SEMARAK ILMU PUBLISHING
202103268166(003316878-P)

Journal of Advanced Research in Fluid Mechanics and Thermal Sciences

Journal homepage:
https://semarakilmu.com.my/journals/index.php/fluid_mechanics_thermal_sciences/index
ISSN: 2289-7879

Design and Optimization of Heatsink for an Active CPU Cooler using Numerical Simulations

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ARTICLE INFO

Article history:
Received 13 August 2023
Received in revised form 4 November 2023
Accepted 15 November 2023
Available online 30 November 2023

ABSTRACT

Heat generation in CPUs with high computing performance has been a very serious issue that deteriorates their performance. To ensure that CPUs operate at their maximum potential, it is essential to maintain their temperature below 80 °C. Forced convection coolers comprising of a heatsink and fan are considered the most effective means of satisfying operating temperature requirements for a CPU in order to ensure its maximum performance. A heatsink design paired with a fan with airflow speed of 80 cubic feet per minute (CFM) for a CPU Cooler targeted at CPUs with maximum heat generation of 380 watts operating in ambient temperature of 25 °C, is developed using numerical methods of Computational Fluid Dynamics (CFD) and topology optimization using ANSYS Mechanical and ANSYS Fluent. Various fin profiles, fin arrangements, fin numbers and heatsink materials are comparatively analyzed. The best results from the comparative analyses are combined to present a base design capable of keeping the CPU temperature below 80 °C, which is the requirement for ensuring maximum computing performance. A 30 fin heatsink with covered rectangular plate fins in an arc-shaped placement configuration is determined to provide the maximum cooling performance. In materials, Silicon Carbide resulted in the lowest CPU temperature of 78°C, followed by copper at 84 °C. Silicon Carbide heatsink successfully managed to satisfy the requirements for maximum CPU performance. Copper heatsink is unlikely to cause CPU failure, but it fails to meet conditions for maximum CPU performance. In addition, this base design is then optimized for material cost reduction using topology optimization which resulted in 13% reduction in material cost with a negligible cooling performance reduction of only 0.32%. The overall design of the cooler can be improved by incorporating fan design and various CPU load conditions into the design parameters in future research.

Keywords:
Forced convection; heatsink; topology optimization; fin profile

1. Introduction

In order to satisfy the increasing demand for CPUs with high computing performance, processor manufacturers have been ramping up the transistor count on an individual CPU chip. Moore's law, an observation and techno-economic model predicted in 1965, that transistors in a dense integrated electronic circuit will double every two years [1]. For instance, the first personal computer, the IBM Model 5150 had 2900 transistors on its CPU, whereas the transistor count for Intel's 13th generation

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https://doi.org/10.37934/arfmts.111.2.214224

CPUs have about 4 billion transistors [2]. The highest transistor count in a consumer CPU is 114 billion, in Apple's M-1 Ultra CPU. Intel is aiming to have 1 trillion transistors on their CPUs by 2030 by making a single transistor of the size of 1 angstrom. This increase in transistor has enabled the CPUs to possess a very high computing performance, however it has also resulted in high generation in CPU, which consequently deteriorates their performance [3]. There is a need for robust cooling mechanisms for maintaining CPU temperatures below the 80 °C, which is the limit above which CPU performance tends to deteriorate, whereas 90 °C is considered the limit for CPU safety [4]. For this purpose, various means of cooling the CPUs have been suggested and implemented [5]. In CPUs, this generated heat is not only damaging to the processor itself, but if left unchecked, it has the potential to harm other computer components such as RAM (read-only-memory) or storage drives as well [6]. The most effective mean of CPU cooling is implementing active coolers that incorporate a fan blowing air over a heatsink that absorbs heat from CPU and dissipates it into the ambience [7].

The primary component of an active cooler is the heatsink, and the factors affecting its ability to transfer heat from CPU to ambient environment are its material, fin profile, the configuration in which fins are placed on base of heatsink and finally the number of fins.

Liquid cooling, that utilizes liquid as the working fluid for forced convection is widely considered as the best choice of cooling CPUs with very high heat generation. Various configurations have been explored in the realm of liquid cooling like thermosiphon, liquid jet impingement and direct liquid cooling using liquid block [8]. Most common working fluid in liquid cooling is cold water while other novel liquids such as nanofluids are gaining more and more attention [9]. Aluminum oxide based nanofluids have also been explored [10]. Liquid Cooling although effective, opens up plethora of other problems that may result in computer failure, most notably the potential danger of water leakage on to the computer's motherboard [11].

Fin profile is the one of the most crucial parameters dictating the cooling performance of the heatsink. Various fin geometries have been analyzed for the conjugate heat transfer performance in the forced convection environment. Rectangular plate fins and airfoil fins were determined to be the among the best choice of fin profiles for heatsinks [12]. Square pin fins and elliptical pin fins were comparatively analyzed where the latter were determined to be the better choice [13]. Ease of manufacturability also needs to be accounted for in fin profiles, where rectangular plate fins prove to the most suitable choice.

In case of heatsink materials, Aluminum and Copper have been the materials of choice for a very long time. However, the impact of other novel materials has also been analyzed. For instance, Silver has been determined to possess a very high heat transfer rate but its high cost makes it unsuitable for general consumer market. Lee et. al. explored the cooling performance of Copper-Graphite composite based heatsinks and results are promising, however high cost makes it unsuitable for general consumer market [14]. Possibility of Graphene-based composites for heatsinks has been examined and it has been found that although the Graphene's light weight is highly promising for designing coolers for mobile computers like smartphones and tablets, but once again its high cost makes it unsuitable for desktop PCs and general consumer market [15]. Due to high cost of these novel materials, Aluminum and Copper based heatsinks are still considered as the best option for general consumer market.

Design optimization techniques have been implemented by previous research on heatsinks to provide very promising results. A typical topology optimization formulation implements numerical methods in order to analyze the design performance, and also uses these results to formulate an optimal design by using optimization algorithms such as genetic algorithm or the optimality criteria algorithm [16]. The thermal resistance of heatsink was found to be improved by 15% by implementing topology optimization for a heatsink under natural convection while requiring 26% less material as compared to the conventional heatsink, which signifies the potential of topology optimization [17]. The cooling performance of heatsinks for tablets were found to be hugely improved by implementing topology optimization [18]. Topology optimization of a radial heatsink under natural convection is implemented where cooling performance of heatsink was maintained while mass reduction of about 40% is also achieved [19].

From the aforementioned available literature, it is concluded that the research on performance parameters of a heatsink is generic and that research is not done in the context of the problem of cooling high heat generating CPUs. The focus of this research is, therefore to address that gap with the aim to determine the design of a heatsink for an active CPU cooler using air as its coolant with the optimum fin geometry, heatsink material, fin arrangement and a fin number that can provide an optimal cooling performance for a CPU generating 380 watts of heat.

2. Methodology

Comparative Analyses for thirteen (13) fin profiles, twelve (12) heatsink materials and four (4) fin arrangements are analyzed using ANSYS Fluent 2021 R1. The heat transfer analyses for all configurations of fin profiles, materials, fin arrangements and fin numbering are carried out using ANSYS Fluent. Further, using the best results from these analyses, a base design of heatsink is established which is optimized for fin number and finally a topology optimization analysis is carried out using ANSYS Mechanical on the base design with the aim of reducing material cost.

2.1 Numerical Model

Reynolds-averaged formulation of Navier-Stokes equations (RANS) is implemented for the computation of flow velocity and pattern as implemented in Kepekci and Asma [12], and Joo *et al.,* [19]. These equations are given as follows:

$$\frac{\partial(\rho U)}{\partial t} + \nabla.(\rho UU) = -\nabla p + \nabla.[\mu(\nabla U + (\nabla U)^T] + \rho F - \nabla\left(\frac{2}{3}\mu(\nabla.U)\right) - \nabla.(\rho\overline{U'U'}) \quad (1)$$

The Reynold stresses were evaluated using standard k-epsilon turbulent eddy viscosity model. The continuity equation for the coolant i.e. air can be given as follows:

$$\nabla.\mathrm{u} = 0 \quad (2)$$

Every simulation is iterated to 1000 iterations. Most of the results converged before that number, to the convergence factor of “0.0001” for continuity and velocity equations, whereas 10^{-9} for energy equations for heat transfer calculations.

Three different mesh densities are initially generated to analyze the impact of mesh density on the final results. Element size was kept comparatively lower in the proximity region, where fluid-solid interaction between heatsink and air was taking place. An Aluminum heatsinks with 15 rectangular plate fins of 3 mm width each, is analyzed for these three mesh densities. The details of mesh densities are explained in Table 1.

Table 1
Mesh Density Comparison

Mesh Type	Elements	CPU Temperature (°C)	Analysis Time (sec)
Coarse	1,292,798	118.652	20 min 10 sec
Medium	2,263,990	119.925	27 min 17 sec
Fine	3,390,790	120.367	35 min 40 sec

Three-dimensional tetrahedral mesh type as shown in Figure 1, was selected for all comparative and topology optimization analyses, due its high efficiency in terms of ease of computation and simulation accuracy.

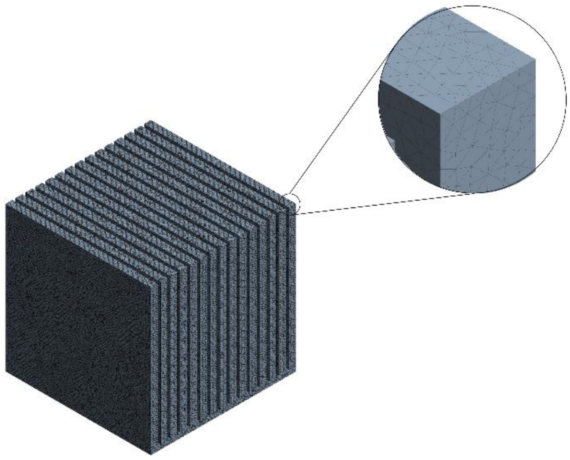

Fig. 1. Heatsink with Tetrahedral meshing

For the analysis, a PC with "Intel (R) Core (TM) i5-4690 CPU @ 3.90 GHz with 4 cores " is used. A fine mesh of approximately 3 million elements, is chosen for all subsequent simulations.

The comparative analyses for heat transfer capability of various heatsink configurations were carried out using ANSYS Fluent Solver. The heatsink design for each configuration was placed in a fluid domain with dimensions of 300 mm of length, 90 mm of height and width as shown in Figure 2.

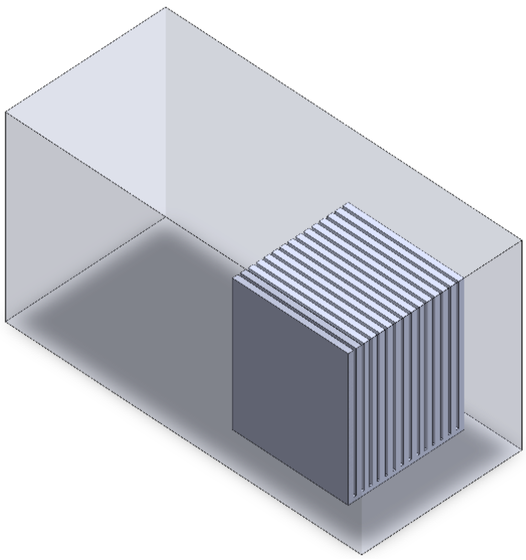

Fig. 2. Heatsink in analysis fluid domain

2.2 Topology Optimization

Topology Optimization is a design optimization technique that optimizes topology of a design by eliminating excess material from an initial design using the finite element method. It assesses each part of the base design and removes non-contributing sections with the aim to reducing both material cost and weight. The resultant design can be readily manufactured through additive manufacturing methods like 3D printing. However, with slight modifications, the design can be refined for traditional manufacturing processes like machining.

A standard topology optimization process includes following steps:

i. Creation of a base design.
ii. Defining boundary conditions and carrying out a prerequisite analysis like structural or heat transfer analysis on the basis of which optimization study is carried out.
iii. Establishing topology density which is the region from which material can be removed and defining exclusion region.
iv. Meshing the geometry and performing the analysis using finite element method.
v. Validation of the design, and potentially adjusting it for conventional manufacturing methods.

For the topology optimization of heatsink, the conjugate heat transfer analysis of the base design from the best options from comparative analyses served as the prerequisite analysis. Topology density was selected on the basis of boundary conditions. The heatsink base which takes up heat from the CPU constitutes the exclusion region from where material is not to be removed, whereas the rest comprises the topology density. Prerequisite analysis and the topology optimization analysis is carried out using the fine mesh of about 3.3 million elements with the goal of reducing material with as little reduction of cooling performance as possible.

Figure 3 displays the heatsink's topology density where optimization region, from where material can be removed is highlighted with blue colour. The regions highlighted with red colour comprises the exclusion region from where the material cannot be removed as in accordance to boundary conditions.

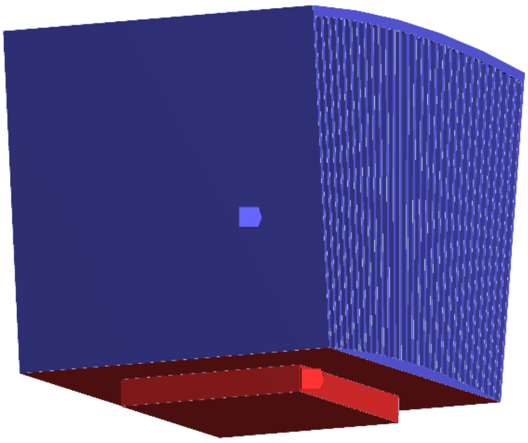

Fig. 3. Optimization region for the heatsink

Topology optimization yields a lightweight design that is expected to fulfil the requirements established in the prerequisite analysis. To confirm this, a validation study is carried out to assess the optimized design under the same loads as in the prerequisite analysis.

2.3 Design Constraints

The CPU cooler design has to confine to constraints, both in terms of available space inside the Desktop PC Case and reasonable airflow speed. These design constraints are described in Table 2.

Table 2
Design Constraints for heatsink

Parameter	Value
Heatsink Size (L x W x H)	75mm x 75mm x 75 mm
Fan Size (L x W x H)	20mm x 90mm x 90mm
CPU Size (L x W x H)	37mm x 37mm x 5mm
Airflow Speed	80 CFM
CPU Heat Generation	380 watts
Ambient Temperature	25 ^{o}C

3. Results and Discussion

3.1 Fin Profiles

Total of thirteen (13) fin profiles and configuration are analyzed under a conjugate heat transfer simulation in ANSYS Fluent. All fin profiles are analyzed for an Aluminum heatsink with fifteen (15) fins. The results from the selected profiles are shown in Table 3.

Table 3
CPU Temperatures for various fin profiles

Fin Profile	CPU Temperature (^{o}C)
Rectangular Plate Fins	120.367
Square Pin Fins	138.264
Wedge Shaped Fins	154.201
Double Sided Wedge Fins	135.168
Wavy Fins	123.244
Wavy Fins (Mirrored Config)	131.854
Concave Fins	120.695
Convex Fins	133.894
Triangular Plate Fins	146.615
Square Perforated Fins	131.958
Covered Square Pin Fins	133.462
Covered Rectangular Plates with Central Plate	119.111
Covered Rectangular Plate	119.133

Covered configuration of rectangular plates obtained the lowest CPU temperature and was selected for the base design. Although, the covered configuration between central plate provides a slightly better temperatures, but it’s not enough to warrant increase in material cost for central plate.

3.2 Heatsink Materials

A total of twelve (12) materials are analyzed for heatsink. The results for heatsink materials are not only considered for their cooling performance only, but are also their material cost which also needs to be considered in selecting the best material for the heatsink of a CPU Cooler. The results for the selected materials are provided in Table 4.

Table 4
CPU Temperatures for various heatsink materials

Heatsink Material	CPU Temperature (°C)	Material Cost (USD per Kg)	Heatsink Cost (USD)
Aluminum	119.133	2.40	1.36
Copper	102.094	8.22	14.80
Silver	100.25	729.5	1623.80
Gold	107.789	57941.11	222770
Nickel	159.778	26.93	46.32
Cadmium	119.541	1.98	3.46
Iron	170.024	0.08	0.12
Silicon	131.833	1.91	0.89
Magnesium	129.441	2.26	0.77
Tin	186.33	23.1	34.118
Tungsten	124.814	25.52	98.09
Silicon Carbide	98.11	36	23.40

According to Table 4, the obvious choice for material seems to be Silicon Carbide which clearly outperforms other material. However, Copper is slightly behind Silicon Carbide whereas, the lightweight nature and low cost of Aluminium cannot be overlooked as well.

Therefore, three materials including Silicon Carbide, Copper and Aluminium are kept in consideration until further analysis of fin configuration and fin number optimization are carried out.

3.3 Fin Arrangement

An Aluminum heatsink with fifteen (15) covered Rectangular Plate fins are analyzed for four (4) different fin arrangements. The results are shown in Table 5. Table 5 details the CPU temperatures obtained for various fin configurations and it can be seen that the arc-shaped fin placement highly improves the cooling performance of heatsink by 9.47%, which is more than enough to consider this configuration as the better choice for further analyses namely fin number optimization and topology optimization.

Table 5
CPU Temperatures for various fin arrangements

Fin Profile	CPU Temperature (°C)
Regular Arrangement	119.133
Staggered Placement	121.771
Inline-Interrupted Placement	124.092
Arc-Shaped Placement	107.840

3.4 Fin Number Optimization

Aluminum, Copper and Silicon carbide were selected to be the best material choices for the heatsink of the CPU Cooler, which yielded the CPU temperatures with 380 watts to be 107.840 °C, 90.368 °C and 86.125 °C respectively for a heatsink with 15 covered rectangular plates in arc-shaped placement.

However, as it is clear from these results that they are still not enough to fulfil the temperature requirement of 80 °C for maximum computing performance from CPU. Therefore, fin number optimization from fifteen (15) to thirty (30) fins for each of these three materials is carried out.

The results of fin number optimization for Aluminum, Copper and Silicon Carbide are shown in Figure 4 to 6.

It is evident from Figure 4 to 6 that improvement rate in the cooling performance for every additional fin to the heatsink decreases with each fin and begins to really converge after 19 fins, and becomes negligible after 30 fins.

Lowest CPU temperature is obtained to be 78.336 °C for Silicon Carbide heatsink with 30 fins. Copper obtained 84.149 °C and aluminium obtained 102.897 °C, for a CPU generating 380 W.

Silicon Carbide heatsink with thirty (30) covered rectangular plate fins of 0.78 mm width each, is found to satisfy the temperature requirements for maximum CPU performance (T<80 °C). Copper heatsink could not qualify to meet the performance requirements. However, it can be used safely without the risk of CPU failure. Aluminium heatsink failed to satisfy both performance and safety criteria of below 90 °C and is therefore, not recommended to be used for CPU with high heat generation.

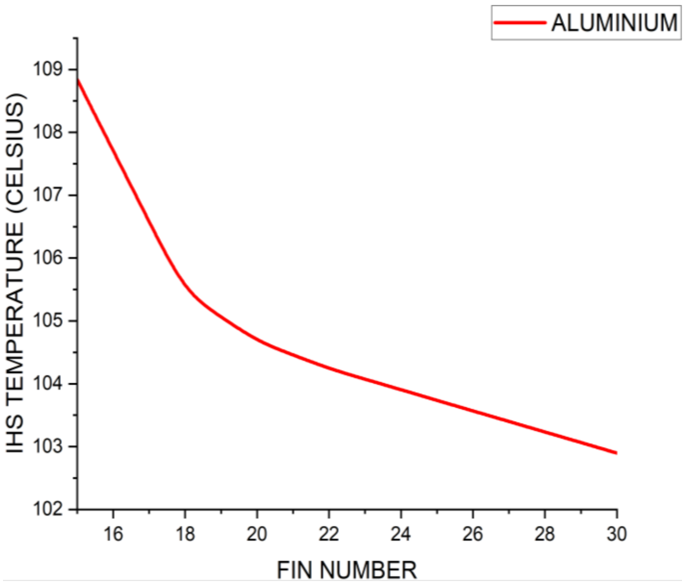

Fig. 4. Fin number optimization for Aluminium heatsink

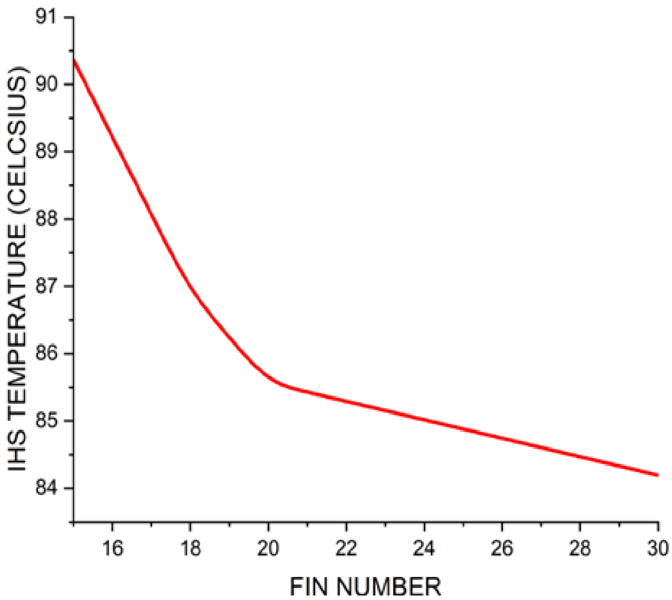

Fig. 5. Fin number optimization for Copper heatsink

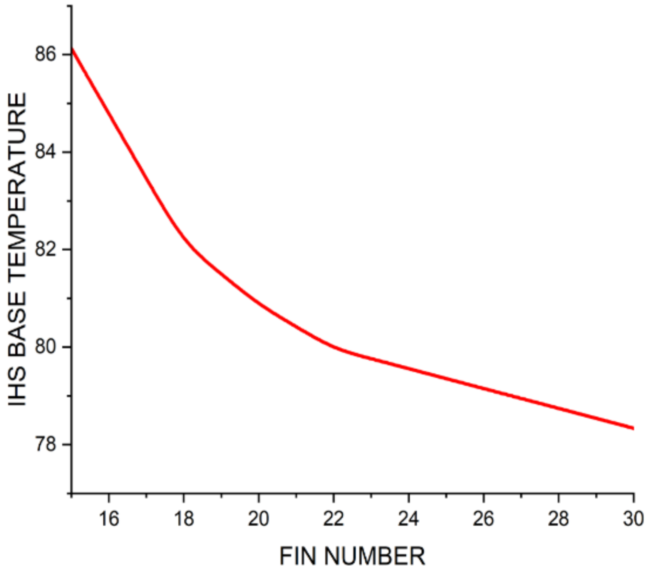

Fig. 6. Fin number optimization for Silicon Carbide heatsink

3.5 Topology Optimization

A base design is established by combining best options from four comparative analyses, comprising of thirty (30) covered rectangular fins in arc-shaped arrangement yielding the CPU temperature of 84.149 °C. The topology density is selected on the basis of boundary conditions.

This base design was used for the topology optimization analysis. The final design from topology optimization is shown in Figure 7.

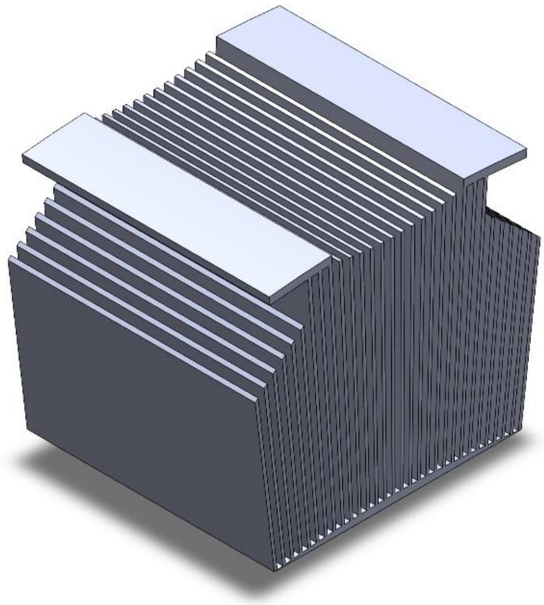

Fig. 7. Optimal design after topology optimization

Topology optimal design obtained the CPU temperature of 84.425 ^{o}C, with a negligible 0.32% increase as compared to the CPU temperature of 84.149 °C for the base design. In terms of material cost, the 20% reduction is achieved which resulted in 13.6% material cost reduction.

3.6 Results Validation

The best fin profile in this research is determined to be the rectangular plate fins, which is supported by research carried out in Kepekci and Asma [12], where five (5) fin profiles were compared for their cooling performance and declared airfoil fins to be the best fin profile for heat dissipation from electronic components. The research in that paper is expanded by analysing more types of fin profiles, thirteen (13) to be exact and performing subsequent analyses such as fin number optimization and heatsink materials. Incorporation of more fin profiles in the analyses resulted in about 10.4 % increase in cooling performance, specifically provided by introducing a top covering for fins and arranging the fins in an arc-shaped configuration.

Lee *et al.,* [20] implemented topology optimization for heatsinks under forced convection and achieved 9% reduction in weight while maintaining their cooling performance. The weight reduction of 20% for rectangular plate heatsinks was achieved in the research in this paper which consequently led to material cost reduction of 13.6% for Copper heatsinks.

4. Conclusions

Computational Fluid Dynamics and Finite Element Analysis provide a robust framework for designing heatsinks for various cooling requirements. This research used CFD and FEA for developing an optimal heatsink design capable of maintaining the CPU temperature below 80 ^{o}C, which is required to obtain maximum computing performance from CPUs. It is achieved by comparatively analyzing four key design parameters dictating cooling performance of a heatsink and performing topology optimization for the sake of cost reduction. Rectangular Plate fins are found to be the best option in terms of fin profiles in terms of cooling performance. Cooling performance of any fin profile can be further improved by covering the fins at the top. Silicon Carbide is found to be the most efficient heatsink material. Copper is also a good option as it has only slightly lower cooling performance as compared to Silicon Carbide but it is significantly inexpensive. Therefore, a Copper heatsink is able to avoid CPU failure, but is unable to satisfy temperature requirements for maximum CPU computing performance. Hence, it depends on consumer's decision to make a trade-off between cost and maximum computing performance. Implementing the rectangular plate fins in an arc-shaped configuration provides 10% higher cooling performance. The improvement rate in cooling performance decreases exponentially with every further addition of fins and gain in cooling performance becomes negligible after 30 fins. For CPUs with very high heat generation, a Silicon Carbide based heatsink with 30 covered rectangular plate fins placed in arc-shaped configuration is capable of maintaining below 80 ^{o}C, which is required to yield maximum computing performance from the CPU. Copper heatsinks can be used safely; however, it would not be possible to yield maximum computing performance from CPU. Aluminum should not be used for CPUs with very high heat generation of about 380 watts, as it is neither able to satisfy safety nor maximum performance requirements. Mass reduction of 20% was achieved with a negligible 0.32% cooling performance reduction using topology optimization. Topology Optimization proved to be viable option for developing designs with comparatively less material and hence, lower material cost as compared to traditionally developed designs.

Acknowledgement
This research was not funded by any grant.

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