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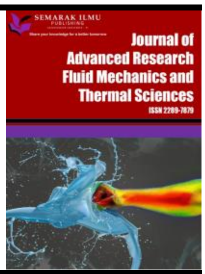

Artificial Neural Networks Solutions for Solving Differential Equations: A Focus and Example for Flow of Viscoelastic Fluid with Microrotation

Abdullah[1], Ibrahima Faye[1,*], Laila Amera Aziz[2]

[1] Department of Fundamental and Applied Sciences, Universiti Teknologi PETRONAS, 32610 Seri Iskandar, Perak Darul Ridzuan, Malaysia
[2] Centre for Mathematical Sciences, Universiti Malaysia Pahang, Lebuhraya Tun Razak Gambang, 26300 Kuantan, Pahang, Malaysia

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ABSTRACT

Physics-informed neural networks (PINN) are an artificial neural network (ANN) approach for solving differential equations. PINN offers an alternative to classical numerical methods. The paper discusses the applications of PINN in various domains by highlighting the advantages, challenges, limitations, and some future directions. For example, PINN is implemented to solve the differential equations describing the Flow of Viscoelastic Fluid with Microrotation at a Horizontal Circular Cylinder Boundary Layer. The differential equations resulting from a nondimensionalization process of the governing equations and the associated boundary conditions are solved using PINN. The obtained results using PINN are discussed and compared to other state-of-the-art methods. Future research might aim to increase the precision and effectiveness of PINN models for solving differential equations, either by adding more physics-based restrictions or multi-scale methods to expand their capabilities. Additionally, investigating new application domains like linked multi-physics issues or real-time simulation situations may help to increase the reach and significance of PINN approaches.

1. Introduction

PINNs have attracted much interest in physics and machine learning [1]. With the help of these networks, complicated physical processes may be modeled accurately and effectively by combining physics-based information with neural networks' learning capabilities. Scientists and engineers create mathematical models based on fundamental equations and rules to describe the behavior of physical systems in conventional physics-based modelling [2]. These models help provide insights but frequently make simplifying assumptions and may need help to reflect real-world occurrences' complexity fully. It might be difficult and time-consuming for complicated systems to solve the equations computationally [3].

On the other hand, machine learning methods—particularly neural networks—perform exceptionally well at identifying patterns and correlations in massive volumes of data and have

* *Corresponding author.*
E-mail address: ibrahima_faye@utp.edu.my

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succeeded in various fields, including recommendation systems, picture recognition, and natural language processing [4]. However, standalone neural networks are less suited to modelling physical systems and adhering to fundamental principles because of the lack of explicit knowledge of physical laws. The value of PINNs resides in their capacity to connect machine learning with physics-based modelling. PINNs are neural network architectures with physical rules, restrictions, or governing equations, allowing them to learn from data while still adhering to the fundamental laws of physics. This integration enables precise simulations, the identification of hidden patterns and relationships in intricate physical systems, and accurate predictions [5]. The combination of physics-based knowledge and machine learning in PINNs opens new possibilities in various fields of physics, such as fluid dynamics, materials science, astrophysics, and quantum mechanics. PINNs have the potential to enhance our understanding of complex physical phenomena, improve predictions, optimize designs, and enable real-time control in a wide range of applications [3].

The motivation behind creating PINNs arose from recognizing that while neural networks are proficient at learning complex patterns from data, and often lack explicit consideration of the physical laws and constraints governing physical systems [6]. PINNs aim to bridge this gap by integrating physics-based information into neural networks, combining the benefits of physics-based modeling and machine learning. Physics-based models rely on established physical laws and equations to explain system behavior, offering insightful explanations but often being computationally expensive. On the other hand, neural networks excel at extracting patterns from large datasets but may lack interpretability and reliability when applied to physical systems [7].

By incorporating physics-based information into neural networks, PINNs address these limitations and modify the neural network architecture to include physical laws, constraints, or governing equations directly. This integration ensures that the learned models adhere to fundamental physics principles while benefiting from the data-driven learning capabilities of neural networks. PINNs improve predictability, precision, and generalizability by utilizing physical constraints. Incorporating physics-based information into neural networks is crucial in domains where accurate modeling of complex physical processes is essential, such as quantum physics, materials science, and fluid dynamics [8]. By fusing physics-based knowledge with machine learning, PINNs offer a potential approach to tackle complex problems, enhance understanding, and improve predictions across various fields [7].

The theory and concept of a neural network (NN) is motivated by the structure and operation of the human brain. It comprises linked nodes or neurons that process inputs, assign weights, and generate outputs [9]. The network can learn and express complex patterns in data because the nodes are arranged into layers to form a hierarchical design. Several NN topologies include deep belief networks, convolutional networks, feedforward networks, and recurrent networks. The most straightforward network is a feedforward network, in which information moves from the input layer to the output layer in one direction without needing feedback loops. Recurrent networks can recognize temporal relationships in data because of feedback connections [4]. Convolutional networks are made to perform image recognition tasks by extracting information from pictures using convolutional and pooling layers. Deep belief networks, which can learn complicated data representations, comprise numerous layers of linked nodes. Optimization techniques are used in the learning process of NN to reduce the discrepancy between projected and actual outputs. These algorithms modify the weights and biases of the connections between nodes. The three basic kinds of NN learning algorithms are supervised, unsupervised, and reinforcement learning; supervised learning requires labelled data with defined target outputs for the algorithm to map inputs to matching outputs. Unsupervised learning evaluates unlabeled data, and the algorithm discovers hidden patterns and structures. Reinforcement learning involves an agent interacting with the environment and learning through rewards or punishments to improve behaviour [10].

Normalizing the inputs and dividing the dataset into training, validation, and test sets are common first steps in the NN learning process. The exemplary NN architecture is then chosen, considering the task's difficulty, the data's volume, and the available computing power. Backpropagation, gradient descent, or other optimization methods are used to iteratively update the network after initializing it with random weights and biases [11]. The network steadily improves performance throughout the training by learning to anticipate outputs based on inputs. Finally, the test set evaluates the trained network's accuracy and generalizability. Adjusting learning rates, hyperparameters, or regularization methods may be required to avoid overfitting. In conclusion, neural networks are an effective tool for modeling and resolving complicated issues in various fields. Designing and training effective NN models for various applications, from image classification to natural language processing, requires understanding the various NN topologies, learning methods, and the learning process [10].

To combine the benefits of physics-based modelling and machine learning, PINNs provide a robust framework for incorporating physics-based information into neural networks [1]. By embedding physics-based restrictions or equations into the neural network design, PINNs enable accurate modelling of complicated physical systems while gaining the advantages of data-driven learning and have been effectively utilized in several physics fields and increase the predictability, precision, and generalizability of results—the design of the network architecture and scalability present problems for PINNs [12]. The capabilities and uses of PINNs are being expanded by ongoing research that tries to solve these constraints and investigate hybrid strategies that pair deep learning methods with physics-based models. The research aims to discuss the PINN to solve the differential equation with the example of the viscoelastic fluid flow with microrotation [13].

2. PINN Applications in Different Domains

PINNs have been applied in various domains, demonstrating their versatility and potential. Here is a description of their applications in different fields.

PINNs are used to solve various equations, including partial differential equations (PDEs), fractional, integral-differential, and stochastic PDEs. This application showcases the ability of PINNs to encode model equations as components of the neural network itself, making them practical tools for solving complex mathematical equations [14].

Integrating data and mathematical physics models is crucial to physics-informed machine learning. PINNs enable the seamless integration of data and physics-based knowledge, even in contexts where the understanding is partial, uncertain, or high-dimensional. This application highlights the capability of PINNs to handle complex and challenging scenarios, allowing for enhanced data-driven simulations and predictions [6]. PINNs have been effectively used in many sectors, showcasing their adaptability and power to handle challenging issues. This section will review some of the crucial uses of PINN from earlier papers and research, emphasizing its usefulness in various contexts.

i. **Fluid Mechanics:** PINN has been used to solve a variety of fluid mechanics issues, including forecasting turbulent flow through pipes, channels, and around objects, to compare the outcomes of their PINN simulation of turbulent flow in a channel to actual data and conventional numerical techniques. The investigation proved that PINN was accurate and effective at modelling turbulent flows [3].

ii. **Electromagnetism:** PINN has been used to simulate Maxwell's equations to examine electromagnetic wave propagation, among other electromagnetic issues. The precision and adaptability of PINN in handling complicated geometries by using it to handle the scattering problem of electromagnetic waves by a convex polyhedron. The wave propagation in a photonic crystal fiber was examined using PINN, demonstrating good agreement with numerical simulations [15].

iii. **Thermodynamics:** PINN has been used to represent heat transport, phase transition processes, and other thermodynamic issues, validating the predictions against experimental data using PINN to forecast the temperature distribution in a heat exchanger. Using PINN to simulate vanadium dioxide's phase transition behaviour showed the software's capacity to handle intricate thermal dynamics [16].

iv. **Chemical Engineering:** Chemical engineering issues, including mass transport and reaction kinetics modelling, have been tackled with PINN, validating the predictions against experimental data using PINN to forecast the concentration profiles in a tubular reactor [16].

v. **Climate Modeling:** PINN has been used to solve issues with climate modelling, such as forecasting ocean currents and atmospheric circulation patterns, and evaluation of PINN's efficacy in climate modeling applications by simulating the Navier-Stokes equations for incompressible fluid flow using PINN [17].

vi. The examples provided show the adaptability and power of PINN in tackling a broad range of issues across several disciplines. PINN offers a potential framework for tackling forward and inverse issues in science and engineering by using the strength of deep learning and the physical principles regulating a system. It is conducive in cases where standard approaches fail since it can tackle complex, nonlinear issues with scant or noisy data [18]. PINN will be more significant in expediting scientific discoveries and technological innovation as more research is done and technology develops. These demonstrate that PINNs have been applied in different domains, including mathematics, physics, engineering, and other interdisciplinary fields [19]. Their ability to incorporate physics-based knowledge into neural networks enables more accurate modeling, simulations, and predictions in complex systems. PINNs offer a flexible and powerful approach to tackling various problems and promise to advance scientific understanding and decision-making processes in various domains [3].

3. Advantages

PINN models can be used to de-noise and reconstruct clinical magnetic resonance imaging (MRI) data of blood velocity while constraining this reconstruction showcases how PINNs can effectively de-noise and reconstruct clinical MRI data, leveraging the constraints provided by physics-based models [6]. The quantity and diversity of methodologies available for data-driven modelling of physical processes have increased during the past ten years. PINNs are among the most promising methods because PINNs combine data from sensors or numerical solvers with physics knowledge expressed as partial differential equations. The advantage of PINNs is that they can integrate data-driven information with physics-based knowledge, allowing for more accurate simulations and predictions of physical phenomena [19].

In the context of inverse design issues in several engineering fields, such as acoustics, mechanics, thermal/electronic transport, electromagnetism, and optics, PINNs with strict restrictions are examined. The work shows how PINNs may handle complex restrictions and optimize for inverse design issues [15]. These references highlight the benefits of PINNs, such as their capacity for data de-noise and reconstruction, the integration of data-driven and physics-based knowledge, the improvement of simulations and predictions, and the management of complex constraints for inverse design problems. This also offers insightful information about the benefits of adopting PINNs in many fields and applications [13].

4. Challenges and Limitations

PINNs have shown great promise in integrating physics-based knowledge with data-driven learning. However, like any approach, PINNs also have challenges and limitations. Below are some of the challenges and limitations associated with PINNs:

i. **Network Architecture Selection:** Selecting the best neural network design for a given issue might be difficult. Complexity and generalizability should be balanced in the design, and experimentation may be necessary to identify the best architecture for a particular issue [1].
ii. **Data Selection and Preprocessing:** PINNs are data- and physics-based knowledge-based systems. However, choosing and preparing data that faithfully depicts the underlying physical system can be challenging when working with sparse or noisy data. Ensuring the training data accurately represents the system's fundamental properties requires careful attention [1].
iii. **Training Complexities:** Due to the inclusion of physics-based limitations, PINNs can be computationally taxing. The training procedure can be more difficult since the optimization process may call for longer training durations and can be sensitive to hyperparameters [20].
iv. **Handling Physical Constraints:** It is essential to ensure that the learned neural network abides by the physical restrictions and laws imposed by the underlying physics. A vital factor is balancing data-driven learning and adherence to physical rules [1]. Implementing these limitations inside the neural network design can be difficult.
v. **Scalability to Large-scale Problems:** It can be challenging to scale PINNs to more extensive and sophisticated systems. The computer resources needed for training and inference may become unaffordable as the system's complexity rises. Research is still being done on creating scalable methods for PINNs to solve large-scale issues [1].
vi. **Interpretability Issues:** Since neural networks, including PINNs, are frequently viewed as "black box" models, it can be challenging to interpret the learned representations and comprehend the underlying physical principles. It is a current field of study to create methods to make PINNs more interpretable and to derive insightful conclusions from their forecasts [1].
vii. **Applicability to Novel Physics Phenomena**: PINNs are often created using physics equations and current information. To accurately capture the underlying physics and make predictions, using PINNs for unique or new physics phenomena may call for extra considerations and changes [1].

It is significant to note that many of these difficulties and restrictions are addressed by current research and developments in PINNs. Researchers are continually working on creating new methods, increasing training methods, and expanding the use and efficiency of PINNs across various areas [6]. Despite the difficulties currently faced, PINNs still have much potential for improving our knowledge of and resolving complex physical systems in the future.

5. Application

Researchers in various fields actively explore the application of PINNs. For example, utilizing the PINNs in Julia 1.8.5, one may arrive at numerical solutions for the flow of viscoelastic fluid with microrotations at a boundary layer of a horizontal circular cylinder to solve problems involving ordinary differential equations effectively [21]. A deep learning framework uses PINNs with a hyperparameters discretization of 0.01, an Adam optimizer, a learning rate of 0.01, and a maximum iteration of 2000. The architecture of the neural network is described in Figure 1.

The ODE is given below [22]:

$$\left(1+\frac{K_1}{2}\right)g'' + fg' - f'g - K_1(2g + f'') = 0 \tag{1}$$

$$(1+K_1)f''' + ff'' - f'^2 + 1 + K_1 g' - M(f'-1)\sin^2\alpha + K(2f'f''' - ff^{iv} - f''^2) = 0 \tag{2}$$

Corresponding boundary conditions

$$f(0) = f'(0) = 0,\ \ g(0) = -\frac{1}{2}f''(0)\ \text{ at }\ y = 0$$

$$f'(\infty) = 1,\ f''(\infty) = 1,\ g(\infty) = 0\ \text{ as }\ y \to \infty$$

where k is the viscoelastic parameter, M is the microinertia density, and K_1 is the material parameter.

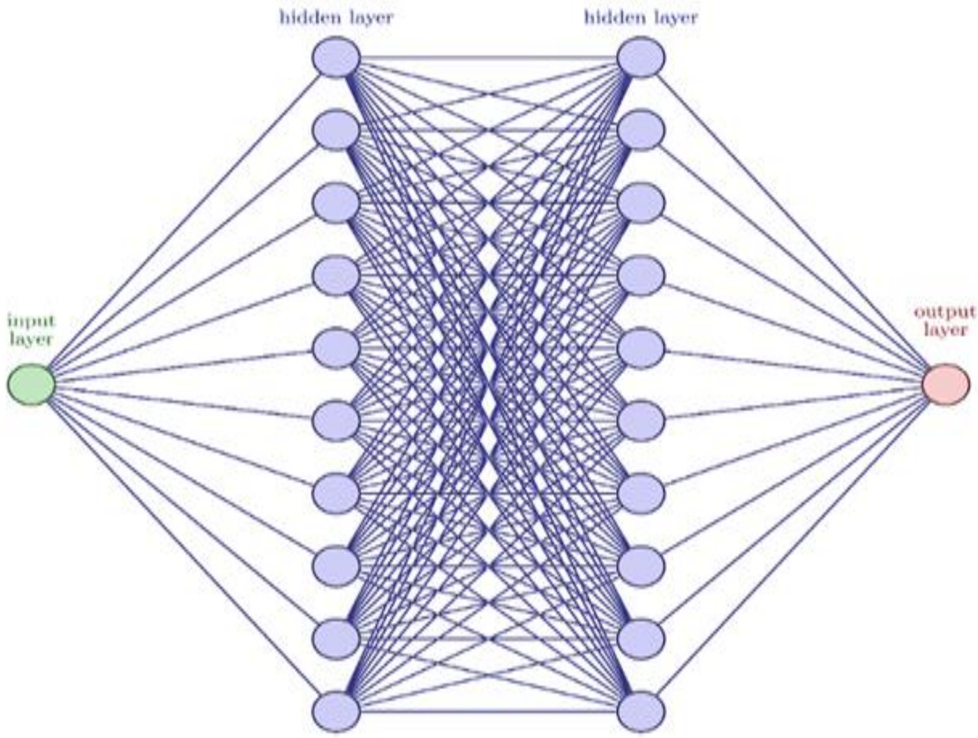

Fig. 1. Artificial Neural Network Architecture

The study solved the non-Newtonian flow model using the PINN approach. However, the PINN method results were less encouraging than those from other approaches, as shown in Table 1. Despite the potential benefits of PINNs, including their use in modelling complex fluid flows and their capacity to learn unknown physics from sparse and indirect data, there is still room for improvement in the method. These include improving hyperparameters, the neural network architecture, and adding additional data sources or constraints to direct the learning process to improve the results. We intend to overcome the constraints and improve the precision of the PINN approach for solving non-Newtonian flow models by extending our study and improving the methodology.

Table 1
The value of $f''(0)$ at different values of K, when $M = K_1 = 0$

K	Exact solution (Ariel [16])	Previous (Lu *et al.*, [15])	Viscoelastic model [8]	PINNs
0	1.232588	1.232657	-	1.079401
0.01	-	1.221447	1.222693	1.073383
0.05	1.179830	1.179893	-	1.048076
0.1	1.134114	1.134172	1.135982	1.017785
0.2	1.058131	1.058180	1.045412	0.966326
0.3	0.996844	0.996886	0.960922	0.920227
0.4	0.945869	0.945907	0.882512	0.922295
0.5	0.902500	0.902535	0.810182	0.844998
0.6	-	0.864985	0.743933	0.814564
0.7	-	0.832019	0.683763	0.787106
0.8	-	0.802749	0.629673	0.761609
0.9	-	0.776511	0.581664	0.739604
1	0.752766	0.752803	-	0.718390
100	0.099515	0.100783	-	0.099264
500	0.044677	0.045487	-	0.034661
1000	0.031607	0.032229	-	0.014113

6. Conclusion

PINNs provide a robust framework for integrating physics-based knowledge with data-driven learning and offer several advantages, including combining data-driven learning and physics-based modeling, handling nonlinear dynamics, utilizing limited and noisy data efficiently, and improving interpretability. However, PINNs also face challenges such as network architecture selection, training complexities, and scalability. In this work, we have discussed and compared the solution of a differential equation with the example of focus on viscoelastic fluid flow with microrotation. Future directions for PINNs include hybrid approaches, uncertainty quantification, handling noisy data, scalability to large-scale systems, enhanced interpretability, and advanced optimization strategies. Ongoing research in PINNs continues to improve their effectiveness and applicability, offering great potential for advancing scientific knowledge and solving complex physical systems.

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