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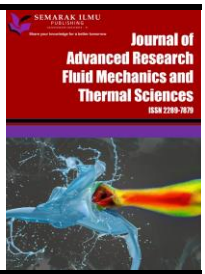

Flow and Thermal Characteristics of Couple Stress Fluid over a Stretching Surface with Hybrid Nanoparticles

Iskandar Waini[1,*], Najiyah Safwa Khashi'ie[2], Nurul Amira Zainal[2], Khairum Hamzah[1], Abdul Rahman Mohd Kasim[3], Anuar Ishak[4], Ioan Pop[5]

[1] Fakulti Teknologi dan Kejuruteraan Industri dan Pembuatan, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, Durian Tunggal 76100, Melaka, Malaysia
[2] Fakulti Teknologi dan Kejuruteraan Mekanikal, Universiti Teknikal Malaysia Melaka, Hang Tuah Jaya, Durian Tunggal 76100, Melaka, Malaysia
[3] Centre for Mathematical Sciences, Universiti Malaysia Pahang Al-Sultan Abdullah, Gambang, 26300 Kuantan, Pahang, Malaysia
[4] Department of Mathematical Sciences, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 43600 UKM, Bangi, Selangor, Malaysia
[5] Department of Mathematics, Babes-Bolyai University, 400084 Cluj-Napoca, Romania

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ABSTRACT

In the current research, a mathematical analysis of couple stress fluid flow and heat characteristics through a stretchable permeable surface with hybrid nanoparticles is conducted. The solid nanoparticles of the aluminium alloys (AA7072 and AA7075) are suspended in methanol to create the hybrid nanofluid. The similarity approach is used to reduce the governing equations into the similarity equations. Then, MATLAB's bvp4c function is employed to solve the resulting equations. The solutions for the flow and temperature fields, as well as the skin friction coefficient and Nusselt number are presented in table and graphical forms. The results demonstrate that hybrid nanofluids excel as thermal conductors, significantly augmenting the heat transfer rate. The heat transfer rate is increased by 0.38% for the nanofluid, while 0.89% increment for the hybrid nanofluid compared to the base fluid. Furthermore, a larger couple stress parameter is found to be associated with a decrease in the fluid temperature and an enhancement in fluid velocity.

1. Introduction

In recent years, there has been a notable surge in the interest of researchers and scientists towards the advancement of heat transfer fluids. While conventional fluids widespread use in engineering and industrial applications, their heat transfer capabilities are limited due to their low thermal conductivity. To overcome this limitation, researchers introduced a solution known as 'nanofluid' in 1995, pioneered by Choi and Eastman [1]. This innovative approach involves incorporating nanosized particles into the aforementioned fluids. Then, Khanafer *et al.,* [2], and Oztop and Abu-Nada [3] studied the nanofluid flow in a rectangular enclosure. However, in order to

* *Corresponding author.*
E-mail address: iskandarwaini@utem.edu.my

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further enhance the thermal properties of nanofluids, researchers have developed a novel type of fluid known as 'hybrid nanofluid'. Early studies focusing on the utilization of hybrid nano-composite particles can be attributed to Jana *et al.,* [4]. Unlike conventional nanofluids, hybrid nanofluids consist of multiple types of nanoparticles, which synergistically enhance the heat transfer rate [5,6]. By combining or hybridizing appropriate nanoparticles, the desired level of heat transfer can be achieved [7]. Takabi and Salehi [8] conducted a study on enhancing the heat transfer efficiency of a sinusoidal corrugated enclosure using a hybrid nanofluid. Numerous papers on hybrid nanofluids have been published and are accessible in the literature [9-16].

The theory of couple stress fluids serve as an extension of the classical Newtonian fluids, allowing for the inclusion of polar effects such as body couples and couple stresses [17]. Couple stress fluids find practical applications in various fields, including biomechanics, chemical and petroleum industries, geothermal energy extraction, geohydrology, and medicine. Several researchers have conducted simulations and investigations pertaining to couple stress flows in different contexts. For instance, Turkyilmazoglu [18] investigated the flow and heat characteristics of couple stress fluid over surfaces that can stretch or shrink. Khan *et al.,* [19] computationally studied flow of couple stress with several surface temperature conditions. Additionally, Das *et al.,* [20] reported the effects of the Darcy-Forchheimer on couple stress fluid past an inclined sheet.

In recent times, numerous types of nanomaterial have been reported in the literature. Notably, aluminum alloy nanoparticles AA7075 and AA7072 stand out as exceptional nanomaterials due to their remarkable thermal, chemical, and physical properties [21]. The special features and superior properties make them highly desirable for various industrial applications, including aerospace. Also, these alloys find extensive applications in the manufacturing of transportation equipment such as glider aircraft and rocket frames [21]. Different from the previous study of Turkyilmazoglu [18], the objective of this endeavor is to investigate the effects of hybrid nanoparticles over a stretched surface in a couple stress fluid. The hybrid nanoparticles considered in this study are aluminum alloy nanoparticles AA7075 and AA7072, while the base fluid is methanol. The obtained results are presented in graphical and tabular forms for several physical parameters. It should be mentioned that, this problem has not been explored. Hence, this study holds immense importance as a future point of reference for practitioners, scientists, engineers, and fluid mechanists.

2. Methodology

In this study, we examine the behavior of an electrically conducting quiescent couple stress fluid interacting with a continuously stretching surface. The schematic model is shown in Figure 1. We consider the influence of hybrid nanoparticles and a uniform external magnetic field, with the coordinates *x* and *y* denoting the surface and normal directions, respectively.

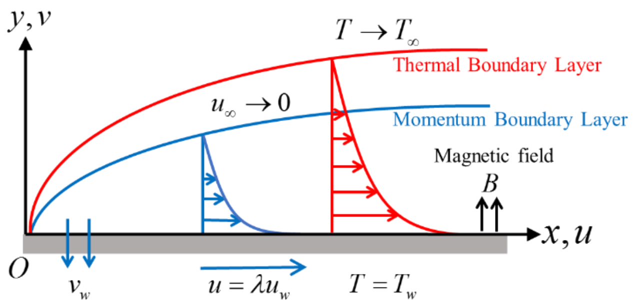

Fig. 1. Schematic Model

The governing equations are then given by [18]

$$u_x + v_y = 0 \tag{1}$$

$$uu_x + vu_y = \frac{\mu_{hnf}}{\rho_{hnf}} u_{yy} - \frac{\eta_0}{\rho_{hnf}} u_{yyyy} - \frac{\sigma_{hnf}}{\rho_{hnf}} B_0^2 u \tag{2}$$

$$uT_x + vT_y = \frac{k_{hnf}}{(r C_p)_{hnf}} T_{yy} \tag{3}$$

subject to

$$\begin{aligned} &v = v_w, \quad u = \lambda u_w, \quad T = T_w = T_\infty + T_0 x^2 \quad \text{at} \quad y = 0 \\ &u \to 0, \quad u_y \to 0, \quad u_{yy} \to 0, \quad T \to T_\infty \quad \text{as} \quad y \to \infty \end{aligned} \tag{4}$$

where h_0 is the material constant for the couple stress fluid, r is the density, and m is the dynamic viscosity. The properties of hybrid nanofluid are provided in Table 1 and Table 2. The following similarity transformations are considered [18].

$$\psi = x\sqrt{a\nu_f}\, f(\eta), \quad u = axf'(\eta), \quad v = -\sqrt{a\nu_f}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = y\sqrt{\frac{a}{\nu_f}} \tag{5}$$

Here, y signifies the stream function with $u = ¶y/¶y$ and $v = -¶y/¶x$ which satisfied the Eq. (1). Then, using Eq. (5), the governing equations (2) and (3) are reduced to the similarity form

$$-Cf''''' + \frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f''' + ff'' - f'^2 - \frac{\sigma_{hnf}/\sigma_f}{\rho_{hnf}/\rho_f} Mf' = 0 \tag{6}$$

$$\frac{1}{\Pr} \frac{k_{hnf}/k_f}{(\rho C_p)_{hnf}/(\rho C_p)_f} \theta'' + f\theta' - 2f'\theta = 0 \tag{7}$$

subject to

$$\begin{aligned} &f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1 \\ &f'(\eta) \to 0, \quad f''(\eta) \to 0, \quad f'''(\eta) \to 0, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{8}$$

with l represents the stretching parameter. Further, Prandtl number Pr, the mass flux parameter *S*, the couple stress parameter *C*, and the magnetic parameter *M* are defined as

$$\Pr = \frac{(mC_p)_f}{k_f}, \quad S = -\frac{v_w}{\sqrt{an_f}}, \quad C = \frac{h_0 a}{r_{hnf} n_f^2}, \quad M = \frac{s_f B_0^2}{ar_f} \tag{9}$$

Table 1
The properties of AA7075, AA7072, and methanol [21]

Properties	Nanoparticles		Base fluid
	AA7075	AA7072	Methanol
$\rho\ (kg/m^3)$	2810	2720	792
$C_p\ (J/kgK)$	960	893	2545
$k\ (W/mK)$	173	222	0.2035
$\sigma\ (S/m)$	26.77×10^{6}	34.83×10^{6}	0.5×10^{-6}
Prandtl number, Pr			7.38

Table 2
The thermophysical properties of hybrid nanofluid [10]

Thermophysical properties	Hybrid Nanofluid
Dynamic viscosity	$\mu_{hnf} = \dfrac{\mu_f}{(1-\phi_{hnf})^{2.5}}$
Heat capacity	$(\rho C_p)_{hnf} = (1-\phi_{hnf})(\rho C_p)_f + \phi_1(\rho C_p)_{n1} + \phi_2(\rho C_p)_{n2}$
Density	$\rho_{hnf} = (1-\phi_{hnf})\rho_f + \phi_1\rho_{n1} + \phi_2\rho_{n2}$
Thermal conductivity	$\dfrac{k_{hnf}}{k_f} = \dfrac{\dfrac{\phi_1 k_{n1}+\phi_2 k_{n2}}{\phi_{hnf}} + 2k_f + 2(\phi_1 k_{n1}+\phi_2 k_{n2}) - 2\phi_{hnf}k_f}{\dfrac{\phi_1 k_{n1}+\phi_2 k_{n2}}{\phi_{hnf}} + 2k_f - (\phi_1 k_{n1}+\phi_2 k_{n2}) + \phi_{hnf}k_f}$
Electric conductivity	$\dfrac{\sigma_{hnf}}{\sigma_f} = \dfrac{\dfrac{\phi_1 \sigma_{n1}+\phi_2 \sigma_{n2}}{\phi_{hnf}} + 2\sigma_f + 2(\phi_1 \sigma_{n1}+\phi_2 \sigma_{n2}) - 2\phi_{hnf}\sigma_f}{\dfrac{\phi_1 \sigma_{n1}+\phi_2 \sigma_{n2}}{\phi_{hnf}} + 2\sigma_f - (\phi_1 \sigma_{n1}+\phi_2 \sigma_{n2}) + \phi_{hnf}\sigma_f}$

The local Nusselt number Nu_x and the skin friction coefficient C_f are given as

$$Nu_x = -\frac{xq_w}{k_f(T_c - T_w)}, \qquad C_f = \frac{\tau_w}{\rho_f u_w^2} \tag{10}$$

where the surface heat flux q_w and the surface shear stress τ_w are defined by

$$q_w = -k_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}, \qquad \tau_w = \mu_{hnf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{11}$$

Thus, we have

$$\mathrm{Re}_x^{-1/2} Nu_x = -\frac{k_{hnf}}{k_f}\theta'(0), \qquad \mathrm{Re}_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0) \tag{12}$$

where $\mathrm{Re}_x = u_w x/\nu_f$ denotes the local Reynolds number.

3. Results and Discussion

This section presents a comprehensive analysis of the numerical results obtained from solving Eq. (6) to Eq. (8) using the MATLAB software's bvp4c package [22]. Detailed discussions are provided regarding the impact of various physical parameters associated with the proposed model. The values of the parameters are referred from Turkyilmazoglu [18]. For the limiting cases, the models of Turkyilmazoglu [18] are equivalent to the present model. The validation processes yielded excellent agreement across all aspects, demonstrating a high level of consistency, see Table 3.

Table 3
Values of -$f''(0)$ and -$\theta'(0)$ for various values of C and M when $f_1 = f_2 = 0$, and $\Pr = S = l = 1$

C	M	Present results (bvp4c)		Turkyilmazoglu [18] (Exact Solution)	
		$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$
0.01	0	9.39253498	1.28320643	9.39253499	1.28320641
	1	9.32620475	1.28495495	9.32620476	1.28495495
	3	9.18544032	1.28873652	9.18544033	1.28873650
0.02	0	6.40060694	1.39035128	6.40060694	1.39035128
	1	6.28611983	1.39601185	6.28611983	1.39601185
	3	6.01265407	1.41018617	6.01265405	1.41018617

The variations of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ against f_1 and f_2 are presented in Figure 2 and Figure 3. The increasing values of f_1 and f_2 tend to decline the values of $\mathrm{Re}_x^{1/2} C_f$. However, the values of $\mathrm{Re}_x^{-1/2} Nu_x$ enhanced with the rising of nanoparticle but it is intensified when hybrid nanoparticles are considered. Note that the heat transfer rate is increased by 0.38% for the nanofluid ($f_1 = 0, f_2 = 0.01$) while 0.89% increment for the hybrid nanofluid ($f_1 = f_2 = 0.01$) compared to the base fluid ($f_1 = f_2 = 0$).This finding supports the notion that hybrid nanofluids exhibit superior thermal characteristics compared to both the base fluid and nanofluids containing single nanoparticles, owing to the synergistic effects observed. Additionally, the numerical values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ which corresponded to Figure 2 and Figure 3 are given in Table 4.

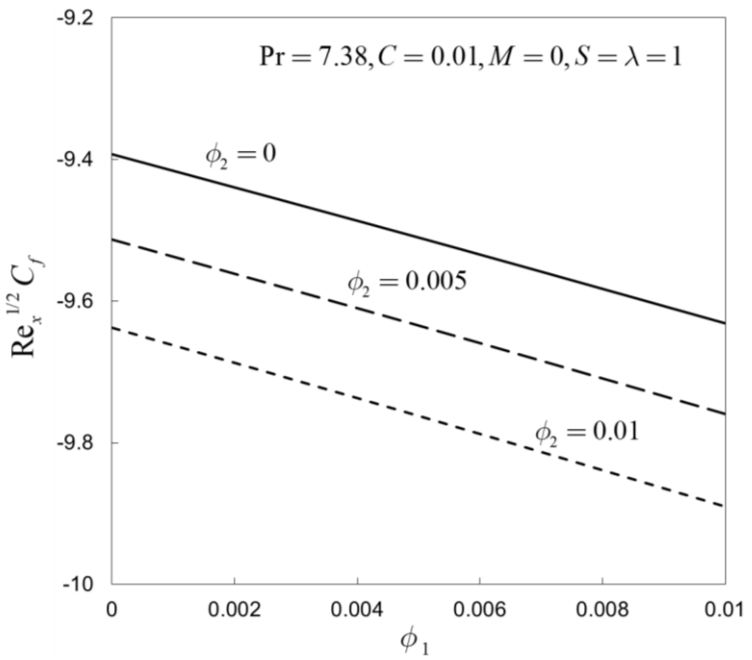

Fig. 2. Variations of $\mathrm{Re}_x^{1/2} C_f$ with f_1 and f_2

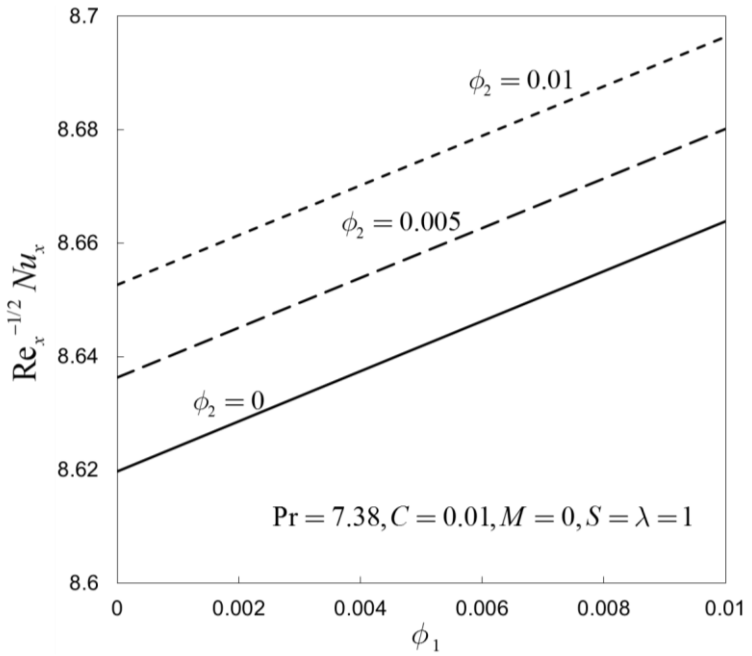

Fig. 3. Variations of $\mathrm{Re}_x^{-1/2} Nu_x$ with f_1 and f_2

Table 4

Values of $\mathrm{Re}_x^{1/2} C_f$ and $\mathrm{Re}_x^{-1/2} Nu_x$ for various values of f_1 and f_2 when $\mathrm{Pr} = 7.38$, $C = 0.01$, $S = l = 1$, and $M = 0$

f_1	$\mathrm{Re}_x^{1/2} C_f$			$\mathrm{Re}_x^{-1/2} Nu_x$		
	$f_2 = 0$	$f_2 = 0.005$	$f_2 = 0.01$	$f_2 = 0$	$f_2 = 0.005$	$f_2 = 0.01$
0	-9.392542	-9.513503	-9.637854	8.619724	8.636259	8.652682
0.005	-9.510239	-9.634587	-9.762404	8.641835	8.658264	8.674580
0.01	-9.631323	-9.759137	-9.890503	8.663845	8.680167	8.696374

Figure 4 illustrates the impact of the couple stress parameter, *C* on the velocity field. As *C* increases, the viscous forces decrease, leading to a significant increase in the velocity field. In order to understand the influence of the couple stress parameter, *C* on the temperature field, we present Figure 5. It is evident from the figure that higher values of *C* result in a slight decrease in temperature and thermal boundary layer thickness. Physically, this behaviour is attributed to the friction force that counteracts and controls the fluid motion. Moreover, as the value of *C* increases, the rotational of fluid particles becomes ineffective which fails to retard the fluid flow.

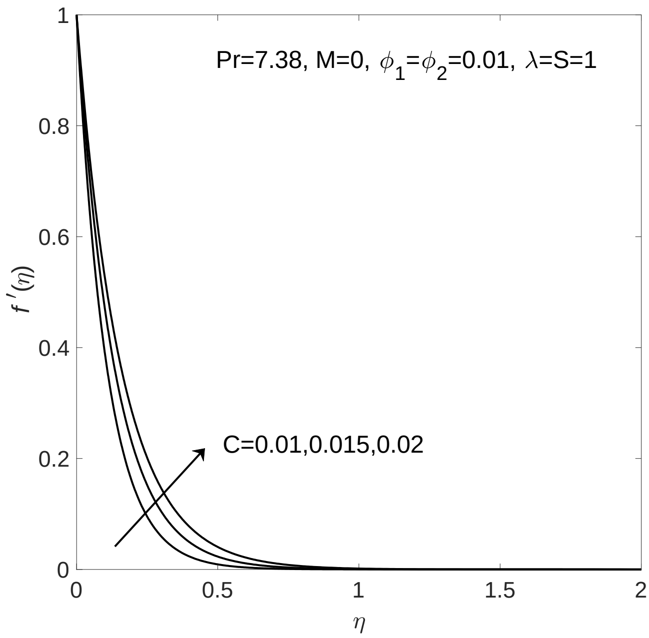

Fig. 4. Velocity profiles $f'(\eta)$ for different values of C

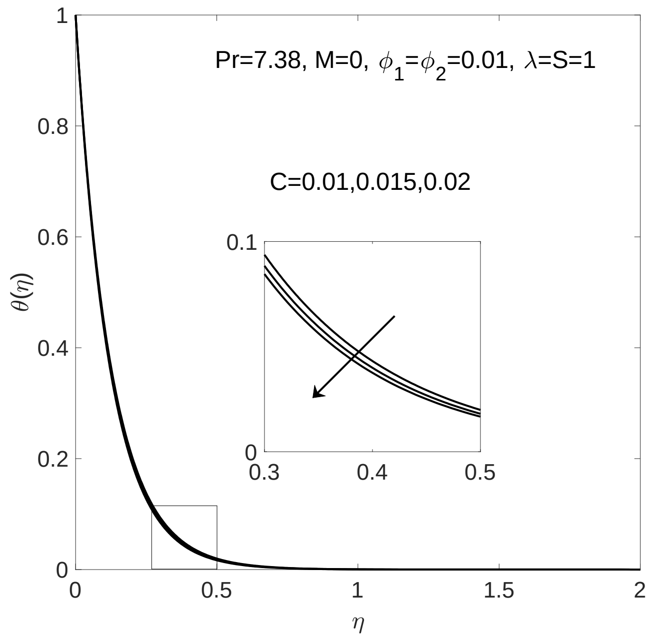

Fig. 5. Temperature profiles $\theta(\eta)$ for different values of C

4. Conclusions

This study focuses on investigating the steady flow of a couple stress fluid over a stretching surface in the presence of hybrid nanoparticles. The governing problem is transformed using similarity transformation, and the numerical solution is obtained utilizing the bvp4c solver. The findings suggest that hybrid nanofluids exhibit exceptional thermal conductivity and play a vital role in augmenting the rate of heat transfer. It is reveals that the heat transfer rate is increased by 0.38% for the nanofluid, while 0.89% increment for the hybrid nanofluid compared to the base fluid. Moreover, a reduction in the fluid temperature is observed while the fluid velocity enhances with larger couple stress parameter.

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