in the Hartmann number is to blame for the increased Lorentz forces associated with magnetic fields since this is pretty apparent. Because of the amplified Lorentz forces, the passage of fluid through the channel is met with an increasing amount of resistance. Consequently, it is to be anticipated that the velocity field will become less intense as the value of the Hartmann number rises. As a result, a tendency toward cross-flow may be seen at the channel's core part due to these differences in the axial velocity at the boundary. On the other hand, the temperature profile (θ) and the fluctuations seen in it for various Ha values are shown in Figure 12. It can be seen by looking at this figure that when the value of Ha goes greater, there is a muted effect on the temperature field. Because of an increase in the Hartmann number, there is a decrease in the elasticity stress variable. This decrease in elasticity stress variable is responsible for the deterioration of the thermal field in the flow region, which is responsible for observing a thinner temperature boundary layer. Additionally, the temperature field is a function of Ha that goes up as you increase it. In addition, it is incredibly essential to observe that, as shown in Figure 13, all of the concentration curves that regulate the concentration field are consistent. This is because Ha does not significantly influence the concentration profile. Ha does not appear explicitly in the concentration equation (see Eq. (11)).

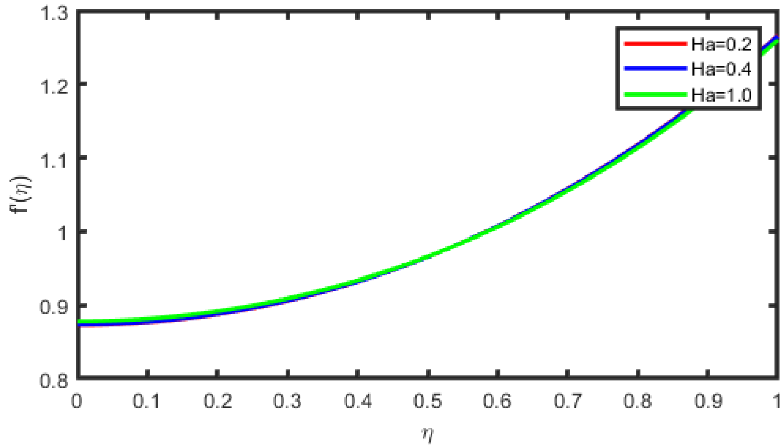

Fig. 11. The velocity profiles for f(η) against Ha

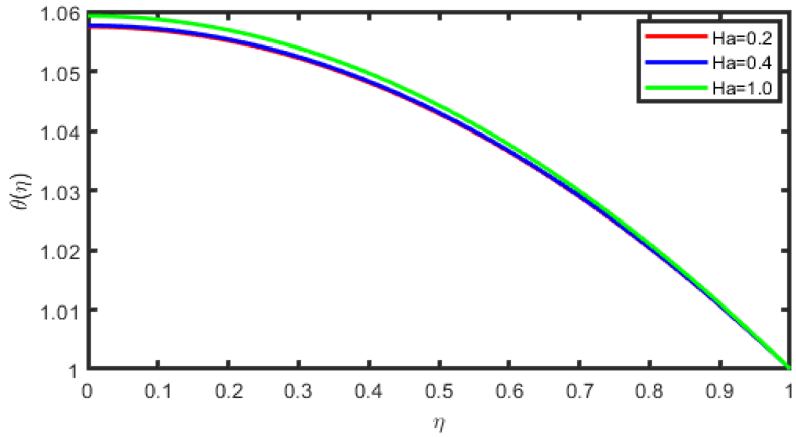

Fig. 12. The temperature profiles for θ(η) against Ha

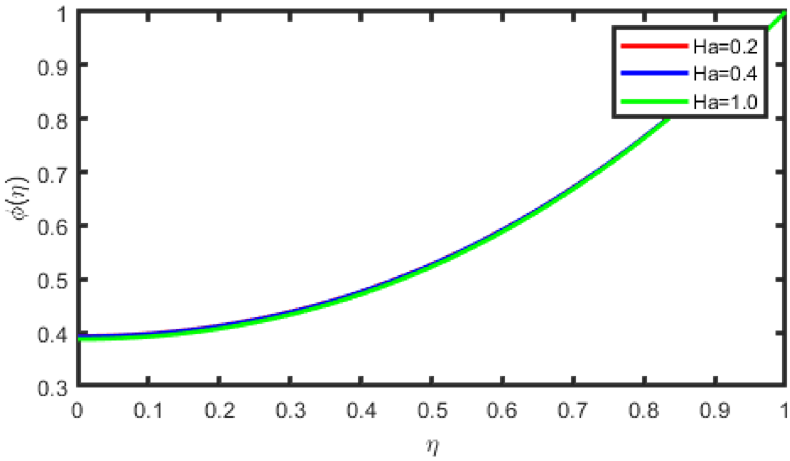

Fig. 13. The concentration profiles for ϕ(η) against Ha

The effect of radiation parameter on temperature profile is described in Figure 14 with fixed values of S=Ha=0.5, β=Q=Kr=0.1, Sc=0.7and Ec=1.0, Pr=0.7. Figure 14 clearly portrays that, an upsurge in thermal radiation parameter eventually diminishes on the temperature profile in the flow region. This fact can be depicted through the relation $R_d = \frac{4\sigma T_\infty^3}{k_f k_1^*}$ Thus, in view of the relation $R_d = \frac{4\sigma T_\infty^3}{k_f k_1^*}$, a small upsurge in Rd causes the decay of absorption coefficient k_1^*, hence temperature profile decreases. Also, the slope of temperature curves close to the wall indicates that the heat flows from the surface plates to the fluid. Thus, from Figure 14 it is conclude that, the numerical results obtained in this figure are reasonable and are acceptable. Also, physically, an increment in thermal radiation parameter gives the greater temperature value which may be useful in many of the thermodynamic industries.

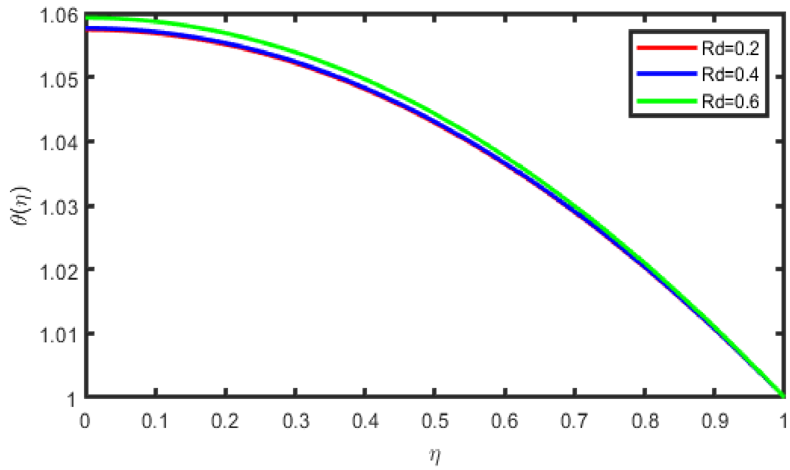

Fig. 14. The temperature profiles for θ(η) against Rd

The influence of heat generation or absorption parameter (Q) on temperature profile is illustrated in Figure 15 with fixed values of S=Ha= 0.5, Pr=0.7, Sc=0.7 and β=R=kr=0.1, Ec=1.0. It is observed from Figure 15 that as Q increases, the temperature field increases. Also, the thickness of thermal boundary layer increases for increasing values of Q. It is expected that, during the heat generation process more temperature is usually released into the working fluid. Owing to this reason, the temperature profile upsurges as heat generation parameter increases. Also, due to the exothermic chemical reactions, the temperature field increases.

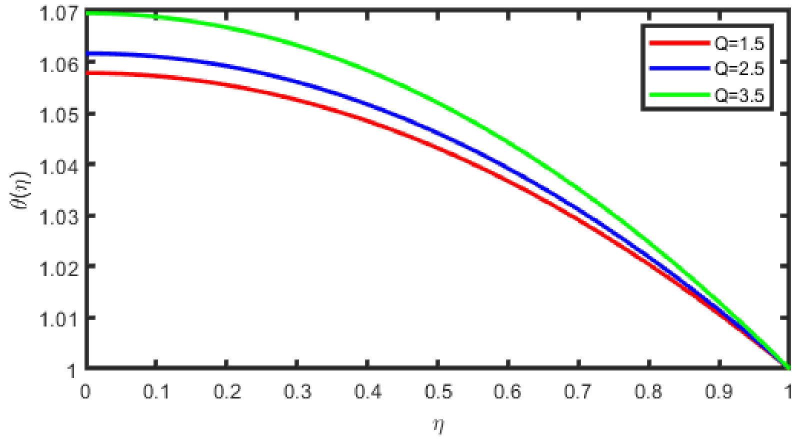

Fig. 15. The temperature profiles for θ(η) against Q

The influence of Pr on temperature profile is described in Figure 16 with fixed values of S=Ha=0.5, β=R=Q=Kr=0.1, Sc=0.7. It is observed from Figure 16 that, temperature field increases for increasing values of Pr. This increment in temperature profile is mainly due to decrease in the thermal conductivity values. However, dissipation effects are also responsible for the increase of temperature field in the flow. Further, the thickness of temperature boundary layer decreases for increasing values of Pr [44]. This decrement is mainly due to the fact that, the magnified Pr values greatly decrease the thermal diffusivity which in turn causes the decreasing of the temperature boundary layer thickness. Generally, it is known that, Pr1 corresponds to the high-viscosity materials like oils etc. From this observation it is concluded that, temperature field is an increasing function.

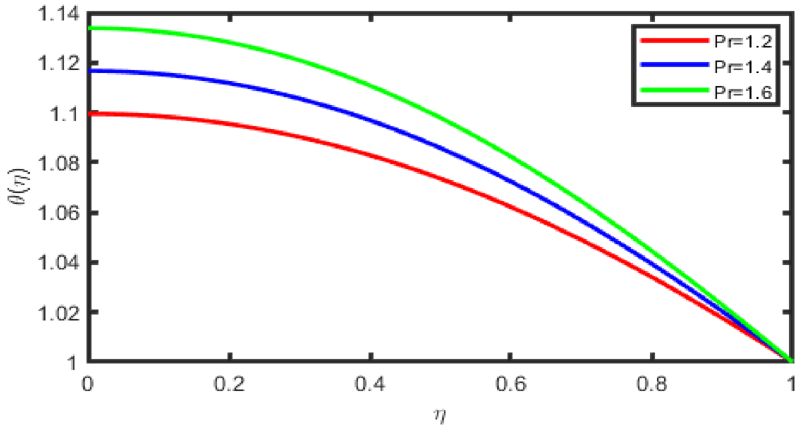

Fig. 16. The temperature profiles for θ(η) against Pr

The effect of chemical reaction parameter on concentration profile is illustrated in Figure 17. with fixed values of S=Ha=0.5, β=R=Q=Ec=0.1 and Pr=0.7, Sc=0.7. Generally, it is observed that, in many of the cases the decreased concentration field is noticed for destructive chemical reaction. Thus, Figure 17 clearly illustrates that, for the destructive chemical reaction concentration distribution decreases.

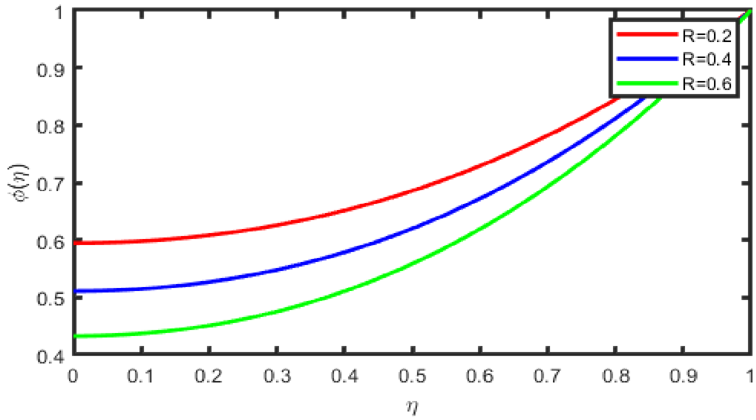

Fig. 17. The concentration profiles for $\phi(\eta)$ against R

The influence of Schmidt number (Sc) on concentration profile with fixed values of S=Ha=0.5, β=R=Q=Ec=Kr=0.1 and Pr=0.7 is described in Figure 18. Figure 18 clearly illustrates that the increasing Schmidt number decreases the concentration field in the flow region. This fact can be justified as follows: a small upsurge in Schmidt number diminishes the coefficient of mass diffusion, which in turn causes the decreases of concentration field in the flow region. Further, it is observed that, the concentration field is a decreasing function of Sc. Also, the concentration boundary layer thickness decreases for increasing values of Sc.

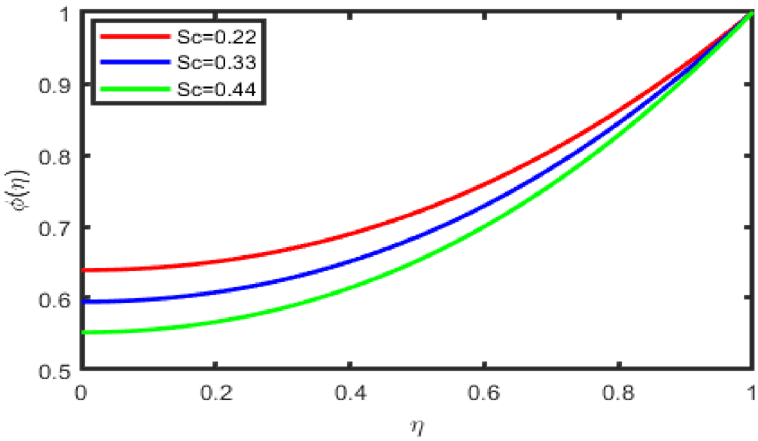

Fig. 18. The concentration profiles for $\phi(\eta)$ against Sc

4. Conclusions

In view of industrial use, the power required to generate the movement of the parallel plates is considerably reduced for the negative values of squeezing number. The influence of negative and positive squeezing parameter on velocity field is observed to be reversed.

(i) The higher values of squeezing number diminish the squeezing force on the fluid flow, which in turn reduces the thermal field.
(ii) The thinner temperature boundary layer corresponds to the lower values of thermal diffusivity and it shows the higher values of temperature gradient for the increasing values of β.
(iii) Due to the presence of stronger Lorentz forces the temperature and velocity fields behave like decreasing functions of Hartmann number.
(iv) Temperature field is suppressed for the increasing values of thermal radiation parameter.

(v) The destructive chemical reaction intensifies the concentration field and constructive chemical reaction decreases the concentration field.
(vi) The current numerical investigation has provided some remarkable insights of squeezing flow with non-Newtonian Casson fluid between two parallel plates.
(vii) The numerical results presented in this paper may be useful in theoretical and experimental studies related to the squeezing flow phenomena.

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